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PROCEEDINGS

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TECHNICAL PAPERS AND DISCUSSIONS

EDITED BY
ALFRED N. GOLDSMITH, Ph.D.

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Correction: On page 159 of volume 6, number 3 of the PROCEEDINGS, line 7 from the bottom of the page should read:
"terminals of a current supply by means of a self-inductance"

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THE INSTITUTE OF RADIO ENGINEERS
announces with regret the death of Messrs.

Edward J. Baskin
Guy E. Morse
William D. Woodcock

Edward J. Baskin was born in Lynn, Massachusetts in 1899. He was a graduate of the Chelsea Grammar School, the Northeastern Preparatory School, the Eastern Radio School, and the Gillespie Aviation School of Boston. During 1917 he was employed as operator on coastwise ships plying to the West Indies by the Marconi Wireless Telegraph Company of America. In 1918 he enlisted in the Naval Reserve, Aviation Section. After being stationed at Charleston, South Carolina, he contracted pneumonia, and died on October 8, 1918. He was a member of The Institute of Radio Engineers.

Guy E. Morse was the son of Mr. and Mrs. Ernest R. Morse of Kansas City, Missouri. He was born in 1895 at Wolfville, Nova Scotia, his parents being then Canadians. He was a graduate of the Kansas City public schools, and had completed two years of his studies at the University of Illinois where he was preparing himself for electrical engineering. He enlisted in the first Officers' Training Camp, at Fort Sheridan, Illinois in 1917, and was transferred to Fortress Monroe. He was there commissioned as Second Lieutenant in the Coast Artillery, and stationed at Key West. On application, he was transferred to the air service as an aerial observer, and took his ground training at Austin, Texas. In 1918, he was sent to France, and was trained there at Saumur, Tours, and Cazaux, being then attached to the 135th Aero Squadron. In August, 1918 he was sent to the front, and was killed in aerial combat at St. Mihiel, September 12, 1918. He died at the age of twenty three, having been cited for gallantry before his death. He was well known as an excellent student and a fearless young man.

William D. Woodcock was born in 1896. He was graduated from the Lafayette High School, and was a member of the class of 1919 in analytical chemistry at the University of Buffalo. Before the war, he was a well known radio amateur, a member of an organization of radio operators, a member of The Institute of Radio Engineers, holder of a first class communication license, and of a special station license. He was the Buffalo operator who transmitted the President's message sent from coast to coast on October 27, 1916. After war was declared, he enlisted in the Naval Reserve, was sent to Great Lakes in 1917, and appointed a first-class operator, first at Cleveland, and then at Buffalo. After several service changes, he was made an instructor in the Radio School at Great Lakes, and then advanced to the radio laboratory. There he contracted pneumonia, which caused his death.

RESONANCE MEASUREMENTS IN RADIOTELEGRAPHY WITH THE OSCILLATING AUDION*

By

Louis W. Austin

(United States Naval Radiotelegraphic Laboratory, Washington, D. C.)

For purposes of rough tuning, many workers have doubtless made use of the click heard in the telephones of an oscillating audion circuit when it is brought into resonance with another circuit at proper coupling. As the resonance click has apparently not been mentioned in any of the publications on radio frequency measurements, it seems probable that it is not generally known that this click offers by far the quickest and simplest means of making nearly all measurements depending on the determination of resonance. The accuracy is quite equal to that obtainable with sensitive thermo-elements, and greatly superior to the accuracy of the detector and telephone method.[1]

Since the audion circuit itself is not suited to exact calibration, the substitution method is generally used. The following examples illustrate the procedure:

Capacity of an Antenna by Substitution

The antenna is loaded with inductance so as to give a wave length of five to ten times the fundamental, then the oscillating audion circuit is coupled to the antenna inductance, and the audion tuning condenser varied until a click is heard in the telephones. In general, if the coupling is close, the click will be heard at different points with increasing and decreasing condenser capacity. The coupling should then be loosened until both clicks appear at the same condenser setting, or, if this is impossible, the mean setting is taken provided the points are less than a degree apart. Next, leaving the audion condenser on the resonance point, the ground and antenna are discon-

* Received by the Editor, July 24, 1918.

[1] Care must be taken regarding harmonics in all measurements in which bulbs are used for excitation.

nected from the antenna inductance and replaced by the calibrated variable substitution condenser. This last is adjusted to resonance with the audion circuit exactly as described above, and the capacity of the condenser is then equal to that of the antenna, subject to a small correction for the natural antenna inductance.

Wave Length of a Distant Station

The receiving antenna and secondary oscillating circuit are first tuned exactly to the distant station, preferably at loose coupling, the audion tuning condenser being adjusted to give the dead point of the beats in the case of continuous wave reception. Next, without changing anything in the antenna or secondary, a wave meter is coupled to the secondary and adjusted to resonance by the click method. The reading of the wave meter gives at once the wave length for the sending station.

In a similar way, wave meters can be compared and condensers and inductances calibrated, either by substitution or by making use of the well-known relation existing between the product of inductance and capacity and the wave length.

Besides the simplicity and quickness of this method, it has the advantage that it does away with the necessity for all auxiliary apparatus in the wave meter, and enables measurements of the highest accuracy to be taken on shipboard and in other places where the use of sensitive galvanometers is impossible.

U. S. Naval Radiotelegraphic Laboratory.
June, 1918

SUMMARY: The telephone click in an oscillating audion circuit when a coupled circuit is brought into tune with it is utilized to measure quickly and accurately antenna capacity, wave length of distant stations, capacities, inductances, and wave lengths.

A BRIEF TECHNICAL DESCRIPTION
OF THE
NEW SAN DIEGO, PEARL HARBOR, AND CAVITE HIGH POWER NAVAL RADIO STATIONS*

(Supplementing Captain Bullard's Paper)

By

LEONARD F. FULLER

(CHIEF ELECTRICAL ENGINEER, FEDERAL TELEGRAPH COMPANY)

As mentioned in Captain Bullard's article, 11,000-volt, 3-phase, 60-cycle power will be delivered to the new San Diego high power radio station. The power equipment is being manufactured by the General Electric Company and will consist of the usual oil switches, switchboards, and motor-generators used in power work.

The motor-generators, which are in duplicate, will be 2-unit, 4-bearing sets, consisting of 300 horse-power, 1,200 revolutions per minute, 2,200-volt, 3-phase, 60-cycle, squirrel cage induction motors, direct connected to 200 kilowatt, 1,200 revolutions per minute, 950-volt, compound wound, direct current generators with 2-kilowatt, 125-volt, over-hung exciters.

The temperature rises of the motors will be 40° C. for continuous full load operation and not over 55° C. for a continuous series of duty cycles of 1.5 hours on and 1 hour off at 125 per cent. of load.

The generator temperature rises will be 40° C. for continuous full load operation and not over 55° C. for a continuous series of duty cycles of 1.5 hours on and 1 hour off at 250 kilowatt output. The sets as a whole can withstand a 100 per cent. overload for short periods.

Duplicate 14-kilowatt, 125-volt, direct current, motor-generators with a complete set of spare parts will be installed also, for furnishing power for plant auxiliaries.

The Federal-Poulsen arc converter will be of the oil-immersed, water-cooled type capable of furnishing 150 amperes radiation continuously, and 170 amperes for periods of 1.5 hours on and 1.5 hours off. The temperature rises will be 40° C. and 50° C. respectively for all current-carrying or electrical parts.

* Received by the Editor, May 20, 1916.

At Pearl Harbor, power will be supplied, as Captain Bullard states, at 2,200 volts, 3-phase, 60 cycles. This plant is similar to that at San Diego but of higher power. The large motor-generators are provided in duplicate with a complete set of spare parts including spare armatures. They are 2-unit, 4-bearing sets, manufactured by the General Electric Company, consisting of 750 horse-power, 900 revolutions per minute, 2,200-volt, 3-phase, 60-cycle, wound rotor, induction motors, direct connected to 500 kilowatt, 900 revolutions per minute, 1,430-volt, compound wound, direct current generators with 3-kilowatt, 125-volt, compound wound, over-hung exciters.

The temperature rises of both motors and generators will be 50° C. on a continuous series of duty cycles of 2 hours on and 1 hour off. High voltage direct current switchboards and control apparatus of a type used in electric railway work will be provided.

The Federal-Poulsen arc converter will be of the oil-immersed, water-cooled type, capable of furnishing 200 amperes radiation continuously and 250 amperes for periods of 1.5 hours on and 1.5 hours off with temperature rises of 40° C. and 50° C. as specified for San Diego.

On account of the oil immersion and water cooling of all coils and arc converter windings, this type of apparatus is extremely rugged and reliable. The arc transforms or converts the 1,500-volt direct current power into radio frequency energy without the use of rotating parts or radio frequency magnetic circuits.

The antenna loading coil will be of litzendraht cable supported entirely by porcelain. This cable will be 1.75 inches (4.45 cm.) in diameter, and the loading coil diameter will be 14 feet (4.26 m.).

The wave changer will be of the rotary type capable of throwing the plant onto any one of five wave lengths when operating at full power. This feature holds for San Diego and Cavite as well as Pearl Harbor and in all three the usual antenna grounding switches, transfer switches, etc., will be provided.

The generating equipment at Cavite will be identical with that at Pearl Harbor except that the power supply will be 220-volt direct current and no over-hung exciter will be used. The 500-kilowatt, 1,500-volt, generator excitation will be supplied from the 220-volt direct current buses. At both Pearl Harbor and Cavite, special insulation and fittings for tropical conditions will be provided.

The arc and the remainder of the radio set at Cavite will be in every way identical with that at Pearl Harbor.

The arcs at Pearl Harbor and Cavite will be approximately 9 feet 2.5 inches (2.81 m.) by 7 feet 4 inches (2.23 m.) wide by 12 feet (3.66 m.) long, and will each weigh approximately 60 tons (54,000 kg.) under operating conditions. The San Diego arc is somewhat smaller and will weigh 21 tons (19,000 kg.) under operating conditions. The larger arcs will present an outside appearance very similar to certain types of vertical shaft hydro-electric generators.

San Francisco, California, May 12, 1916.

SUMMARY: The new high power Federal-Poulsen arc stations of the United States Navy are described briefly. The motor-generator sets, temperature rises, field excitation, and operating characteristics are given. The arc-converter, antenna loading inductance, and wave changing switch are described shortly.

(The distance from Cavite, Philippine Islands to Pearl Harbor, Hawaii, is 5,300 miles (8,500 km.), from Pearl Harbor to San Diego 2,600 miles (4,200 km.), and from Cavite to San Diego 7,800 miles (12,500 km.), altogether over water. The approximate power used to cover 4,000 kilometers is 70 kilowatts in the antenna, and for 8,500 kilometers is 160 kilowatts. The former value may be compared with that given by Mr. John L. Hogan, Jr., on page 419 of the October, 1916, issue of the PROCEEDINGS, namely 72 kilowatts.—EDITOR).

HYSTERESIS AND EDDY-CURRENT LOSSES IN IRON AT RADIO FREQUENCIES*

By

Christian Nusbaum

(Jefferson Physical Laboratory, Harvard University, Cambridge, Massachusetts)

Introduction

The study of the heat losses due to hysteresis and eddy-currents in iron, when subjected to an oscillating magnetic field, is of practical interest in radio telegraphy and radio telephony. In this paper will be described a calorimetric method for the determination of these losses, in a certain specimen of soft iron, at frequencies between the limits of 150,000 and 1,000,000 cycles per second, and the results obtained by this method of measurement will also be stated.

Various methods of measurement were used by previous investigators for the determination of these losses, and the results obtained are quite at variance with each other. The results of Warburg and Honig,[1] Tanakadate,[2] and Weihe[3] indicate that the loss per cycle, at frequencies of three cycles and sixty cycles per second, is less than for static magnetization. The results of Hopkinson,[4] Maurain,[5] Gray,[6] and Guye and Herzfeld[7] indicate that the loss per cycle is independent of the frequency, while those of Steinmetz,[8] Niethammer,[9] M. Wien,[10] Krogh and Rikla,[11] Dina,[12] and Corbino,[13] lead to the conclusion that the loss per cycle increases with the frequency.

*Received by the Editor, July 10, 1918.

1 Warburg and Honig, Wied. Ann., 20, p. 814, 1883.
2 Tanakadate, A., Phil. Mag., 28, p. 207, 1889.
3 Weihe, F. A., Wied. Ann., 61, p. 578, 1897.
4 Hopkinson, Elektrotechn. Zeitschr., 13, p. 642.
5 Maurain, C., Ecl. Electr., 15, p. 499, 1898.
6 Gray, Phil. Trans. 174 A, p. 351.
7 Guye and Herzfeld, Compt. Rend., 136, p. 957, 1903.
8 Steinmetz, Elektrotechn. Zeitschr., 13, p. 957.
9 Niethammer, Wied. Ann., 66, p. 29, 1898.
10 Wien, M., Wied. Ann., 66, p. 859, 1898.
11 Rikla and Krogh, Elektrotechn. Zeitschr., p. 1083, 1900.
12 Dina, A., Elektrotechn. Zeitschr., p. 41, 1902.
13 Corbino, O. M., Atti. Assoc. Electr., Ital., 1903.

Alexanderson,[14] using a radio frequency generator, obtains results which may be stated as follows: (1) that the permeability is the same at high as at low frequencies, and (2) that a higher induction than 1,200 lines per square centimeter cannot be reached on account of the "shielding effect" due to eddy-currents. The loss per cycle is less for higher than for lower frequencies, the observations extending from 40,000 to 200,000 cycles per second. Recently, Fassbender and Hupka[15] outlined an experimental method for the determination of the permeability and the form of the hysteresis cycle for very high frequencies. Preliminary observations clearly show the change in the form of the hysteresis cycle due to the "shielding effect" of eddy-currents. The results also show, that even for a frequency of 200,000 cycles per second, it is impossible to obtain wire of sufficiently small diameter to prevent disturbing effects due to eddy-currents.

Method and Apparatus

If an undamped oscillatory current is sent thru the winding of a toroid containing a core of soft iron wires, the iron core will, during each oscillation, pass thru a complete hysteresis loop. On the other hand, if the current is damped, however little, there will be a series of non-closed hysteresis loops for each train of oscillations. The rapid change in the magnetic induction will induce eddy-currents, which in turn cause a non-uniform distribution of the magnetic flux thru the cross-section of the iron. These processes manifest themselves in heat, and are generally designated as "heat losses."

For the purpose of the measurement of these losses, two toroids, both being alike in every respect except that only one contains a specimen of the iron the losses of which are to be determined, are connected in series with each other in the secondary of an oscillating circuit. In Figure 1, T_o and T, respectively, represent the toroids without and with iron. Each of these toroids is placed in a separate calorimeter. If the heat capacity of the calorimeters, as well as of the toroids, is the same, and the $I^2 R$ losses in the winding of the two toroids are equal, then the heat produced by the iron specimen in one calorimeter may be balanced in the other one by means of a direct-current heating coil ($H C$, Figure 1).

Each calorimeter consists essentially of two parts, an inside vessel C_i, Figure 2, and an outside one, C_o. The inside vessel

[14] Alexanderson, Elektrotechn. Zeitschr., 32, p. 1078, 1911.

[15] Fassbender and Hupka, Jahrbuch der Drahtlosen, Telephonie und Telegraphie., 6, p. 133, 1913.

is supported by the brass cones *c* and the ebonite disc *r*, and separated from the outside vessel by an air space and the three equally-spaced insulators *i*, only one of which is shown in the drawing. Both vessels have their entire surfaces nickel-plated and polished. By such an arrangement the loss of heat by conduction and radiation is reduced to a minimum. The cover of each calorimeter is fitted to the outside vessel by a ground joint, and is provided with six vertical tubes. The central tube, 1, admits

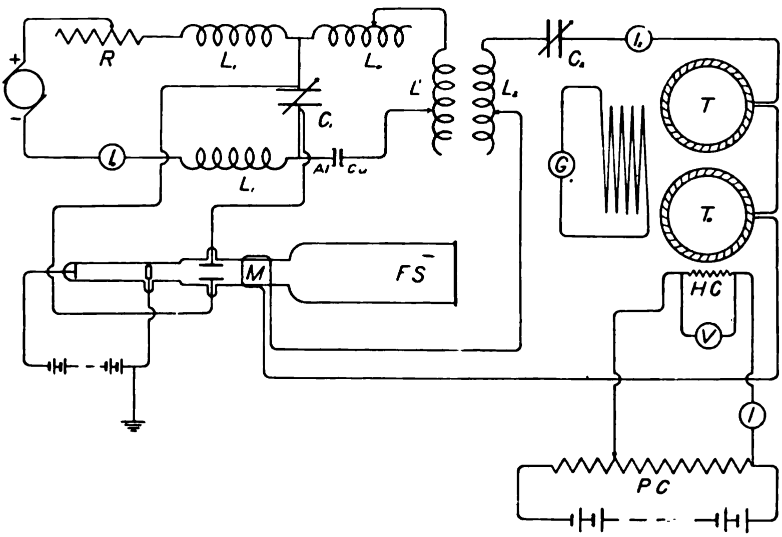

FIGURE 1—Diagram of Connections

one end of the thermo-pile; tubes, 2, admit the supports for the stirrers, *s*, and tubes, 3, the leads to the toroids. The remaining tube, 4, admits the leads of the direct-current heating coil which is placed in the calorimeter containing the toroid without iron. Each lead wire is carefully insulated from the tube which admits it. The stirrer in each calorimeter consists of two parallel discs, equal in diameter and held together by four uprights. Each disc has a series of circular openings arranged in the form of a circle. Its internal diameter is large enough easily to permit motion of the stirrers past the toroid, which is supported in the central portion of the calorimeter. Both of the calorimeters are mounted on a common base and entirely immersed in a bath of kerosene, so that the liquid surface is above the ground joints but below the upper ends of the vertical tubes. The temperature of this external bath is maintained approximately the same as that of the calorimetric liquid within the calorimeters by means of a heating coil and stirring apparatus. The turns of the toroids are wound on two glass cylinders, about four centimeters (1.6 inch) in diameter and one centimeter (0.4 inch) in width, both cylinders having been cut from the same glass tubing.

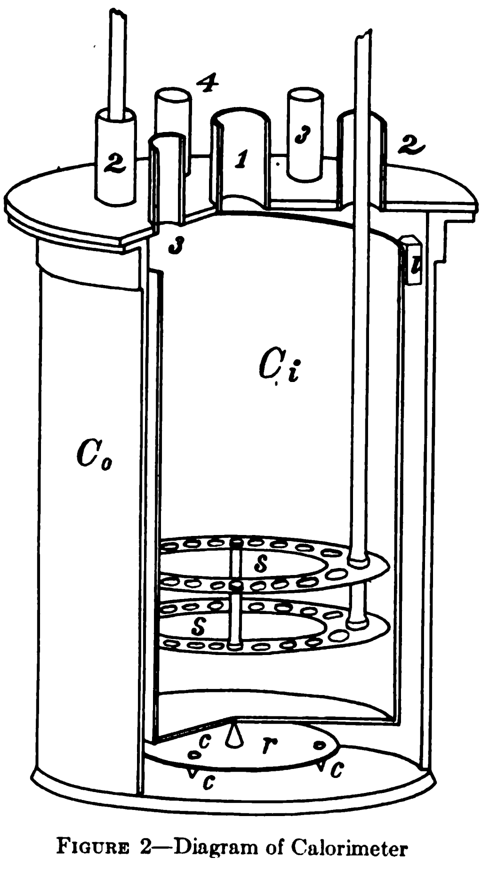

FIGURE 2—Diagram of Calorimeter

Around one of these cylinders the specimen of soft iron wire had been previously wound in the form of a helix of small pitch. With this type of core the individual turns of the specimen are well insulated from each other and their direction is nearly parallel to the direction of the magnetic field. A small quantity of iron is used in order that the inductance due to the presence of the iron shall be a small fraction of the total inductance of the secondary circuit, and that there may not be an excessive rise in temperature within the calorimeter.

The heating coil, made from number 30 constantan wire,* is wound non-inductively on an ebonite support. The current thru the coil can be varied by "taking off potentials" from the potentiometer PC, Figure 1. The ammeter I is in the circuit of the heating coil, and the voltmeter is connected across its terminals.

To test for the equality of temperature between the two calorimeters, a thermo-pile consisting of 12 copper-constantan junctions is used. These junctions are arranged in two groups of six junctions each. With such an arrangement the two groups may be balanced against each other and thus the circuit tested for stray electromotive forces. To reduce the latter to a minimum all terminal contacts are of copper, as well as all switches, and the lead wires are well sheathed and protected.

The Chaffee [16, 17] gap, used as the source of the oscillations, possesses the especial advantage of not only producing continuous oscillations of a definite wave form at radio frequencies, but also of operating in such a manner that the oscillogram of its current wave can readily be obtained. On account of its method of operation, the damping of the current is generally very small. It consists, essentially, of an aluminum cathode and an anode of copper, both terminals being surrounded by an atmosphere of hydrogen. The general diagram of the circuit is shown in Figure 1. The primary circuit consists of an e m f. of 500 volts, direct-current, connected in series with a variable resistance R, two choke coils L_1, the gap G, and two variable inductances L' and L_o. In parallel with the inductances L' and L_o and the gap itself is placed a variable condenser C_1. The secondary circuit consists of the variable inductance L_2, air condenser C_2, hot-wire ammeter I_2, the two toroids T and T_o, and the magnetic deflecting coil M. The variable inductance L' of the primary and the variable inductance L_2 of the secondary circuit form a closely connected oscillation transformer. The linkages of the primary thru the secondary may be changed by varying the number of turns of the primary inductance. The Braun tube is used in this experiment to indicate the condition of complete syntony between the primary and

*Diameter of number 30 wire = 0.0100 inch = 0.025 cm.
[16] Chaffee, E. L., Proc. Am. Acad. Arts and Sci., 47, p. 267, 1911.
[17] Washington, Bowden, PROC. INST. RADIO ENGRS., 4, p. 341, 1916.

the secondary circuits, and also to indicate the form of the current wave in the secondary circuit. The terminals of the primary capacity C_1 are connected to the electrostatic plates of the tube. The magnetic deflecting coil M, Figure 1, is a part of the secondary circuit, and is used to deflect the cathode beam at right angles to the deflection of the electrostatic field.[18] This coil consists of sixteen turns of annunciator wire (number 18, cotton covered*), half of the turns being on opposite sides of the tube. Electrostatic deflection of the cathode beam by the coil is avoided by completely surrounding the tube inside the coil by a split solenoid.[19] The fluorescent screen $F\,S$, Figure 1, made from finely powdered calcium tungstate, was used for visual observation and for photographic purposes, and was found very satisfactory. The camera used in photographing the oscillogram was placed below the screen, so that its axis coincided with the axis of the tube. By this arrangement distortion of the picture of the oscillogram is prevented.

Method of Operation and Procedure

The operation of the gap can be most readily understood by tracing the sequence of phenomena from the instant at which the system is started. When the potential of the primary condenser C_1 has attained a value sufficient to break down the high resistance of the gap, the discharge of C_1 and the main current I_o rush across the gap and thru the primary inductance L'. After the discharge, the main current remains constant and flows into C_1 at a practically constant rate, neutralizing the inverse charge on the condenser and charging it again in the initial direction. The secondary thus receives periodic impulses from the primary. The frequency of these impulses depends on the main current and the capacity in the primary circuit, and not on the duration of the discharge. During the intervals between the successive discharges of the primary circuit, the secondary circuit oscillates with its own free period and with a damping determined wholly by the conditions existing in it. The amplitude of the oscillations can be maintained nearly constant by making the primary impulses occur at every three or four oscillations of the secondary. Since the primary condenser charges up at a uniform rate, if its terminals are connected to the electro-

*Diameter of number 18 wire =0.0403 inch =0.102 cm.

[18] As drawn, the coil is in the plane of the paper and the deflection caused by it would be parallel to the deflection produced by the electrostatic field.

[19] Chaffee, E. L., above reference.

static plates of the Braun tube, and the oscillatory current in the secondary is run thru the magnetic deflecting coil, an oscillogram will be obtained on the fluorescent screen. This oscillogram indicates the wave form of the secondary current, and also shows the number of its oscillations for each discharge of the primary.

Preparatory to the taking of a set of observations, the capacity of the secondary circuit is varied until its frequency of oscillation has the value at which one wishes to make an observation. If the two circuits are not in syntony, the constants in the primary are varied until such condition is attained. The primary capacity is generally kept as large as possible, consistent with the energy to be transmitted and the number of oscillations of the secondary for each discharge of the primary. The current in the secondary is varied by changing the linkages between the inductances L' and L_2. During a set of observations for a given frequency, generally, the inductance L_2 is not altered, but the number of linkages changed only by varying the number of turns of the inductance L'. Since L' is used as one side of a variable transformer it is convenient to have an additional variable inductance L_0 in the primary.

The number of oscillations of the secondary per second is determined by means of a wave meter. The resonance between the secondary circuit and the wave meter was in most of the observations very sharp.

In the taking of any given observation, the current thru the direct-current heating coil is varied until the rate of rise of temperature in the two calorimeters is the same, as indicated by zero deflections of the galvanometer in the thermo-pile circuit, if the calorimeters are at the same temperature, or by a constant deflection of the galvanometer if the calorimeters are at a slightly different temperature. Generally, two preliminary observations are taken, one which will give a more rapid rise of temperature in the calorimeter with the heating coil, and a second which will give a less rapid rise of temperature. By interpolating between these two values of the current, a new and more accurate value can now be obtained. The calorimeters are then brought to the same temperature, the current set at the interpolated value, and the observation again repeated. If it is satisfactory, the heat developed by the heating coil in one calorimeter, and by the specimen of iron in the other calorimeter, can be calculated from the readings of the ammeter and voltmeter in the heating coil circuit.

DATA, RESULTS, ERRORS AND CONCLUSIONS

The galvanometer, G_o, Figure 1, used in the thermo-pile circuit was a high resistance D'Arsonval instrument. Tho it was not especially adapted for thermo-pile work, and for the particular junctions used, yet a difference of temperature of 0.004°C. between the two calorimeters could be read directly, when the scale was at a distance of two meters (78.7 inches) from the galvanometer. During any given set of observations, there was in the majority of cases a rise of from 4 to 6 degrees within each calorimeter.

The thermal-junctions were arranged in two sets of six pairs each. By such an arrangement, even tho the calorimetric fluids were at different temperatures, there should be no deflection of the galvanometer if the two sets were opposed to each other, provided there is no other source of emf. in the circuit. At the beginning of every set of observations, the two sets of thermal-junctions were balanced against each other, as well as at the end of each set of observations.

The hot-wire ammeter I_2, in the secondary circuit, was

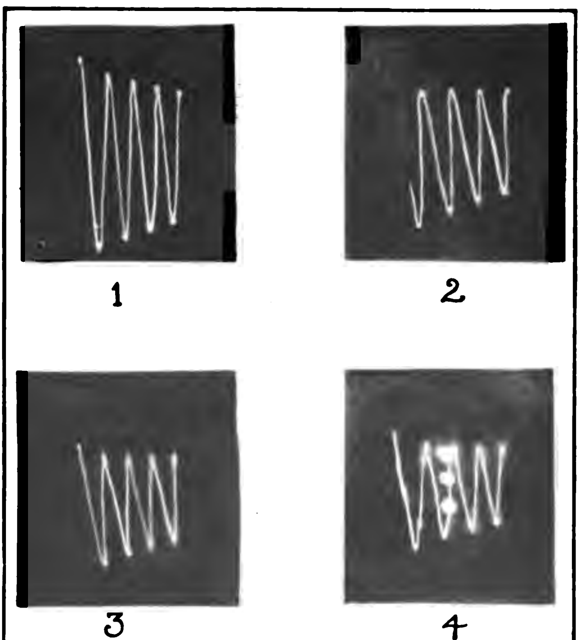

PLATE 1

calibrated by comparison with direct-current values. In the calibration it was assumed that the resistance of the hot-wire ammeter does not vary appreciably with the frequency, so that the calibration curve will hold equally well for radio and for audio frequencies. This condition holds only when the wire of the ammeter is not more than 0.35 mm. (0.014 inch) in diameter. To serve as a check, comparisons were made on the Braun tube between the deflections produced by a direct-current of a given value and that produced by a radio frequency current. Figure 4 of Plate 1 shows such a comparison. The central spot indicates zero deflection.

The volume of the specimen of iron on the toroid *T* is 0.0298 cc. (0.0019 cu. inch). Both toroids are 4.078 cm. (1.61 inch) in diameter and the current-bearing winding of each consists of

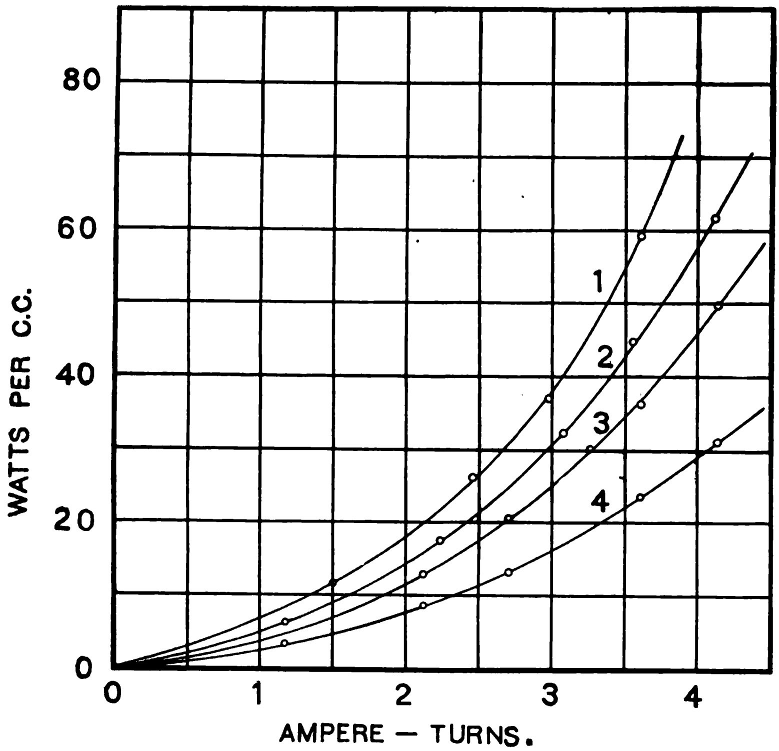

FIGURE 3—Curves Showing Heat Developed in Watts Per Cc. Plotted Against the "Effective Ampere-turns Per Centimeter"

Curve 1, 1,000,000 cycles
2, 550,000 "
3, 300,000 "
4, 150,000 "

72.5 turns. The term "effective ampere-turns per centimeter" is used to designate the product NI, where N is the number of turns per centimeter in the toroids and I the "square-root mean-square" values of the current as indicated by the hot-wire ammeter.

Figure 3 represents the results of Table I. The "effective ampere-turns per cm." as abscissas are plotted against the heat developed in watts per cc. as ordinates for the various frequencies.

TABLE I

Current Values

Deflection of hot-wire ammeter	Square-root mean-square values	Effective ampere-turns per cm.	Energy developed in watts	Energy developed in watts per cc.	Frequency
2.20	0.266	1.505	0.359	11.97	1,000,000
4.00	.433	2.451	.790	26.33	1,000,000
5.00	.526	2.977	1.112	37.07	1,000,000
6.20	.638	3.611	1.776	59.20	1,000,000
1.60	.207	1.172	0.184	6.13	550,000
3.60	.395	2.236	.528	17.60	550,000
5.20	.545	3.085	.974	32.47	550,000
6.10	.629	3.560	1.349	44.97	550,000
7.10	.724	4.097	1.850	61.67	550,000
3.40	.376	2.128	0.390	13.00	300,000
4.50	.479	2.711	9.626	20.87	300,000
5.60	.582	3.294	0.908	30.27	300,000
6.20	.638	3.617	1.089	36.30	300,000
7.20	.733	4.148	1.496	49.87	300,000
1.60	.207	1.172	0.101	3.35	150,000
3.40	.376	2.128	0.259	8.65	150,000
4.50	.479	2.711	0.404	13.45	150,000
6.20	.638	3.611	0.709	23.65	150,000
7.20	.733	4.148	0.937	31.25	150,000

In Table II are given the heats developed in ergs per cycle per cc., for definite values of the "effective ampere-turns per cm." These values are represented by the graphs of Figure 4.

Plate I shows the current-wave forms for various frequencies. The current in the secondary is thus not undamped, but the value of the damping can be made very small by making the primary impulses take place at every four or five oscillations of the secondary. This quantity is called by Chaffee [20] the "in-

[20] Chaffee, E. L., previous reference.

TABLE II

Effective ampere-turns per cm.	Energy developed in watts per cc.	Energy developed in ergs per cycle per cc.	Frequency
1	2.13	1.42×10^2	150,000
2	7.42	4.94×10^2	150,000
3	16.46	10.97×10^2	150,000
4	29.45	19.60×10^2	150,000
1	3.73	1.24×10^2	300,000
2	11.64	3.88×10^2	300,000
3	24.92	8.31×10^2	300,000
4	45.70	15.23×10^2	300,000
1	5.13	0.93×10^2	550,000
2	14.25	2.59×10^2	550,000
3	30.42	5.53×10^2	550,000
4	58.62	10.66×10^2	550,000
1	7.13	0.71×10^2	1,000,000
2	18.50	1.85×10^2	1,000,000
3	37.90	3.79×10^2	1,000,000
4	81.20	8.12×10^2	1,000,000

verse charge frequency." The value of this constant was 5 thruout the entire experiment. Table III gives the values of the frequency for the different figures of Plate I.

TABLE III

Figure.............	1	2	3	4
Frequency..........	150,000	550,000	300,000	1,000,000

Tests were made for the relative heat capacities of the calorimeters, differences of heat losses of the calorimeters due to radiation and losses due to heat conduction along the leads, and relative $I^2 R$ losses in the windings of each toroid.

These tests were made by sending a direct-current of constant value thru the windings of the two toroids and then observing the difference of the rate of increase in temperature between the two calorimeters. These tests indicated that the errors thus introduced were in the majority of the observations less than one-tenth of one per cent. The largest errors entering into the observations are the variations of the current values in the secondary circuit. During the time of an entire set of observations the current may vary from three to five per cent. of its mean value.

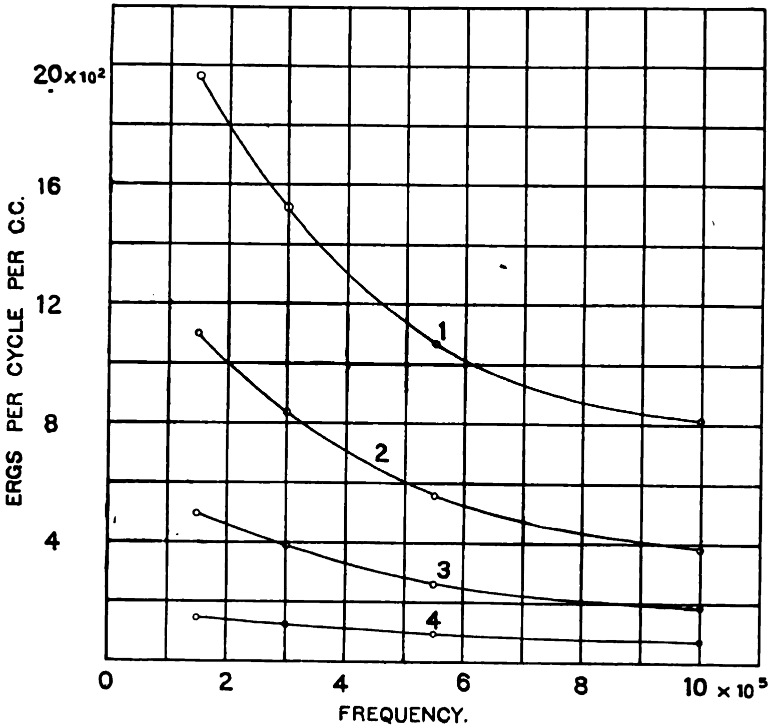

FIGURE 4—Curves Showing the Heat Developed in Ergs Per Cycle Per CC. Plotted Against the Frequency

Curve 1, 4 "effective ampere-turns per cm."
2, 3 " " " " "
3, 2 " " " " "
4, 1 " " " " "

The results of the experiment show that the heat developed per cycle by magnetic hysteresis and eddy-currents in iron decreases with an increase in the frequency of the oscillating magnetic field. This fact indicates a decrease of the magnetic induction on account of the "shielding-effect" produced by the eddy-currents.

Jefferson Physical Laboratory,
Harvard University,
June 1, 1918.

SUMMARY: After reviewing the bibliography of heat losses per cycle at various frequencies, the author describes a comparison calorimetic method whereby the losses in the soft iron wire core of a toroid are measured (against a similarly wound toroid without an iron core). The Chaffee gap radio frequency generator is used. Braun tube oscillograms of the cycle are shown, together with experimental data on the iron losses. It is found that the loss per cycle decreases as the frequency increases.

THE MEASUREMENT OF RADIO FREQUENCY RESISTANCE, PHASE DIFFERENCE, AND DECREMENT*

By
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Various methods of measuring radio (high) frequency resistances, the power factor or phase difference, of radio condensers, the sharpness of a resonance curve, and the decrement of a wave have been given. It is proposed in this paper: (1) to show that these quantities all express essentially the same physical magnitude and hence a single process of measurement gives them all; (2) to derive and classify the methods of measurement; (3) to describe improvements in these measurements.

Relations of the Quantities

The principal difference between the phenomena of radio (high) and audio (low) frequency is the importance of capacity and inductance at radio frequency as compared with the predominance of resistance in audio frequency phenomena. Nevertheless, resistance is the measure of power consumption, since any dissipation or loss of electrical power is expressible in terms of a resistance. Furthermore, resistances change rapidly with frequency at radio frequencies. The change can be calculated for certain very simple cases, but in most practical cases the resistance can be determined only by measurement. Thus the measurement of resistance at radio frequencies is a necessary and important operation.

Certain related quantities also express power dissipation. The results of a measurement can be expressed in terms of any of these quantities when their relations are clearly established.

In a simple series circuit, Figure 1, when the emf. is a sustained sine wave, a maximum of current is obtained when the inductive reactance is just equal to the capacitive reactance, i. e., when the equation

$$I = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

* Received by the Editor, April 17, 1918.

reduces to $I_r = \frac{E}{R}$. This condition of resonance obtains when

$$\omega L - \frac{1}{\omega C} = 0$$

or

$$\omega = \frac{1}{\sqrt{CL}} \tag{1}$$

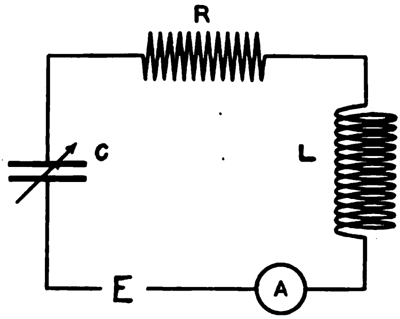

FIGURE 1

For any variation of either ω, L, or C, from this condition, as at A for the curves of Figure 2, the current is smaller than the value for which this relation holds.

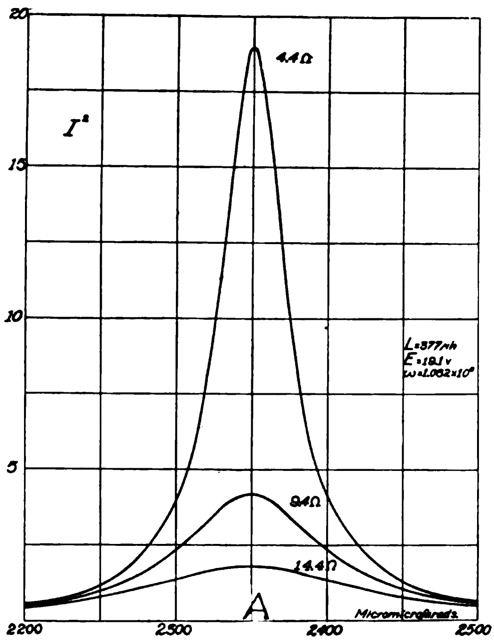

FIGURE 2

The phase angle of the circuit is zero at resonance as shown in Figure 3. The expression, phase angle of the circuit, means the same thing as phase angle of the current in the circuit.

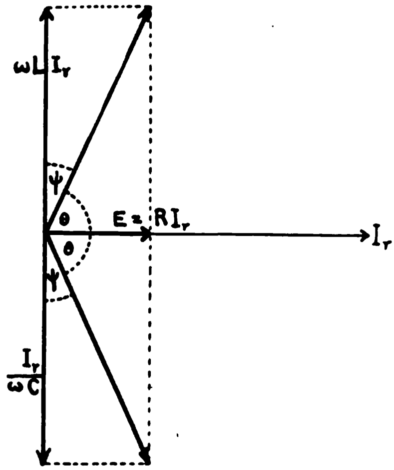

FIGURE 3

The phase angle θ of the coil, considering the resistance R to be associated with the coil L, is equal to the phase angle θ of the condenser, considering the resistance to be associated with the condenser. The complement of the phase angle θ is the phase difference ψ. From Figure 3,

$$\tan\psi = \frac{R}{\omega L} = R\,\omega C$$

When R and ψ are small, as usually in radio circuits, the tangent equals the angle and

$$\psi = \frac{R}{\omega L} = R\,\omega C \qquad (2)$$

Phase difference is thus a ratio of resistance to reactance. It follows also that $\psi = \sin\psi = \cos\theta$, or phase difference is equal to power factor. The same relation is brought out by multiplying in (2) by I^2.

$$\psi = \frac{R I^2}{\omega L I^2}$$

or

$$\psi = \frac{R I^2}{\dfrac{I^2}{\omega C}}$$

= ratio of power dissipated to power flowing.

Thus

$$\psi = \textit{power factor.} \tag{3}$$

Another quantity of importance in connection with the expression of power dissipation is the sharpness of resonance. This is the quantity which measures the fractional change in current for a given fractional change in either the capacitive or inductive reactance from its value at resonance. (Practically the same quantity has been known as persistency, selectivity, and resonance ratio.) It may be defined in mathematical terms for a variation of C by the following ratio

$$S = \frac{\sqrt{\dfrac{I_r^2 - I_1^2}{I_1^2}}}{\dfrac{\pm(C_r - C)}{C}}$$

where the subscript "$_r$" denotes value at resonance, and I_1 is some value of current corresponding to a capacity C which differs from the resonance value. The numerator of this expression is somewhat arbitrarily taken to be the square root of the fractional change in the current-square instead of taking directly the fractional change of current. This is done partly because of the convenience in actual use of this expression (since the deflections of the usual detecting instruments are proportional to the square of the current), and also because of mathematical convenience. It is easily seen qualitatively that sharpness of resonance is large when phase difference is small; as when R is small compared with ωL, a given fractional change in ωL changes the impedance, and therefore the current, by a relatively large fractional amount. Quantitatively, from the relations,

$I_1^2 = \dfrac{E^2}{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$, $I_r^2 = \dfrac{E^2}{R^2}$, and $\omega L = \dfrac{1}{\omega C_r}$, it follows at once that

$$S = \frac{\sqrt{\dfrac{I_r^2 - I_1^2}{I_1^2}}}{\dfrac{\pm(C_r - C)}{C}} = \frac{1}{R\,\omega C_r} = \frac{\omega L}{R} = \frac{1}{\psi} \tag{4}$$

= ratio of power flowing to power dissipated. Thus,

Sharpness of resonance = reciprocal of power factor. (5)

Another related quantity is the logarithmic decrement. For free oscillation in a simple circuit, the value of the decrement or napierian logarithm of the ratio of two successive current maxima in the same direction is readily shown to be

$$\delta = \pi R \sqrt{\frac{C}{L}} = \frac{\pi R}{\omega L} = \pi R \omega C \qquad (6)$$

Comparing with (2) and (4),

$$\delta = \pi \psi = \frac{\pi}{S} \qquad (7)$$

That is, altho defined originally in connection with damped oscillations, the decrement is a constant of the circuit and can be dealt with in the same way as resistance and the other energy-determining quantities. It is similarly possible to speak of the decrement of a part of a circuit in the same manner as phase difference and resistance.

The decrement is definable in terms of an energy ratio. Thus

$$\delta = \pi \psi = \frac{\pi R}{\omega L} = \frac{\pi R I^2}{\omega L I^2} = \frac{\frac{\pi R I^2}{2 \pi f}}{L I^2} = \frac{1}{2} \frac{\frac{R I^2}{f}}{L I^2}$$

For sustained sinusoidal oscillations, $\frac{R I^2}{f}$ = energy dissipated per cycle, and $L I^2 = \frac{1}{2} L (2 I^2) = \frac{1}{2} L I_o^2$ = magnetic energy associated with the current at the maximum of the cycle. It follows that the decrement is one-half the ratio of the energy dissipated per cycle to the energy associated with the current at the maximum of the cycle. This relation holding for each cycle, it holds for the average of all the cycles.

That the same definition of decrement applies to the natural damped oscillations of a circuit may be shown as follows. Suppose the circuit to be set oscillating, so that a train of natural oscillations takes place, N times per second, and that the energy of each train is practically all dissipated before the next one begins. N is the group frequency or number of complete trains of oscillations per second. The energy dissipated during a train of waves equals the energy input at the beginning of each train $= \frac{1}{2} L I_o^2$, where I_o is the first maximum of current. The average energy dissipated per cycle must equal the energy dissipated during a train of waves divided by the number of cycles in a train. The number of cycles in a train is the ratio of the frequency of oscillations f to the group frequency N. Therefore, the average energy dissipated per cycle $=$ $\frac{\frac{1}{2} L I_o^2}{\frac{f}{N}} = \frac{N L I_o^2}{2f}$. The average energy associated with the current at the maximum of each cycle may be shown to be $L\,I^2$, where I^2 = root-mean-square current, just as in the case of undamped currents, provided the decrement is not large. Applying the energy-ratio definition of decrement,

$$\delta = \frac{1}{2}\frac{\dfrac{N L I_o^2}{2f}}{L I^2} = \frac{N I_o^2}{4 f I^2} \tag{8}$$

This checks the familiar equation for the root-mean-square value of natural oscillations,

$$I^2 = \frac{N}{4 f \delta} I_o^2 = \frac{N}{4 a} I_o^2 \tag{9}$$

This definition of decrement, one-half the ratio of the average energy dissipated per cycle to the average energy associated with the current at the maximum of each cycle, is a valuable conception. It has here been shown to apply to both undamped oscillations and to natural damped oscillations.

It is thus evident that resistance, phase difference, sharpness of resonance, and decrement are all constants of a circuit expressing energy dissipation or power factor. Their relations are given in equations (2) to (7). Using these relations, a measurement of any one of them can be made to give all the others. Since resistance is the simplest quantity, specific consideration is given in the following to resistance measurement. Nevertheless in some cases one of the other quantities is the more convenient or more useful one in terms of which to express the results of measurement.

Methods of Measurement

General.—There is considerable difficulty in attaining high accuracy in measurements at radio frequencies. Much of this is due to the fact that the quantities to be measured or upon which the measurement depends are generally small and sometimes not definitely localized in the circuits. Thus the inductances and capacities used in the measuring circuits are so small that the effect upon these quantities of lead wires, indicating instruments, surroundings, etc., must be carefully considered. The capacity of the inductance coil and sometimes even the inductance within the condenser are of importance. In order to minimize these various effects, it is generally best to use measuring circuits and methods which are the least complicated. On this account simple circuits and substitution methods in which the determination depends upon deflections are usually used in preference to more complicated methods.

In addition to the uncertainty or the distributed character of some of the quantities to be measured, there are other limitations upon the accuracy of radio measurements. The usual ones are the variation with frequency of current distribution, of inductance, resistance, and so on, and the difficulty of supplying radio frequency current of sufficient constancy. The latter limitation is entirely overcome by the use of the electron tubes as a source of current, but is troublesome when a buzzer, spark, or arc is used. As to the other difficulty: the variations of inductance, and so on, with frequency, while these variations have a profound effect, they are generally subject to control; the quantities have definite values at a particular frequency under definite conditions, and their effect can usually be determined by calculation or measurement.

It is not always possible to determine the effects of the capacities of accessory apparatus and surroundings, nor to eliminate them, and thus they remain the principal limitation upon the accuracy of measurements. These stray capacities include the capacities of leads, instrument cases, table tops, walls, and the observer. They may not only be indeterminate but may vary in an irregular manner.

Classification of Methods

On account of the requirement of simplicity in radio measurements, the methods available are quite different, and are fewer in number, than in the case of audio-frequency or direct-current measurements.

The methods of measuring radio-frequency resistance may be roughly classed as:

(1) Calorimeter method.
(2) Substitution method.
(3) Resistance-variation method.
(4) Reactance-variation method.

The fourth has frequently been called the "decrement method," but it is primarily a method of measuring resistance rather than decrement, exactly as the resistance variation method is. Either may be used to measure the decrement of a wave under certain conditions, and in fact the results of resistance measurement by any method may be expressed in terms of decrement.

All four methods may be used with either damped or undamped waves, tho in some of them the calculations are different in the two cases. They are all deflection methods, in the sense of depending upon the deflections of some form of radio-frequency ammeter. In the first and second, however, it is only necessary to adjust two deflections to approximate equality, while in the third and fourth the deflections may have any magnitude.

CALORIMETER METHOD

This method may be used to measure the resistance either of a part or the whole of a circuit. The circuit or coil or other apparatus, the resistance of which is desired, is placed in some form of calorimeter, which may be a simple air chamber, an oil bath, or other suitable form. The current is measured by an accurate radio-frequency ammeter, and the resistance R_x is calculated from the observed current I and the power, or rate of heat production, P

$$P = R_x I^2 \qquad (10)$$

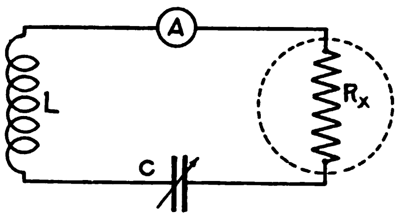

FIGURE 4

While P might be measured calorimetrically, in practice it is always measured electrically by an auxiliary observation in terms of audio-frequency or direct current Thus it is only necessary to observe the temperature of the calorimeter in any arbitrary units when the radio-frequency current flows, and then cause audio-frequency current to flow in the circuit, adjusting its value until the temperature becomes the same as before. Denoting by the subscript "$_o$" the audio-frequency values

$$P_o = R_o I_o^2$$

$$\frac{P}{P_o} = \frac{R_x I^2}{R_o I_o^2}$$

$$\frac{R_x}{R_o} = \frac{P}{P_o} \frac{I_o^2}{I^2}$$

For $P=P_o$, $$R_x = R_o \frac{I_o^2}{I^2} \tag{11}$$

From the known audio frequency value of the resistance, therefore, and the observed currents, the resistance is obtained.

The radio and audio frequency observations are sometimes made simultaneously, using another resistance of the same magnitude as that of the apparatus, the resistance of which is desired, placed in another calorimeter as nearly identical with the first as possible. High (radio) frequency current is passed thru one, low (audio) frequency thru the other, and the calorimeters kept at equal temperatures by means of some such device as a differential air thermometer or differential thermoelement. To compensate for inequalities in the two sets of apparatus, the radio and audio frequency currents are interchanged. This method may be found more convenient in some circumstances, but the extra complication of apparatus is usually not worth while, and the value of the measurement depends upon the accurate observation of the radio frequency current I, just as the simpler method does.

The calorimeter method, while capable of high accuracy, is slow and less convenient than some of the other methods. It has been used by a number of experimenters to measure the resistance of wires and coils.

Substitution Method

This method is applicable only to a portion of a circuit. Suppose that in Figure 4 the coil L is loosely coupled to a source of oscillations The capacity C is varied until resonance is obtained, and the current in the ammeter is read A resistance standard is then substituted for the apparatus R_x and varied until the same current is indicated at resonance. If the substitution has changed the total inductance or capacity of the circuit, the retuning to resonance introduces no error when undamped or slightly damped electromotive force is supplied, provided the change of condenser setting introduces either a negligible or known resistance change. In the case of a rather highly damped source, however, the method can only be used when the resistance substitution does not change the inductance or capacity of the circuit. The unknown R_x is equal to the standard resistance inserted, *provided* the electromotive force acting in the circuit has not been changed by the substitution of the standard for R: this condition is discussed below.

The resistance standards usually used are not continuously variable, and hence the standard used may give a deflection of the ammeter somewhat different from the original deflection. To determine the resistance in this case, three deflections are required, all at resonance. In one application, the apparatus of unknown resistance R_x is inserted and the current I_x observed; then a similar apparatus of known resistance R_n is substituted for it and the current I_n observed; and finally a known resistance R_1 is added and the current I_1 observed. The relations between these quantities and the electromotive force involve the unknown but constant resistance of the remainder of the circuit R, thus,

$$R_x+R=\frac{E}{I_x}$$

$$R_n+R=\frac{E}{I_n}$$

$$R_1+R_n+R=\frac{E}{I_1}$$

from which

$$R_x-R_n=R_1\frac{\frac{I_n}{I_x}-1}{\frac{I_n}{I_1}-1} \qquad (12)$$

This method is closely related to the resistance variation method; see formula (13) below.

The substitution method is very convenient and rapid and is suitable for measurements upon antennas, spark gaps, etc., and for rough measurements of resistances of condensers and coil. In radio laboratory work, however, using delicate instruments and with loose coupling to the source of oscillations, it is found that it is not a highly accurate method, except for measuring small changes in resistance of a circuit. The reason for this is that there are other electromotive forces acting in the circuit than that purposely introduced by the coupling coil, viz., emf.'s electrostatically induced between various parts of the circuit. When the apparatus under measurement is removed from the circuit, these emf.'s are changed, and there is no certainty that when the current is made the same the resistance has its former value. Something of the same difficulty enters into the question of grounding the circuit in the following method, as discussed below.

Resistance Variation Method

This method measures primarily the effective resistance of the whole circuit, including that due to condenser losses and to radiation. The principle may be readily understood from the diagram of the simple circuit, Figure 5.

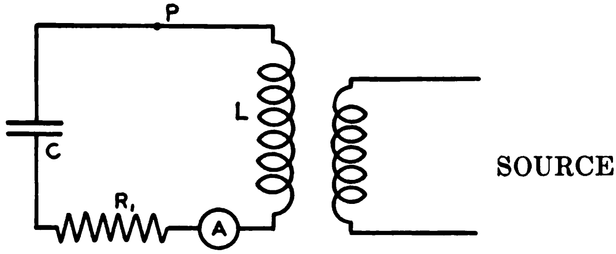

FIGURE 5

If the resistance of some particular piece of apparatus, inserted at P for example, is to be found, the resistance of the circuit is measured with it in circuit and then re-measured in the same way with it removed or replaced by a similar apparatus of known resistance; and the resistance of the apparatus is obtained by simple subtraction.

The results of a measurement may be expressed in terms of ψ, S, or δ by equations (2) to (7) above. The method, however, is particularly convenient where resistance is the actual quantity the value of which is wanted.

The measurement is made by observing the current I in the ammeter A when the resistance R_1 has its zero or minimum value, then inserting some resistance R_1 and observing the current I_1. Let R denote the resistance of the circuit without added resistance. Suppose that a sine-wave electromotive force E is introduced into the circuit by induction in the coil L from a source of undamped waves, and that the two observations are made at resonance. For the condition of resonance,

$$I = \frac{E}{R}$$

$$I_1 = \frac{E}{R+R_1}$$

from which the resistance of the circuit is given by

$$R = R_1 \frac{I_1}{I-I_1} = \frac{R_1}{\frac{I}{I_1}-1} \tag{13}$$

The same method can be employed using damped instead of continuous waves, and can even be used when the current is supplied by impulse excitation, but the equations are different; see (59) and (16) below. When the damping of the supplied emf. is very small, equation (13) applies.

PRECAUTIONS

A limitation on the accuracy of the measurement is the existence of the emf.'s electrostatically induced that were mentioned above. In the deduction of (13) it is assumed that E remains constant. The virtue of this method is that these emf.'s may be kept substantially constant during the measurement of resistance of the circuit. They will invariably be altered by the insertion of the apparatus, the resistance of which is desired, but the resistance of the circuit is measured accurately in the two cases and the difference of the two measurements gives the resistance sought. In order to keep these stray electromotive forces unchanged when R_1 is in and when it is out of circuit, particular attention must be paid to the grounding of the circuit. The shield of the condenser and the ammeter (particularly if it is a thermocouple with galvanometer) have considerable capacity to ground and are near ground potential. A ground wire, if used, must be connected either to the condenser shield or to one side of the ammeter. If connected to the high-potential side of the inductance coil, absurd results will be obtained. The resistance R_1 also must be inserted at a place of low potential, preferably between the condenser and ammeter.

USE OF THERMOCOUPLE

Another necessary precaution is to keep the coupling between source and measuring circuit so loose that there is no reaction. This necessitates the use of a sensitive device for current measurement. As regularly carried out at the Bureau of Standards, in the resistance variation method, a pliotron is used as a source of undamped emf., and current is measured with a thermocouple in series in the measuring circuit. The currents corresponding to given deflections of the thermocouple galvanometer are obtained from a calibration curve, or from the law $d \propto I^2$, where d = deflection, if the instrument follows this law sufficiently closely. When the deflections follow this law, equation (13) becomes

$$R = \frac{R_1}{\sqrt{\frac{d}{d_1}} - 1} \tag{14}$$

Several values of resistance R_1 are usually inserted in the circuit and the corresponding deflections obtained; the resulting values of R are averaged.

When the thermocouple follows the square law accurately, the quarter deflection method may be used, which eliminates all calculation. When the deflection d_1 is $\frac{d}{4}$, equation (14) becomes

$$R = R_1 \tag{15}$$

This method requires a variable resistance standard such that R_1 can be varied continuously in order to make d_1 just equal to $\frac{d}{4}$. Practically the same method is used if the resistance is varied by small steps, as in a resistance box, and interpolating between two settings of R_1.

USE OF IMPULSE EXCITATION

The procedure for the resistance variation method is the same when the current is damped as when undamped. When the circuit is supplied by impulse excitation, so that free oscillations are produced, the theory of the measurement is very simple. The current being I when the resistance is R, and I_1 when the resistance R_1 is added, the power dissipated in the circuit must be the same in the two cases because the condenser in the circuit is charged to the same voltage by each impulse which is impressed upon it, and there is assumed to be no current in the primary after each impulse.

Therefore

$$R I^2 = (R + R_1) I_1^2$$

whence,

$$R = R_1 \frac{I_1^2}{I^2 - I_1^2} \tag{16}$$

It is difficult to obtain high accuracy by the method in practice because of the difficulty of obtaining pure impulse excitation.

The method is specially convenient when an instrument is used in which the deflection d is proportional to the current squared. Then (16) becomes

$$R = R_1 \frac{d_1}{d - d_1} \tag{17}$$

This is still further simplified if the resistance R_1 is adjustable so that d_1 can be made equal to one-half d. The equation then reduces to

$$R = R_1 \tag{18}$$

This is commonly known as the half-deflection method.

Use of Damped Excitation

The resistance variation method has already been shown to be usable with either undamped or free oscillations. It can also be used when the supplied emf. is damped so that both forced and free oscillations exist in the circuit. The equations (56) to (65) below show how the decrement of a circuit is obtained from such measurements. Resistance is then readily calculated by equation (6). As already stated, when the damping of the supplied emf. is extremely small, equation (13) applies. The decrement of the supplied emf. may itself be obtained by such measurements whether the emf. be due to a nearby circuit or to a wave travelling thru space.

Application of Method

This method is used in precision measurements upon condensers, coils, wavemeters, etc. The accurate measurement of resistance of a wavemeter circuit is of particular importance because the wavemeter is frequently used to measure the resistance, phase difference, or decrement of other apparatus. It is the calibration of a resistance-measuring standard.

The resistance of a wavemeter is not a single constant value. It varies with frequency and with the detecting or other apparatus connected to the wavemeter circuit. Usually both the resistance and the decrement of the circuit vary with the condenser setting. It is usually desirable to express either resistance, sharpness of resonance, or decrement in the form of curves for the several wavemeter coils, each for a particular detecting apparatus or other condition.

When a pliotron, arc, or other source of undamped wave is used, formula (13) above is used. When the current-measuring device is a current-square meter, thermocouple or crystal detector with galvanometer, or other apparatus which is so calibrated that deflections are accurately proportional to the square of the current, and when in addition a continuously variable resistance standard is used, the quarter-deflection method may be employed eliminating all calculation.

When a buzzer or other source is used, arranged to give impulse excitation, equation (16) above gives the resistance.

When the current indicator is calibrated in terms of the square of the current and the resistance standard is continuously variable, the measurement is conveniently made by the half-deflection method.

REACTANCE VARIATION METHOD

This has been called the decrement method, a name which is no more applicable to this than to the other methods of resistance measurement since all measure decrement in the same sense that this does. That the method primarily measures resistance rather than decrement is seen from the fact that in its simple and most accurate form it utilizes undamped current, which has no decrement.

The method is analogous to the resistance variation method, two observations being taken. The current I_r in the ammeter (Figure 6) is measured at resonance, the reactance is then varied and the new current I_1 is observed. The total resistance of the circuit R (including that due to condenser losses, radiation, etc.) is calculated from these two observations. The reactance may be varied by changing either the capacity, the inductance, or the frequency, the emf. being maintained constant. The reactance is zero at resonance and it is changed to some value X_1 for the other observation. With undamped emf. E, the currents are given by

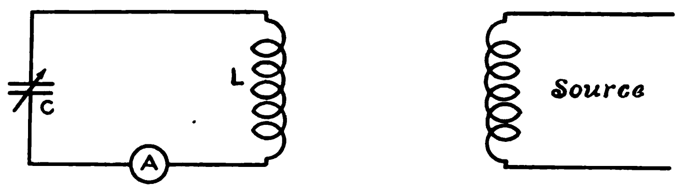

FIGURE 6

$$I_r^2 = \frac{E^2}{R^2}$$

$$I_1^2 = \frac{E^2}{R^2+X_1^2}$$

From these it follows that

$$R = X_1\sqrt{\frac{I_1^2}{I_r^2-I_1^2}} \qquad (19)$$

This has a similarity to $R = R_1 \frac{I_1}{I - I_1}$, the equation (13) for the resistance-variation method. It is also interesting that when the reactance is varied by such an amount as to make the quantity under the radical sign equal to unity, the equation reduces to

$$R = X_1 \tag{20}$$

This is similar to $R = R_1$, which is the equation for the quarter-deflection and half-deflection resistance-variation methods.

Resistance Measurement

When the reactance is varied by changing the setting of a variable condenser,

$$X_1 = \pm \left(\frac{1}{\omega C} - \frac{1}{\omega C_r} \right)$$

and the equation (19) becomes

$$R = \frac{\pm (C_r - C)}{\omega C_r C} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \tag{21}$$

For variation of the inductance, (19) becomes

$$R = \pm \omega (L - L_r) \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \tag{22}$$

and for variation of the frequency

$$R = \frac{\pm L (\omega^2 - \omega_r^2)}{\omega} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \tag{23}$$

This equation is equivalent to

$$R = \frac{\pm 6 \pi \times 10^8 L (\lambda_r^2 - \lambda^2)}{\lambda \lambda_r^2} \sqrt{\frac{I_1^2}{I_r^2 - I_1^2}} \tag{24}$$

for λ in meters, R in ohms, and L in henrys.

It must be noted that variation of the frequency or wave length requires some alteration in the source of emf., and the greatest care is necessary to insure that the condition of constant emf. is fulfilled. This is discussed below in connection with equations (30) and (31).

In the use of equation (22), some error is introduced into the measurement if the variable inductor is also used as the coupling to the source, on account of the variation thus introduced into the E supplied. The per cent. error, however, is usually not more than the per cent. variation of L.

A convenient method which differs slightly from those just described is to observe two values of the reactance both corresponding to the same current I_1 on the two sides of the resonant value I_r. For observation in this manner of two capacity values C_1 and C_2,

$$R=\frac{1}{2\omega}\frac{C_2-C_1}{C_2C_1}\sqrt{\frac{I_1^2}{I_r^2-I_1^2}} \tag{25}$$

The simple derivation here given for these formulas is much shorter than the usual treatments, and at the same time is more comprehensive. These formulas are all rigorous, involving no approximations, provided the applied emf. is undamped. They also apply for damped emf. when the damping is negligibly small.

It is customary to reduce the labor of computation by varying the reactance by such an amount that $I_1^2=\frac{1}{2}I_r^2$, making the quantity under the radical sign equal to unity, so that formulas (21) to (25) are much simplified.

Measurements of Phase Difference, Sharpness of Resonance, and Decrement

Measurements by the reactance variation method are very conveniently expressed in terms of phase difference, sharpness of resonance, and decrement. The formulas are in fact simpler for any of these quantities than for resistance. Thus, utilizing equations (2) to (7) it is readily found that (21) is equivalent to:

$$\psi=\frac{\pm(C_r-C)}{C}\sqrt{\frac{I_1^2}{I_r^2-I_1^2}} \tag{26}$$

$$S=\frac{C}{\pm(C_r-C)}\sqrt{\frac{I_r^2-I_1^2}{I_1^2}} \tag{27}$$

$$\delta=\pi\frac{\pm(C_r-C)}{C}\sqrt{\frac{I_1^2}{I_r^2-I_1^2}} \tag{28}$$

Equation (27) is identical with (4) above, thus suggesting that the definition of sharpness of resonance itself contains inherently this method of measurement. The equations corresponding to (22) to (25) are obtained for ψ, S, and δ, in the same manner as (26) to (28). Those for phase difference, expressed in radians, are

$$\psi=\frac{\pm(L-L_r)}{L_r}\sqrt{\frac{I_1^2}{I_r^2-I_1^2}} \tag{29}$$

$$\psi=\frac{\pm(\omega^2-\omega_r^2)}{\omega\,\omega_r}\sqrt{\frac{I_1^2}{I_r^2-I_1^2}} \tag{30}$$

$$\psi=\frac{\pm(\lambda_r^2-\lambda^2)}{\lambda\,\lambda_r}\sqrt{\frac{I_1^2}{I_r^2-I_1^2}} \tag{31}$$

$$\psi=\frac{C_2-C_1}{C_2+C_1}\sqrt{\frac{I_1^2}{I_r^2-I_1^2}} \tag{32}$$

Phase difference is a particularly convenient constant in terms of which to express the results of measurements upon condensers, since the phase difference of most condensers is usually a constant with respect to frequency at radio frequencies. These formulas are rigorous provided ψ is small, as it usually is in radio circuits, when the emf. is sustained, and hold also for damped emf. when the damping is negligibly small. The use of the method when the applied emf. has a moderate damping is discussed in the last section of this paper.

A convenient way to utilize the method indicated in (30) and (31) is to vary the wave length by means of a variable condenser or inductor in the source circuit. An incorrect formula has sometimes been given for decrement measurement by this method. The following are rigorous:

$$\psi=\frac{\pm(C_r'-C')}{\sqrt{C_r'C'}}\sqrt{\frac{I^2_1}{I_r^2-I_1^2}}$$

$$\delta=\pi\frac{\pm(C_r'-C')}{\sqrt{C_r'C'}}\sqrt{\frac{I_1^2}{I_r^2-I_1^2}}$$

where the capacities are those of the condenser in the source circuit. This method must be used with great caution because constancy of E, the applied emf., is required. The source circuit is necessarily disturbed by the variation of its condenser setting; when the variation is small, and a pliotron is used as the source, the current I' in the source circuit may not be appreciably changed. It is desirable to use a sensitive indicating instrument and actually observe I'. Constancy of I', however, would not mean that the emf. acting on the measuring circuit was constant, for $E=\omega\,M\,I'$, and thus E varies by the amount of the ω variation. The per cent. error in the resulting value of ψ or δ equals the per cent. change of C', when the method is made by the familiar procedure which reduces the current ratio under the radical to unity.

DIRECT-READING PHASEMETERS AND DECREMETERS

A phasemeter as used in radio work is a wavemeter conveniently arranged for measurements of phase difference. A decremeter is a wavemeter similarly arranged for measurements of decrement. While, of course, resistance and sharpness of resonance can be calculated from measured values obtained by either of these instruments, the principal application of a phasemeter is in measurements of phase difference of condensers and of dielectric materials and the principal use of a decremeter is in the measurement of the decrement of a wave. The forms of these instruments usually employed make use of the reactance-variation method. Any such instrument may be used either as a phasemeter or a decremeter by merely changing the instrument scale by a constant factor. While decremeters have been more commonly used, the phasemeter is a somewhat more direct application of the underlying theory. In the development of the theory of the instrument, undamped (sustained) sine-wave emf. is assumed.

Determination of the Scale of a Phasemeter or Decremeter

Any wavemeter, the circuit of which includes some form of ammeter, may be fitted with a special scale from which phase difference or decrement may be read directly. The procedure for a wavemeter having any sort of variable condenser is given here.

The usual use of the reactance-variation method is in accordance with equation (32), the currents being adjusted so as to make the quantity under the radical unity. That is, the current-square meter is first observed at resonance, the variable condenser is reset to a value C_1 on one side of resonance such that the current-square is reduced to one-half, and then set to another value C_2 on the other side of resonance giving the same current-square. The phase difference is calculated by

$$\psi = \frac{C_2 - C_1}{C_2 + C_1} \qquad (33)$$

A certain value of phase difference, therefore, corresponds to that displacement of the condenser's moving plates which varies the capacity by the amount $(C_2 - C_1)$. The displacement for a given phase difference will, in general, be different for different values of C, the total capacity in the circuit. At each point of the condenser scale, therefore, any displacement of the moving plates which changes the square of current from $\frac{1}{2} I_r^2$ on one side of resonance to the same value on the other side means a certain value of ψ.

A special scale may, therefore, be attached to any variable condenser, with graduations upon it and so marked that the difference between the two settings on the two sides of resonance is equal to the phase difference. The spacing of the graduations at different parts of the scale depends upon the relation between capacity and displacement of the moving plates. When this relation is known, the scale can be predetermined. A scale may, therefore, be fitted to any condenser, from which phase difference may be read directly, provided the capacity of the circuit is known for all settings of the condenser. The scale may be attached either to the moving plate system or to the fixed condenser top. It is usually convenient to attach it to the unused half of the dial opposite the capacity scale.

The scale for such an instrument is determined as follows. When the change of capacity setting is small, as usually in radio work, (33) may be written

$$\psi = \frac{dC}{2C} \tag{34}$$

Letting s denote readings on the required scale, the value of ψ is the difference of two s readings, or

$$\psi = ds$$

$$ds = \frac{dC}{2C}$$

The readings of the scale are then given by

$$s = \int_C^{C_a} \frac{dC}{2C}$$

$$s = \tfrac{1}{2}\,(\log_\epsilon C_a - \log_\epsilon C) \tag{35}$$

C_a is the arbitrary capacity chosen as the zero point of the scale. Thus the scale can begin anywhere. Such a scale gives ψ in radians.

A wavemeter in which the inductance is variable and the capacity fixed is also convertible into a phasemeter in similar manner. The instrument is operated in just the same way, and the equation, corresponding to (33), is

$$\psi = \frac{L_2 - L_1}{L_2 + L_1} \tag{36}$$

and the direct-reading phasemeter scale is given by

$$s = \tfrac{1}{2}\,(\log_\epsilon L_a - \log_\epsilon L) \tag{37}$$

where L_a is the arbitrary inductance chosen as the zero point of the scale.

A direct-reading decremeter is made in precisely the same way as a phasemeter. The decrement is π times the phase difference in radians, hence the equations for a decrement scale on a wavemeter with a variable condenser or variable inductor are respectively

$$s = \frac{\pi}{2}(\log_e C_a - \log_e C) \tag{38}$$

$$s = \frac{\pi}{2}(\log_e L_a - \log_e L) \tag{39}$$

The phase difference or decrement measured by such an instrument, using undamped sine-wave emf., is the phase difference or decrement of the measuring circuit itself. Its application to measuring the decrement of a wave is explained in the last section below. When the instrument is used as the measuring circuit with undamped emf., the variable condenser or inductor must be one having zero effective resistance or in which the resistance for each setting and each wave length is accurately known, in order that the resistance or ψ or δ of other apparatus connected in the circuit may be obtained. When a variable condenser is used as the phasemeter it is thus convenient for measurements upon coils, and when a variable inductor is the phasemeter it is a convenient means for measurements upon the R or ψ or δ of any condenser connected to it.

The direct-reading phasemeter or decremeter may also be used in the source circuit, to vary the ω or λ supplied to the measuring circuit, as described in connection with equations (30) and (31) above. In this use it is not necessary to make correction for the resistance of the phasemeter or decremeter itself, whether it be variable condenser or variable inductor, as it has no effect upon the measuring circuit, except insofar as it may effect the value of ω, which would be a second-order effect. This use of the direct-reading phasemeter is now being exhaustively studied by Messrs. G. C. Southworth and J. L. Preston at the Bureau of Standards.

Simple Direct-reading Phasemeter or Decremeter

It is particularly easy to make a phasemeter or decremeter out of a condenser with semi-circular plates. Such condensers follow closely the linear law,

$$C = a\beta + C_o \tag{40}$$

where β is the angle of rotation of the moving plates and a and C_o are constants. It can be shown that the phase difference scale applicable to such a condenser is one in which the graduations vary as the logarithm of the angle of rotation. Furthermore, the same scale applies to all condensers of this type. This scale has been calculated and is given in Figure 7 for values of phase difference in degrees.

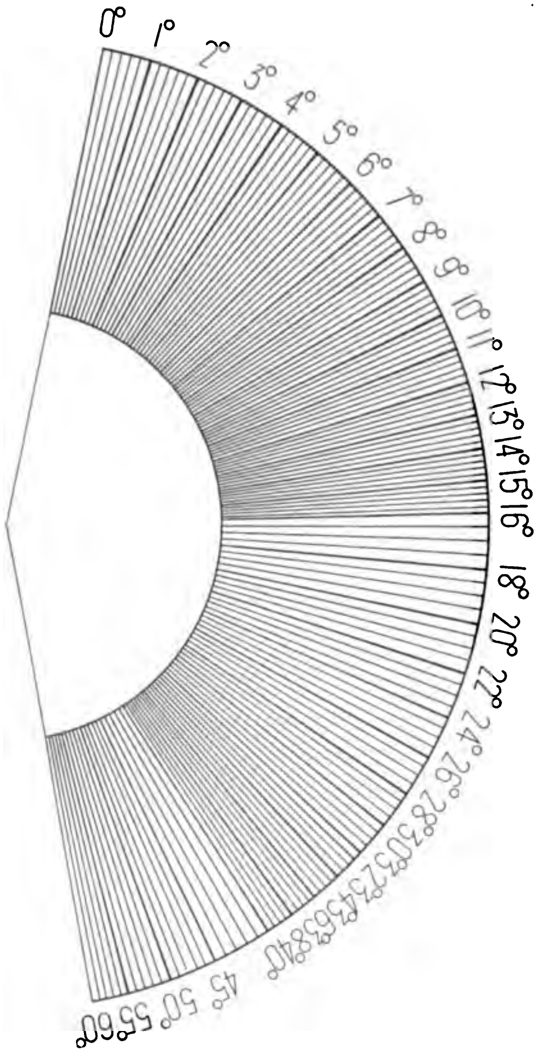

FIGURE 7

The scale was calculated in the following manner. Inserting equation (40) in (35)

$$s = \tfrac{1}{2}\left[\log_{\epsilon}(a\,\beta_a + C_o) - \log_{\epsilon}(a\,\beta + C_o)\right] \qquad (41)$$

Let $C_o = a\,\beta_o$,

$$s = \frac{2.303}{2}\left[\log_{10}(\beta_a + \beta_o) - \log_{10}(\beta + \beta_o)\right] \tag{42}$$

For $C_o = \beta_o = 0$,

$$s = 1.151\ (\log_{10}\beta_a - \log_{10}\beta) \tag{43}$$

$$\log_{10}\beta = \log_{10}\beta_a - \frac{s}{1.151}$$

For s expressed in degrees rather than radians, and for $\beta_a = 180°$, the angular separation in degrees on the ψ scale is given by

$$\log_{10}\beta = \log_{10}180 - \frac{s}{57.3\ (1.151)} \tag{44}$$

This scale may be used as it stands on any variable condenser with semi-circular plates, regardless of the kind of capacity scale on the condenser or even if the condenser has no scale whatever on it. The phase difference scale may, if desired, be cut out and trimmed at such a radius as to fit the dial and then affixed to the condenser, with its zero point approximately in coincidence with the graduation which corresponds to maximum capacity. This usually puts it on the unused half of the dial opposite the capacity scale. If the figures are trimmed off they can be added over the lines in red ink. This scale will then give accurate results if the capacity varies linearly with the setting, a condition which holds closely enough in the ordinary condensers. This same scale may also be affixed to the dial of any variable inductor and used without error if the variation of inductance with setting is linear. Also on either condenser or inductor, the same scale is used either with moving pointer and stationary dial or with moving dial.

A measurement of phase difference is made by first observing the current-square at resonance, then reading the scale at a setting on each side of resonance for which the current-square is one-half its value at resonance. The difference between the two readings on the scale is the value of ψ in degrees. The value of power factor in per cent. may be obtained from the result if desired by multiplying by 1.75.

A similar scale is readily made to read decrements directly. The readings of Figure 7 are all divided by 18.24, or the scale is independently calculated by equation (38). The scale shown in Figure 8 is thus obtained, which may be used on any condenser with semi-circular plates. Since writing this paper, the author has been informed that a scale constructed on this principle was devised for use in a decremeter by Mr. Waterman of the Marconi Company. This is the only instance known in which the method has been used. It has here been shown to be convenient for measurements of other quantities than decrement, and is worthy of wide application, since it converts any wavemeter into a direct-reading phasemeter or decremeter at no additional cost.

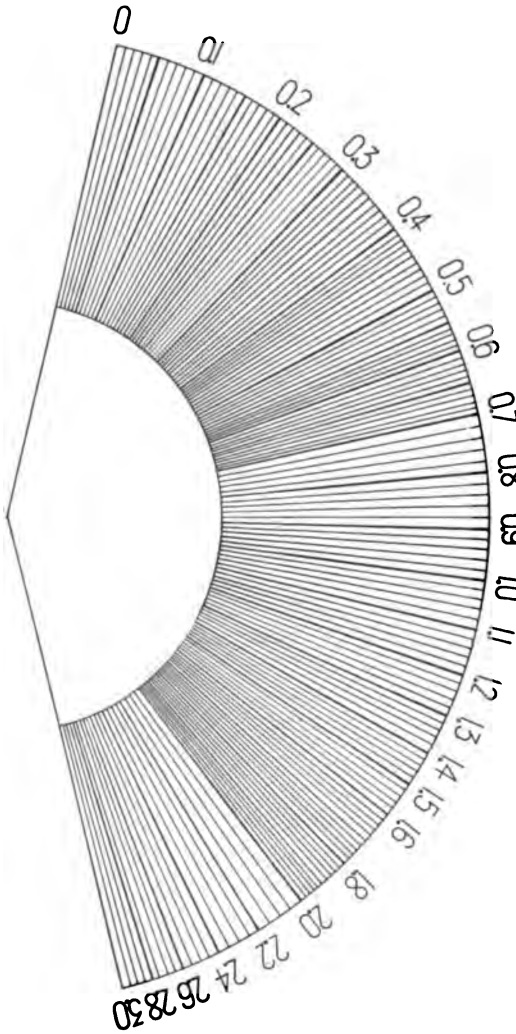

FIGURE 8

LOCATION OF SCALE FOR ACCURATE MEASUREMENTS

The scale gives accurate results only when $C_o=\beta_o=0$. In many semi-circular plate condensers C_o or β_o has a small positive or negative value. This can always be reduced to zero by shifting the β scale. Thus for a particular position of the scale suppose

$$C=a\beta'+C_o \qquad (45)$$
$$=a(\beta'+\beta_o)$$

Define a new β such that

$$\beta=\beta'+\beta_o \qquad (46)$$

This reduces (45) to $C=a\,\beta$, and it is accomplished by shifting the dial toward zero by the amount β_o. The value of β_o is determined by two measurements of capacity. Suppose C_1 and C_2 are the values for β_1' and β_2'. The constant a is the change of capacity per degree and is given by

$$a=\frac{C_2-C_1}{\beta_2'-\beta_1'} \qquad (47)$$

The angle β_o is therefore given by

$$\beta_o=\frac{C_1}{a}-\beta_1' \qquad (48)$$

The scale of Figure 7 is accurately placed as follows. It is first placed on the dial by eye, and the capacity in the circuit accurately observed at the two points marked 5 and 36 on the scale. The amount by which the scale is to be shifted toward zero is then the angle in degrees,

$$\beta_o=\frac{100\,C_{36}}{C_5-C_{36}}-51.2 \qquad (49)$$

The capacity concerned is the total capacity in the circuit, which consists mainly of the capacity of condenser and of the inductance coil in parallel with it. Since the coils of a wavemeter do not all have the same capacity, it is desirable to mount the phase difference scale in such a way that its angular position can be varied a few degrees on the dial, to correspond to the different coils that are used. A fiducial mark can be placed on the scale for each coil.

MEASUREMENT OF SMALL PHASE DIFFERENCE OR DECREMENT

These scales permit accurate measurement of fairly large phase differences or decrements, but offer no precision in the measurement of very small values, particularly at the low-capacity end of the scale. They are thus of particular value in tests upon condensers or other apparatus having fairly large phase differences. The method can, however, be extended to the precise measurement of small values in several ways. One method is to use a gear to open out the scale. The scale can then be in the form of a spiral on the rapid-motion gear shaft, and be spaced by a factor equal to the gear ratio. This device does not have the simplicity of merely attaching a scale to the condenser dial. Another method is to place a condenser of fixed capacity in parallel with the variable and use a different scale on the variable. This narrows the range of capacity variation, but for some kinds of work the method is very satisfactory, and any desired precision of measurement may be obtained. The scale suitable for the use in parallel with the variable of a fixed capacity equal to 10 times that at the middle of the scale of the variable, is obtained as follows. The fixed capacity is C_o in (40), and its value must be equal to 10 times that at the 90° point on the variable condenser (or 19.86 on the scale of Figure 7). Expressing β in degrees, $C_o = a\ \beta_o = a \times 900$. Equation (41) becomes

$$s = \frac{2.303}{2}\left[log_{10}\left(\beta_a + 900\right) - log_{10}\left(\beta + 900\right)\right]$$

For the ψ scale beginning at the upper end of the capacity scale and s expressed in minutes, this becomes

$$s = \frac{60\,(57.3)\,(2.303)}{2}\left[log_{10}\,1080 - log_{10}\left(\beta + 900\right)\right]$$

or,

$$log_{10}\left(\beta + 900\right) = 3.033424 - \frac{s}{3957.9} \qquad (50)$$

The scale thus calculated is given in Figure 9. It may be used on any linear scale condenser, as in the previous cases, and permits measurements of phase difference to closer than one minute. A similar scale is obtained for decrement by dividing the scale readings by 1094., permitting the measurement of decrement to better than 0.001. These scales have the additional advantage of almost uniform spacing.

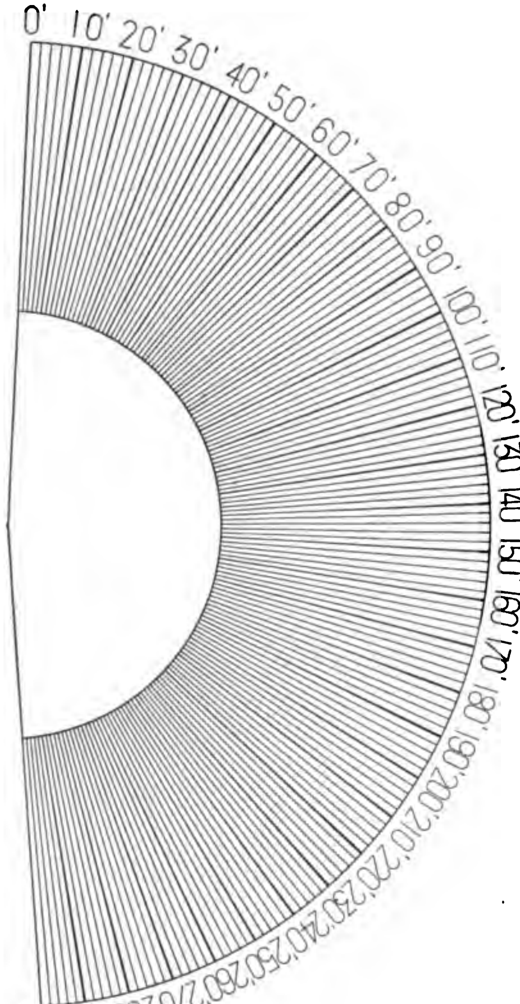

FIGURE 9

DECREMETER OR PHASEMETER WITH UNIFORM SCALE

Just as it is possible to determine a ψ or δ scale to fit a condenser having any sort of law of capacity variation, it is equally possible to design a condenser with capacity varying in such a way as to fit any specified ψ or δ scale. A uniform scale, i. e., one in which the graduations are equally spaced, is particularly convenient, and is the kind used in the Kolster decremeter.

A uniform scale of either ψ or δ requires in accordance with equation (34) that the condenser plates be so shaped that for any small variation of setting the ratio of the change in the capacity to the total capacity is constant. The condenser required to give this uniform scale has its moving plates so shaped that the logarithm of the capacity is proportional to the angle of rotation of the plates.

This decremeter is fully described in "Bulletin of the Bureau of Standards," **11**, page 421, 1914, Scientific Paper Number 235.* By the use of a separate shaft geared to the moving plates at a 6-to-1 ratio, the decrement scale is opened out so that very precise measurements may be made. This decremeter is used in the inspection service of the Bureau of Navigation of the Department of Commerce and by radio engineers elsewhere. On account of the uniform scale of decrements its use is more convenient than the instruments with specially shaped scales, but, on the other hand, the adjustment of the instrument to read decrements accurately is more difficult as this requires the adjustment of a small auxiliary condenser in parallel with the variable condenser. It is, of course, a more costly instrument because of the specially shaped condenser plates. It can be made to read phase difference directly in degrees by replacing the decrement scale with another in which the readings are multiplied by 18.24.

Use of Damped Oscillations

When damped oscillations are used in a measurement of resistance or one of the related quantities, there are two distinct decrements concerned, that of the circuit and that of the emf. supplied to the circuit. To determine either of these, in general two measurements are required. In special cases, however, one measurement only is necessary. For example, when the decrement of the supplied emf. is very small, the measurement of resistance of the circuit is made exactly the same as when the emf. is undamped and the equations are unchanged, in all the methods. Also, when impulse excitation is used for the resistance-variation method, the procedure is the same as with undamped emf.; the equations are different in this case, as shown in equation (16) above. In any of these cases, of course, the results of measurement can be expressed in terms of ψ, s, or δ of the circuit as well as R.

When the emf. supplied to the circuit has a moderate de-

*See also "Proc. Inst. Radio Engrs.," volume 3, number 1, page 29, 1915.

crement, damped oscillations flow in the circuit. Calculation of the current is very difficult except when the decrement of both the emf. and the circuit are small. The definition of decrement that has been given in terms of an energy ratio furnishes some interesting relations in this connection. Suppose the emf. is produced in the measuring circuit (Figure 5) by coupling to the source circuit so loosely that there is no reaction upon the source. If I' = the root-mean-square current of small decrement in the source circuit and M is the mutual inductance between the two circuits, it may be shown that the r.m.s value of emf. induced in the measuring circuit at resonance is

$$E = \omega M I'$$

and the maximum amplitude is

$$E_o = \omega M I_o'$$

From equation (9), $(I')^2 = \frac{N}{4f\delta'}(I_o')^2$,

therefore

$$E^2 = \frac{N}{4f\delta'} E_o^2$$

where δ' is the decrement of the current in the source circuit and hence of the emf. induced in the measuring circuit. Using $\frac{E^2}{R}$ as a definition of power consumption, the average power dissipated is

$$\frac{E^2}{R} = \frac{NE_o^2}{4f\delta' R}$$

Average energy dissipated per cycle $= \frac{NE_o^2}{4f^2\delta' R}$.

Average energy associated with current at maxima $= LI^2$, provided the decrement is small, as before. Assuming now that decrements are additive, and applying the energy-ratio definition given just after equation (9) to the sum of δ', the decrement of the applied emf. and δ, the decrement of the circuit,

$$\delta' + \delta = \frac{1}{2} \cdot \frac{NE_o^2}{4f^2 \delta' R L I^2}$$

$$I^2 = \frac{NE_o^2}{8f^2 R L \delta' (\delta' + \delta)}$$

$$I^2 = \frac{NE_o^2}{16 f^3 L^2 \delta' \delta (\delta' + \delta)} \qquad (51)$$

This is the correct relation between I^2 and E_o^2 at resonance, as obtained from the elaborate rigorous proofs. The short demonstration just given involves the assumption that decrements are additive, which seems reasonable since energies are additive.

Resistance-Variation Method

Measurements made with damped waves are most conveniently expressed in terms of decrements. Resistances and the other related quantities can then be calculated from the values of decrement.

The equation for the resistance variation method is obtained from (51). Suppose the resistance of the circuit to be increased by an amount R_1 changing δ to $\delta+\delta_1$, and the original resonance current I to some other value I_1; then

$$I_1^2=\frac{NE_o^2}{16f^3L^2\delta'(\delta+\delta_1)(\delta'+\delta+\delta_1)}\cdot$$

$$\frac{I^2}{I_1^2}=\frac{(\delta+\delta_1)(\delta'+\delta+\delta_1)}{(\delta'+\delta)} \tag{52}$$

This is the equation for the resistance-variation method of measurement, using damped waves. It applies only when the decrements are small, and when the coupling to the source is so loose that the emf. is not affected by the current in the measuring circuit.

It is possible to solve either for δ' if δ is known or vice versa. When the method is thus used to obtain δ', the decrement of the applied emf., the result of the measurement really gives the shape of the trains of waves which are acting on the circuit. Decrement measurement may thus accomplish something similar at radio frequencies to what is done at low frequencies by wave analysis.

Determination of Decrement of Wave

The solution for δ' is

$$\delta'=\frac{2\,\delta\,\delta_1+\delta_1^2-\dfrac{I^2-I_1^2}{I_1^2}\delta^2}{\dfrac{I^2-I_1^2}{I_1^2}\delta-\delta_1} \tag{53}$$

This may be simplified by choosing the resistance inserted such that $\delta_1=\delta$; then

$$\delta'=\delta\frac{4\,I_1^2-I^2}{I^2-2\,I_1^2} \tag{54}$$

Another convenient simplified procedure is to vary the inserted resistance until the square of the current is reduced to one-half its previous value, then $\frac{I^2-I_1^2}{I_1^2}=1$, and

$$\delta'=\frac{2\,\delta\,\delta_1+\delta_1^2-\delta^2}{\delta-\delta_1} \tag{55}$$

This equation expresses the method presented by L. Cohen in "PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS," **2,** page 237, 1914.

DETERMINATION OF DECREMENT OF CIRCUIT

When δ' is the known quantity, the direct solution of (52) for δ, the decrement of the measuring circuit, is

$$\delta=\frac{1}{B}\delta_1-\frac{1}{2}\delta'\pm\frac{1}{2B}\sqrt{(B\,\delta')^2+4\,\delta_1^2+4B\,\delta_1^2} \tag{56}$$

where
$$B=\frac{I^2-I_1^2}{I_1^2}$$

This complicated form of solution is of very little use. Equation (52) is itself a more convenient expression than this explicit solution. The following formula has been found useful in certain cases as discussed below.

$$\delta=\delta_1\frac{K\,I_1^2}{I^2-K\,I_1^2} \tag{57}$$

where
$$K=1+\frac{\delta_1}{\delta'+\delta} \tag{58}$$

It is sometimes advantageous to express this in terms of resistance or the related quantities. Thus the solution for R of the circuit, where R_1 is the inserted resistance, is

$$R=R_1\frac{K\,I_1^2}{I^2-K\,I_1^2} \tag{59}$$

This is, of course, not an explicit solution for R, since K involves δ and, therefore, R, but gives a ready means for finding R or δ when the sum of the two decrements $(\delta'+\delta)$ is known from some other measurement, such as the reactance-variation method described below. Thus a combination of the two methods gives both δ' and δ, or δ' and R.

An interesting special case occurs when δ and δ_1 are both very small compared with δ'. K becomes unity and equation (59) reduces to

$$R=R_1\frac{I_1^2}{I^2-I_1^2} \tag{60}$$

This happens to be the same as equation (16) above, the equation for the use of impulse excitation. The proof given here can not, however, be regarded as a deduction of equation for impulse excitation, as it has been by some writers; since equation (51) is involved, which assumes that δ' and δ are both small.

Reactance-Variation Method

The procedure when the supplied emf. is damped is the same as when undamped, two observations of current being taken, one at resonance and the other after varying the reactance. The equations for decrement are only slightly different from those applying to undamped current.

Bjerknes' classical proof shows that the sum of the decrements of the emf. and of the measuring circuit is given by the same expression as that which gives the decrement of the measuring circuit when the emf. is undamped. Thus (28) becomes (61) below, and the equations for decrement corresponding to (21) to (25) become

$$\delta'+\delta=\pi\frac{\pm(C_r-C)}{C}\sqrt{\frac{I_1^2}{I_r^2-I_1^2}} \tag{61}$$

$$\delta'+\delta=\pi\frac{\pm(L-L_r)}{L_r}\sqrt{\frac{I_1^2}{I_r^2-I_1^2}} \tag{62}$$

$$\delta'+\delta=\pi\frac{\pm(\omega^2-\omega_r^2)}{\omega\,\omega_r}\sqrt{\frac{I_1^2}{I_r^2-I_1^2}} \tag{63}$$

$$\delta'+\delta=\pi\frac{\pm(\lambda_r^2-\lambda^2)}{\lambda_r\,\lambda}\sqrt{\frac{I_1^2}{I_r^2-I_1^2}} \tag{64}$$

$$\delta'+\delta=\pi\frac{C_2-C_1}{C_2+C_1}\sqrt{\frac{I_1^2}{I_r^2-I_1^2}} \tag{65}$$

These formulas are correct only when: (1) the coupling between the source of emf. and measuring circuit is so loose that the latter does not appreciably affect the former; (2) δ' and δ are both small compared with 2π, and (3) the ratio $\frac{(C_r-C)}{C}$ and the corresponding ratios are small compared with unity. From any of these δ' is obtained if δ is known and vice versa. If a separate measurement is made by the method of equation (57) above, both decrements are obtained.

The appearance of the sum $(\delta+'\delta)$ in the equations does not mean that the current flowing in the measuring circuit actually has a decrement equal to $(\delta'+\delta)$. As a matter of fact the actual decrement of the current is a value nearly equal to whichever of the two, δ' or δ, is the smaller. For this reason the equations involving $(\delta'+\delta)$ can not be extended to the measurement of the sum of the decrements of two loosely coupled circuits by coupling to one of them a third measuring circuit, as has sometimes been tried.

As mentioned earlier, the reactance-variation method is simplified if the reactance is varied by such an amount as to make $I_1^2 = \frac{1}{2} I_r^2$. This is done very easily when the current measuring instrument is graduated in terms of current squared. The quantity under the square root sign in all the preceding equations becomes unity, greatly simplifying the formulas. Calculation may be entirely eliminated by use of direct-reading decremeters as previously described. Such instruments when thus used with damped waves give directly $(\delta'+\delta)$.

SUMMARY: The methods of measuring resistance and related quantities at radio frequencies are fewer in number and necessarily different from those at low frequencies. The conditions of such measurements and the relations of various methods have not previously been given in comprehensive fashion. This paper shows the relations between resistance, phase difference, sharpness of resonance, and decrement. The methods of measurement are derived and classified. Most of the valuable methods are comprised under the resistance-variation and reactance-variation methods. Special direct-reading methods of measuring phase difference and decrement are presented.*

*Extra copies of the special scales of figures 7, 8, 9 can be obtained from The Institute of Radio Engineers by addressing the Editor, The College of the City of New York.

SYMBOLS USED IN THIS PAPER

C = capacity of condenser in measuring circuit.
C' = capacity of condenser in source circuit.
d = deflection of current measuring instrument.
d_1 = deflection when known resistance is inserted in circuit.
E = effective electromotive force.
E_o = maximum electromotive force.
f = frequency of alternation.
I = effective current.
I_o = maximum current.
I_r = current at resonance.
I_1 = current when either resistance or reactance of circuit is increased.
L = self-inductance of circuit.
M = mutual inductance.
N = number of trains of oscillations per second.
P = average power.
r = subscript used to denote resonance.
R = resistance of circuit.
R_1 = known resistance inserted in circuit.
R_x = resistance of apparatus under measurement.
s = scale setting.
S = sharpness of resonance
X = reactance.
X_1 = change of reactance.
W = average energy.
α = damping factor.
δ = logarithmic decrement of circuit.
δ_1 = increase in decrement caused by adding resistance.
δ' = decrement of applied emf.
ε = base of napierian logarithms $= 2.71828$.
θ = phase angle of C or of L, considering the resistance to be associated with it.
λ = wave length.
ψ = phase difference of C or L, considering the resistance to be associated with it.
$\omega = 2\pi \times$ frequency.

NOTE ON LOSSES IN SHEET IRON AT RADIO FREQUENCIES*

By

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(Paris, France)

I. The study of Foucault currents in iron sheets at high frequency was made first by Oliver Heaviside, and later by J. J. Thomson.[1]

Recently, Mr. Bethenod[2] has considered the complication caused by the phenomenon of hysteresis, and has introduced that phenomenon in his calculations by utilizing the method of procedure first employed by Ferraris (1888), according to which hysteresis is supposed to cause a constant lag τ of phase-angle between the magnetic induction $\boldsymbol{B}$ and the magnetizing field $\boldsymbol{H}$. In seeking to determine the final phase-lag between the emf. and the current in a coil having a closed magnetic circuit, when the frequency is increased indefinitely, Mr. Bethenod has shown that this limiting phase-lag, instead of being equal to $\frac{\pi}{4}$, as indicated by the formulas of J. J. Thomson, is diminished by an angle equal to $\frac{\tau}{2}$ by the effect of hysteresis. In reality, Mr. Bethenod's conclusion, according to which the limiting angle of phase-lag, between the emf. and the current, is diminished thru hysteresis by an angle equal to $\frac{\tau}{2}$, does not seem to be related to any particular interpretation of the phenomenon of hysteresis. It is natural, in fact, that the losses due to hysteresis, as with the losses due to Foucault currents, should tend to increase the "watt" current absorbed, and, consequently, should tend to bring the current more nearly in phase with the emf.

Before proceeding to any calculation, it is easy to under-

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[1] "The Electrician," volume 28, 1892, page 599.

[2] "La Lumière Electrique," July 22, 1916, volume 34, 2nd Series, page 73.

stand how hysteresis influences Foucault currents, and to understand how Foucault currents may influence hysteresis losses. If we assume that hysteresis introduces a phase-lag between magnetic induction and ampere-turns, we can understand at once that the introduction of Foucault currents into the equations is thereby affected. On the other hand, the presence of Foucault currents causes the magnetic induction in iron sheets to vary from the center to the external surface; and since the losses due to hysteresis increase according to a power of the magnetic induction which is higher than the first power, the losses will be higher than if the magnetic induction in the sheet were assumed to remain uniform. The hysteresis losses, therefore, depend on the influence of Foucault currents on the distribution of magnetic induction.

It will be principally necessary to take into consideration this distribution of magnetic induction in the sheet in order to calculate the losses due to hysteresis. In particular, if we assume these losses to be proportional to the square of the magnetic induction, which is all the more likely because, at high frequency, the magnetic induction is always low, it will be necessary to know, somehow, the effective spatial distribution of the magnetic induction in the sheet.

The purpose of the author is to revise the methods of dealing mathematically with Foucault currents, and to study the influence of hysteresis on losses due to Foucault currents, as well as to study the losses due to hysteresis itself. His purpose also is to obtain formulas which are useful for the study of radio frequency apparatus.

II. To establish the equations for Foucault currents, the author will not introduce directly the general equations of Maxwell, as is usually done; these equations can be established by equivalent considerations which are much more familiar to engineers.

Let us consider (Figure 1) a sheet of thickness $2a$. Let us take as origin the median plane XX, parallel to the two faces of the sheet, S and S'. On account of symmetry, the Foucault currents which go thru the sheet will give rise to currents of equal densities, and of opposite polarities at symmetrical points x and x', situated at equal distances, Ox and Ox', from the origin O. The Foucault currents which tend to form a shield against the magnetic flux which is passing thru the sheet, in a direction parallel to the faces S and S', follow the directions indicated by arrows in the plane of the figure. It should be noted, moreover, that the ampere-turns acting to produce a magnetic field at the point x, are those due to currents circulating in the region at the right of Ox and at the left of Ox'. The currents circulating

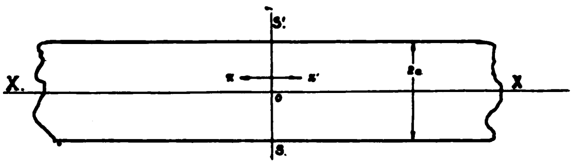

FIGURE 1

within the space bounded by the planes parallel to XX, which pass thru x and x', produce, in fact, no field external to that space. Let us designate by δ the current-density in the sheet at the point x. The decrease in ampere-turns per centimeter, which results in passing from a thickness dx toward the external surface S of the sheet, is $\delta\, dx$. The magnetic induction $\boldsymbol{B}$ at the point x will be decreased by a corresponding amount $d\boldsymbol{B}$ such that

$$d\boldsymbol{B} = -4\pi\mu\delta\, dx$$

whence

$$\frac{d\boldsymbol{B}}{dx} = -4\pi\mu\delta \qquad (1)$$

In reality, the current density δ and the magnetic induction B, being both harmonic functions of time, can be expressed as follows:

$$\delta = \delta_1 \sin \omega t - \delta_2 \cos \omega t$$

$$\boldsymbol{B} = \boldsymbol{B}_1 \sin \omega t - \boldsymbol{B}_2 \cos \omega t$$

in which equations any origin can be taken arbitrarily for time values (t). Under those conditions equation (1) corresponds to the two following equations:

$$\left.\begin{aligned} \frac{d\boldsymbol{B}_1}{dx} &= -4\pi\mu\delta_1 \\ \frac{d\boldsymbol{B}_2}{dx} &= -4\pi\mu\delta_2 \end{aligned}\right\} \qquad (2)$$

If we assume that hysteresis introduces an angle of phase-lag, τ, between magnetic induction and ampere-turns, we can say that everything happens, as far as the ampere-turns are concerned, as if the current density, δ, corresponded to a fictitious density, δ, having a phase-lag, τ, such that

$$\begin{aligned}\delta' &= \delta_1 \sin(\omega t-\tau) - \delta_2 \cos(\omega t-\tau)\\ &= (\delta_1 \cos\tau - \delta_2 \sin\tau)\sin\omega t - (\delta_2\cos\tau + \delta_1 \sin\tau)\cos\omega t\end{aligned}$$

Equations (2) then become

$$\begin{aligned}\frac{d\boldsymbol{B}_1}{dx} &= -4\pi\mu(\delta_1\cos\tau - \delta_2\sin\tau)\\ \frac{d\boldsymbol{B}_2}{dx} &= -4\pi\mu(\delta_2\cos\tau + \delta_1\sin\tau)\end{aligned} \tag{2'}$$

The choice between equations (2) and (2′) will then depend on whether the phenomenon of hysteresis is to be taken into consideration or not, in the equations for Foucault currents.

We now proceed to establish a second equation by a simple consideration which follows.

The emf. induced per centimeter in the direction XX between the two planes which pass thru the abscissa points x and $x+dx$ is

$$-\frac{d}{dt}\boldsymbol{B}\,dx = -\omega(\boldsymbol{B}_1\cos\omega t + \boldsymbol{B}_2\sin\omega t)\,dx$$

This emf. must be exactly balanced by the difference between the ohmic drop per centimeter in the plane passing thru x and the ohmic drop per centimeter in the plane passing thru $x+dx$. This difference is equal to $\varrho\, d\,\delta$, where δ designates the resistivity of the sheet. We therefore have

$$\begin{aligned}-\omega(\boldsymbol{B}_1\cos\omega t + \boldsymbol{B}_2\sin\omega t)\,dx &= \varrho\, d\,\delta\\ &= \varrho\left(\frac{d\delta_1}{dx}\cdot\sin\omega t - \frac{d\delta_2}{dx}\cos\omega t\right)dx\end{aligned} \tag{3}$$

From this we obtain the two following equations:

$$\begin{aligned}\omega\boldsymbol{B}_1 &= \varrho\frac{d\delta_2}{dx}\\ \omega\boldsymbol{B}_2 &= -\varrho\frac{d\delta_1}{dx}\end{aligned} \tag{4}$$

From equations (2′) and (4) we obtain the two following equations of the second order:

$$\begin{aligned}\frac{d^2\delta_1}{dx^2} &= \frac{4\pi\mu\omega}{\varrho}(\delta_2\cos\tau + \delta_1\sin\tau)\\ \frac{d^2\delta_2}{dx^2} &= -\frac{4\pi\mu\omega}{\varrho}(\delta_1\cos\tau - \delta_2\sin\tau)\end{aligned} \tag{5}$$

Taking

$$\frac{4\pi\mu\omega}{\varrho}=2m^2, \quad \sqrt{1+\sin\tau}=\alpha, \quad \sqrt{1-\sin\tau}=\beta$$

we obtain, by integration, the following:

$$\begin{aligned}\delta_1&=-A\,\frac{e^{m\alpha x}-e^{-m\alpha x}}{2}\cos m\beta x=-A\sinh m\alpha x\cos m\beta x\\ \delta_2&=A\,\frac{e^{m\alpha x}+e^{-m\alpha x}}{2}\sin m\beta x=A\cosh m\alpha x\sin m\beta x\end{aligned} \tag{6}$$

in which A is a constant of integration.[3]

From these and from equation (4), the values of $\boldsymbol{B}_1$ and $\boldsymbol{B}_2$ are obtained:

$$\begin{aligned}\boldsymbol{B}_1&=\frac{A\varrho m}{\omega}(\alpha\sinh m\alpha x\sin m\beta x+\beta\cosh m\alpha x\cos m\beta x)\\ \boldsymbol{B}_2&=-\frac{A\varrho m}{\omega}(\beta\sinh m\alpha x\sin m\beta x-\alpha\cosh m\alpha x\cos m\beta x)\end{aligned} \tag{7}$$

From the above solution (6) we obtain the maximum current density value δ_{max} at the point x:

$$\delta_{max}=\sqrt{\delta_1^2+\delta_2^2}=\frac{A}{\sqrt{2}}\sqrt{\cosh 2m\alpha x+\cos 2m\beta x} \tag{8}$$

From solution (7) we obtain the maximum value of magnetic induction $\boldsymbol{B}_{max}$ at the point x:

$$\boldsymbol{B}_{max}=\frac{A\varrho m}{\omega}\sqrt{\cosh 2m\alpha x+\cos 2m\beta x} \tag{9}$$

We can now determine the constant A by starting either from the apparent, or the mean magnetic induction in the sheet, or from the external ampere-turns per centimeter (J) which act on the sheet.

When starting from the apparent magnetic induction $\boldsymbol{B}_{app}$ it is to be noted that the emf. per centimeter, which must equal the ohmic drop due to Foucault currents along the external surface of the sheet, is $\omega\boldsymbol{B}_{app}a$. We therefore have:

$$\omega\boldsymbol{B}_{app}a=\frac{\varrho A}{\sqrt{2}}(\cosh 2m\alpha a-\cos 2m\beta a)^{\frac{1}{2}}$$

Whence:

$$A=\frac{\sqrt{2}\,\omega\,a\,\boldsymbol{B}_{app}}{\varrho(\cosh 2m\alpha a-\cos 2m\beta a)^{\frac{1}{2}}} \tag{10}$$

[3] The general integral would require a second constant, but it is found that this constant must be equal to zero in order that the condition of symmetry implying $\delta_1=\delta_2=0$ should be satisfied for $x=0$.

When starting from the external ampere-turns, J, per centimeter, it must be noted that the magnetic induction at the surface of the sheet must be equal to $4\pi\mu J$. We therefore have:

$$4\pi\mu J = \frac{A\varrho m}{\omega}(\cosh 2m\alpha a + \cos 2m\beta a)^{\frac{1}{2}}$$

Whence:

$$A = \frac{\omega}{\varrho m}\frac{4\pi\mu J}{(\cosh 2m\alpha a + \cos 2m\beta a)^{\frac{1}{2}}} \qquad (11)$$

Apparent Permeability.—Equating (10) and (11) we have:

$$\frac{\sqrt{2}\,a\,B_{app}}{(\cosh 2m\alpha a - \cos 2m\beta a)} = \frac{4\pi\mu J}{m(\cosh 2m\alpha a + \cos 2m\beta a)^{\frac{1}{2}}}$$

From this we obtain immediately an expression for the apparent permeability:

$$\mu_{app} = \frac{\mu}{\sqrt{2}\,m\,a}\frac{(\cosh 2m\alpha a - \cos 2m\beta a)^{\frac{1}{2}}}{(\cosh 2m\alpha a + \cos 2m\beta a)^{\frac{1}{2}}} \qquad (12)$$

This expression for the apparent permeability becomes identical with that given by J. J. Thomson when we assume $\tau = 0$, that is to say $\alpha = \beta = 1$.

It is necessary to know the expression for the apparent permeability, in order to determine the given or apparent magnetic induction, as a function of the available ampere-turns, in any given radio frequency apparatus.

III. We now proceed to determine the Foucault current and hysteresis losses as a function of the mean or apparent magnetic induction.

Foucault Current Losses.—In order to determine the Foucault current losses, it is necessary to determine in some manner the effective value in space of the current density δ_{max}, that is to say, the value of:

$$\frac{1}{a}\int_0^a \delta^2_{max}\,dx$$

We have

$$(\delta^2_{max})_{av} = \frac{A^2}{2a}\int_0^a (\cosh 2m\alpha x - \cos 2m\beta x)\,dx$$

$$= \frac{A^2}{4ma}\left(\frac{\sinh 2m\alpha a}{\alpha} - \frac{\sin 2m\beta a}{\beta}\right)$$

Bearing in mind that the effective current density, as a function of the time, is $\sqrt{2}$ times lower than the maximum density, the Foucault current losses per cubic centimeter will be:

$$W_F = \frac{A^2 \varrho}{8\, m\, a}\left(\frac{\sinh 2\, m\, \alpha\, a}{\alpha} - \frac{\sin 2\, m\, \beta\, a}{\beta}\right).$$

If we replace the constant A by its value obtained from (10), we have,

$$W_F = \frac{\omega^2 a}{4\, m\, \varrho} \frac{\dfrac{\sinh 2 m\alpha a}{\alpha} - \dfrac{\sin 2\, m\, \beta\, a}{\beta}}{\cosh 2\, m\, \alpha a - \cos 2\, m\, \beta\, a} B^2_{app} \tag{13}$$

The effect of hysteresis on Foucault current losses is readily seen. Leaving out hysteresis, the expression of these losses takes the simple form:

$$W_F = \frac{\omega^2 a}{4 m \varrho} \frac{\sinh 2\, m\, a - \sin 2\, m\, a}{\cosh 2\, m\, a - \cos 2\, m\, a} B^2_{app}. \tag{13'}$$

HYSTERESIS LOSSES.—To determine the hysteresis losses, we must bear in mind that the assumption of a constant phase-lag τ of the magnetic induction B behind the magnetizing field H supposes the hysteresis losses to be proportional to the square of the maximum induction. It is therefore necessary to determine in some way the effective value of B_{max} in space.

We have:

$$\begin{aligned}(B^2_{max})_{av} &= \frac{A^2 \varrho^2 m^2}{\omega^2 a} \int_0^a (\cosh 2\, m\alpha x + \cos 2\, m\beta x)\, d\, x \\ &= \frac{A^2 \varrho^2 m^2}{2\, \omega^2 a}\left(\frac{\sinh 2\, m\, \alpha\, a}{\alpha} + \frac{\sin 2\, m\, \beta\, a}{\beta}\right).\end{aligned}$$

We also know that the same hypothesis of constant phase-angle between the magnetic induction B and the magnetizing field H implies that, in the formula

$$W = \eta\, B^2_{max}$$

which gives the hysteresis losses per cycle, the value of the coefficient η is equal to $\dfrac{\sin \tau}{4\, \mu}$. Under those conditions, the hysteresis losses per cubic centimeter are equal to:

$$\begin{aligned}W_H &= \frac{\sin \tau}{4\, \mu} \frac{\omega}{2\, \pi} \frac{A^2 \varrho^2 m}{2\, \omega^2 a}\left(\frac{\sinh 2\, m\, \alpha\, a}{\alpha} + \frac{\sin 2\, m\, \beta\, a}{\beta}\right) \\ &= \frac{\sin \tau}{8} \frac{A^2 \varrho}{m\, a}\left(\frac{\sinh 2\, m\, \alpha\, a}{\alpha} + \frac{\sin 2\, m\, \beta\, a}{\beta}\right)\end{aligned}$$

If we replace the constant A by its value from (10), we have:

$$W_H = \frac{\omega^2 a \sin\tau}{4 m \varrho} \frac{\frac{\sinh 2 m \alpha a}{\alpha} + \frac{\sin 2 m \beta a}{\beta}}{\cosh 2 m \alpha a - \cos 2 m \beta a} B^2_{app} \tag{14}$$

RATIO OF LOSSES.—The ratio of hysteresis losses to Foucault current losses can be readily determined. We find:

$$\frac{W_H}{W_F} = \frac{\frac{\sinh 2 m \alpha a}{\alpha} + \frac{\sin 2 m \beta a}{\beta}}{\frac{\sinh 2 m \alpha a}{\alpha} - \frac{\sin 2 m \beta a}{\beta}} \sin\tau \tag{15}$$

This ratio tends toward *versin* τ in proportion as the frequency increases.

TOTAL LOSSES.—The expression for the total losses takes the form:

$$W_F + W_H = \frac{\omega^2 a}{4 m \varrho} \frac{a \sinh 2 m \alpha a - \beta \sin a m \beta a}{\cosh 2 m \alpha a - \cos 2 m \beta a} B^2_{app} \tag{16}$$

If the calculation were not complicated by introducing the phase-angle τ, and by seeking to determine the hysteresis losses according to the formula for losses per cycle (10), equation (14) could be simplified by replacing sin τ by its value $4 \mu \eta$, and by making $\tau = 0$, that is to say, by assuming $\alpha = \beta = 1$ everywhere else in the equation; which would give the following value for W_H,

$$W_H = \frac{\mu \eta \omega^2 a}{\varrho m} \frac{\sinh 2 m a + \sin 2 m a}{\cosh 2 m a - \cos 2 m a} B^2_{app}. \tag{14'}$$

Taking into consideration the uncertainty which exists in regard to the exact value of hysteresis losses per cycle, and owing to the circumstances that all authors differ as to the value which should be given to the exponent of $\boldsymbol{B}$, to which the losses are proportional, the simplified expression for W_H, given in equation (14′), may often be utilized.

Under those conditions, the total losses derived from (13′), and (14′), will be as follows:

$$W_F + W_H = \frac{\omega^2 a (1 + 4 \eta \mu)}{4 m \varrho} \frac{\sinh 2 m a}{\cosh 2 m a - \cos 2 m a} B^2_{app}. \tag{16'}$$

MINIMUM LOSSES.—It is possible to determine the losses per cubic centimeter in a total volume containing the iron sheets and the insulation between them.

Let ε be the thickness of the insulation between the sheets, and B'_{app} the mean magnetic induction in the total section composed of the sheets and of the insulation between them. The apparent induction in the sheet itself will be:

$$\frac{2a+\varepsilon}{2a} B'_{app}$$

On the other hand, the space occupied by the iron will be reduced in the proportion $\frac{2a}{2a+\varepsilon}$. Finally, the losses per cubic centimeter of total space will be, from (16), as follows:

$$W_F + W_H = \frac{\omega^2 (2a+\varepsilon)}{8m\varrho} \frac{\alpha \sinh 2m\alpha a - \beta \sin 2m\beta a}{\cosh 2m\alpha a - \cos 2m\beta a} B'^2_{app} \quad (17)$$

The thickness of sheets $2a_{opt}$ which will give the minimum of loss in a given volume for a given magnetic induction B'_{app} will be that which will always give the minimum value to the preceding expression. This minimum value is obtained by taking the derivative of that expression with respect to "a." In other words, it will be that thickness which represents the solution of the following transcendental equation:

$$2m(2a+\varepsilon)\frac{1-\cosh 2m\alpha a \cos 2m\beta a}{\cosh 2m\alpha a - \cos 2m\beta a} + \alpha \sinh 2m\alpha a - \beta \sin 2m\beta a = 0 \quad (18)$$

As a numerical illustration, let us take:

$$\mu = 2000$$
$$\varrho = 4(10)^4$$
$$\omega = 2\pi \times 30{,}000$$
$$\varepsilon = 0.003$$

From this we have:

$$m = \sqrt{\frac{2\pi\mu\omega}{\varrho}} = 243.5$$

We then find for $2a_{opt}$ the following values:

With $\sin\tau = 0.2$ $2a_{opt} = 0.027$ mm. (0.001 inch)
With $\sin\tau = 0.3$ $2a_{opt} = 0.0315$ mm. (0.0012 inch)
With $\sin\tau = 0.5$ $2a_{opt} = 0.0375$ mm. (0.0014 inch)

Phase-Angle Between EMF. and Current in an Inductance Having a Closed Magnetic Circuit

The losses have been evaluated directly without seeking to determine, as is usually done, the phase-angle ϕ between the emf. and the current. When the losses are known this phase-angle can be determined quite easily by an inverse process.

The maximum emf. "V," induced per turn per square centimeter will be ωB_{app}, and the power consumed per cubic centimeter will, therefore, be:

$$\frac{VJ}{2} \cos \phi = \frac{\omega B_{app} J}{2} \cos \phi$$

On the other hand, we know this power from equation (16) and we also know the value of B_{app}, as a function of J, from that of the apparent permeability. Equating these two expressions for losses, the value of $\cos \phi$ may be readily obtained. We will have:

$$\cos \phi = \frac{1}{\sqrt{2}} \frac{a \sinh 2 m a a - \beta \sin 2 m \beta a}{(\cosh^2 2 m a a - \cos^2 2 m \beta a^{\frac{1}{2}}}$$

When m tends toward infinity $\cos \phi$ tends toward $\frac{a}{\sqrt{2}}$; or $\sqrt{\frac{1+\sin \tau}{2}}$; that is to say, ϕ tends toward the angle $\frac{\pi}{4} - \frac{\tau}{2}$, as Mr. Bethenod has shown. We have in fact:

$$\sqrt{\frac{1+\sin \tau}{2}} = \cos \left(\frac{\pi}{4} - \frac{\tau}{2}\right)$$

Phase-Angle Between EMF. and Current in a Coil with Open Magnetic Circuit

In a coil having an open magnetic circuit, which includes an air-gap that multiplies the apparent reluctance of the magnetic circuit by k, the magnetic induction for the same ampere-turns, J, will be divided by k. Consequently, the losses are divided by k^2, while the emf. induced at the terminals is divided by k. Under those conditions it will be found that the phase-angle ϕ' becomes such that:

$$\cos \phi' = \frac{\cos \phi}{k}$$

in which $\cos \phi$ retains the value indicated in the preceding paragraph.

In proportion as the air-gap is increased, the emf. and the current tend more and more to assume the quarter-phase relation. As a rule, it is $\tan \phi'$ which should be given as high a value as possible.

The author will return later to the important question of the construction of inductance coils with low losses (with or without iron) for a given frequency.

SUMMARY: In this article there is determined the power dissipated separately by Foucault currents and by hysteresis, in a sheet of iron, on the assumption that there exists a constant angle of lag between the magnetic induction in the sheet and the magnetizing field producing it. There is deduced the thickness which should be given to the iron sheets of apparatus supplied with radio frequency current, in order that the total power expended shall be a minimum. A calculation is made of the angle of lag between the voltage and the current in the circuit of an inductance coil.

THE NATURAL FREQUENCY OF AN ELECTRIC CIRCUIT HAVING AN IRON MAGNETIC CIRCUIT*

By

H. G. Cordes

(Research Engineer, Bremerton, Washington)

The natural frequency of a circuit depends upon inductance and capacitance and is modified by resistance and conductance in the circuit. When the inductance consists of a coil which has an iron magnetic circuit, the effective value of the inductance of the coil becomes a variable. The change in the value of the inductance is attributed to eddy currents induced in the iron. The eddy currents vary with the conductivity and permeability of the iron and the thickness of the laminations.

Other factors remaining constant, the frequency of an oscillatory circuit depends upon the value of the inductance and the effective inductance depends upon the frequency of the oscillating current in the circuit. The problem to be solved consists in finding an expression showing a relation between these two interdependent quantities.

The application of such an expression is illustrated by the following figure.

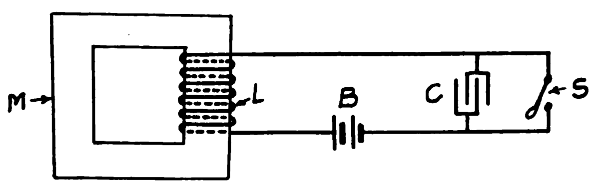

Figure 1

Magnetic flux is induced in the iron magnetic circuit M when current flows thru the coil L. Let B represent a battery, C a condenser and S a switch. When S is closed current passes thru

* Received by the Editor, April 16, 1918.

B, L, and S. Let r = the resistance of the circuit, and assume the potential of C to be zero.

Open the switch S quickly so that no sparking takes place. In order to determine the rate at which the potential rises in C and the maximum potential to which C will become charged, it is necessary to know the inductance of L, the capacitance of C, the resistance r and the initial current I flowing in the circuit.

Consider the potential of B and the potential drop $I\,r$ to be negligible compared with the emf. of self-induction. The electromagnetic energy initially stored in the coil is $\frac{1}{2} L_1 I^2$, where I is the initial current and L_1 is the inductance (coefficient of self-induction) of the coil at a low rate of change of the current. The value of L_1 decreases to an effective value L_m, as the average rate of change of current is increased due to an increase of eddy currents induced in the iron. Part of the initial electromagnetic energy is transformed into electrostatic energy, and the remainder is dissipated in the form of heat by eddy currents. The effective inductance of the coil is directly proportional to the total flux induced by the current thru the coil.

A method developed by Dr. C. P. Steinmetz for determining the ratio of the magnetic flux density due to a continuous current, to the effective magnetic flux density at a given frequency of alternating magnetic flux, is as follows:

Figure 2 shows the section of three laminations the thickness of which $= 2\,l_o$ and length $= S$. The thickness $2\,l_o$ is negligible compared to the length S. Current flows thru the conductor Z and in the direction of the arrow at the instant considered. The dotted lines show the path along which magnetic flux is induced

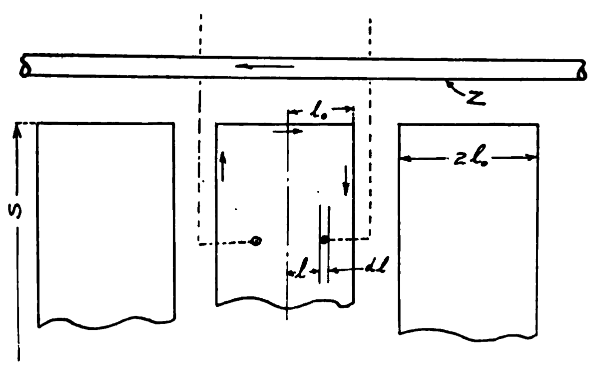

FIGURE 2

by the current. The flux induced in the iron is large compared with that induced in the air between the surface of the lamination and the surface of the conductor. The latter will be neglected and the total flux induced will be considered to generate an emf. in the iron at the surface of the lamination.

To obtain an expression for the flux density which is induced by the resultant magnetomotive force of the current in the conductor and the eddy current in the iron the instantaneous direction of which is shown by arrows, the following is quoted from Steinmetz's "Transient Electric Phenomena and Oscillations," Chapter VI (first edition):

"Let μ = the magnetic permeability, λ = the electric conductivity, l = the distance of a layer dl from the center line of the lamination, and $2l_o$ = the total thickness of the lamination. If then I = the current density in the layer dl and E = the emf. per unit length generated in the zone dl by the alternating magnetic flux, we have

$$I = \lambda E \tag{1}$$

The magnetic flux density B_1 at the surface $l = l_o$ of the lamination corresponds to the impressed or external mmf. The density B in the zone dl corresponds to the impressed mmf. plus the sum of all the mmf.'s in the zones outside of dl or from l to l_o.

The current in the zone dl is

$$I\,dl = \lambda E\,dl \tag{2}$$

and produces the mmf.

$$H = 0.4\,\pi\,\lambda E\,dl \tag{3}$$

which in turn would produce the magnetic flux density

$$dB = 0.4\,\pi\,\lambda\,\mu E\,dl \tag{4}$$

that is, the magnetic flux density B at the two sides of the zone dl differs by the magnetic flux density dB (equation (4) produced by the mmf. in zone dl, and this gives the differential equation between B, E and l,

$$\frac{dB}{dl} = 0.4\,\pi\,\lambda\,\mu E \tag{5}$$

The emf. generated at distance l from the center of the lamination is due to the magnetic flux in the space from l to l_o. Thus the emfs. at the two sides of the zone dl differ from each other by the emf. generated by the magnetic flux $B\,dl$ in this zone.

Considering now $\dot{B}, \dot{E}$ and $\dot{I}$ as complex quantities, the emf. $d\dot{E}$, that is, the difference between the emf.'s at the two sides of the zone $d\,l$, is in quadrature ahead of $\dot{B}\,d\,l$, and thus denoted by

$$d\dot{E} = -j\,2\,\pi f \dot{B}\,10^{-8}\,d\,l \qquad (6)$$

where f is the frequency of the alternating magnetism.

This gives the second differential equation,

$$\frac{d\dot{E}}{d\,l} = -j\,2\,\pi f \dot{B}\,10^{-8} \qquad (7)$$

The reader is referred to the text quoted for the steps by which the expression for the average flux density in the iron is obtained. It is briefly as follows:

From (5) and (6)

$$\dot{B} = \frac{\dot{B}_1}{2}\,\frac{\varepsilon^{(1-j)cl} + \varepsilon^{-(1-j)cl}}{\varepsilon^{(1-j)cl_o} + \varepsilon^{-(1-j)cl_o}} \qquad (8)$$

where

$$c = \sqrt{0.4\,\pi^2 f \lambda \mu\,10^{-8}} \qquad (9)$$

The average value of the flux density in the iron is

$$\dot{B}_m = \frac{1}{l_o}\int_o^{l_o} \dot{B}\,d\,l \qquad (10)$$

Equation (8) in (10) gives

$$\dot{B}_m = \frac{\dot{B}_1}{(1-j)\,c\,l_o} \cdot \frac{\varepsilon^{(1-j)c\,l_o} - \varepsilon^{-(1-j)c\,l_o}}{\varepsilon^{(1-j)c\,l_o} + \varepsilon^{-(1-j)c\,l_o}} \qquad (11)$$

The absolute value of $\dot{B}_m$ is the square root of the sum of the squares of the real and imaginary terms in equation (11), which, substituting hyperbolic and circular functions, is

$$B_m = \frac{B_1}{c\,l_o\sqrt{2}} \cdot \sqrt{\frac{\cosh 2\,c\,l_o - \cos 2\,c\,l_o}{\cosh 2\,c\,l_o + \cos 2\,c\,l_o}} \qquad (12)$$

The inductance of a conductor or coil is directly proportional to the total magnetic flux induced by the current in the conductor or coil, therefore

$$\frac{L_m}{L_1} = \frac{B_m}{B_1} \qquad (13)$$

The inductance L_1 is due to the flux induced in the iron by the mmf. of the current in the conductor or coil, while the inductance L_m is due to the flux induced in the iron by the resultant mmf.'s of the current in the conductor or coil and the eddy currents in the iron.

When the conductor Z is placed at an appreciable distance from the iron or the distance between the laminations is not negligible compared to the thickness of the laminations, then an inductance L_o, which is due to the flux induced in these spaces, must be added to L_m to get an expression for the total inductance of a conductor or coil at very high frequencies. The value of L_o will be a fractional part of the inductance of the conductor or coil in air, and its value may be calculated when these distances are known.

The natural frequency of a circuit is

$$f=\frac{1}{2\pi}\sqrt{\frac{1}{(L_m+L_o)C}-\left(\frac{r}{2(L_m+L_o)}-\frac{g}{2C}\right)^2} \qquad (14)$$

where r is the resistance of the oscillatory circuit and g is the conductance of the condenser dielectric. The quantities L_o and g will be considered negligible so that (14) becomes

$$f=\frac{1}{2\pi}\sqrt{\frac{1}{L_mC}-\frac{r^2}{4L_m^2}} \qquad (15)$$

Solve (15) for L_m,

$$L_m=\frac{1}{4\pi^2f^2C}\left(\frac{1+\sqrt{1-(2\pi frC)^2}}{2}\right) \qquad (16)$$

From (12) and (13)

$$L_m=\frac{L_1}{a\,l_o f^{\frac{1}{2}}}\left(\frac{\cosh 2cl_o-\cos 2cl_o}{\cosh 2cl_o+\cos 2cl_o}\right)^{\frac{1}{2}} \qquad (17)$$

where $a=\sqrt{0.8\pi^2\lambda\mu 10^{-8}}=0.000281\sqrt{\mu\lambda}$ (18)

For brevity, let $q=2\pi frC$ (19)

$$\text{“}\quad s=\left(\frac{1+\sqrt{1-q^2}}{2}\right)^{\frac{2}{3}} \qquad (20)$$

$$\text{“}\quad v=\left(\frac{\cosh 2cl_o+\cos 2cl_o}{\cosh 2cl_o-\cos 2cl_o}\right)^{\frac{1}{3}} \qquad (21)$$

From (16) and (17)

$$\frac{S^{\frac{3}{2}}}{4\pi^2f^2C}=\frac{L_1}{a\,l_of^{\frac{1}{2}}v^{\frac{3}{2}}} \qquad (22)$$

from which

$$f=\left(\frac{a\,l_o}{4\pi^2L_1C}\right)^{\frac{2}{3}}v\,s \qquad (23)$$

The factors v and s are both functions of the frequency but do not vary appreciably from unity, except when the frequency is low or the resistance is high.

For comparatively high frequencies and low resistances, equation (23) becomes

$$f_o = \left(\frac{a\, l_o}{4\,\pi^2 L_1 C}\right)^{\frac{1}{3}} \tag{24}$$

The values of v given in Table I were calculated from assigned values of $2\, cl_o$ in equation (21)

From (9) and (18),

$$2\, c\, l_o = a\, l_o \sqrt{2 f} \tag{25}$$

When $2\, cl_o$ is greater than 1.4, the value of f in (25) should be determined by equation (24) but when $2\, cl_o$ is less than 1.4, the value of f to be substituted in (25) is more accurately determined by

$$f_1 = \frac{1}{2\,\pi}\sqrt{\frac{1}{L_1 C} - \frac{r^2}{4 L_1^2}} \tag{26}$$

where r can generally be considered negligible.

From (23) and (26), (assuming $r=0$ and therefore $s=1$),

$$\frac{f_1}{f} = \sqrt{\frac{\sqrt{2}}{2\, c\, l_o\, v^{\frac{3}{2}}}} \tag{27}$$

Substituting in (27) the values of 1.4 and 1.054 of Table I for $2\, cl_o$ and v respectively shows that f_1 cannot be less than 0.966 f when $2\, cl_o$ is equal to or less than 1.4. Table I shows that f_o (equation (24)) cannot differ from f (equation (23)) more than 8.5 per cent when $2\, cl_o$ is greater than 1.4 and this occurs when $2\, cl_o = 2.4$. It is interesting to note that v varies most from unity when $\tanh 2\, c\, l_o + \tan 2\, cl_o = 0$ which occurs in the table when $2\, cl_o = 0$, 2.4 and 5.5.

The resistance of an oscillatory circuit does not appreciably affect its natural frequency unless the resistance is large. The value of s approaches unity as the value of q approaches zero. Equation (19) shows that $q=0$ when $r=0$ and also that $q=0$ when r is so large that the circuit becomes non-oscillatory since then $f=0$. There must therefore be a value of r for which q is a maximum.

From (15) and (18)

$$q = r\sqrt{\frac{C}{L_m} - \left(\frac{r}{2} \cdot \frac{C}{L_m}\right)^2} \tag{28}$$

which gives a maximum value of q when

$$r = \sqrt{\frac{2\, L_m}{C}} \tag{29}$$

TABLE I

$2cl_o=$	0.2	0.4	0.6	.8	1.0	1.2	1.4	1.6	1.8	2.0
$v=$	3.6832	2.322	1.774	1.470	1.277	1.145	1.054	0.9925	0.9524	0.9286
$2cl_o=$	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0
$v=$	0.9173	0.9149	0.9187	0.9264	0.9363	0.9472	0.9579	0.9678	0.9767	0.9842
$2cl_o=$	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0
$v=$	0.9903	0.9950	0.9985	1.0010	1.0026	1.0035	1.0038	1.0038	1.0036	1.0032

TABLE II

$q=$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$s=$	0.9983	0.9933	0.9845	0.9719	0.9548	0.9322	0.9022	0.8618	0.8018	0.6300

which may be compared with the resistance $r=\sqrt{\frac{4L_m}{C}}$, which renders the circuit non-oscillatory.

Substituting (29) in (28) shows that

$$q_{max.}=1 \tag{30}$$

The value of q depends upon the product $r\,f$, and f decreases as r increases, therefore q and s do not vary greatly with changes in the value of r. Table II gives values for q and the corresponding values of s as computed by means of equation (20).

The determination of q is made in a manner similar to $2\,cl_o$ The value of f to be substituted in (19) is computed from either (24) or (26). When both v and s have been determined, the natural frequency of the circuit can be evaluated by means of equation (23). If $v\,s$ differs much from unity, then the value of f in (23) can be used to determine new values of $2\,cl_0$ and q from which v and s will give a more accurate solution for f in (23).

The value of L_1 in (23) and (24) can be calculated from the equation

$$L_1=\frac{4\,\pi\,\mu\,A\,N^2}{d} \tag{31}$$

for a coil having a magnetic circuit the mean length of which $=d$, cross-sectional area of iron core $=A$, and a total of N turns of wire forming a layer over a large part of the length d.

To illustrate the practical application of equation (23) an example will be given.

Assume $d=50$ cm., $l_o=0.03$ cm., $\lambda=1.2\times10^4$, $\mu=2000$, $A=100$ sq. cm., $N=20$, and $C=0.1$ microfarad.

Then from (28)

$$L_1=\frac{4\,\pi\times2000\times100\times400}{50}=6.4\,\pi\times10^6\text{ cm.}$$

Substitute in (18)

$$a\,l_o=0.000281\times0.03\times\sqrt{2000\times1.2\times10^4}=0.0413$$

From these values in (24)

$$f_o=\left(\frac{0.0413}{4\,\pi^2\times6.4\,\pi\times10^6\times0.1\times10^{-15}}\right)^{\frac{2}{3}}=8{,}793\text{ cycles.}$$

To find the value of v substitute in (25), which gives

$$2\,c\,l_o=0.0413\sqrt{2\times8793}=5.477$$

Referring to Table I, $v=1.0038$ when $2\ c\,l_o=5.477$. This correction factor adds 33 cycles to f_o or

$$f=f_o\,v=8{,}826\text{ cycles.}$$

The natural frequency of the circuit, if L_1 is substituted for L_m (equation (26)), is

$$f_1 = 3{,}550 \text{ cycles,}$$

or the presence of eddy currents in the iron increases the natural frequency by the factor 2.5.

The circuit was considered to have negligible resistance. A resistance of 45 ohms is required to decrease the frequency one per cent. It is interesting to determine the resistance which will give the maximum value of q.

Substitute $f_o s$ for f in (25) which is equivalent to multiplying the former value of

$2 c l_o$ by $\sqrt{s}$, or $2 c l_o = 5.477 \sqrt{0.63} = 4.35$ or $v = 0.9938$, therefore

$$f = 8793 \times 0.63 \times 0.9938 = 5{,}505 \text{ cycles.}$$

In (19), put $q = 1$ and $f = 5{,}505$, and solve for r; then $r = 289$ ohms.

It will be noted that the quantity L_m may be defined by equations (16) and (17), or by the equation,

$$\tfrac{1}{2} L_m I^2 = \tfrac{1}{2} C E^2 \qquad (32)$$

in which, referring to Figure 1, I is the initial current, E is the potential to which the capacitance C becomes charged, the resistance of the circuit is considered negligible, and the potential of the battery is considered negligible compared to the potential E. If the value of L_m is considered the same in both instances, then the potential E can be calculated. A consequence of this assumption is that the eddy current loss, W, during the first quarter oscillation is

$$W = \tfrac{1}{2} I^2 (L_1 - L_m) \qquad (33)$$

where $\tfrac{1}{2} L_1 I^2$ is the electromagnetic energy stored in the coil when the switch S is opened. The voltage E may be assumed to rise sinusoidally, from which the speed required in opening the switch S without a spark can be estimated.

The effective inductance in the direct current circuit of a Poulsen arc generator depends upon the natural frequency of the magnet coils and generator in series with the antenna capacitance, which form an oscillatory circuit during the interval of each cycle of the radio frequency current that the arc is extinguished. A large loss occurs in the iron due to radio frequency eddy currents.

Increase of permeance due to the presence of iron is accompanied by an increase of conductance. This inherent property of iron will not allow its use in the magnetic circuit of a radio frequency oscillatory electric circuit where efficiency is an essential requirement.

SUMMARY: In radio or high audio frequency circuits containing iron-core inductances, the eddy currents induced in the iron cause a marked change of effective inductance which is dependent on the frequency.

Following the Steinmetz procedure, the author finds the magnetic flux density in a laminated iron core with alternating current excitation. He then derives expressions and tables for determining the natural frequency of circuits containing iron-core inductances. The results obtained are numerically illustrated.

DISCUSSIONS

on

'THE ELECTRICAL OPERATION AND MECHANICAL DESIGN OF AN IMPULSE EXCITATION MULTI-SPARK GROUP RADIO TRANSMITTER'† BY BOWDEN WASHINGTON

FIRST DISCUSSION*

By

LIEUTENANT ELLERY W. STONE, U.S.N.R.F.

(DISTRICT COMMUNICATION SUPERINTENDENT, NAVAL COMMUNICATION SERVICE, SAN DIEGO, CALIFORNIA)

Mr. Washington's paper is an excellent contribution to a subject which has been of considerable interest to me for the last four years. However, I cannot subscribe to his statement in the first paragraph of his article that impact excitation has not "been put into thoroly practical operation before." To do so is to ignore the published accounts of various other types of prior impulse excitation transmitters.

In particular, I have reference to the impulse excitation transmitter of the C. Lorenz Company of Berlin, described in volume 1, number 4, of the PROCEEDINGS of this Institute, the impulse excitation transmitter described by me in volume 4, number 3 and volume 5, number 2, of the PROCEEDINGS, and the impulse transmitters of the Kilbourne & Clark Manufacturing Company which have been described in United States letters patent owned by that company and in other technical publications.

The extensive use and thoroly practical features of the "Multitone" (Lorenz) system are too well known to require further discussion here.

In connection with the impulse excitation transmitter designed by me, I may state that following the publication of my gap and gap circuit designs in the two papers noted above, with a view toward allowing any interested party to use them gratis, this system of transmitter was adopted by the Haller Cunningham

* Received by the Editor, December 30, 1918.

† Published in the PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS, volume 6, number 6.

Electric Company of San Francisco. Certain mechanical improvements in the original design have been made by them from time to time. The practical adoption which has been accorded this type of transmitter is evidenced by that company's advertisements, appearing monthly in the PROCEEDINGS.

The Kilbourne & Clark Company, who were pioneers in this work, have on the market two types of impulse transmitters, known respectively as the "Simpson" and "Thompson" transmitters (after their designers). A novel type of gap together with a gap circuit of aperiodic constants is employed in each. It does not appear necessary, in the face of their extensive sales, to comment on the practicability of their operation.

It may be of interest to note that the first impulse excitation transmitter was designed by Lodge in his United States Letter Patent 609,154, inasmuch as he provided for gap and antenna circuits without resonant tuning, and for a gap of high resistance ("polished and protected from ultra-violet light, so as to supply the electric charge in as sudden a manner as possible"). With the expiration of this patent in 1915, the field was opened for its adoption by those companies not desiring to make use of the Marconi four-tuned-circuit patent.

In this connection, interest attaches itself to a statement of somewhat prophetic tone made by Mr. Robert Marriott in 1913, in his discussion of the Lorenz paper noted above, which I quote below:

"It is interesting to note that in the case of 'ideal' impulse excitation, where the primary and secondary circuits need not be syntonized, various patents covering such tuning are avoided."

The use of an untuned secondary receiver with an impulse transmitter, which procedure has been adopted by Kilbourne & Clark, Haller Cunningham, and apparently by Cutting and Washington, is a further reversion to the disclosures of the Lodge patent, in which such a receiver is described. I am in hearty agreement with Mr. Washington as to the advantages of this type of receiver over the coupled tuned circuit type, when used commercially with a crystal detector.

The almost undamped or continuous nature of the antenna current due to partial discharges, noted by Mr. Washington, is characteristic of true impulse excitation, as observed in my papers in which comment was made on the increase in signal strength with tikker and beat reception.

Mr. Washington's "concentration circuit" appears to play the same rôle as the "tone" circuit of other impulse transmitters.

SECOND DISCUSSION

(In Answer to the Preceding Discussion)

By

ENSIGN BOWDEN WASHINGTON, U. S. N. R. F.

I would like to say in answer to Lieutenant Stone's interesting criticism of my statement, that to the best of my knowledge, "Impact excitation has not been put in a thoroly practical operation before." I was speaking only from such data as I had available on this subject, and it is more than possible that my remarks were too broad in scope. The inapplicability of Lieutenant Stone's criticism appears to be somewhat dependent on the definition of "impact excitation." I have always felt that this expression should be taken to mean *no oscillations in the primary*, that is, the transfer of energy should occur during one current pulse. Otherwise it is only a matter of degree, and this seems objectionable for a definition.

It was my impression, gathered in Europe in 1913, that very few Lorenz, or "Multitone," transmitters were put in practical commercial operation, and that these few gave considerable trouble. Among other things mentioned were commutator troubles, usually present in high voltage direct current machines; and it was stated that the tone circuit was extremely critical to the condition of the gap. I found this latter to be true of the original Chaffee gap. I also have been told by British and French signal officers that the German Army purchased a considerable number of these sets which had to be discarded later as highly unreliable.

I am strongly of the impression that the standard Kilbourne & Clark transmitter, as marketed, is not true impact excitation, but is highly quenched. I had an opportunity to make quite extensive tests on a 2 kilowatt commercial set of their manufacture at the Cruft Laboratory of Harvard University, using a Braun tube in these tests. I was never able to get less than 2.5 complete oscillations in the primary circuit, and the set displayed one characteristic which strongly prejudiced me towards the belief that it was not operating as a true impact set, apart from the evidence obtained with the Braun tube. It is extremely critical as to tuning, more so, it seems, than the average quenched set. In this connection I would like to state that I have seen some Braun tubes oscillograms taken on this type of transmitter purporting to show impact excitation. These were taken, I believe, of the primary condenser voltage with no time axis deflection. A single heavy line on one side of the zero was the result, supposedly showing potential in but one direction. This can be easily explained;—as the condenser charges the spot moves very slowly, allowing the fluorescent screen to be exposed to the spot for some time. The actual oscillations occur in a very much shorter time, and unless every effort is made to have the spot exceedingly bright, there will be no lighting of the path of the spot due to the oscillations. The gaps appeared very little different in general construction from the ordinary quenched gap, except for what seemed rather inadequate cooling surfaces, and the gap length, 0.010 inch (0.4 mm.), seems long for this type of excitation.

The writer did considerable work on the "Hytone" gap, as manufactured by the Clapp Eastham Company, at the Jefferson Physical Laboratory, at Harvard University, with Dr. Chaffee in the spring of 1914. The Braun tube was used continuously; and on direct current, using a smooth disk it was possible to obtain pure impact excitation under ideal conditions. With a segmented gap, the following difficulty was encountered both on direct and alternating current. When the gap was opened, that is, when the faces of the segments were not opposite to each other, the condenser was given an opportunity to charge to considerable potential. On the approach of the segments a long, stringy spark would pass, followed by several small discharges. The only way to obtain the energy of this first spark appeared to be to tune for resonance, and adjust for proper coupling. It was, however, possible to quench entirely the succeeding minor sparks by properly proportioning the circuits, but in either case some energy was lost, and the apparatus was far from efficient. Alcohol vapor was tried, but never pressures of any real magnitude. Sixty cycle alternating current and a tone circuit were also tried on the smooth gap, but both the tone quality and tone efficiency seemed poor. The only thoroly satisfactory combination as to efficiency and functioning appeared to be the smooth gap, direct current, and a tone circuit. This was open to the practical objections of a high voltage direct current generator. Also, tho several gaps were constructed, it would seem difficult to obtain a gap which would keep its adjustments for spark length thruout the range of temperature encountered during a long run starting with a cold gap.

I should like to take exception to the term "partial discharges." I cannot conceive of a discharging condenser being stopped in "mid air," so to speak, and if it is meant that the condenser is only partially charged, what constitutes a full charge?

It may also be pointed out that the "concentration circuit" does not function in the same manner, or rather to the same purpose, as a tone circuit. This type of transmitter is entirely dependent on the generator frequency as to the radio frequency of the emitted groups. The concentration circuit is merely to group the discharges so as to give the proper form to the antenna envelope for maximum tone efficiency.