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ALFRED N. GOLDSMITH, Ph.D.

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NOTE ON "THE MEASUREMENT OF RADIOTELEGRAPHIC SIGNALS WITH THE OSCILLATING AUDION"*

By

L. W. Austin

(United States Naval Radiotelegraphic Laboratory,
Washington, D. C.)

I regret very much that I did not have an opportunity to take part in the discussion of my paper which appeared in the August number of the "Proceedings." In this discussion there seems to have been a certain amount of misapprehension regarding the real object of the work, for which undoubtedly the wording of the paper was in part responsible.

The primary purpose of the work was not, strictly speaking, to determine the power in the antenna with the audion circuit coupled for reception, but rather to make it possible to exterpolate the readings of the contact detector to cover the range of weak signals beyond the sensibility of the detector. As was stated in the paper, the resistance of the antenna with the contact detector coupled is the resistance used in the calculation of power, and the power itself is usually useful in our work only for calculating the strength of field produced at the receiving point by the sending station.

As the contact detector is generally accepted as a proper instrument for measuring received signals, it is only necessary to show that the audion used as described gives audibilities proportional to the square root of the deflections of the detector galvanometer. The first part of the paper shows that this proportionality exists, at least for the loosely coupled condition. A large number of experiments with a distant station sending with various antenna currents show that within the limits of experimental error, the received audibilities are proportional to the sending current, when the receiving audion is coupled to the antenna, and the observations taken as described in the Nauen-Eilvese experiments.[1]

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* "Proceedings of The Institute of Radio Engineers," 5, page 239.

[1] "Journal of the Franklin Institute," 1916, page 605.

Mr. Englund's proposal of a comparison method for measuring distant signals is very attractive, and I have spent some time in attempting to carry it out experimentally, but have in the end concluded that the chances of error due to the varying interaction at different wave lengths between the circuits are even greater than in the shunted telephone method, altho the observational accuracy may be somewhat better.

On account of changes in the strength of the telephone pulses with changing resistance in the circuit, the usual law of shunts is useless[2] for calculating audibility and the only method of being reasonably sure of correct results is to make laborious calibration experiments. The difficulties are considerably greater with the telephones in the "B" battery circuit than when they are used in the circuit in parallel with the "B" battery as shown in Figure 1 of the paper.

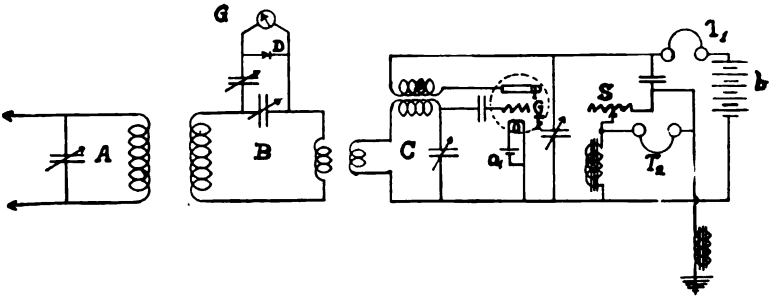

FIGURE 1

I have attempted to eliminate the variations in audion sensibility by using the same type of bulb in all the experiments, and also by having two audions ready for use, either one of which can be thrown in by a switch so that when one burns out, a new one of the same sensibility can be chosen by comparison with the one remaining.

I have recently been making experiments with the calibrated variable coupling telephone transformer, without iron, and it seems possible that the strength of signal may be measured with the variable coupling more accurately than by the shunted telephone method.

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[2] Using an audibility resistance box which according to the law of shunts should be used with telephones of 5,000 ohms impedance, the best linear relation is actually obtained by using telephones of nearly 30,000 ohms impedance.

I am very ready to admit that the absolute measurements of received signals at great distances hitherto made, may very possibly be considerably in error. The subject is of such great importance scientifically, that I hope that others will take it up as soon as war conditions permit.

THE EFFECT OF COMMERCIAL CONDITIONS ON SPARK TRANSMITTER CONSTRUCTION*

By

JULIAN BARTH

(Radio Engineer, Marconi Wireless Telegraph Company of America, Aldene, New Jersey)

A radio transmitter is essentially a generator of electromagnetic waves. A spark transmitter, in its usual form, consists essentially of an arrangement like Figure 1.

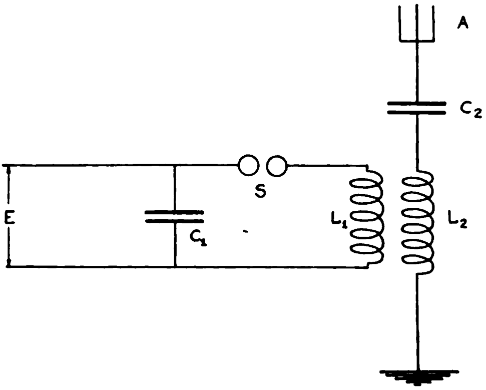

Figure 1—Typical Spark Transmitter

In fundamental terms, E is a source of electric energy, which charges condenser C_1 to a potential at which it causes a breakdown in spark gap S, permitting the stored energy to oscillate in the primary circuit composed of C_1, S, and inductance L_1 at a frequency determined by the constants of the circuit. The energy is transferred thru the coupled inductances L_1 and L_2 from the primary to the antenna circuit, consisting of L_2, condenser C_2 (used only for short waves), and the antenna A. This circuit, from which the energy is radiated, is tuned to the primary circuit.

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* Presented before The Institute of Radio Engineers, New York, February 7, 1917.

We have here stated the fundamental action of a spark transmitter. To put to practical use these fundamental scientific ideas requires the art of radio engineering. Actual apparatus must be constructed and installed to meet varying commercial conditions, and it is this phase of radio which it is the aim of this paper to present.

In bringing up a topic of this nature, there are so many factors which might enter into the discussion that it is found desirable at once to limit the discussion to a few important conditions determining the kind of apparatus to be used for a given high note, spark transmitter installation, and furthermore, to limit it to the description of apparatus already designed and constructed by my colleagues in the Marconi Wireless Telegraph Company of America and myself, while bringing out the pertinent features bearing upon these conditions.

The chief items for consideration in planning a transmitter are

1. The nature of the traffic to be handled;
2. The available source of initial power;
3. Space limitations;
4. Permissible expenditure.

The first factor involves such factors as required distances and direction of transmission, volume of business to be handled, traffic schedules, interference, and reliability of service.

The second factor includes the requirements for the production of electric energy to charge the primary condenser. It takes into account the availability of D. C. or A. C. or the need for a prime mover as part of the apparatus.

The third factor determines simply the size of the whole transmitter and the size and shape of its separate parts.

Of the fourth factor it may well be said that it is "last but not least." It means to the engineer in his work what a man's income means to him in everyday life and requires as strict a proportioning of income and expenditure. Economy is, of course, always essential, but at times actual inexpensiveness is required; and yet at times outlay for the little niceties of operation and for symmetry and neatness is required.

Without doubt, as far as quantity manufacture goes, ship installations demand first attention. To meet this kind of business, the company has standardized installations of three kinds, two differing from each other only to a comparatively small extent, while they differ from the third quite markedly. Mr. Harry Shoemaker has minutely described, in a previous paper,[1] the 2 and 0.5 kilowatt quenched gap sets for ship use known as types P-4 and P-5. What I have to say about them adds nothing to his description, but the way in which they illustrate good construction for the conditions met with in operation makes them fit aptly into this paper.

Figures 2 and 3 are a front and rear-side of the 2 K. W. quenched gap panel set which, with a key, antenna switch, antenna, and antenna series condenser, comprises a complete

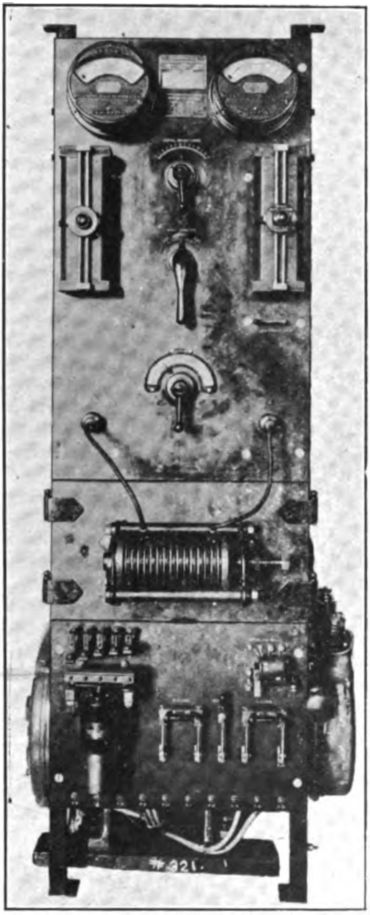

FIGURE 2—Front of Marconi Company 2 K. W. Panel Type Quenched Spark Set

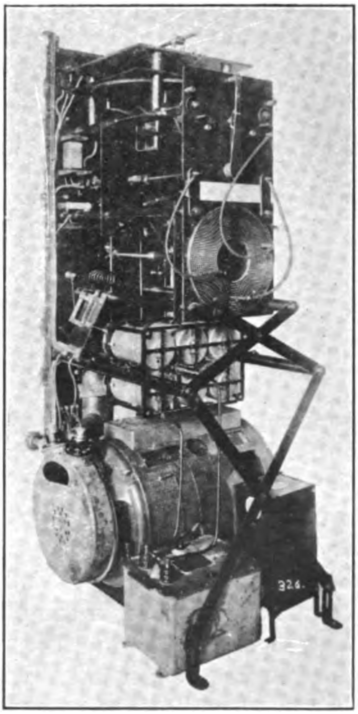

FIGURE 3—Rear and Side of 2 K. W. Panel Type Quenched Spark Set

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[1] "PROCEEDINGS OF THE INSTITUTE OF RADIO ENGINEERS," 1916, volume 4, number 4, page 313.

transmitter. The conditions which this installation was made to meet and how it meets them follow in the order in which the factors were previously arranged:

1. NATURE OF TRAFFIC. One of the chief considerations on passenger vessels is the protection afforded by radio. For vessels crossing the ocean or making trips to South America, where distances between vessels or between vessels and coast stations may average 50 miles (80 km.), a range of a few hundred miles (or km.) will cover the zone occupied by several ships, thus providing both protection and the convenient handling of ordinary traffic. A 2 K. W., 500 cycle, quenched gap set was decided on as being able to ensure this range most economically. To ensure against the set being out of commission because of failure of the quenched gap (which is probably the least invulnerable point), a synchronous rotary gap was added with a throw-over switch enabling it to be substituted for the quenched gap during repairs to the latter. The rotary gap is mounted directly on the motor-generator; and the switch is mounted above the rotary gap, which latter also acts as a blower for the quenched gap.

The volume of traffic handled on the seas by boats within each other's range is considerable, and must all be carried on with a range of wave lengths between 300 and 600 meters. Evidently some means for minimizing interference is imperative. The set has two distinctive features and a third, auxiliary feature for effecting this. There is provided a wave changing device for instantly throwing over to any of the three wave lengths: 300, 450, and 600 meters, after the set has once been tuned. To do this, the wave length switch, located in the center of the top front panel, operates two switch arms mounted on a common shaft. These arms make contact with tap points on the primary, the secondary, and the loading coil, all of which (except the primary points, which are set in manufacturing), are set once and for all when the set is installed. Thereafter, except for slight adjustments now and then, the turning of the switch handle accomplishes a complete wave length shift. The second feature is the low power arrangement. On the front panel, just under the generator rheostat, is a switch marked "Low Power Open." When closed, it short-circuits an added resistance in the generator field; when opened, it inserts the resistance, which is of just the proper value to give a suitable field excitation when only one gap is included in the oscillation circuit. This arrangement is used when traffic is handled between ships up to distances of 20 to 50 miles (30 to 80 km.) depending on atmospheric conditions. The whole arrangement accomplishes the purpose of getting traffic thru to nearby stations without interfering with distant ones. The third and auxiliary interference minimizer is the quenched gap, permitting of low decrements and sharp tuning, which, in conjunction with the three wave length shifts, reduces interference by two-thirds. Adding to this the power reducer cuts down interference by about 80 per cent. when all these features are handled as intelligently as they are by our modern operators.

The volume of business handled involves a necessity for speed of operation. The speeding-up is helped along by an automatic motor starter mounted on the lower front panel and controlled from the operator's bench. Merely putting the antenna switch into the "Send" position starts the motor and throws the generator field in after the machine has come up to speed. Throwing the antenna switch into "Receive" opens the generator field and sets into operation a magnetic brake on the motor, thus stopping it quickly so that the noise of running is eliminated while receiving. If the starting and stopping feature is not needed for good receiving conditions, a switch on the operator's table can short-circuit the contacts on the antenna switch which accomplish it.

2. Available Source of Power. On shipboard, D. C. at from 90 to 125 volts is the one source of power always available. Hence the method of charging the condensers, taking into account also the advantages of a 500 cycle, quenched spark transmitter, is to have a D. C. driven motor generator with low tension, 500 cycle output supplied to a high tension transformer.

3. Space Limitations. The space allotment for radio sets on shipboard is notoriously small and doors and hatchways are narrow. These are the reasons for a narrow panel arrangement of small floor space, utilizing height to as great an extent as possible. The set stands nearly six feet (2 m.) high and has adjustable extension supports to reach the ceiling of the operating room.

4. Permissible Expenditure. This set is intended for use on passenger vessels where it is open to public inspection. Hence it was considered advisable to make the set as neat as sound, rugged construction would permit. A glance at the front panel, Figure 2, will immediately impress one with its symmetry.

The finish on all parts is uniform, everything being either black dilecto or metal with a black nickel plate, except, of course, switches, which are copper. In addition, because of the volume of traffic to be handled, nicety of operation was considered more essential than low cost.

This set as a whole offers a very good example of the gigantic strides made in radio in the last few years, both as to reliability in the handling of traffic and in the organization of the manufacturing of apparatus. As a significant illustration of the latter, Figure 4 shows a few of these sets awaiting shipment.

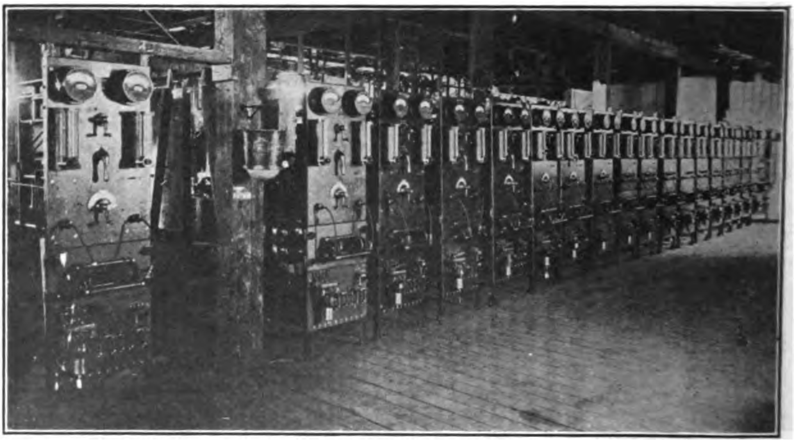

FIGURE 4—Group Manufacturing and Testing of 2 K. W. Quenched Spark Sets at Aldene Factory of Marconi Company

Figures 5 and 6 show a 0.5 K. W., 500 cycle, quenched gap set built to meet very nearly the same commercial conditions as the 2 K. W. sets. The one difference is in the nature of the traffic. The sets are intended for coastwise passenger vessels. The ships are never far from some coast station and it was therefore considered that 0.5 K. W. was ample power. The smaller set has all the features of the large set but its range is smaller, and it is naturally smaller also in physical dimensions.

The third type of ship installation is shown in Figures 7 and 8. The over-all dimensions are roughly 23 inches by 23 inches by 3 feet high (58 cm. by 58 cm. by 91 cm.). The motor-generator, rotary gap, transformer, primary reactance coil, primary condenser, and hand starter are mounted underneath the frame; the oscillation transformer and loading coil unit (of which the primary is movable), the antenna series condenser, and the tuning indicator are mounted on top. The set is intended for use on small cargo vessels only. It differs markedly from the two other types of ship installations in two of the four

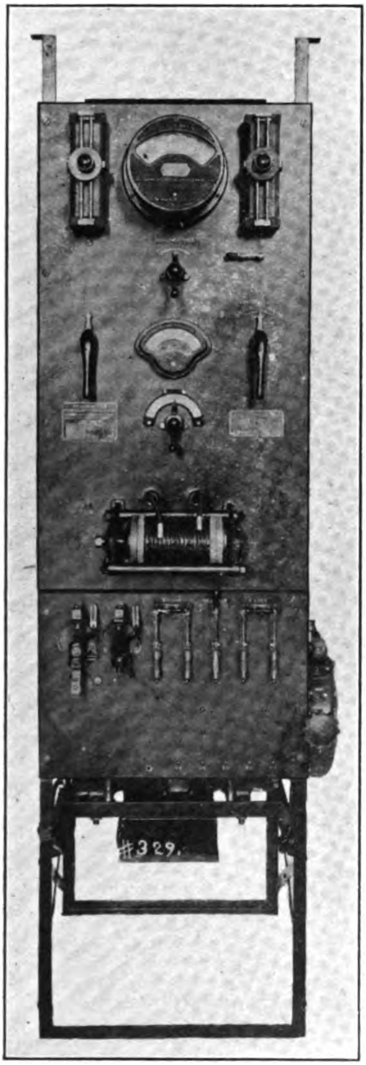

FIGURE 5—Front of Marconi Company 0.5 K. W., Quenched Spark, Panel Type Set

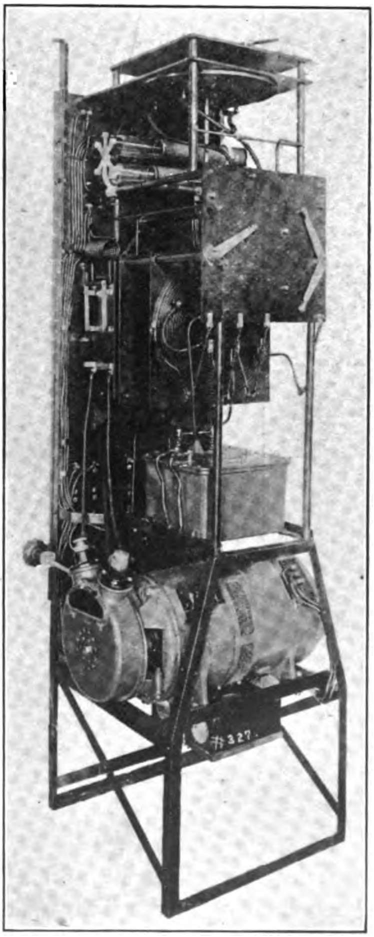

FIGURE 6—Rear and Side View of 0.5 K. W., Quenched Spark Set

factors we have been discussing, namely, the nature of traffic and the permissible expenditure. As to the nature of traffic, small cargo vessels naturally have very little traffic to handle, and such business as they do handle is not so urgent as that of passenger vessels. A power of 0.25 K. W. was considered ample for the range required. Such aids to efficient handling of a large volume of business as quick wave length shifts, very low power regulators, and automatic starters are not needed. There are not even the usual motor and generator rheostats. The motor speed and the power will never vary 10 per cent., and the set is not critical of adjustment for note. The only adjustment necessary is the synchronizing of the gap.

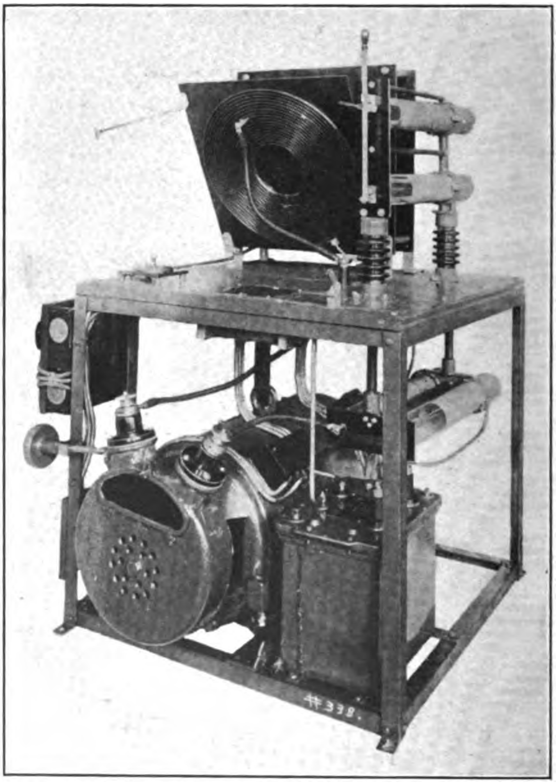

FIGURE 7—Front View of Marconi Company 0.25 K. W., Rotary Synchronous Spark, Cargo Ship Set

As to expenditure allowance, ruggedness and stoutness have been substituted for any attempt at beauty. There will be no passengers to view the apparatus critically; hence finish has been made of secondary importance. Actual inexpensiveness is a feature of this outfit. The more rugged rotary gap has been substituted for the quenched gap. Because of this, the tuning is less critical, and a tuning indicator consisting of a small inductance in the ground circuit, with a small lamp shunted across a variable portion of it, is used in place of the more expensive aerial ammeter. The coupling adjustment is a simple hinged swing of the primary coil, guided and locked on a rod extending from the fixed secondary.

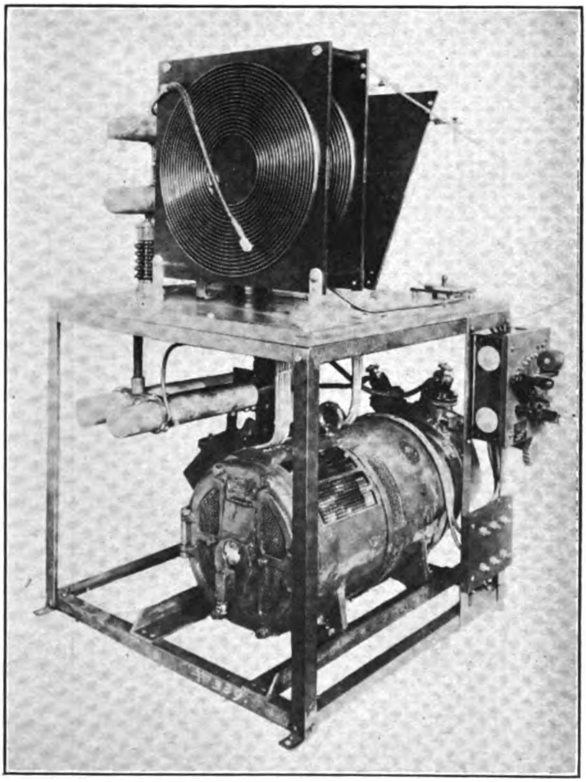

Figure 8—Rear and Side View of 0.25 K. W., Rotary Synchronous Spark, Cargo Ship Set

The set, as a whole, is a model of rugged construction at small cost for short range work. We are building a very large number of these sets now on one order, a few of which have been completed; another example of modern manufacturing methods applied to radio.

Figure 9 is a transmitting panel whose reason for being is the second factor under discussion, namely, availability of initial power. It is built for coast station work where the only power obtainable is single phase, 60 cycle, 110 or 220 volt current. A single phase, A. C. driven motor-generator of any power gives

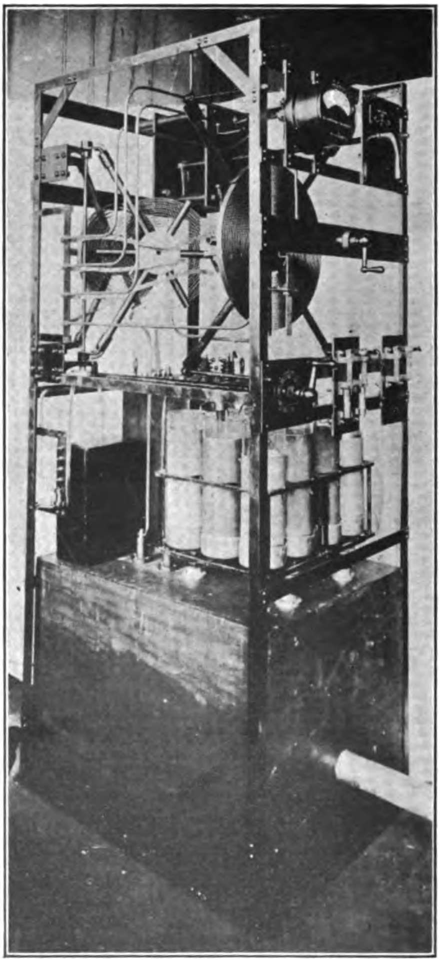

FIGURE 9—Marconi Company 3 K. W., Non-Synchronous Rotary Gap Set

poor satisfaction when it is started and stopped frequently. We have, therefore, substituted for the 2 K. W. motor-generator, quenched gap, ship sets, a 3 K. W., 1170 spark-per-second, non-synchronous gap set. The initial input and the antenna output in both cases are practically identical, the motor-generator losses about equalling the extra losses in the non-synchronous gap, giving the same over-all efficiency; but in the latter case a complete motor-generator, an exciter, and a quenched and synchronous rotary gap are saved, and in their places are a non-synchronous rotary gap, driven by a single phase induction motor, and a silencing cabinet to deaden the spark. The latter is a double walled wooden box with a 3 inch (7.5 cm.) thick lining of "tinofelt." The note obtained is not as clear and high pitched as a quenched spark note, but operators claim it is less shrill and more pleasant to read, and carries at least as far. The set has a quick shift wave length switch for quick handling of traffic just as have the ship sets. The parts, starting at the top are antenna ammeter, wave length switch, loading coil, secondary coil, primary coil, coupling adjuster, control switches, condenser, primary reactance coil, transformer, and silencing cabinet in which the rotary gap and motor are placed.

As far as expense is concerned, the set has been robbed of all ornament, but everything necessary to make the set convenient for operating has been provided. A set of this type is now operating at Sea Gate (near New York City).

Figures 10 and 11 illustrate the power end of a set which is built especially to meet the initial power problem. It covers the case of no local source of current supply and is therefore a general type of set. There is a 10 H. P., 4 cylinder, marine Sterling gasolene engine, with high tension ignition (shown in Figure 10), which is run at 2,000 revolutions per minute with a fly-ball governor acting on the carburetor butterfly valve to maintain constant speed. The large vertical tank in the corner is a water tank for cooling, from which the water is pumped thru the engine. The horizontal tank on the wall is a 27 gallon (102 liters) gasolene tank. The small horizontal cylinders above the engines are mufflers. The engine drives a 2 K. W., 500 cycle generator (with characteristics like those of the generator of the ship equipment) and a 32 volt, shunt wound exciter, thru a 2-to-1 gear at a speed of 2,000 R. P. M. The exciter is a 24-volt series motor for starting duty being fed by a 24 volt, 160 ampere hour, starting and lighting battery, and after starting becomes an exciter. The method of control, Figure 11, of this outfit is interesting. The handle and wire just to the left of the center of the board are pulled down, advancing the spark for starting. The handle is attached to a guard, which upon pulling the handle down, *and only then,* discloses a push button for starting up the outfit. The push button closes the starting clapper switch immediately to the right of the spark control. The series starter draws about 200 amperes from the battery for about a second and then falls off to zero in about eight seconds, at which time the engine has reached full speed and the series motor has become a series generator giving about 30 volts.

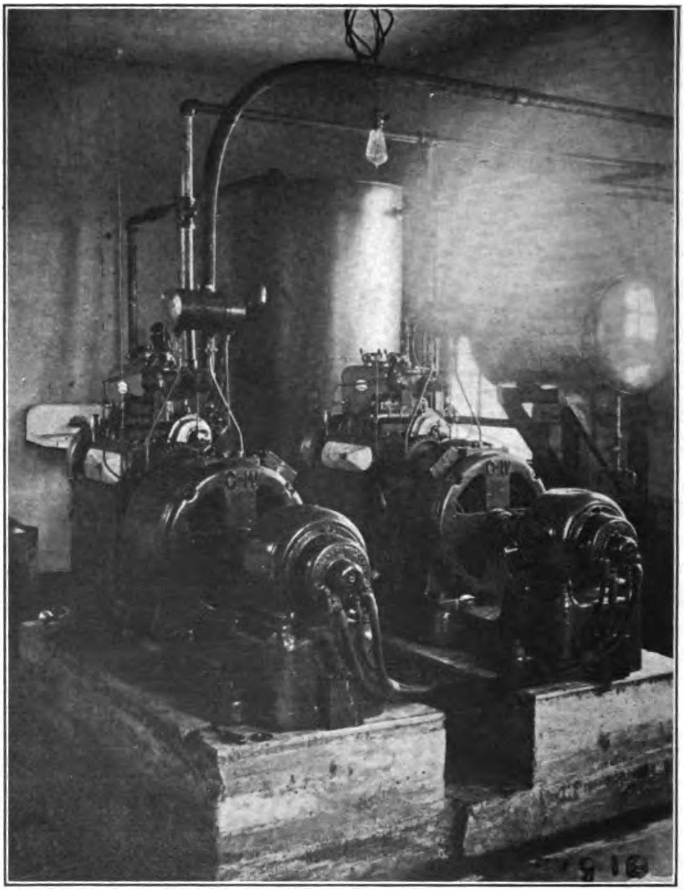

FIGURE 10—Duplicate Gasoline Engine, Alternator, and Exciter at Siasconset

This is enough voltage to actuate a relay immediately above the starting clapper switch. This in turn closes the clapper switch, which causes a shift in the connections of the exciter to the shunt arrangement and throws in the generator field and all auxiliary apparatus such as gap blowers. The closing of this shift switch automatically trips the starting switch. At the same time, the third clapper switch in the row of four closes the battery charging circuit, which connects the battery to the exciter thru an ammeter (in the upper left hand corner) and thru the 15 ampere charging resistance located between the clapper switches. The fourth clapper switch automatically opens the charging circuit when the batteries are fully charged. A voltmeter in the upper right hand corner gives the battery voltage when wanted. The batteries are never used enough to become run down, so that, except for looking after the electrolyte, the control panel takes complete care of the batteries.

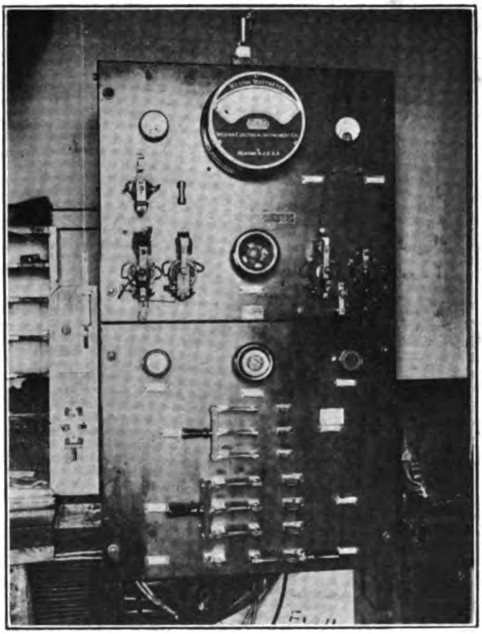

FIGURE 11—Starting, Battery, and Exciter Control Panels at Siasconset

As to space limitations, it is obvious that a set of this kind is feasible only for stations having plenty of room; in fact, a special engine room is necessary.

To meet the heavy traffic and ensure continuous service, the complete generating outfit and the battery are made in duplicate and the lower half of the control panel makes possible the use of either battery in conjunction with either generating unit.

The radio part of the set has no distinctive features, but the set as a whole offers a very good example of a set constructed for heavy traffic and in such a way that no dependency is placed on outside power plants. A set of this kind is installed at Siasconset, Massachusetts.

As an example of how a "semi-high power" station is made to meet commercial conditions, there will be described a set which is now being constructed for Juneau, Alaska.

The traffic which Juneau is expected to handle is between Juneau and Ketchikan, Alaska, or Juneau and Astoria, Washington. The former is 200 miles (320 km.) south with very mountainous, rocky country between, thus offering poor transmitting conditions, and Astoria is 1,000 miles (1,600 km.) south with less mountainous country for the last 800 miles (1,300 km.). We expect regular thru traffic to Astoria in winter, but in summer it would take enormous power to accomplish the same result. In summer, Ketchikan will act as a relay station. A 10 K. W., 500 cycle, quenched gap set was decided on as being sufficient for the work. The transmission will all be done on one wave length of from 1,500 to 4,000 meters, which will be determined by trial and there will be no interference, so that quick wave length shifts are unnecessary. The station will also handle a small amount of ship traffic, so that a separate antenna suited to short waves will be used in conjunction with a 2 K. W., 500 cycle, quenched gap set.

Figure 12 is a plan of the station showing the location of apparatus. No special compactness of apparatus is required since a special building houses the equipment. The entire set itself is very little different from a smaller set, each unit simply being larger. The incoming 2,300 volt, 3 phase, 60 cycle supply comes thru oil switch (1), thru the 2,300-220 volt transformers (2) to the motor of the motor-generator exciter, which is stationed farthest from the operating room to minimize noise. The control panel (27) is in the operator's room. The 10 K. W. oscillation transformer and loading coils (8) and (7) are hinged spider coils suspended from a post. (16) and (17) are the

2 K. W. oscillation transformer and loading coil. (14) is the 10 K. W. condenser and (22) is the 2 K. W condenser. The quenched gaps (15) can be thrown by switches into either the

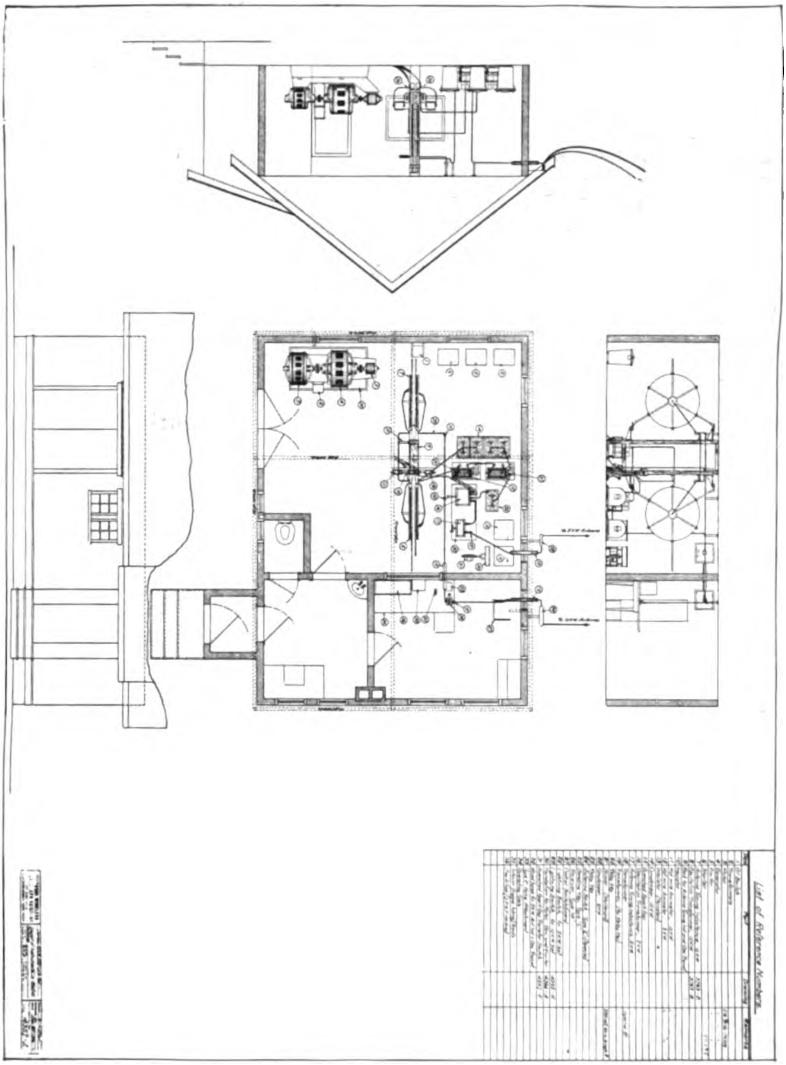

FIGURE 12—Plan of Marconi Company, 10 K. W., Quenched Spark Station at Juneau, Alaska

10 K. W. or 2 K. W. oscillation circuits which are always tuned ready for use. For 2 K. W. working an added reactance coil is inserted in the generator circuit and the 10 K. W., 500 cycle

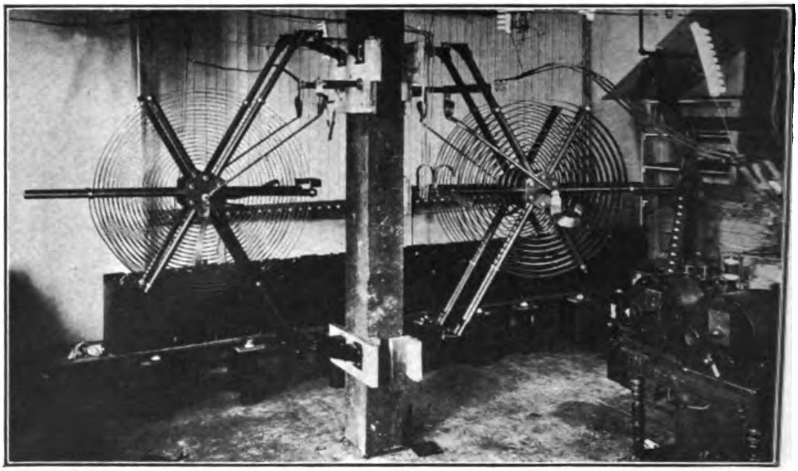

FIGURE 13—Radio Frequency Inductances for 10 K. W., Quenched Spark Set at Juneau, Alaska

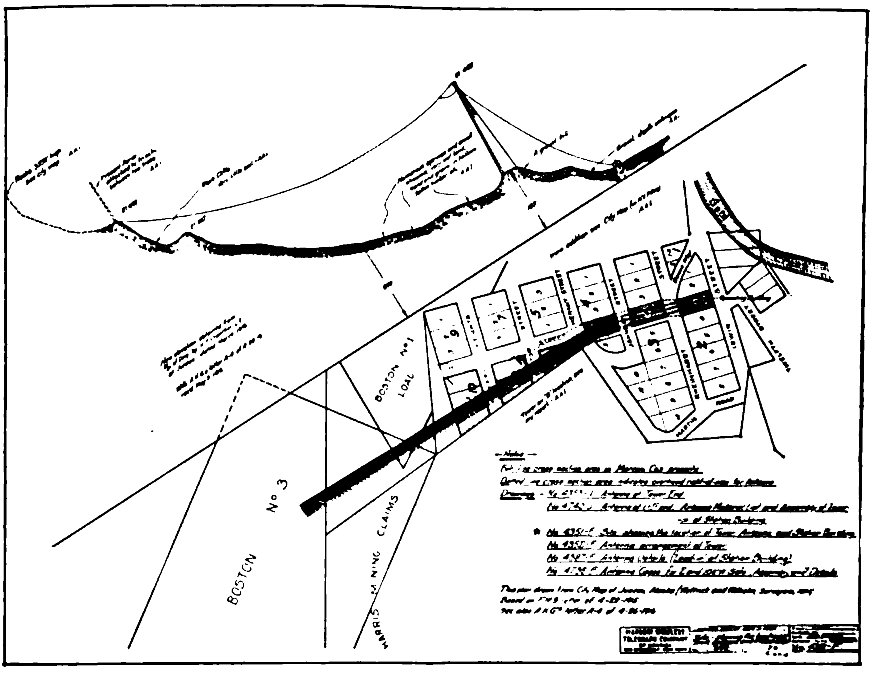

FIGURE 14—Antenna at Juneau, Alaska Station

transformer and machine are used for 2 K. W. working. The 10 K. W. radio frequency coils are interesting types of inexpensive construction for high power work and are shown in Figure 13. The picture shows them set up at the Aldene factory.

The 10 K. W. antenna, shown in Figure 14, is of special interest and carries out ideas based on the space factor. The line of communication is south so that a site was bought in that direction to give whatever directional aid is possible. The entire town lies on a hill side, and the site was so chosen as to give the proper direction to whatever reflection would occur. The antenna will terminate about 200 or 300 feet (60 or 100 meters) short of the bluff itself to prevent any closed loop effect.

It is hoped that in placing before the reader the chief factors controlling the construction of transmitters there have been brought out those other features which must be taken into consideration in the application of the radio art, namely, the practical commercial conditions.

SUMMARY: Six types of commercial sets are considered:
1. A 2 k.w., Quenched or Rotary Synchronous Spark, Ship Set;
2. A 0.5 k.w., Quenched or Rotary Synchronous Spark, Ship Set;
3. A 0.25 k.w., Rotary Non-Synchronous Spark, Cargo Ship Set;
4. A 3 k.w., Rotary Non-Synchronous Spark, Land Station Set;
5. A 2 k.w., Quenched Spark, Gasoline engine driven, Land Set;
6. A 10 k.w., Quenched Spark Land Set for Juneau, Alaska.

Photographs and descriptions of these are given, with special reference to nature of traffic, power source, space limitations, and expense.

DISCUSSION

George S. Davis: One of the most important features dealt with in Mr. Barth's paper is that which permits an operator, either skilled or unskilled, instantly and accurately to shift to any one of a number of predetermined wave lengths. While neither the idea nor the mechanism by which the shifting is accomplished is new or novel, the fact that it was included in the design of a "standard" ship equipment testifies not only to the thoroness of the designing engineer but to the fact that such a device has come to be a necessity in the practical operation of a radio transmitter.

Unfortunately, both the London Convention and the Act of August 13, 1912 limit the application of this very practicable idea to such an extent that it is impossible to use it to the best advantage. In effect, these laws require that all "general public correspondence" between ships, and between ship and shore, be handled on wave lengths of 300, 600, or 1,800 meters. Now, bearing in mind that practically all correspondence with ships is "general public correspondence," and also that the 300 and 1,800 meter wave lengths cannot be employed to advantage on the average ship, it follows that the 600 meter wave length is employed almost altogether, thereby causing a great deal of interference. In other words, these laws require practically all ships and all shore stations, when communicating with ships, to use one wave length—600 meters and their practical application therefore defeats the purpose for which they were enacted, i. e., to promote safety of life at sea by regulations which would minimize interference. I do not mean to infer that we should not have radio laws and regulations, but they should be framed in such a manner and should be flexible enough to permit the Government and commercial companies to take the fullest advantage of new inventions or devices such as the one under discussion to the end that interference from all sources can be overcome and new methods devised and adopted which will permit the handling of a greater volume of traffic over a given circuit at a given time. This will result in an even greater protection to passengers and ships than they now have.

It certainly is not good engineering practice, nor to the interest of the public or to the radio art itself, to enact laws which, in effect, prohibit the use of new developments, particularly in a new field, without providing a means whereby regulations can be revised so as to meet new conditions and permit the use of newer and more highly developed apparatus which will better accomplish the purposes for which the laws were enacted. I refer particularly to the regulations of the London Convention, which are wholly unreasonable when applied to continuous wave transmitters such as the "arc." True, several ships have been equipped with arc apparatus, but from all accounts their over-all efficiency is seriously impaired on account of having to comply with these regulations.

It is quite possible in my mind that had these regulations been flexible enough to permit the use of wave lengths more suitable for continuous waves, the development of moderate-power continuous wave sets for use on board ships would have been seriously undertaken and accomplished long ago.

The advantages offered by *continuous* over *damped* wave transmitters are such that the use of the former on shipboard would, by virtue of the increased range of transmission and reduction of interference, accomplish more in the promotion of safety of life at sea than all the regulations that could be devised covering the use of damped wave transmitters. Damped wave transmitters seem to have reached their limit of usefulness and it is to be hoped that at the next International Radio Convention the regulations will be revised in such a manner as will permit operating companies to take full advantage, particularly with respect to shipboard equipment, of developments such as continuous wave transmitters, quick wave changing devices and other devices which promote accuracy and efficiency in communication.

J. B. Elenschneider: Reports from the operator on the "Baltic" state that Seagate can be read 400 miles (650 km.) from New York because of the distinctive note emitted by the non-synchronous spark set. Altho produced by 1,167 breaks per second at the rotary spark gap, the note of the received signals is more in the vicinity of a frequency of 700 per second. The range given is over water and in an easterly direction. Inland, and particularly over Long Island Sound, the range is greatly diminished.

The antenna at Seagate is suspended from a wooden mast 125 feet (37 meters) high and is 175 feet (58 meters) long. It consists of six wires spaced 3 feet (1 meter) apart and runs from northwest to southeast at an angle of about 45 degrees. The southeast or lower end is connected to the apparatus.

The fundamental wave length is 390 meters. The transmitter is tuned for wave lengths of 600, 450, and 300 meters.

The antenna currents and decrements are 9 amperes and 0.095 respectively for the 600 meter wave, 8.7 amperes and 0.1 for the 450 meter wave, and 4.8 amperes and 0.11 for the 300 meter wave.

The change of wave lengths is effected by manipulating a single switch. The coupling is held constant for the three wave lengths as far as the position of the coupling coils is concerned.

The Seagate Station is one of the busiest stations and messages follow each other so rapidly that it would not pay to stop the rotary gap motor. The motor is often kept running for eight hours at a time.

The Siasconset generating plant is the first one of its kind employed in shore stations and its principal advantage is economy. Heretofore the shore stations have been equipped with stationary engines, belt driven generators, large storage batteries and motor generators, with considerable loss of energy between these different units. In the new type generating plant, as described in the paper, the losses caused thru belt slipping, friction, heat, low efficiency of storage batteries and motor generators are reduced to a minimum, and the energy generated by the engine is directly applied to the alternator whence it is led in the form of alternating current to the power transformer.

The generating plant consists of a 10 H.P., 4-cylinder, 4 cycle marine engine geared to a 2 K.W., 500 cycle generator, the latter being flexibly coupled to a 32-volt, direct current exciter. The exciter also serves as a 24-volt motor which in connection with a 24-volt, 160 ampere-hour storage battery serves as a starter for the engine. The engine is equipped with high tension magneto ingition. The cooling is effected by a rotary water pump which sends the water from a large storage tank thru the water jackets and back again to the tank.

Another pump propels the oil from a tank thru an oil-sight into the crank and gear cases. The amount of oil is regulated by means of an adjusting screw on the pump.

The fly ball governor maintains the speed of the engine under all changes of the load on the generator-exciter set within 2 per cent. above and below normal speed.

A wire runs over pulleys from the operating to the engine room, a distance of 30 feet (9 m.). The wire is connected to the high tension magneto, which, while the engine is at rest, is held in advanced position by a spring. To start the engine, the operator pulls down the handle of the wire which exposes a push button for closing the starting circuit. This arrangement makes it impossible for the operator to start the engine with advanced spark and cause back firing and eventual damage to the engine or gears. The engines are run at a speed of 1,000 revolutions per minute which, by means of the gear at a ratio of 2-to-1, gives the generator and exciter 2,000 revolutions per minute, which is their normal speed. A 2-to-1 reduction is of advantage insofar as the load on the starter is greatly reduced and thus permits the use of a small starter motor and batteries.

At first, straight spur gears made of cast iron and rawhide were used, but it was soon found out that under the continuous impact which is caused by the variation in the generator load while the operator is transmitting, this type of gears would not answer the purpose. The teeth of these gears wore away rapidly, and, after several weeks' use, the gears failed entirely. Owing to the peculiar conditions which are brought about by the constant change in the generator load, it was necessary to substitute straight spur gears with gears of the herringbone type.

Considerable annoyance was also caused by the noise produced by open gears. It was found that open gears, running at a pitch line speed of 2,000 feet (650 m.) per minute, even if constructed of cast iron and rawhide or fiber would produce a strong shrieking sound varying in intensity as the load on the generator was thrown on and off by the operator's sending. The solution of this problem was found in the use of a tight gear case filled with soft grease.

The starting current for the engine is approximately 250 amperes at 20 volts for the first fraction of a second. This current shows a rapid linear decay to zero amperes within 6 to 8 seconds. After this time, the engine attains its full speed and the starter motor automatically becomes a generator.

The station is located on Nantucket Island and gasolene has to be transported from time to time from the mainland in small boats. In order that the station is supplied with sufficient fuel for at least two weeks' operation, a large fuel tank is installed underground and outside of the station. This tank is connected by means of a pipe system and a rotary hand pump with a small fuel tank located near the engines. The fuel consumption of these engines is approximately one pint per horsepower hour.

The original intention was to use one generating set until it developed faults and then to use the other set for emergency, giving the latter a running test once a week. This plan had worked out well for several weeks until the first engine developed serious gear trouble. After the straight spur gears and the standard bearings on the second set had also failed, both sets were dismantled for the purpose of changing the gears to the herringbone type, running in grease, and also to increase the size of the shafts and bearings on the generator.

AN AUTOMATIC TRANSMITTER FOR DISTRESS SIGNALS*

By

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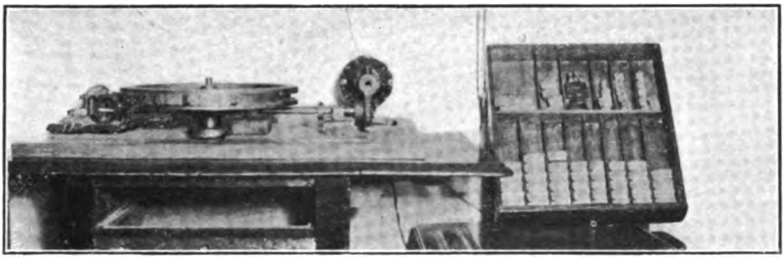
(Sacramento, California)

The instrument here described is principally for the purpose of giving ships that are not required by law to carry a radio apparatus a means of notifying other ships by radio telegraphy of their need for assistance, and of their position, in case of disability or disaster. This is accomplished without any assistance from a skilled operator, or one having any knowledge of radio telegraphy, or of the code used. There are at this moment many ships plying the oceans unprotected insofar as radio telegraphy may be considered a protection. They are unable to call for assistance and to advise other ships of their position in case of disaster or disability.



The apparatus mentioned consists of a brass disc, and a set of brass type blocks to be used to form a message. The disc is arranged to revolve at a speed of about 2.25 revolutions per minute, being driven by a small electric motor. The disc has a channel cut around its circumference with marginal internal flanges extending into the channel for the purpose of holding the type blocks.

The blocks are curved so as to conform to the curvature

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* Received by the Editor, February 16, 1917.

of the channel of the disc. They have two longitudinal grooves on opposite sides to receive the flanges, so that the blocks cannot escape. The grooves on the blocks are of such depth that a portion of the blocks projects radially outward beyond the outer edge of the disc. On one side of each type block is stamped the information, or its abbreviation, which the block is able to transmit.

The blocks shown in the figure consist of:—

Two—S O S blocks.
One—Ship's call letters.
Abbreviation of word "longitude" (lng).
Abbreviation of word "latitude" (lt.)
Abbreviation of word "east" (E).
Abbreviation of word "west" (W).
Abbreviation of word "north" (N).
Abbreviation of word "south" (S).
Signal (D) meaning "fire."
Signal (K) meaning "disabled."
Signal (C) meaning "in life boats."
Signal (L) meaning "on rocks."
The numerals.
Space blocks marked "space."

The non-conducting portions of the blocks are made by cutting grooves crosswise on the surface about 0.125 inch (3 mm.) deep. These cuts are filled with a composition of wax (such as shoemaker's wax) and rosin. This composition is fairly hard and will adhere perfectly if in the proper proportion.

The surface of the blocks having the conducting and non-conducting portions, will be brushed by a needle when the blocks are carried by the disc as it rotates, causing the opening and closing of a battery circuit operating a relay. This acts as a key in the primary circuit of a transformer.

Electrical connection between the disc and the relay is made by the needle brush and also by making connection at the base of the perpendicular axle upon which the disc turns. Grease or oil for lubrication is not used on this axle.

To form a message, it is necessary to select the desired blocks, placing them in the channel of the disc. The flanges of the disc at a certain point are broken away, so that the blocks can be inserted into the channel This break in the flanges is long enough to receive the longest block. The blocks can be placed on the flanges only on the right side of the entering place, because of the obstruction of a permanent stop member at the left end of the entrance. As one flange extends further out into the channel than the other, and as one groove of the block is made deeper than the other, it is impossible to insert blocks into the channel backwards.

This apparatus is designed to transmit automatically the position of a ship in case of distress, therefore the message must be arranged according to a certain form, and the spacing between words and letters, etc., is so arranged on the blocks as to comply with this form.

Form of message:—

(SOS) (SOS) (Ship's name) (longitude) 148° 17′ (W) (latitude) 24° 51′ (N) (Fire).

Words between parentheses are single blocks.

Rule:—Place a space block (marked "space") between degrees and minutes in both longitude and latitude: lng 148 17 W. lt 24 51 N.

The spacing between words, letters, or numbers, etc., is arranged on the blocks in such manner that they space automatically with the exception given in the above rule. It can therefore be seen that it is a very simple matter to set up a message. Altho this apparatus has blocks only for sending a position in latitude and longitude, other blocks may be added, such as the alphabet, so that a brief message could be formed giving the position of a ship as "so many miles from a certain point." This, of course, is not necessary.

When forming a message, the first block as it is placed in the channel should be pushed along until it is stopped by a projection in its path, at the point where the blocks were inserted into the channel. Each following block should be pushed along likewise until it is stopped by the block preceding it. When all the blocks forming the message have been placed on the flanges, the space between the last block and the part where the flanges are omitted should be filled with blank blocks having a non-conducting outer surface. These blocks are used to make the disc almost evenly balanced, so that when the ship rolls or if there is a heavy list, the motor will have a continuous even pull. If these blank blocks are not used, and the apparatus is not on a level, the motor will have a tendency to slow up and then speed as the disc revolves.

A block having a set screw is placed on the flanges at the end of the row of blocks, its purpose being to prevent the blocks from having longitudinal movement. The opposite end from the set screw block is held by means of the permanent stop member projecting into the channel.

SUMMARY: A rotating disc is arranged to carry curved blocks on its periphery. Each block permits transmitting a certain sign. Simple distress messages can thus be sent by unskilled persons.

HARMONIC METHOD OF CALIBRATING A WAVE METER*

BY

E. LEON CHAFFEE

(DEPARTMENT OF PHYSICS, HARVARD UNIVERSITY, CAMBRIDGE, MASSACHUSETTS)

The following paper describes a scheme which the writer has found extremely useful as an aid in the calibration of a wave meter. The scheme makes use of the fact that a non-sinusoidal current is resolvable into a fundamental and a series of harmonics the frequencies of which are integral multiples of the frequency of the fundamental. If the frequency or the wave length of any one of the series of harmonics including the fundamental is known, then the frequency or wave length of each of the other members of the harmonic series is accurately determined.

The scheme may be useful in checking the accuracy of a wave meter already calibrated or the method may be of service in extending the calibration of a wave meter either up or down in wave lengths if a certain small range of an octave of the calibration has already been made by some other method.

In detail the method is as follows: Continuous oscillations are excited in circuit (1) of the diagram by means of an audion, Pierce mercury bulb[1], or other form of electron relay using any one of the familiar connections. A radio frequency alternator would also serve as a source of the continuous oscillations. The connections used by the writer are shown in the diagram where L_1 is a single-layer solenoid provided with a tap at its middle point; C_1 is a variable air condenser forming with L_1 the oscillatory circuit (1); R_1 is the electron relay; C_o is a stoppage condenser; B is the high-voltage battery which supplies the energy for the oscillations; and S_1 is the source of current which heats the filament of the electron relay.

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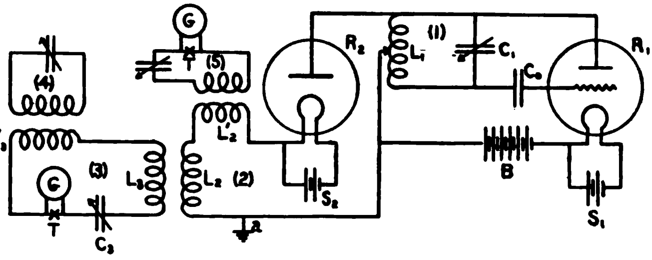
* Received by the Editor, January 15, 1917.

[1] The Pierce mercury bulb described in its original form in U. S. patent number 1,112,549, October 6, 1914, but later improved by extending the grid across the whole tube, acts as a powerful source of continuous oscillations when connected in any of the ways of connecting an audion for generating oscillations.

The oscillations in circuit (1) may, under certain conditions, show the presence of weak harmonics altho usually the oscillations are very closely sinusoidal in form. Strong harmonics may be produced by the addition of circuit (2)

Circuit (2) consists of a rectifier R_2 in series with coils L_2 and L_2' and a portion of L_1. The rectifier may be an audion with the plate and the grid connected. L_2 is a coil of 50 to 100 turns. L_2' is a coil of a few turns and may, for the present, be omitted from consideration. Circuit (2) may be inductively connected to L_1 instead of being directly connected as shown in the diagram. The current thru L_2 consists of a series of impulses corresponding roughly to the rectified pulses obtained if all half loops in one direction of the current in circuit (1) are suppressed. Because of the non-linear resistance characteristic of the rectifier, the pulses are not sinusoidal in form. Whatever their shape, the current in L_2 can be expressed by a Fourier's series of the form



$$i=b_0+b_1\cos\omega t+b_2\cos 2\,\omega t+b_3\cos 3\,\omega t+b_4\cos 4\,\omega t+\cdots.$$

A third circuit (3), consisting of inductances L_3 and L_3', a variable air condenser C_3, a thermocouple T shunted by a galvanometer G, is loosely coupled to circuit (2). Circuit (2) can be tuned to any of the harmonics of the current in L_2. The tuning is exceedingly sharp and is best done by means of some micrometer attachment on the condenser or by means of a long wooden handle attached to the moving element of the condenser.

The thermocouple T consists of a one mil (0.001 inch = 0.025 mm.) platinum wire rolled flat and fused at one corner to a small piece of tellurium. Its resistance is about 2 ohms. Such a thermocouple has been described by Austin.[2] G is a Leeds and Northrup 5-ohm portable galvanometer.

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[2] Austin, "Bulletin of Bureau of Standards," 7, 1911, page 301.

A fourth oscillatory circuit (4) represents the wave meter under test and is shown loosely coupled to circuit (3) thru the inductance L_3'. When circuit (3) is tuned to any one of the harmonics of the current in L_2, the galvanometer shows a deflection. If now the wave meter circuit (4) be tuned so that it has the same natural period as circuit (3), the current in circuit (3), and hence the galvanometer deflection, decreases because of the absorption of energy from circuit (3). The same indicating device, namely the thermocouple and galvanometer, therefore serves to tell first, when circuit (3) is tuned to one of the harmonics, and, second, when circuit (4) is in resonance with circuit (3).

The procedure adopted in calibrating a wave meter is as follows: Condenser C_1 is adjusted so that either the fundamental or one of the harmonics falls within the previously calibrated range of the wave meter. Condenser C_3 is then varied until circuit (3) is tuned for the fundamental as shown by a maximum deflection of the galvanometer. Circuit (4) is next tuned to circuit (3). Resonance is indicated by a decrease of the galvanometer deflection to a minimum. The reading of the wave meter is observed. Circuit (3) is then tuned to the next harmonic which has double the frequency of the fundamental and circuit (4) adjusted again to reduce the deflection to a minimum. This process is repeated for several harmonics or for all that are sufficiently intense to be of use. The adjustments of the condensers for resonance in both circuits (3) and (4) are easily made with a deviation of less than 0.1 degree in 180.

It has been found usually desirable to ground point (*a*) to prevent resonance of coil L_2 when excited by the fluctuations in potential of the middle point of L_1 to which L_2 is connected. This precaution ensures that the excitation of L_1 comes only thru the rectifier. Even with this precaution, L_2 may oscillate if the natural period of the coil approximates the period of one of the harmonics of the impulses which pass thru the rectifier. This resonance for one harmonic is undesirable because of the resulting magnification of the corresponding amplitude in circuit (3). This great difference in amplitudes causes inconvenience because of the widely different galvanometer deflections, and may produce slight inaccuracies in the data due to the differing degrees of reaction on the oscillations of circuit (1). The oscillations of coil L_2 were successfully eliminated and properly proportioned amplitudes of the harmonics of the series were obtained by winding coil L_2 with about 100 turns of fine high resistance wire. This added resistance is small compared with the resistance of the audion rectifier and consequently does little harm.

Unless the absorption of energy by circuit (3) is considerable, no detectable change in the frequency of the oscillations can be observed. Full scale deflections of the galvanometer were obtained with no harmful reaction on the fundamental oscillations. In case there is any doubt as to the constancy of the frequency of the oscillations of circuit (1) while circuit (3) is tuned to the series of harmonics, it is well to couple loosely to circuit (2) thru L_2' a control circuit (5) which may be tuned to the fundamental and used to detect any slight change in frequency.

A typical series of harmonics and the corresponding galvanometer deflections are given in the following table:

Harmonic	Deflection	Harmonic	Deflection
1	72.	5	3.6
2	66.	6	2.2
3	12.	7	0.8
4	11.		

The scheme described above was used by the writer in the calibration of a certain wave meter which has a range from 100 meters to 10,000 meters. A part of the scale from about 500 meters to about 1,800 meters was calibrated by the rotating-mirror method. The accuracy of this calibration was checked and the calibration extended both up and down in wave lengths to cover the entire range of the instrument.

Cruft Laboratory, Harvard College.

SUMMARY: The output of an electron relay oscillator is passed thru a non-resonant, rectifying circuit. The rectified radio frequency is rich in upper harmonics. These are used to calibrate a wave meter after a limited range thereof has been calibrated by another and absolute method. Details of the requisite procedure are given.

DISCUSSION

Julius Weinberger (communicated): This method of calibrating a wave meter also forms an extremely easy means of determining an electrical constant which has heretofore been rather a difficult one to measure—namely, the effective capacity of a coil, when used as a wave meter inductance (that is, connected in circuit with a variable condenser).

Suppose the wave meter condenser is calibrated, for capacity, say at audio frequency. Now, if we tune the wave meter (circuit 4 in the paper) to the fundamental wave length and note the condenser capacity (say C_1), and then to the second harmonic, again noting the condenser capacity (say C_2), we have:

(1) For the fundamental,

$$59.6\sqrt{L\,(C_1+C_L)}=\lambda_1$$

(2) For the second harmonic,

$$59.6\sqrt{L\,(C_2+C_L)}=\frac{\lambda_1}{2}$$

where C_L is the effective capacity of coil L.

Therefore, dividing, we get

$$\sqrt{\frac{C_1+C_L}{C_2+C_L}}=2$$

$$\frac{C_1-4\,C_2}{3}=C_L.$$

For example, if $C_1=2276$ $\mu\mu f$ and $C_2=548$ $\mu\mu f$,
Then

$$C_L=\frac{2276-2182}{3}=32\,\mu\mu f.$$

The beauty of this method is that it is extremely rapid, and that it gives the actual value of the coil capacity under operating conditions. It is certainly much shorter than any other method I know of; and is independent of any other standard except the capacity of the variable condenser. The latter can be determined with great accuracy at audio frequency; and, if the inductance of the coil is measured at audio frequency, the combination of these audio frequency calibrations with the coil capacity measured at radio frequency as above, will give an unimpeachable standard wave meter calibration.

I have found that the method is applicable to any of the high power bulb oscillators, such as the large pliotrons, without the use of the rectifying scheme of the paper. These bulbs usually give oscillations with a strong second harmonic, and running one of these on the local 235 volt circuit, I have found it quite practicable to make calibrations of various sorts, with a "current square meter" of 80 milliamperes maximum scale deflection as indicating instrument in the wave meter circuit.

THE COUPLED CIRCUIT BY THE METHOD OF GENERALIZED ANGULAR VELOCITIES*

By

V. Bush

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Abstract of Paper

In an oscillating-current circuit there is no impressed electromotive force and the sinusoids which are involved are damped.

In the alternating-current circuit, a certain function, called the impedance, may be used for the purpose of generalizing Ohm's law to apply to such circuits.

In order further to generalize Ohm's law so that it may be applied to oscillating-current circuits, an initial voltage must be used instead of an impressed voltage. The function which may be used to change this voltage amplitude into a current amplitude may be called the "*threshold impedance.*"

The alternating-current involves an angular velocity. In the oscillating-current circuit this angular velocity may be generalized to include the decrement of the circuit, and it then becomes a complex quantity. From this complex generalized angular velocity may be formed by analogy a generalized impedance. This generalized impedance is always zero for free oscillations. This law enables us to determine the generalized angular velocities, and hence the frequencies and decrements, present in the free oscillation.

The threshold impedance is derived by a single differentiation from the generalized impedance. The use of the threshold impedance furnishes a second law to be used in the determination of the amplitudes of oscillation.

These two laws completely solve the oscillating-current circuit. They are of importance only when there are several generalized angular velocities simultaneously present.

The inductively coupled circuit furnishes an example of the

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*Received by the Editor November 29, 1916.

utility of the method. In order to render this solution of greater practical value, a short approximate method is given in an appendix, for solving a fourth degree algebraic equation which appears.

A list of symbols will be found at the end of the paper.

INTRODUCTORY

An oscillating-current circuit is one which oscillates in the absence of impressed electromotive force. The quantities involved are thus damped sinusoids. When there is only a single frequency of oscillation, and a single decrement to correspond, such a circuit may be readily solved by the use of differential equations. When, however, there are several frequencies and decrements simultaneously present, such a solution soon becomes cumbersome, particularly as regards the determination of the constants of integration in accordance with initial conditions.

Similar difficulties were experienced in the solution of alternating-current networks. The simple series circuit was readily solved by differential equations; but complicated networks presented difficulties.

A practical method was obtained for the alternating-current case by the introduction of the concept of impedance. This impedance was simply the function of the constants of the circuit which divided into the amplitude of voltage would give the amplitude of current. The solution of a network, then, required simply a knowledge of the rules for forming the impedance of the network from the several impedances of the branches.

If we attempt to generalize this law in a similar manner so that we may apply it to the oscillating-current circuit, we are confronted with the fact that here we have no impressed electromotive force. We must then use some other voltage; and for this purpose the initial voltage present in the circuit offers itself. This initial voltage may be due to an initial charge present in a condenser; or it may be due to an initial current thru a resistance.

We seek, then, a function of the constants of an oscillating network such that it may be divided into an initial voltage to give an initial amplitude of current oscillation.

ANGULAR VELOCITIES

In forming the impedance of an alternating-current circuit we encounter the "*angular velocity*" of the circuit. This is the time rate of change of the argument in the impressed voltage expression:

$$e = E \cos (\omega t) \qquad \text{volts.}$$

It receives the name "angular velocity" because of the usual representation of such a quantity by means of a revolving plane vector.

If we make use also of the symbolic operation j, such a vector may be represented by the expression:

$$e = E\, \varepsilon^{j\omega t} \qquad \text{volts.}$$

In an oscillating-current circuit we have present also logarithmic decrements.

In the expression:

$$A\, \varepsilon^{nt} \qquad \angle$$

where

$$n = -a + j\,\omega \qquad \text{hyp./sec.} \ \angle$$

we have both the usual angular velocity and the decrement present, for the expression may be rewritten in the form:

$$A\, \varepsilon^{-at}\, \varepsilon^{j\omega t} \qquad \angle$$

when it is seen to consist of the alternating-current term, multiplied by a damping factor.

It will be convenient to call the complex quantity n, which includes both the angular velocity and the decrement, the "generalized angular velocity" of the circuit.

Using this generalized angular velocity, we may form generalized impedances, admittances, etc., by analogy with the alternating-current case.

FIRST LAW OF OSCILLATING-CURRENT CIRCUITS

If an alternating current of angular velocity ω passes thru a branch of impedance Z, the voltage across the branch is given by the product of the current and impedance.

It may readily be shown that this is also true if the current is oscillatory with a generalized angular velocity n[1].

Hence it follows, since the impressed voltage in an oscillating-current circuit is zero, that *the generalized impedance of the entire circuit must be zero.*

This fact, which appears in Rayleigh's Theory of Sound, in Heaviside's "Electrical Papers," and in Helmholtz's works, may be taken as the first law of oscillating-current circuits.

The application of the law enables us to determine the un-

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[1] "The Impedances, Angular Velocities, and Frequencies of Oscillating-Current Circuits"—A. E. Kennelly, "Proc. I. R. E.," 1915.

known generalized angular velocities of any given oscillating circuit, for the equation obtained on equating the generalized impedance to zero, may be solved for n. There may, of course, be several values of n, the generalized angular velocity, which appear as roots.

Upon separating these values of n into their real and imaginary portions, the decrements and angular velocities may be respectively found. In this way we may determine the damping factors and frequencies present in the free oscillation of any network.

The application of this law to practical circuits has been investigated by Eccles, Campbell and Kennelly.[2]

SECOND LAW OF OSCILLATING-CURRENT CIRCUITS

When the circuit oscillates at a single frequency, the amplitude of oscillation may be found by inspection. When several frequencies are simultaneously present, there is needed a law which will determine the amplitude of the various terms of the current expression.

If the equation from the first law:

$$z=0 \qquad \text{ohms } \angle$$

yields as roots

$$n_1, n_2, \cdots \qquad \text{hyp./sec. } \angle$$

these are the generalized angular velocities of free oscillation; and it follows that the current in the circuit will be of the form:

$$i=A_1\varepsilon^{n_1t}+A_2\varepsilon^{n_2t}+\cdots \qquad \text{amperes.}$$

It is our problem to determine the A's. If E is the initial voltage of the circuit, we seek a function which will divide into E to give A.

Such a function will be found in the expression:

$$n\frac{dz}{dn} \qquad \text{ohms } \angle$$

and this expression may appropriately be called the "*threshold impedance*" of the circuit. This fact is here given without proof; as a formal proof is necessarily too long for a paper of this character.

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[2] Eccles, "Electrician," 1915; "Phys. Soc. Proc.," **24**, 1912.
Campbell, "Proc. A. I. E. E.," 1911.
Kennelly, "Proc. I. R. E.," 1915.

Heaviside[3] gives without proof the following formula for the current in a network when a voltage E is suddenly applied:

$$i = \frac{E}{z(0)} + \sum_{r=1}^{m} \frac{E}{n_r \left(\frac{dz}{dn}\right)_{n_r}}$$

$$= \frac{E}{z(0)} + \frac{E}{n_1 \left(\frac{dz}{dn}\right)_{n=n_1}} \varepsilon^{n_1 t} + \frac{E}{n_2 \left(\frac{dz}{dn}\right)_{n=n_2}} \varepsilon^{n_2 t} + \cdots$$

$$+ \frac{E}{n_m \left(\frac{dz}{dn}\right)_{n=n_m}} \varepsilon^{n_m t}$$

where $n_1, n_2 \cdots n_m$ are the roots of $z(n) = 0$.

z is the generalized impedance of the circuit, a function of n.

$z(0)$ is the value of the generalized impedance obtained on inserting $n = 0$.

$\left(\frac{dz}{dn}\right)_{n_r}$ is the value of $\frac{dz}{dn}$ obtained on inserting n_r for n.

Wagner[4] proves this formula by the use of the function theory. A summary of Wagner's proof is given in Appendix B of this paper.

In circuits in which the charged element is a condenser, $z(0)$ is ∞; so that the first term disappears. The second term may also be considered the current on discharge for such a case, since the charge and discharge currents are equal and opposite. If the charged element is not a condenser, the first term may not disappear, and the full formula should be used.

Heaviside's formula applies to any system, physical or electrical, of any number of degrees of freedom, in which the relation between the magnitudes involved may be expressed by linear differential or algebraic equations. The oscillating-current circuit is a special case to which the formula applies. For this case the proof as given by Wagner is valid without qualification.

The use of this formula is particularly advantageous in finding oscillating-current solutions; because of the fact that z may be formed by rules already familiar from the alternating-current circuit, without referring to the differential equations of the circuit.

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[3] Heaviside, "Elec. Papers," Vol. 2, page 373; "Electromagnetic Theory," Vol. II.

[4] Karl Willy Wagner, "Archiv für Elektrotechnik," 1916, IV Band.

We may then state as a second law of oscillating-current circuits:

The initial amplitude of current oscillation equals the initial voltage of the circuit divided by the threshold impedance.

There will be a value of n, a value of $n\frac{dz}{dn}$, and hence a term in the current expression, corresponding to each root of the equation: $z=0$.

In forming the threshold impedance, it is necessary to form the generalized impedance z of the circuit considering the initially charged branch of the circuit as the main branch.

SOLUTION OF CIRCUITS

If a network contains an initial store of energy in one branch, corresponding to an initial voltage E, the current of free oscillation in the circuit may be found by the following steps:

1. Form the generalized impedance z of the circuit, considering the initially charged branch as the main branch.
2. Equate to zero, and solve for n. Call the roots of the equation n_1, n_2, ...
3. Form the threshold impedance $n\frac{dz}{dn}$
4. Write the current expression in the form;

$$i=\left(\frac{E}{n\frac{dz}{dn}}\varepsilon^{nt}\right)_{n=n_1}+\left(\frac{E}{n\frac{dz}{dn}}\varepsilon^{nt}\right)_{n=n_2}+\cdots \quad \text{amperes}$$

or:

$$i=\sum\frac{E}{n\frac{dz}{dn}}\varepsilon^{nt} \quad \text{amperes.}$$

In this expression the generalized angular velocities, and the amplitudes are in general complex quantities.

Upon reducing by the use of the identity:

$$\varepsilon^{j\omega t}=\cos\omega t+j\sin\omega t$$

the imaginary portions of the expression will cancel out, leaving a real expression for i.

If there are several stores of energy initially present, they may be considered separately and the results added.

The current or voltage in a distant portion of the network may be found by combining the generalized impedances of the elements of the circuit, in the manner of simple resistances.

A case of suddenly applied electromotive force may be considered as the inverse of discharge from the final state attained.

ILLUSTRATION

As a simple example to show the method, consider a condenser of capacitance C, initially charged to voltage E, and discharging thru resistance R (Figure 1)

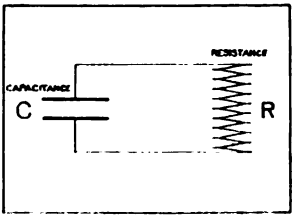

FIGURE 1

Here the generalized impedance is:

$$z = R + \frac{1}{Cn} \qquad \text{ohms.}$$

Equating to zero we obtain:

$$n = -\frac{1}{RC} \qquad \text{hyp./sec.}$$

The threshold impedance is:

$$n\frac{dz}{dn} = -\frac{1}{Cn} \qquad \text{ohms.}$$

Hence the current in the circuit is:

$$i = \frac{E}{n\frac{dz}{dn}}\varepsilon^{nt} = \left(\frac{E}{-\frac{1}{Cn}}\varepsilon^{nt}\right)_{n=-\frac{1}{RC}} = \frac{E}{R}\varepsilon^{-\frac{t}{RC}} \qquad \text{amperes.}$$

This result may be checked by inspection.

THE COUPLED CIRCUIT

The method is very useful for the solution of the circuits which occur in radio work.

It will be illustrated on the simple inductively coupled circuit.

In the circuit of Figure 2, R_1, L_1, C_1, are the primary, and $R_2\, L_2$, C_2 the secondary constants. M is the coefficient of mutual induction. The primary condenser is considered as discharging from an initial voltage E.

Form the generalized impedance of the circuit, considering the primary as the main branch.

For an alternating-current of angular velocity ω, the impedance of such a circuit is well known to be:

$$z_1 - \frac{(M\omega)^2}{z_2} \qquad \text{ohms}$$

where z_1 is the impedance of the primary and z_2 of the secondary alone.[5]

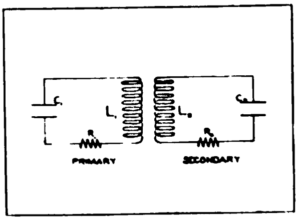

FIGURE 2

Hence, by analogy, we have as our generalized impedance:

$$z = R_1 + L_1 n + \frac{1}{C_1 n} - \frac{M^2 n^2}{R_2 + L_2 n + \dfrac{1}{C_2 n}} \qquad \text{ohms.} \angle$$

Equate to zero, and clear of fractions and we obtain:

$$C_1 C_2 (L_1 L_2 - M^2) n^4 + C_1 C_2 (R_1 L_2 + R_2 L_1) n^3 + (C_1 L_1 + C_2 L_2 + C_1 C_2 R_1 R_2) n^2 + (C_1 R_1 + C_2 R_2) n + 1 = 0.$$

Solve this fourth degree equation for n; and we obtain as roots the four values of the free generalized angular velocity:

$$n_1, n_2, n_3, n_4 \qquad \text{hyp./sec.} \angle$$

Since these four roots are in general all complex, the solution of this equation is often laborious. It may, of course, be solved to any desired degree of accuracy by straightforward algebraic methods. If it is wished to avoid this labor, the approximate method given in appendix A may be used. This method gives results sufficiently accurate for most engineering purposes. The exact method may, however, be used if desired.

The threshold impedance may be found from z by a simple differentiation, and becomes on simplifying:

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[5] "Impedance of Mutually Inductive Circuits," A. E. Kennelly, "The Electrician," London, Vol. XXXI, 1893, page 699.

$$n\frac{dz}{dn}=L_1 n-\frac{1}{C_1 n}-M^2\frac{L_2 n^3+2R_2 n^2+\frac{3n}{C_2}}{\left(R_2+L_2 n+\frac{1}{C_2 n}\right)^2}\quad\text{ohms. }\angle$$

Into this expression we may insert the four values of n found above.

The primary current is then given by the expression:

$$i_1=\sum\frac{E}{n\frac{dz}{dn}}\varepsilon^{nt}\quad\text{amperes }\angle$$

where the summation extends over the roots of n found from $z=0$.

The voltage induced in the secondary is found by multiplying i_1 by $-M\,n$, and is:

$$e_2=\sum -M_n\,n\frac{E}{n\frac{dz}{dn}}\varepsilon^{nt}\quad\text{volts. }\angle$$

The secondary current is then found by dividing this voltage by the generalized secondary impedance:

$$i_2=\sum\frac{-M\,n}{R_2+L_2 n+\frac{1}{C_2 n}}\;\frac{E}{n\frac{dz}{dn}}\varepsilon^{nt}\quad\text{amperes. }\angle$$

NUMERICAL EXAMPLE

This solution was applied to a test circuit at the Massachusetts Institute of Technology, and the results were checked by oscillograms and by comparison with the usual approximate methods of solution.

The constants of the circuit were:

$R_1=1.937$ ohms
$R_2=2.531$ ohms
$L_1=7.52\times10^{-3}$ henries
$L_2=7.63\times10^{-3}$ henries
$C_1=13.51$ microfarads
$C_2=24.62$ microfarads
$M=3.475\times10^{-3}$ henries
$E=7.2$ volts, initial

Inserting these values in the equation $z=0$, and reducing we obtain:

$$n^4+7.45\times10^2 n^3+1.930\times10^7 n^2+5.88\times10^9 n+6.635\times10^{13}=0$$

On solving this algebraic equation by the method of the appendix, there was obtained:

$$\left.\begin{matrix} -249.2 \pm j\,3827 \\ -123.3 \pm j\,2129 \end{matrix}\right\} \qquad \text{hyp./sec.} \angle$$

for the four free generalized angular velocities.

These four values of n, and the values of the constants inserted in the expression of $n \frac{dz}{dn}$, give four values for the threshold impedance. Dividing each of these values into the values of E gave the four amplitudes:

$$\left.\begin{matrix} -\,0.00396 \mp j\,0.1460 \\ 0.00396 \mp j\,0.0228 \end{matrix}\right\} \qquad \text{amperes.} \angle$$

Thus the primary current can be written:

$$\begin{aligned} i_1 = & (-0.00396 - j\,0.1460)\ \varepsilon^{(-249.2 + j\,3827)t} \\ & + (-0.00396 + j\,0.1460)\ \varepsilon^{(-249.2 - j\,3827)t} \\ & + (\ \ 0.00396 - j\,0.0228)\ \varepsilon^{(-123.3 + j\,2129)t} \\ & + (\ \ 0.00396 + j\,0.0228)\ \varepsilon^{(-123.3 - j\,2129)t} \quad \text{amperes} \angle \end{aligned}$$

Reducing the exponential terms to their trigonometric forms, and combining, this expression becomes:

$$\begin{aligned} i_1 = & \,\varepsilon^{-249.2t}(-0.00792 \cos 3827\,t + 0.292 \sin 3827\,t) \\ & + \varepsilon^{-123.3t}(0.00792 \cos 2129\,t + 0.0456 \sin 2129\,t) \text{ amperes.} \end{aligned}$$

or better:

$$\begin{aligned} i_1 = & \,0.292\,\varepsilon^{-249.2t} \sin(3827\,t - 0.0272) \\ & + 0.046\,\varepsilon^{-123.3t} \sin(2129\,t + 0.1719) \qquad \text{amperes.} \end{aligned}$$

Here we have the amplitudes, phase relations, and damping factors for the two terms of the primary current.

To obtain the secondary current amplitudes, we have to multiply the primary amplitudes by the ratio

$$\frac{M n}{R_2 + L_2 n + \dfrac{1}{C_2 n}} \qquad \text{numeric.} \angle$$

Inserting the values of the constants and of the roots for this ratio takes the four values:

$$\begin{aligned} -0.732\ & \underline{\backslash 3^\circ 32.6'} \\ -0.732\ & \overline{\backslash 3^\circ 32.6'} \\ 2.528\ & \overline{\backslash 6^\circ 14.3'} \\ 2.528\ & \underline{\backslash 6^\circ 14.3'} \qquad \text{numeric.} \angle \end{aligned}$$

Multiplying the four primary amplitudes by these respective ratios, gives the four secondary amplitudes:

$$\begin{array}{ll} -0.00370+j\,0.1069 & \\ -0.00370-j\,0.1069 & \\ +0.00370-j\,0.0585 & \\ +0.00370+j\,0.0585 & \qquad \text{amperes.} \angle \end{array}$$

The secondary current expression may now be written in the same manner as was the primary current expression. It reduces to the form:

$$\begin{aligned} i_2 = {} & \varepsilon^{-249.2t}(-0.00740 \cos 3827\,t - 0.2138 \sin 3827\,t) \\ & + \varepsilon^{-123.3t}(+0.00740 \cos 2129\,t + 0.1170 \sin 2129\,t) \qquad \text{amperes} \end{aligned}$$

or:

$$\begin{aligned} i_2 = {} & -0.214\,\varepsilon^{-249.2t} \sin(3827\,t + 0.0346) \\ & + \ 0.117\,\varepsilon^{-123.3t} \sin(2129\,t + 0.0632) \qquad \text{amperes.} \end{aligned}$$

SUMMARY: The generalized impedance z of an oscillating circuit may be formed from the generalized angular velocity of oscillation, n, by analogy with the alternating-current circuit.

Equating this generalized impedance to zero, and solving for n, gives the free generalized angular velocities of oscillation. The real and imaginary portions of these free generalized angular velocities are used to find respectively the damping factors and frequencies of oscillation of the circuit.

From z may be found the threshold impedance of the circuit $n \frac{dz}{dn}$. Dividing the initial voltage of the circuit by this threshold impedance gives the initial amplitudes of current oscillation.

The use of these two rules determines the complete expression for the oscillating current in any oscillating-current network.

The method applies to the simple inductively coupled circuit. A complete exact solution for the case of primary condenser discharge may be readily obtained. The method is of particular service in numerical problems.

An approximate method of solving the biquadratic obtained when coupled circuits are considered is given in an appendix to the paper.

APPENDIX A.

Approximate method for the solution of the fourth degree algebraic equation occurring in the coupled circuit problem.

This equation:

$$C_1 C_2 (L_1 L_2 - M^2)\, n^4 + C_1 C_2 (R_1 L_2 + R_2 L_1)\, n^3 + (C_1 L_1 + C_2 L_2 + C_1 C_2 R_1 R_2)\, n^2 + (C_1 R_1 + C_2 R_2)\, n + 1 = 0 \qquad \text{numeric} \angle$$

may be written in the form:

$$n^4 + \propto n^3 + \beta n^2 + \gamma n + \delta = 0 \qquad (\text{hyp./sec.})^4 \angle$$

where

$$\propto = \frac{L_1 R_2 + L_2 R_1}{L_1 L_2 - M^2}$$

$$\beta = \frac{L_1 C_1 + L_2 C_2 + R_1 R_2 C_1 C_2}{C_1 C_2 (L_1 L_2 - M^2)}$$

$$\gamma = \frac{R_1 C_1 + R_2 C_2}{C_1 C_2 (L_1 L_2 - M^2)}$$

$$\delta = \frac{1}{C_1 C_2 (L_1 L_2 - M^2)}$$

The approximate method depends upon the fact that the absolute values of the roots of this equation are not greatly different from the absolute values of the roots of the equation found for $R_1 = R_2 = 0$.

The equation for the circuit without resistance will be:

$$n^4 + \lambda n^2 + \delta = 0 \qquad (\text{hyp./sec.})^4 \angle$$

where

$$\lambda = \frac{L_1 C_1 + L_2 C_2}{C_1 C_2 (L_1 L_2 - M^2)}.$$

This equation is readily solved; and will yield as roots a pair of imaginary values:

$$j x_1 \text{ and } j x_2 \qquad \text{hyp./sec.} \angle$$

Now if the desired roots of our complete equation are:

$$\begin{gathered} -a \pm j b \\ -c \pm j d \end{gathered} \qquad \text{hyp./sec.} \angle$$

we may express these also in polar form as:

$$\begin{gathered} y_1 \, 7 \, \theta_1, y_1 < \theta_1 \\ y_2 \angle \theta_2, y_2 < \theta_2 \end{gathered} \qquad \text{hyp./sec.} \angle$$

and by examining the relations between the roots and coefficient of our algebraic equation, write:

$$(1)\quad a+c=\frac{\alpha}{2}$$

$$(2)\quad y_1^2+y_2^2+4\,a\,c=\beta$$

$$(3)\quad 2\,a\,y_2^2+2\,c\,y_1^2=\gamma$$

$$(4)\quad y_1^2\,y_2^2=\delta=x_1^2\,x_2^2$$

From (4), since x_1 and y_1, x_2 and y_2 are nearly equal, we may write as a first approximation:

$$y_1=x_1\,(1-q)$$

$$y_2=x_2\,(1+q) \qquad \text{hyp./sec.}\,\angle$$

where q is a small quantity.

Also from (1):

$$\frac{\alpha}{4}-a=c-\frac{\alpha}{4}=p \qquad \text{hyp./sec.}$$

Substitute in (2) and (3)

$$x_1^2\,(1-q)^2+x_2^2\,(1+q)^2+4\left(\frac{\alpha}{4}-p\right)\left(\frac{\alpha}{4}+p\right)=\beta$$

$$\left(\frac{\alpha}{4}-p\right)x_2^2\,(1+q)^2+\left(\frac{\alpha}{4}+p\right)x_1^2\,(1-q)^2= {}_{2} \qquad (\text{hyp./sec.})^3$$

Expand, and neglect the square of q in comparison with unity:

$$(x_1^2+x_2^2)-2\,q\,(x_1^2-x_2^2)+\frac{\alpha^2}{4}-4\,p^2-\beta=0 \qquad (\text{hyp./sec.})^2$$

$$\frac{\alpha}{4}\,(x_1^2+x_2^2)+p\,(x_1^2-x_2^2)-2\,p\,q\,(x_1^2+x_2^2)-q\,\frac{\alpha}{2}\,(x_1^2-x_2^2)=\frac{\gamma}{2} \qquad (\text{hyp./sec.})^3$$

Use as abbreviations:

$$x_1^2+x_2^2=s$$

$$x_1^2-x_2^2=t \qquad (\text{hyp./sec.})^2$$

and the equations become:

$$s-2\,q\,t+\frac{\alpha^2}{4}-4\,p^2-\beta=0 \qquad (\text{hyp./sec.})^2$$

$$\frac{\alpha\,s}{4}-2\,q\,p\,s+p\,t-\frac{\alpha\,q\,t}{2}-\frac{\gamma}{2}=0 \qquad (\text{hyp./sec.})$$

These equations may be solved simultaneously for p and q, giving:

$$q=\frac{s\,t^2+\alpha\,\gamma\,s-\gamma^2-\alpha^2\,\delta-\beta\,t^2}{2\,t\,(2\,s^2+t^2+\alpha\,\gamma-2\,s\,\beta)} \qquad \text{numeric}$$

and:

$$p = \frac{2\gamma - \propto s + \propto q t}{4t - 8qs} \qquad \text{hyp./sec.}$$

Since s and β are nearly equal, it is better to write a further abbreviation:

$$u = \beta - s = \frac{R_1 R_2}{L_1 L_2 - M^2}$$

and hence obtain:

$$q = \frac{\propto \gamma s - t^2 u - \gamma^2 - \propto^2 \delta}{2t\,(t^2 + \propto \gamma - 2su)} \qquad \text{numeric.}$$

We may now give the rule by which to obtain the free generalized angular velocities of the coupled circuit with constants $R_1\, L_1\, C_1\; R_2\, L_2\, C_2\; M$.

1. Solve the circuit without resistance. Call the absolute magnitude of the angular velocities obtained: x_1 and x_2.

2. Form the function:

$$q = \frac{\propto \gamma s - t^2 u - \gamma^2 - \propto^2 \delta}{2t\,(t^2 + \gamma \propto - 2su)}$$

where $\propto, \beta, \gamma, \delta$ are as given above,

$$s = x_1^2 + x_2^2$$
$$t = x_1^2 - x_2^2$$

and

$$u = \beta - s = \frac{R_1 R_2}{L_1 L_2 - M_2}$$

3. Write:

$$y_1 = x_1 (1 - q) \qquad y_2 = x_2 (1 + q)$$

and y_1, y_2 are the absolute values of the generalized angular velocities desired.

4. Form the function

$$p = \frac{2\gamma - \propto s + \propto q t}{4t - 8qs}$$

5. Write:

$$a = \frac{\propto}{4} + p$$

$$c = \frac{\propto}{4} - p$$

and a, c are the decrements desired, so that the generalized angular velocities are:

$$-a \pm j\sqrt{y_1^2 - a^2}$$

and

$$-c \pm j\sqrt{y_2^2 - c^2}.$$

As an illustration of the method to show the degree of approximation, a circuit with constants:

$C_1 = 10^{-9}$ farads

$C_2 = 10^{-10}$ farads

$R_1 = 1000$ ohms

$R_2 = 2000$ ohms

$L_1 = 0.025$ henries

$L_2 = 0.040$ henries

$M = 0.020$ henries

where the resistances are purposely assumed large, was solved by the exact and the approximate methods, and gave results agreeing to at least five significant figures.

q, in this case, was 0.000870, so that the assumption that the square of q could be neglected was evidently justified.

APPENDIX B.

SUMMARY OF WAGNER'S PROOF OF HEAVISIDE'S FORMULA

Wagner's proof is general, and applies to physical as well as electrical systems. This abstract of the proof will treat electrical oscillating systems only.

The constant voltage, which is suddenly applied to the network, is 0 when $t<0$, and E when $t>0$. Such a function may be represented by the Fourier integral:

$$f(t) = \frac{E}{2\pi j}\int_{-j\infty}^{j\infty} \frac{\varepsilon^{nt}}{n}\, dn \qquad (1)$$

where n is the complex variable of integration.[6]

This expression may be transformed to one with a closed path of integration as follows: About O, Figure 3, describe a circle of radius R. Examination will show that as R becomes infinite the integral vanishes along BCA for negative values of t, and along BDA for positive values of t. For negative values of t we may hence replace the open path of integration by the closed path of $AOBCA$, and for positive values of t by the path $AOBDA$.

Since the integrand is everywhere regular, except at the origin, it follows that the first of these integrals will be zero,

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[6] Malcolm, "Transients in Submarine Cables," "The Electrician," May 10, 1912.

while the second will have the value $2\pi j$ times the residual of the integrand at the origin. This residual is unity. Hence the expression:

$$\overset{t<0}{f(t) = \frac{E}{2\pi j}\int_{AOBCA} \frac{\varepsilon^{nt}}{n}\,dn} \qquad \overset{t>0}{f(t) = \frac{E}{2\pi j}\int_{AOBDA} \frac{\varepsilon^{nt}}{n}\,dn} \qquad (2)$$

has the value 0 when $t<0$, and the value E when $t>0$; and hence faithfully represents our function.

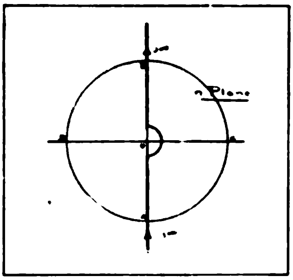

FIGURE 3

The voltage applied to a circuit, and the current in the circuit are always connected by a linear differential equation. This may be written symbolically:

$$e = F(D)\,i \quad \text{or} \quad i = \frac{e}{F(D)}$$

where D represents the differential operator $\frac{d}{dt}$.

If e follows an exponential law of variation with the time, such as:

$$\varepsilon^{kt}$$

it is well known that the current will then be of the form:

$$i = \frac{\varepsilon^{kt}}{F(D)} = \frac{\varepsilon^{kt}}{F(k)}$$

Now in our Fourier integrals above we have expressed the impressed voltage as the sum of terms of the form:

$$\frac{dn}{n}\varepsilon^{nt}$$

and these terms follow the exponential law of time variation.

Hence corresponding to the voltage increment there will be a current increment:

$$\frac{dn}{nZ(n)}\varepsilon^{nt} \tag{3}$$

where $Z(D)$ is the function of the differential operation from the equation of the circuit:

$$e=Z(D)\,i$$

The function Z is thus the generalized impedance of the circuit. Since the relations are linear, the effects of separate increments of the potential add simply to give the total effect.

Thus we obtain for the current in the circuit when $t>0$, the expression:

$$i=\frac{E}{2\pi j}\int_{AOBDA}\frac{\varepsilon^{nt}}{nZ(n)}\,dn \tag{4}$$

The other integral, which gives the current when $t<0$, must be zero. Hence the function $\frac{1}{Z(n)}$ can have no poles, or $Z(n)$ can have no roots, which lie in the positive half of the real plane This readily follows from physical considerations. This fact, and the limitations on $Z(n)$ when more general systems are under consideration, cannot be entered into here.

The value of the expression (4) for the current may now be determined by the evaluation of the integral.

The integrand has poles at $n=0$, and at the roots of $Z(n)=0$. Suppose these roots to be:

$$n_1, n_2, \quad \cdot \; \cdot \; \cdot \quad n_m.$$

Then the value of the integral is $2\pi j$ times the sum of the residuals of the integrand at 0, n_1, n_2, . . n_m. This follows from the fact that, since the integrand is everywhere regular except at these points, the path $AOBDA$, Figure 4, may be deformed into a path consisting of a small circle about each pole, Figure 5. The value of the line integral for one circuit positively about a single pole is $2\pi j$ times the residual of the function of that pole.

If N_r is the residual of the function:

$$\frac{\varepsilon^{nt}}{nZ(n)} \tag{5}$$

at the pole n_r, and N_o at the origin, then the current is given by:

$$i = E N_0 + E \sum_{r=1}^{m} N_r \tag{6}$$

To determine N_r the function (5) must be developed for the region about n_r, into a Laurent series in

$$(n - n_r)$$

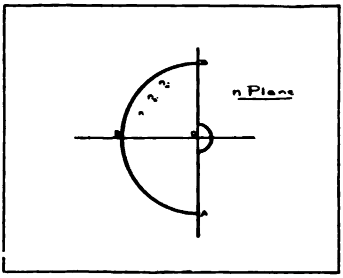

FIGURE 4

For abbreviation put $(n - n_r) = \xi$

We have the following:

$$\left\{\begin{aligned} & \varepsilon^{nt} = \varepsilon^{n_r t}\,\varepsilon^{\xi t} = \varepsilon^{n_r t}\left(1 + \xi t + \frac{\xi^2 t^2}{2\,!} + \quad \cdot \quad \cdot \quad \cdot \quad \right) \\ & n = n_2 + \xi \\ & Z = Z\,(n_r) + \xi \left(\frac{dZ}{dn}\right)_{n_r} + \frac{1}{2}\,\xi^2 \left(\frac{d^2 Z}{dn^2}\right)_{n_r} + \quad \cdot \quad \cdot \quad \cdot \end{aligned}\right. \tag{7}$$

If the expansion of (5) is

$$\frac{\varepsilon^{nt}}{nZ} = \sum_{u=-\infty}^{u=\infty} A_u \xi^u \tag{8}$$

the coefficient A_{-1} is the residual N_r which we seek.

From (7) this may be seen to be:

$$N_r = \frac{\varepsilon^{n_r t}}{n_r \left(\frac{dZ}{dn}\right)_{n_r}} \tag{9}$$

To obtain the residual at the origin we use the expansion in the vicinity of the origin:

$$\frac{\varepsilon^{nt}}{nZ\,(n)} = \frac{\varepsilon^{nt}}{n\,(Z\,(0) + n\left(\frac{dZ}{dn}\right)_0 + \quad \cdot \quad \cdot \quad \cdot \quad)}$$

from which $$N_0 = \frac{1}{Z(0)} \tag{10}$$

Using (9) and (10) in (6) we obtain finally:

$$i = \frac{E}{Z(0)} + \sum_{r=1}^{m} \frac{E}{n_r \left(\frac{dZ}{dn}\right)_{n_r}} \varepsilon^{n_r t} \tag{11}$$

which is the Heaviside formula.

In this expression the first term on the right hand side is the steady state term, and the remaining terms give the transient. In cases where the current is finally zero, the steady state term disappears.

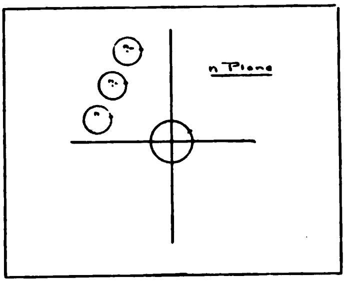

FIGURE 5

From the above derivation certain limitations as to the character of the roots of Z may be noted.

The roots must be negative in real part. Since a positive real part would mean physically, a circuit oscillating with a continuously increasing amplitude, this is of interest only in showing that Heaviside's formula is limited in application to such systems where this occurs; e. g., in the unstable arc.

The roots of Z must be distinct from each other and from zero. The case of multiple roots requires further treatment. This treatment will be found in Wagner's paper.

Singularities in Z do not appear in the treatment of the usual networks

LIST OF SYMBOLS USED

e	Instantaneous electromotive force. Volts.
i	Instantaneous current. Amperes.
E	Maximum or initial value of voltage. Volts.
I	Maximum or initial value of current. Amps.
ω	Angular velocity, $2\pi \times$ frequency. Radians per second.
n	Generalized angular velocity. Hyperbolic radians per second. $\angle$.
α	Logarithmic decrement. Hyps. per second.
j	The pure imaginary $\sqrt{-1}$.
ε	Base of Naperian system of logarithms. 2.718 · · · .
A	A constant amplitude. $\angle$.
z	Impedance. Ohms.
Z	Generalized impedance. Ohms. $\angle$.
R	Resistance. Ohms.
L	Inductance. Henrys.
C	Capacitance. Farads.
M	Mutual inductance. Henrys.
$\alpha, \beta, \gamma, \delta, \lambda$	Coefficients of algebraic equation.
$x_1\ x_2\ y_1\ y_2$	Absolute values of generalized angular velocities.
a, b, c, d	Rectangular components of generalized angular velocity.
$\theta_1\ \theta_2$	Polar angles of generalized angular velocity.
p, q	Correcting factors.
s, t, u	Constants.
$\angle$	Sign of a complex quantity or equation.