VOLUME 1 NUMBER 7 JULY 1964

IEEE spectrum

features

departments

THE INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS, INC.

the cover

This month's cover illustration by General Electric shows a plasma electron beam source consisting of a 20-kV beam in hydrogen. The electron beam is one of three types—photon, electron, and ion—that are being used increasingly as working tools. In the article beginning on page 66 of this issue, each beam form is described from the point of view of its own peculiar advantages and disadvantages as well as its unique applications. All energy beams have the outstanding characteristic of highly localized action. This characteristic is what makes them so outstanding for cutting and welding applications. The localized action minimizes the disturbance to the metallurgical properties of the material being worked.

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Jesse Taub, Harvey Hindin, Gediminas Kurp:s, and Jerome Cohen of our Department of Applied Electronics have been developing quasioptical components for millimeter and submillimeter wavelength operation. Some of their recent work is described this month.

Progress Report on Quasioptical Components

Interest in millimeter and submillimeter radar and communications systems has been stimulated during the past two years by advances in the generation of appreciable amounts of power at these high frequencies. A backward-wave oscillator with 100 kw of peak power in the 97 to 100 Gc region has recently been described (reference 1). Other developments indicate that CW power levels of 1 watt can be expected in the 0.9 to 1 mm region in the near future. Above 100 Gc, the other components for any operational system can not be adequately constructed using standard microwave techniques.

At AIL, we have been developing millimeter and submillimeter components using optical structures housed in oversize waveguide (reference 2); these components incur lower losses and are much easier to construct than conventional waveguide components.

An article appearing in the July 1963 issue of the Proceedings of the IEEE explained AIL's work on directional couplers, variable attenuators, and phase shifters. Since that time, we have concentrated on developing the other components necessary for a complete RF subsystem. However, the following description will be limited to examples of filters, ferrite devices, and diode mounts using quasioptical waveguide techniques that are now under development at AIL.

A two-resonator band-pass filter operating at a center frequency of 33 Gc has been constructed in ten-times-oversize rectangular waveguide (Figure 1). The energy propagates in the TE_{10} mode. Coupling between resonators and input and output loading is determined by the diameters of circular holes in a conducting plate. This method is analogous to iris (single-hole) coupling in standard

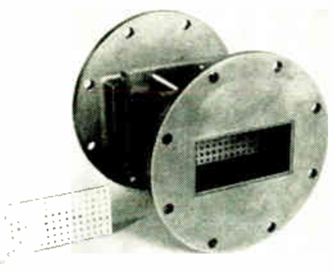

Fig. 1. Quasioptical Waveguide Filter

size waveguide. The interresonator coupling plate has been removed and is shown at the left in Figure 1. The large number of holes is required in order to avoid the generation of higher modes. The filter was constructed using brass waveguide and had a 22.5-Mc 3-db bandwidth and a 1.7-db center-frequency insertion loss. A filter with the same bandwidth constructed in standard-size coin-silver waveguide would have had a 6.6-db loss; standard-size brass waveguide would have yielded losses in excess of 10 db. These results indicate that narrow-band oversize waveguide filters are feasible. A scaled version is being constructed for operation at 330 Gc.

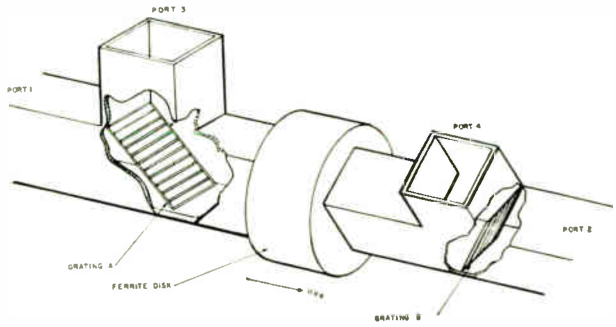

Fig. 2. Quasioptical Circulator

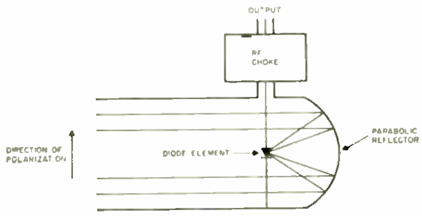

Fig. 3. Parabolically Focused Diode Mount

Ferrite devices in oversize waveguide, using the Faraday rotation principle, are also feasible. Their operation is similar to conventional Faraday-rotation circulators except that optical structures are used. Figure 2 shows the configuration of a circulator using oversize waveguide. A vertically polarized wave entering at port 1 passes through grating A to a ferrite disk that rotates the polarization of the wave by 45 degrees; the wave continues through grating B to port 2. A wave entering port 2 with the same polarization as the previous example would propagate through the ferrite and emerge horizontally polarized. Grating A acts as a mirror on the wave and deflects it to port 3. In a similar fashion, energy is transferred from ports 3 to 4 and from 4 to 1. Calculations indicate that materials such as Ferramic R-1 will give the required 45-degree rotation in a $^{1}/_{8}$-inch thick disk provided that the ferrite is magnetized at 2000 gauss. Furthermore, this rotation is frequency insensitive in oversize waveguide. The exact degree of dissipation loss in the ferrite and matching the ferrite to the waveguide impedance are problems that are currently being investigated. Models of this device are being developed for operation at 30 and 330 Gc. In addition, feasibility models of isolators and ferrite modulators are being designed.

The use of diodes as mixers, detectors, or parametric amplifiers is severely hampered by standard-size wave-guide in the millimeter and submillimeter wavelength regions because of the high losses and the small size of the components. Other successful oversize-waveguide components appear to indicate the need for an arrangement in which the diode is mounted directly in the oversize waveguide. AIL is conducting experiments (currently in the 30-Gc range) in focusing the energy at the diode element (Figure 3). Most of the energy strikes a paraboloid and is reflected toward the diode, which is placed at the focal point. It is expected that sensitivities at least comparable with those of standard-size waveguide mounts will be obtained. We also expect to try similar experiments at frequencies above 100 Gc.

In summary, the continuing work in oversize-waveguide components is yielding a variety of devices. Using this technique, RF subsystems in the low millimeter and submillimeter ranges will soon be a reality.

This work was sponsored by Rome Air Development Center, Griffiss Air Force Base, New York, under Contract AF 30 (602)-3343.

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REFERENCES

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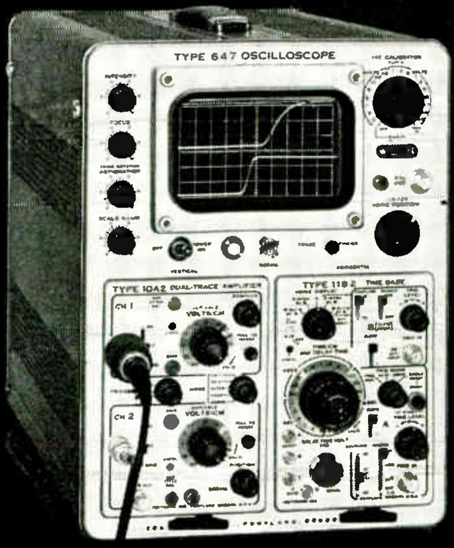

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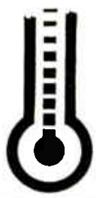

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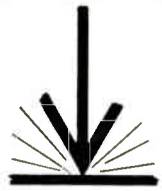

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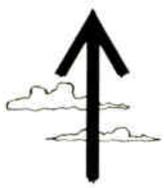

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The ART of engineering

Thermionic-emission electron micrograph of low-carbon (0.2 per cent) steel. The micrograph provides direct evidence of thermally etched contour lines by holding at a temperature of 2250°F. It is part of an "Art in Research" collection of slides collated by the Research Laboratories of General Motors Corporation for availability as an exhibit.

Reflections

75 years ago

A New Type of A-c Motor. "My experience, in common with that of my predecessors, teaches that the alternating current motor has a strong and persistent disposition to stand still, and when persuaded to motion is apt to be a sort of 'go as you please' machine and asserts its inherent right to turn in either direction indifferently, direction of rotation in some cases being purely a matter of chance. I shall not have much to say about efficiency, as my experiments with large machines are not sufficiently advanced to furnish any reliable data, but I will endeavor to give a general solution of the problem designed to meet the following conditions of practice:

"1st. A machine that will start itself independently of the speed of the generator or number of alternations of current per unit of time.

"2nd. A machine that has but one direction of rotation and cannot reverse under any conditions of current alternation.

"3rd. A machine that is not necessarily synchronous with the generator, revolution for revolution.

"4th. A machine in which reversals of current direction do not produce corresponding reversals of magnetism in any iron part, when the machine is in motion at its normal speed and maximum efficiency.

"5th. A machine of simple form having an ordinary continuous wound armature revolving in a single or two pole field."

"... Fig. 5, where the complete machine is shown. There are twenty-four coils in the armature, twenty-four bars in the outside collector and thirty-two bars in the inside one, this latter being composed of twenty-four connecting and eight insulating bars. The connecting bars of the inner ring are numbered to correspond with those of the outer ring around to the right from one to twenty-four:—the insulating bars drawn shaded, separate the others into groups of three. In this machine three segments, 1, 2, 3, in the outer ring, are connected direct to the corresponding segments, 1, 2, 3, of the inner ring, likewise the opposite three, 13, 14, 15, of one ring, are connected direct to 13, 14, 15 of the other. The next group of three is connected inversely, 4, 5, 6, of the outer ring, to the diametrically opposite bars, 16, 17, and 18 of the inner ring, and the corresponding opposite group, 16, 17, 18, of the outer ring, is likewise connected inversely to the diametrically opposite group, 4, 5 and 6 of the inner ring. The remaining segments are connected in the same manner, but the connections are omitted to avoid confusion of the drawing. The operation of the machine is evidently the same as that shown in Fig. 4, except that the required conditions are fulfilled in this instance when three bars of the collector pass under the ring at each alternation of current, and as there are twenty-four segments arranged in groups of three, the machine at its normal speed would make one revolution for every eight alternations of current, and connected in a circuit supplied with 16,000 reversals per minute, its normal speed would be $\frac{16{,}000}{8} = 2000$ per minute, and with 48 segments arranged in groups of three its speed would be 1000 per minute. The blank segments separating the groups of the inner ring are connected to the extremities of a rheostat *Rh. Rh.* which is inclosed inside the commutator and is designed to offer a path for the alternating current such as there may be and prevent its absolute rupture at the period of change from one group of segments to the next; they also serve an important purpose in preventing a dangerous short circuit which would be occasioned by the inner brush bridging two groups of segments oppositely connected.

"It follows as a matter of course that as the machine starts as a direct current motor connected in an alternating circuit, rapid reversals of magnetism will at first be produced in all the iron cores and these should be made of laminated iron to prevent undue loss by heating at the period of starting. The machine in fact starts as a direct current motor and automatically changes at a certain speed to a sort of synchronously alternating motor. It reaches its normal speed at this point, is then self-regulating,

Fig. 5

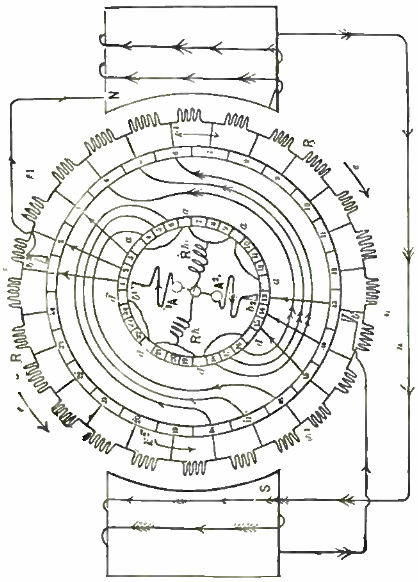
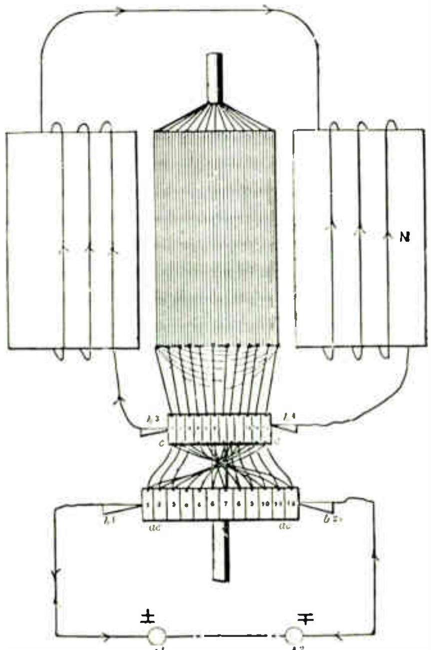


Fig. 6



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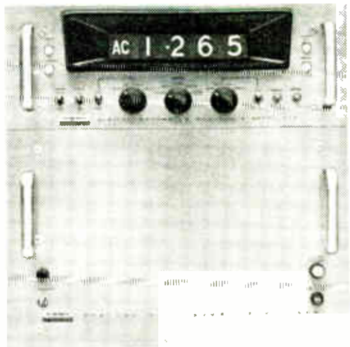
DC Digital Voltmeter and Ratiometer (Model 507D). Measures voltages between ±100 microvolts and ±1000 volts, ratios between ±0.0001:1 and ±999.9:1 with 0.01% (of reading) ±1 digit accuracy. Accessories permit AC/DC and AC/AC ratio measurements. Stepping switches guaranteed for 2 years. Price: $3835.

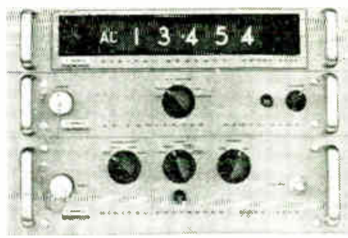
DC Digital Voltmeter (Model 501B). Four-digit, fifth-digit overranging. Measures DC between ± 100 microvolts and ±1000 volts, with 0.01% (of reading) ±1 digit accuracy. Automatic or programmable range; auto polarity. Combines the useful accuracy of a 5-digit voltmeter with the price advantage of a 4-digit voltmeter. Stepping switches guaranteed for 2 years. Price: $2995.

DC Digital Voltmeter (Model 501BZ). Similar to Model 501B (see above). Circuit is automatically and continually calibrated against a Zener diode reference source instead of against an unsaturated mercury-cadmium standard cell. For submarine and other special environment applications. Price: $3160.

AC/DC Digital Voltmeter (Model 502B). Gives you AC accuracy within 0.1% of *reading;* over-ranging on both AC and DC; automatic ranging and remote (programmable) control. Measures DC between ±100 microvolts and ±1000 volts, AC from 30 cps to 10 kc between 1 millivolt and 1000 volts. Five-digit readout. Stepping switches guaranteed for 2 years. Price: $4245.

AC/DC Digital Voltmeter (Model 502BZ). Similar to the Model 502B (see above). Circuit is automatically and continually calibrated against a Zener diode reference source instead of against an unsaturated mercury-cadmium standard cell. Price: $4410.





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"Fig. 6 shows a plan of the machine as constructed; it consists simply of an ordinary closed circuit armature in a single field; *cc* is the ordinary collector really a part of the armature circuit from which the brushes b^3b^4 take a current of constant direction to the field shunt; *ac, ac* is the reversely connected commutator and the brushes b^1b^2 bearing upon this commutator are connected to the terminals of the alternating current circuit." (F. Jarvis Patten, "Alternating Current Motors: The Evolution of a New Type," *AIEE Trans.*, vol. VI, Nov. 1888–Nov. 1889, pp. 388–410.)

50 years ago

The Science of Coding. "It has been my good fortune during the past few years to hear some excellent papers read on radio engineering subjects, and the allied arts, by prominent engineers and scientists of a professional standing recognized here and abroad. I have always been particularly interested in the discussions and I am convinced that no matter what the subject, who the author, nor how convincing the conclusions, there is usually to be found in the audience an honest difference of opinion on at least one point and sometimes several. In this lies the value of these meetings. Then, too, in this day and age of trained minds, the specialist often receives valuable hints and suggestions from interested workers in other lines.

"This leads to the question, 'What is an exact science, and when may one be justified in making a positive statement?' Surely arithmetic may be so classed, altho there may be doubt as to the status of some of the branches of higher mathematics as applied to the arts and sciences. We regret that at present medicine and surgery are not exact sciences.

"The art of 'Radio Communication' is comparatively new. Convinced by the marked consistency and steady agreements of researches conducted in this field during the past few years, I believe we are all satisfied that a certain portion of radio engineering, at least, may be classified under the heading of 'exact science.'

"Radio engineering as a whole may not be termed an 'exact science' so long as the radio operating element enters as such an important factor. This thought occurred to me repeatedly on hearing eminent radio experts lament operating difficulties. . . .

"I am fully satisfied, and I believe experienced inspectors will agree with me, that speed in operating is somewhat of a talent. It is a much more simple engineering problem to force oscillations in a circuit than to force an ungifted individual thru an operator's training school.

"We are all more or less familiar with the comparatively simple duties of the 'land line operator,' which simplicity probably accounts for the excess of average speed of land line transmission as compared to radio transmission.

"The land line operator has to deal with very simple apparatus requiring little or no adjustment. He does not have interference to contend with, nor faint and variable signals. The 'spark frequency' produced by a 'sounder' is the same for all instruments and under all conditions—and, in this country, he uses American Morse code.

"As regards the American Morse code, I may state that I am a strong champion of the American Morse code for radio work, altho I am compelled to bow to the overwhelming majority in favor of the Continental Morse. The necessity for a universal radio code is, of course, apparent to all.

"However, a few comparisons that I have made between the two codes may be of interest.

TABLE

CONTINENTAL MORSE

"One dot is the unit of time.
A dash is equal to three dots.
The space between parts of the same letter is equal to one dot.
The space between two letters is equal to three dots.
The space between two words is equal to five dots.

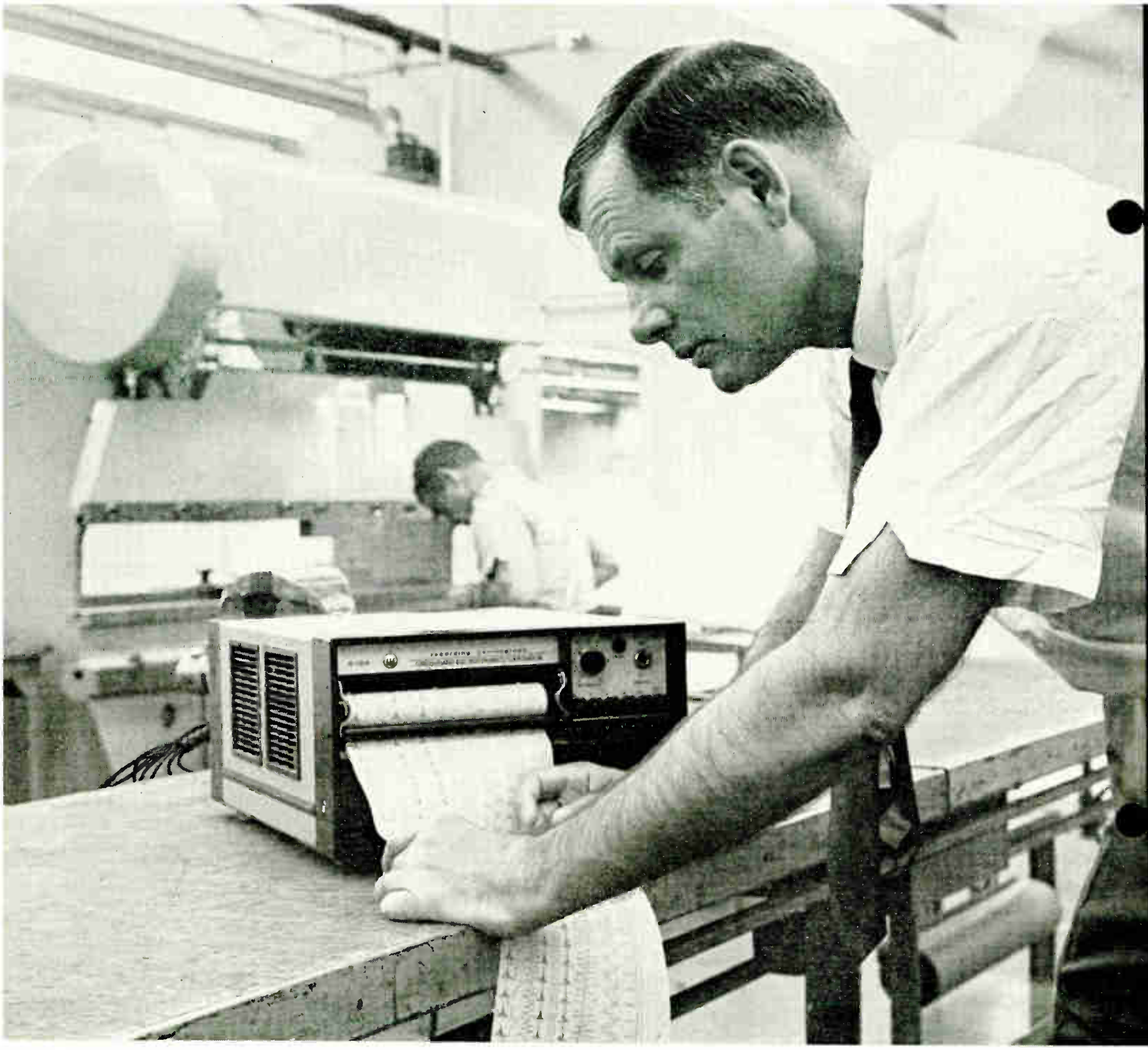
AMERICAN MORSE

"The space in spaced letters is equal to two dots.
The letter L is equal to two dashes.
Zero is equal to three dashes.

"Hence, including the letters and numerals only, and allowing three units space between each letter:

American Morse = 394 dots (for the alphabet).
Continental Morse = 460 dots.

"Therefore, as a whole, the American Morse is about 16.7 per cent faster than Continental for the same degree of skill.



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The rest is industrial history. For although CEC's little 5-124 will never replace its big brothers, it has made *precise* product testing a reality for virtually every company from aircraft and appliances to pharmaceuticals and plastics.

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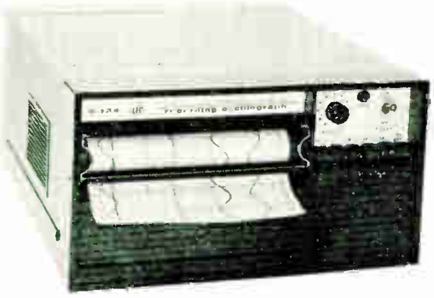

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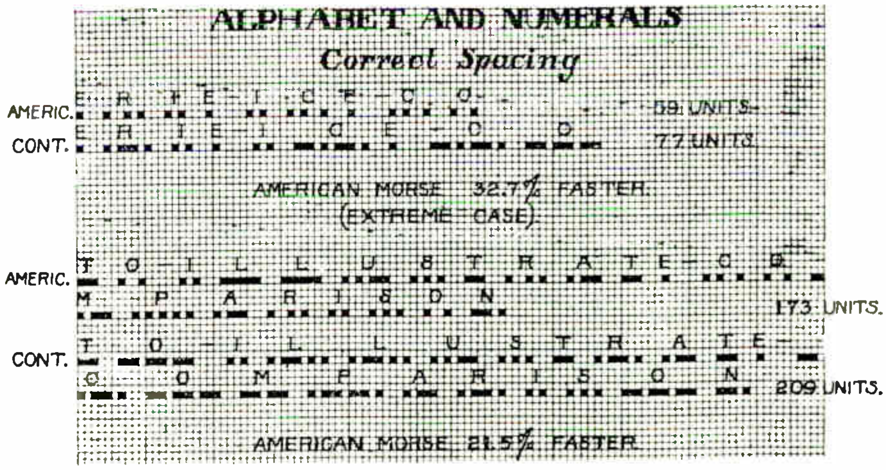


Figure 1. Alphabet and Numerals

In ordinary use it is probably about 20 per cent faster. This is illustrated in Figure 1.

"I believe the principal objection to the American code for radio purposes is the assumed possibility of confusion in the reception of the 'spaced' letters, which may be caused by 'static.' To this objection, I do not agree. It seems to me that the chances are about equal of static making an 'S' out of the American Morse 'O'; or an 'H' out of the American Morse 'C,' or an 'R' and 'S' out of the Continental 'I'; etc. The same reasoning applies to static dashes. . . .

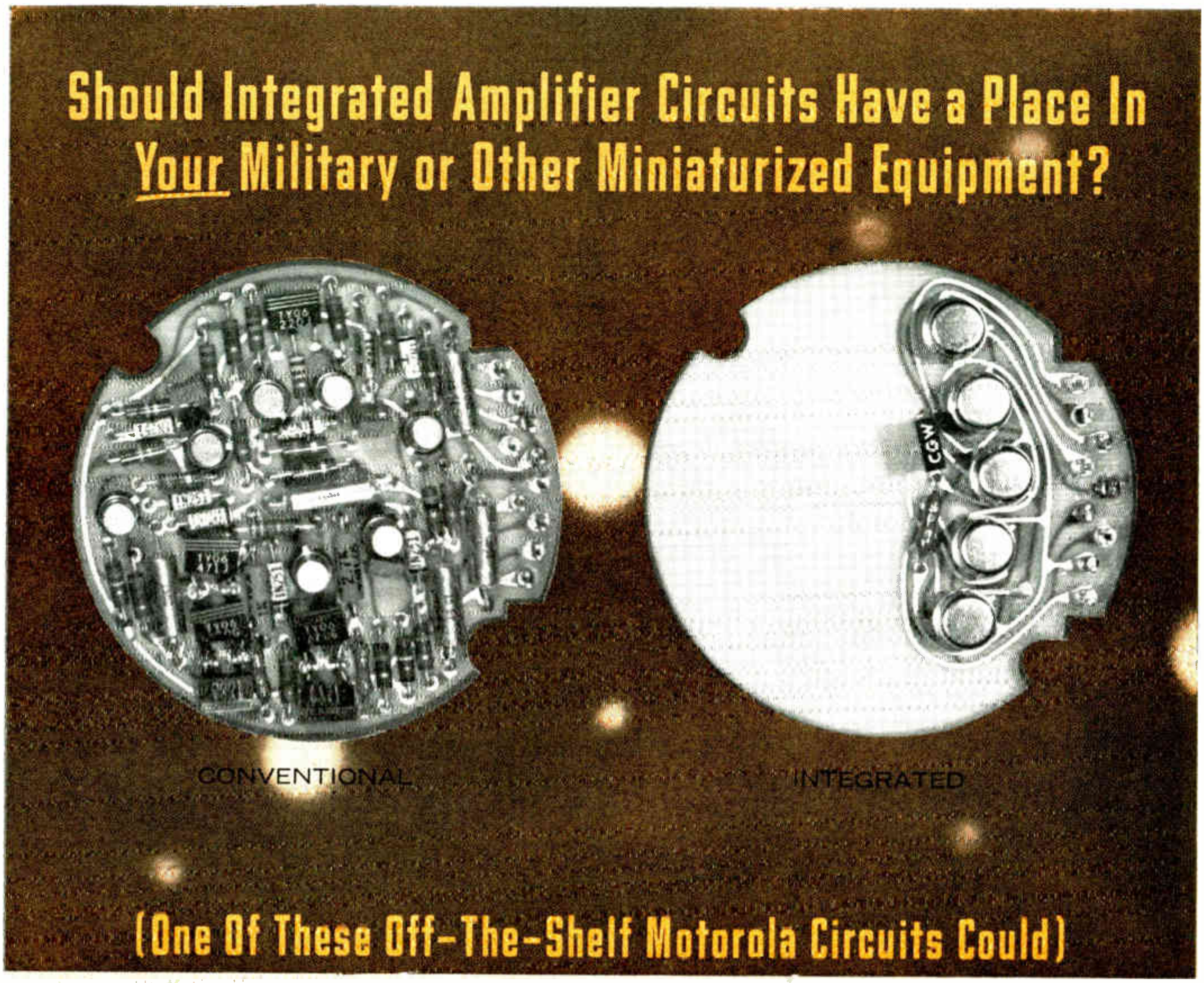
"I have said that I believe good operating is something of a talent. If this be true, I may then be permitted to state my belief in the superior attractiveness of American Morse. To my mind, the American Morse code is more *artistic* than the Continental. It has more rhythm and seems to be more graceful and better balanced. However, I shall not press this point. The law provides for the use of the Continental code, and we are law abiding citizens." (V. Ford Greaves, "The Radio Operator Problems," *Proc. IRE*, Sept. 1914, pp. 195–210.)

Hydroelectric Power. "In 1897 the San Gabriel Electric Co. built at Azusa a hydroelectric plant consisting of four 300-kw. generators, which received their energy from a small stream with a 400-ft. (122-m.) head. The water power converted into electric energy by these generators at a potential of 500 volts was transformed to a potential of 15,000 volts and transmitted at this potential a distance of 25 miles (40 km.) to Los Angeles. Later, four other small hydroelectric plants varying in size from 150 kw. to 1500 kw. were added, and a "standby" steam plant of 2000 kw. was constructed, giving a total capacity of 6000 kw., 4000 kw. of which was water power.

"January 1, 1905, the Kern River plant, a hydroelectric plant made up of five 2000-kw. generators, which transmitted its energy over 125 miles (201 km.) of line at a potential of 60,000 volts, was developed, and two years later, the original Redondo steam plant, which at that time made such a record for efficiency, was completed and its 15,000 kw. contributed to the service of the community. Even this, however, was not sufficient, for in 1911 it was found necessary to add to Redondo two 12,000-kw. turbine units. . . ."

"To transmit electric energy over such a distance with economy requires, of course, the highest practical potential, which in this case was selected as 150,000 volts, and plans for transmission at this voltage were drawn. These plans called for the immediate development of 60,000 kw. maximum, delivered at this voltage to the Los Angeles receiving station known as Eagle Rock."

"This energy is generated by four 17,500-kv-a., 6600-volt generators, two in each power house. At Power House No. 1 the water is delivered to the wheels under an average head of 2050 ft. (624.8 m.) from a reservoir about five miles (8 km.) long and one mile (1.6 km.) wide, with sufficient storage capacity to operate the plant at full load about four months, which is equivalent to six or eight months of operation based upon a 60,000-kw. peak on the two plants. After leaving Power House No. 1 the water enters a second tunnel, whence it is carried 4½ miles (7.2 km.) to Power House No. 2, where it is delivered at an



MECL-A* Linear Amplifier (type MC1110)

- DC to 300 mc performance
- High input and output impedances
- Power gain = 26 db @ 100 mc (typ)

DC Comparator (type MC1514)

- Compares two DC levels and provides a digital output
- Low transition width

High Input Impedance Amplifier (type MC1515 & 16)

- $Z_{in} > 5$ megohm
- FET input stage
- High AC gain stability ±1 db from −55°C to +125°C

A fresh new look for amplifier-equipped electronic gear has dawned with the introduction of three versatile new Motorola linear integrated circuit designs.

Housed in the 10-lead, TO-5 package, their potential for high packaging density, lightweight, and rugged dependability will add a new dimension to IF/RF linear amplifiers (type MC1110), analog-to-digital converters (type MC1514), and small-signal linear amplifier equipment (types MC1515 and 16), for *your* specific end product.

Produced on Motorola's high-speed production line, these three amplifiers are but a limited selection of the linear integrated circuit designs which Motorola is capable of producing. Literally hundreds of other "custom" linear integrated circuits in applications such as:

- Video Amplifiers
- IF Amplifiers
- Small-signal Amplifiers
- Operational Amplifiers
- Sense Amplifiers
- Differential Amplifiers

can be fabricated to meet special requirements . . . and delivered within weeks!

For information on the three circuit types listed here or for an engineering evaluation of your specific integrated amplifier circuit, contact the Motorola semiconductor office nearest you or write: Motorola Semiconductor Products Inc., Dept. TIC-LA, Box 955, Phoenix, Arizona 85001.

**Trademark of Motorola Inc.*

MOTOROLA Semiconductor Products Inc.

BOX 955 • PHOENIX, ARIZONA 85001 • A SUBSIDIARY OF MOTOROLA INC. 92057 average head of 1857 ft. (565 m.)" (Edward Woodbury, *AIEE Trans.*, vol. XXXIII, Part II, 1914, pp. 1283–1289.)

25 years ago

Enter the Orthicon. "A New Television Pickup Tube" was the subject of a paper by Albert Rose and Harley Iams of the RCA Manufacturing Company (Harrison). In it the authors pointed out that the iconoscope, as it is known today, is capable of transmitting clear, sharp pictures, even under unfavorable conditions of illumination. Previous workers with the tube have shown that the good sensitivity is obtained in spite of an efficiency only 5 to 10 per cent of that which is theoretically attainable. An analysis of the operation of the iconoscope suggests that improved efficiency and freedom from spurious signals should result from operating the mosaic at the potential of the thermionic cathode, rather than near anode voltage. The beam electrons then approach the target with low velocity and the number of electrons which land depends upon the illumination.

"Special designs were developed to make sure that the beam of low-velocity electrons was brought to the cathode-potential target in a well-focused state, that the scanning pattern was undistorted, and that the focus of the beam was not materially altered by the scanning process. A strong magnetic field perpendicular to the target was found useful in focusing and guiding the beam. In some of the earlier tubes which were tested, the scanning beam was released by a flying light spot moving over a photocathode. These experiments led to the present form which uses a thermionic cathode to develop the electron beam. A new type of deflection plate was developed too so that the beam could be deflected in the presence of a strong axial magnetic field.

"The new pickup tube, which has been called an Orthiconoscope (or Orthicon), has an output signal over 300 times the noise of a typical television amplifier. The signal is proportional to light intensity. There is no observable spurious signal. Within the accuracy of measurement, the efficiency of conversion of possible photoemission into light is 100 per cent."

"June 7, 1939—President Heising, presiding." (Report on the New York IRE Section Meeting, *Proc. IRE*, July 1939, p. 479.)

News of the IEEE

calendar
people
obituaries

WESCON, scheduled August 25–28 in Los Angeles, has five special sessions

An attendance of more than 45000 persons is anticipated for the Western Electronic Show and Convention in Los Angeles, August 25–28.

Technical program sessions will be held in the Statler Hilton Hotel in downtown Los Angeles. Exhibits will be presented in two major locations—the Los Angeles Sports Arena and Hollywood Park racetrack. Two new hotels, both near the Los Angeles International Airport, will help meet the demand for space. They are the New International Hotel and the Airport Marina.

Wescon's technical program will consist of five special, invited afternoon sessions, about 20 regular morning sessions, and company-sponsored seminars. Special session subjects include: "Microelectronics," "Information Theory," "Electronics in Oceanography," "Apollo Electronics," and "High-Voltage Power Transmission." About one-third of the regular morning sessions will consist of invited papers; the remainder will be contributed papers.

An innovation in Wescon/64 will be the invitation to several individual companies to present full-length technical seminars as part of Wescon's official program. Such seminars, which have often been privately presented during (but apart from) major technical conventions, have not previously been included as part of Wescon.

Papers from all Wescon regular technical sessions, preprinted and bound individually, in advance, will be on sale during the convention.

Wescon is jointly sponsored by the 6th Region of IEEE, represented by the Los Angeles Section and the San Francisco District, and the Western Electronic Manufacturers Association. Wescon's board of directors is made up of eight members, four each representing IEEE and WEMA. Don Larson, the Wescon manager, is also included on the executive committee.

The tentative technical program for the convention follows:

TUESDAY, AUGUST 25

Special Session A. Microelectronics—The Needs, The Approaches, and The Potentials

Chairman: J. G. Linvill, Stanford University

Military Participation—And Objectives. *Richard Alberts, Wright-Patterson AFB*

Which Approach—And When? *W. M. Webster, RCA Labs.*

Commercial Applications—The Challenge. *R. C. Sprague, Sprague Elec. Co.*

Session 1. Microwave Tubes

Chairman: John Mendel, Hughes Aircraft Corp.

Some New Results with High-Power Millimeter-Wave Tubes. *J. F. Heney, Hughes Res. Labs.*

S- and X-Band Traveling-Wave Tube Amplifiers for Satellite Applications. *A. Lubarsky, J. N. Nelson, R. E. Pospisil, O. T. Purl, L. A. Roberts, G. Wada, Watkins-Johnson Co.*

A CW Magnetron for Microwave Cooking. *W. L. Adikes, W. C. Hickman, Amperex Electronic Corp.*

Investigation of Magnetic Materials Technology for PPM Focused Low-Noise Traveling-Wave Tubes. *D. A. Schrumpf, General Elec. Co.*

100-Watt CW Traveling-Wave Tubes With Octave Bandwidths. *Norman Pond, Sylvania Elec. Products*

Session 2. Microelectronics

Chairman: Mort Penberg, Aerojet-General Corporation

An Analysis of the Potential Avionic Reliability Improvement Through Microelectronics. *J. R. Lennon, Boeing Co.*

Integrated High-Frequency DC Amplifiers. *D. R. Breuer, TRW Space Tech. Labs.*

The Minimization of Parasitics in Integrated Circuits by Dielectric Isolation. *D. A. Maxwell, R. H. Beeson, D. F. Allison, Signetics Corp.*

Micro-Integral Monolithic Differential Amplifier-Reducing Offset Voltage and Temperature Variations. *J. R. Nall, H. L. Gorgas, S. Santamaria, Molectro Corp.*

Session 3. Learning Systems

Some Uses and Misuses of Bicomputer Philosophy. *M. L. Babcock, Univ. of Ill.*

Training a Threshold Logic Unit with Imperfectly Classified Patterns. *R. O. Duda, R. C. Singleton, Stanford Res. Inst.*

A Multilayer Learning Network. *R. A. Stanford, Philco Corp.*

Learning Systems In and Out of the Factory. *R. M. Stewart, Space General Corp.*

Generalized Learning Theory. *R. E. Jackson, N. Amer. Aviation*

Session 4. Automatic Control

The Approximate Time Optimal Control of Higher-Order Systems Using a Generalized Model. *W. C. Evans, Taylor Instr. Co.; G. H. Cohen, Univ. of Rochester*

Time-Optimal Control with Adaptive Networks. *J. W. Berkovec, Collins Radio Co.*

A Monitorless Redundance Scheme for Higher Control System Reliability. *A. S. Escobosa, Autonetics*

Optimal Design of Several Reaction Jet Switching Techniques. *J. L. Hinson, M. D. Sarles, Martin Co.*

Compatibility of Impulse Modulation Techniques with Attitude Sensor Noise and Spacecraft Maneuvering. *J. E. Vaeth, Martin Co.*

Session 5. Electron Devices in the Power Industry

Chairman: F. C. Six, The Ralph M. Parsons Company

New Functions in Static Relays for Transmission Line Protection. *W. C. Morris, General Elec. Co.*

Electronics Can Be Effective in Reducing Distribution Circuit Outage. *G. G. Auer, General Elec. Co.*

Static Control and Protective Devices in Electrical Transmission and Distribution. *J. A. Longley, Allis-Chalmers Mfg. Co.*

A New Look at Power Line Carrier Systems. *R. V. Rector, M. C. Adamson, General Elec. Co.*

WEDNESDAY, AUGUST 26

Special Session B. Instrumenting the Sea Floor—Why and How

Chairman: R. A. Frosch, Advanced Research Projects Agency

Organizer: F. B. Spiess, Scripps Institution of Oceanography

Problems Requiring Sea Floor Instrumentation. *R. A. Frosch, Advanced Research Projects Agency*

Vehicles and Stations for Installation and Maintenance of Sea Floor Equipment. *V. C. Anderson, Scripps Institution of Oceanography*

Geophysical Measurements with Sea Floor Instruments. *Hugh Bradner, Univ. of Calif.*

Bottom-Mounted Navigation and Tracking Systems. *D. L. Potter, General Motors Corp.*

Special Session C. Extra-High-Voltage DC Transmission

Chairman: E. C. Starr, Bonneville Power Administration

Design and Development of High-Voltage Plastic Busings for DC Test Facility. *L. I. Gradasoff, Bonneville Power Admin.*

BPA's Extra-High-Voltage DC Test Project Measurements and Instrumentation. *M. G. Poland, M. W. Belsher, A. A. Osipovich, Bonneville Power Admin.*

DC Ground Electrode Behavior. *A. L. Kinyon, Bonneville Power Admin.*

Ground Current in High-Voltage DC Transmission. *T. Cantwell, P. Nelson, J. E. Webb, Geoscience; C. L. Waugh, A. L. Kinyon, R. F. Stevens, Bonneville Power Admin.; A. Orange, U.S. Air Force Cambridge Res. Labs.*

Session 6. Microwaves

Chairman: I. Kaufman, TRW Space Technology Labs.

Spectral lines

The Ladies in Our Midst. Perhaps not many of our members know that the ladies took over last month—they held their own First International Conference of Women Engineers and Scientists at the United Engineering Center in June. Perhaps not many of you know that the ladies have their own national organization in the U.S.—the Society of Women Engineers—with a membership approximating 600. And these women demonstrate that they can be engineers and also beautiful—we have evidence on both counts.

The IEEE can be proud of the manner in which our own women members have been contributing to the work of this sister organization, and in this context sister is doubly meaningful. The first president of the SWE was Dr. Beatrice Hicks, a Senior Member of IEEE, the immediate past president was Patricia Brown, the 1963–1964 president is Aileen Cavanaugh, and the president-elect is Isabelle French, all IEEE members.

In addition, Dr. Hicks received the 1963 SWE Achievement Award, the highest honor conferred by the Society. The Achievement Award for 1964 went to Grace Murray Hopper of the Sperry Rand Corporation, for contributions in the field of automated programming, and Grace is so far the only woman elected to the grade of Fellow in the IEEE. It seems that others recognize the value of electronics in getting things done these days.

While on the subject of women in engineering, it may be well to record a desire to see many more of them; first, because they improve an otherwise drab scene and, much more importantly, because they can and do carry on effectively in today's engineering work. The five eminent members of SWE mentioned above have distinguished themselves in research and development in such well-known organizations as Bell Telephone Laboratories, Newark Controls Company, Texas Instruments, and Sperry Rand.

With the reduced emphasis on hardware and the test floor, with increased importance given to development of concepts and ideas, and with research as a major opportunity, there should no longer be any barrier to keep women out of engineering. With the financial rewards and opportunity available in research and development, mathematical and computer analysis, systems studies and the like, there are many sound reasons why girls should enter the engineering field.

Groups and groups. The Board of Directors, meeting in New York on April 23 took note of the excessive wordiness of the title "Professional Technical Group" and directed that henceforth such groups of members shall be simply known as IEEE Groups. That is, our Professional Technical Group on Widget Technology will now have the more euphonious appellation of the IEEE Widget Group. The lengthy title was a result of merging the Professional Groups of the IRE with the Institute Technical Groups of the AIEE.

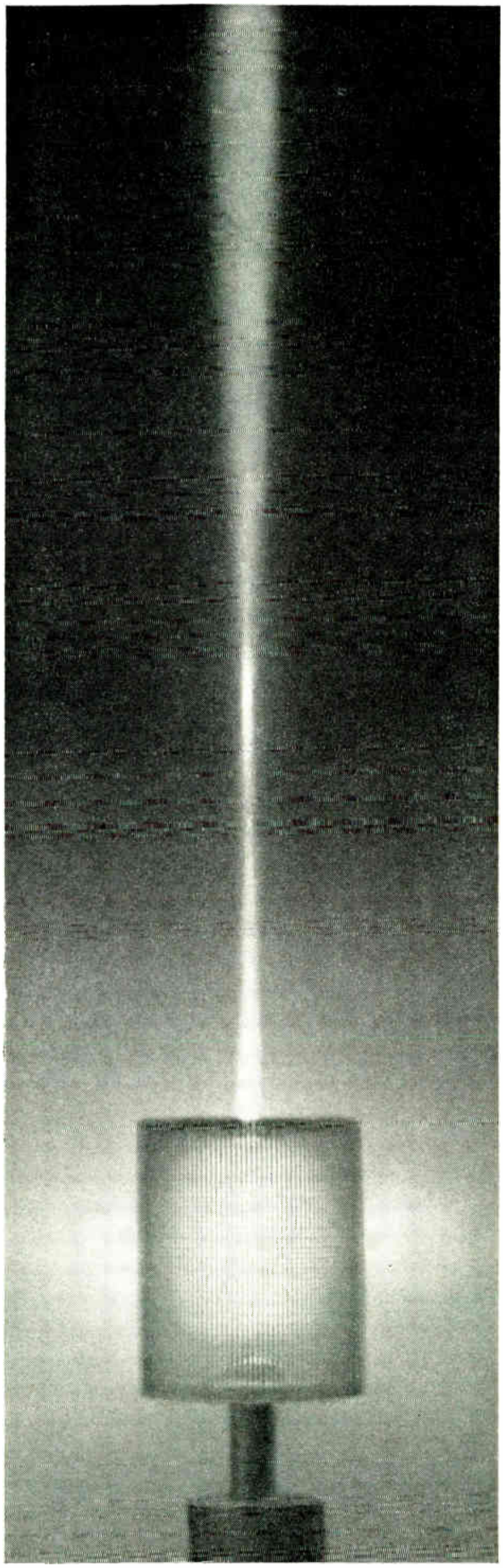
Other changes are occurring: there have been mergers of various Groups and Committees having somewhat overlapping fields of interest. Among those recently accomplished has been one in which the former Communications Division and the Professional Technical Group on Communications Systems merged to form the IEEE Communication Technology Group; the Reliability Group merged with the Subcommittee on Reliability and Quality Control; the Technical Committee on Recording and Controlling Instruments joined with the Industrial Electronics Group to form the IEEE Industrial Electronics and Control Instrumentation Group; the Nuclear Instrumentation Committee and the Nucleonics Committee have merged with the Nuclear Science Group; and the Solid State Devices and New Energy Sources Committees merged into the IEEE Electron Devices Group. Accomplished July first was the transition of the Power Division to the status of the IEEE Power Group, encompassing a number of subfields.

The Computing Devices Committee has merged with the Electronic Computers Group to form a merged IEEE Computer Group, thus broadening the field of interest.

Editorial Matters. To relieve the Editorial Board of some of the detailed editorial planning and to broaden the area of technical contact with new and old fields, thus permitting the Board to exercise more freely its managerial functions, this Editor has asked some of the members of the Editorial Board to assume responsibilities for editorial content of specific publications, supported where desirable by Boards of Consultants. This procedure is not new, having been a form of organization used by the IRE STUDENT QUARTERLY for a number of years—but as has often happened, what was good for the student publication turned out to have advantages also for our other publications.

In carrying out these individual assignments the Editor has asked Dr. F. Karl Willenbrock to assume responsibility for the PROCEEDINGS, aided by a 20-man board of consultants; Dr. Walter MacAdam has been assigned IEEE SPECTRUM, aided by a 7-man board; and Dr. T. F. Jones has agreed to take responsibility for the TRANSACTIONS, aided by the several Group editors. In addition, Dr. A. H. Waynick will interest himself in our foreign translations programs, Dr. S. W. Herwald will follow our activies in indexing, information retrieval and Engineering Index associations, and T. A. Hunter will carry on with the STUDENT JOURNAL.

J. D. Ryder



Energy beams as working tools

Intense beams of light or particles have been increasingly used in the forming and joining of refractory materials and components. Energy beam techniques reduce disturbance of the metallurgical properties to a minimum

C. Q. Lemmond. L. H. Stauffer

General Electric Company

Energy is applied in a variety of forms to cut, shape, and process modern materials. Over the years, wood, stone, and metal have been worked primarily by mechanical and chemical means. Recently, however, concentrated flames, electric arcs, and resistance heating have supplemented mechanical tools for cutting, melting, and joining. With the appearance of harder and more refractory materials the energy used in forming and processing is applied in greater concentration and must be precisely controlled. Automated manufacturing processes have intensified the search for more flexible methods to apply energy with speed and precision in the forming and joining of components.

Energy in the form of intense beams of light or particles is now offered as a solution to some of the problems inherent in this search. While beamed photons, electrons, and ions have been extensively used in the control and instrumentation of industrial processes, they have only recently been applied as primary energy tools. Newly developed sources now provide light or particle beams measured in kilowatts and power concentrations running into many megawatts per square inch. Such beams easily melt or vaporize even the most refractory metals or ceramics and have proved their worth for high-speed cutting and welding processes.

Particles vs. photons

Some of the characteristics of photons or light quanta as energy-carrying entities are comparable with those of charged particles (electrons and ions). All have wave properties characterized by a wavelength λ and each carries energy W related to its associated wavelength. Both particles and photons may be refracted by suitable lenses into either collimated or finely focused beams. Both are absorbed, scattered, and reflected by matter, with attendant energy degradation into heat.

For comparative purposes, some of the properties of particle beams and light beams are displayed in algebraic form:

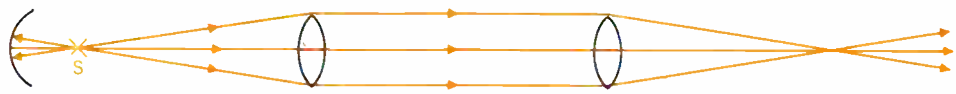
Particles	Photons
$\lambda_p = \dfrac{h}{mv_p}$	$\lambda_L = \dfrac{c}{\nu}$
$W_p = eV = \dfrac{1}{2}mv_p^2 = \dfrac{h^2}{2m\lambda_p^2}$	$W_L = W_1 - W_2 = h\nu = \dfrac{hc}{\lambda_L}$
$v_p = \left(\dfrac{2eV}{m}\right)^{1/2}$	$v_L = c$
$P_p = eV\phi_p = IV$	$P_L = h\nu\phi_L = \dfrac{hc}{\lambda}\phi_L$

where

h	= Planck's constant	ν	= photon frequency
v_p	= particle velocity	c	= velocity of light
e	= electronic charge	v_L	= photon velocity
m	= particle mass	P_L	= photon beam power
P_p	= particle beam power	λ_L	= photon wavelength
λ_p	= particle wavelength	ϕ_L	= photon flux
ϕ_p	= particle flux	W_1, W_2	= atomic energy levels
W_p	= particle energy	W_L	= photon energy
V	= accelerating potential		

It is seen that the quantum mechanical wavelength λ_p of particles is inversely proportional to the momentum

20-kV electron beam from a cold hollow cathode in argon.



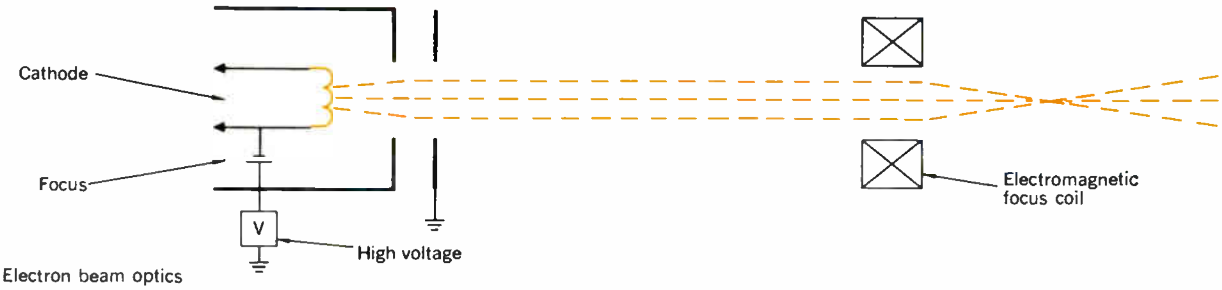


Fig. 1. Analogous light beam and particle beam systems.

mv_p, and therefore is a function of the potential V used to accelerate the particle. Photons, on the other hand, have fixed wavelengths and energies determined by the difference $W_1 - W_2$ between atomic energy states in the emitting atom. In vacuum, their velocities and hence their wavelengths are fixed.

In both particulate beams and light beams the energy flux, or beam power, is proportional to the product of the flux ϕ of carriers by the energy of an individual carrier (photon or particle). Thus it is apparent that the energy of particulate beams is continuously adjustable by varying either the flux ϕ_p from the source or by varying the accelerating voltage V. With photon beams only the flux ϕ_L is conveniently adjustable, as will appear later in this discussion. This last comparison has a fundamental bearing on the ultimate beam power that can be achieved in light and particle beams. Energy can be imparted to particles after they have left the source in proportion to V, the accelerating voltage, while photons have their energies fixed by the nature of the source.

Beam formation by optical systems (for light), and by analogous electron-optical systems (for charged particles), is illustrated by Fig. 1, which compares an electron beam system (B) with a simple optical system (A). In the optical system, light from a concentrated source S at the focal point of the first lens is formed into a parallel beam, which is then focused by a second lens. Analogously, electrons from a hot filament are collected by the electrostatic lensing action of the first aperture plate, shown in Fig. 1(B), and are then accelerated by the potential difference V applied between the two apertures. This electron gun assembly, housed in vacuum, projects a beam that can be condensed to the desired degree by an electromagnetic lens. Ion beams may be formed similarly by replacing the hot cathode by a gaseous discharge and by reversing power supply polarities for positive ions.

If the sources and condensing systems are carefully designed, focal spot diameters measured in microns may be obtained for both light and particle beams. This has resulted in the attainment of continuous electron beam power densities ranging in the tens of megawatts per square inch. For light beams, the focal spot size is independent of the photon flux, or source intensity, but is limited by diffraction effects, which in turn are wavelength dependent. Since the diameter of the first diffraction minimum (ring) is proportional to the wavelength, a smaller spot size is possible with the use of blue light than with longer-wavelength red or infrared light. For ions or electrons with energies measurable in kilovolts the associated quantum mechanical wavelengths are very short compared to the wavelengths of visible light. For such beams, diffraction effects in general play a minor role in determining spot size. Charged particle beams, however, suffer from spreading as a result of mutual electrostatic repulsion between particles. This becomes more pronounced as the beam density increases and the average particle separation becomes smaller.

When energy beams of high density are applied to a metal or other opaque material, they penetrate only to a small depth while most of their energy is being transformed into heat. If the beam power density is high enough, the surface layer may be melted and vaporized before the heat can be dissipated by conduction into the material. In aluminum, for example, electrons of 100-kV energy penetrate only to an average depth of approximately 50 microns, while red light (7000 Å) gives up 95 per cent of its energy in opaque materials after penetrating approximately 0.05 micron.[1] Thus, for equal beam-power densities, the volume energy density in the material is roughly 1000 times greater with photon beams than with electron beams. With ion beams the depth of penetration is much less than with electrons. For example, atomic hydrogen ions of 100-kV energy penetrate to a depth of less than 5 microns in aluminum and heavier ions of the same energy have even less penetrating power. It is thus apparent that the energy beams here considered exert very intense heating effects when they are concentrated on the surface of a solid material.

In addition to heating and attendant changes of state, energy beams readily dissociate, polymerize, etch, optically excite, and ionize elements and compounds. Because a wide range of particle or photon energy is available and because almost any desired power density may be obtained by adjustment of source intensity and beam convergence, such beams are extremely versatile and adaptable to automatic control.

Conventional light beam generation

The various types of light sources may be divided into three main classes: (1) incandescent, thermal sources in which the radiation is the result of temperature; (2) luminescent sources, which depend upon excitation of individual atoms acting independently; and (3) the laser source, which depends on the stimulated emission of radiation. There are also subdivisions of the three major means for producing light, such as injection luminescence (made possible by the development of injection luminescence diodes) and radioactive sources, which have been used for years to illuminate watch hands and numerals. Another classification deals with electroluminescence, bioluminescence, and even chemoluminescence. When light is to be considered as an energy beam, however, the three major categories are the ones to be considered.

Incandescent sources. A large percentage of the light sources in use today are of the thermal type. Tungsten heated in a vacuum enclosure will reach an incandescent temperature and radiate light in all directions. The radiant energy from such a source follows the Stefan-Boltzmann law:

$$E = \sigma e T^4$$

where T = absolute temperature, degrees Kelvin; e = 1 for a black body and less for materials such as tungsten; σ = 5.7 × 10^{-12} watts·cm^{-2}·deg K^{-4}; and E = radiant energy, watts·cm.$^{-2}$

Fig. 2. Sketch of an optical system for collecting and imaging incandescent source.

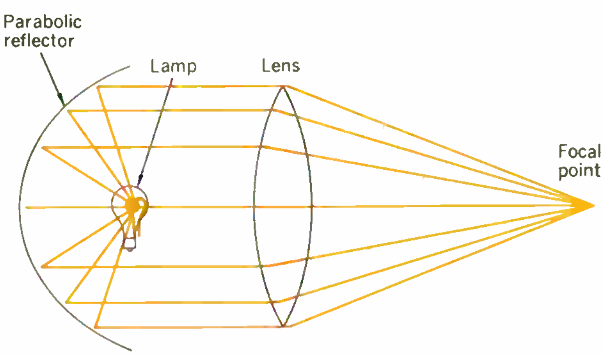
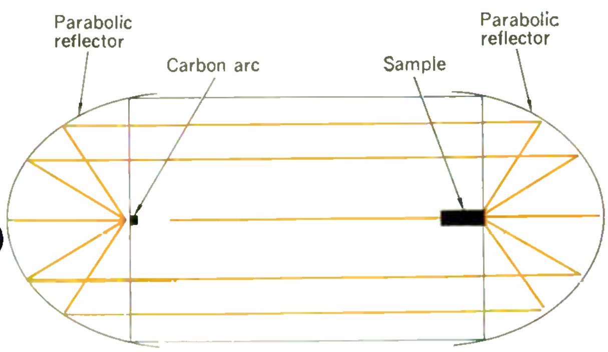


Fig. 3. Arc image furnace.



This means that the total radiant power per unit area of any emitting surface varies as the fourth power of the absolute temperature. Since the light energy so formed radiates in all directions, it is difficult to collect optically all of this radiation and form a useful energy beam. In addition, the radiating surface of most incandescent sources is sizable and therefore cannot be reduced optically to small dimensions, thereby producing a highly concentrated energy beam. Figure 2 illustrates such a source and how the light energy is imaged optically. An optical system of this nature can collect and image approximately 70 per cent of the radiated energy.

Thermal sources are capable of radiating a wide range of wavelengths. The relationship of wavelengths to the absolute temperature of the material is given by Wien's displacement law:

$$\lambda_m = \frac{\rho}{T}$$

where λ_m = peak wavelength, microns; T = temperature, deg Kelvin; and ρ = 2897.9 micron·deg k.

Thus it is possible to produce peak wavelengths in the visible region of the spectrum or, by lowering the temperature, to shift the maximum well into the infrared region. Such energy sources have found considerable application as heat-generating sources, such as infrared lamps.

Infrared lamps are being used for heating or drying, from the drying of codfish to the production-line baking of freshly painted automobile bodies. Where the material is a good absorber of infrared energy, the infrared lamps are particularly efficient.

Luminescent sources. The carbon arc, which has found several applications as an energy radiator, radiates energy in two modes: part of the radiation comes from the incandescence of the electrodes but the major portion comes from the luminescence of the vaporized electrode material formed in the arc. In general, there are three types of carbon arcs:

1. The low-intensity arc, which produces light at the tip of a positive carbon. The tip reaches temperatures of 3700°C, while the gas in the arc is much hotter and reaches temperatures of approximately 6000°C.[2]
2. The flame arc, which is actually a modification of a low-intensity arc wherein a part of the carbon electrode is replaced with certain chemical compounds that vaporize with the carbon and become part of the gaseous mixture in the arc. Various materials are used, depending on the radiation wavelength desired.
3. A high-intensity arc, which is a scaled-up version of the flame arc. Current density in the arc is increased sizably to a point which causes the anode spot to spread over the complete end of the carbon electrode. When this happens, the electrode material is evaporated and a crater is formed. A major portion of the light is produced in the crater and the gaseous region filling it.

Since all the types of carbon arcs produce a compact, intense light source, they can be imaged optically to produce very high energy densities. The output of the carbon arc nearly matches the spectral characteristics of sunlight and for this reason it has been used as the primary light source for solar simulators to test materials and components intended for space vehicles. It is necessary for this application not only to produce the energy equivalent to sunlight but also to maintain the spectral distribution of the energy.

Another application of the carbon arc as an energy beam is as an arc image furnace, shown schematically in Fig. 3. This energy from the carbon arc is collected by a parabolic reflector and is imaged into another reflector symmetrically positioned with respect to the first. The radiation reimages at the focal point of the second reflector. At this point extremely high-energy densities can be realized. An indication of the radiant energy capabilities of arc sources can be given by describing one of the highest power units ever constructed. This was a carbon arc mounted on a 120-inch reflector and used as a searchlight by the British during World War II. The arc employed a 64-mm positive carbon operating at 4000 amperes and dissipating 600 kW at the arc.[3] The peak intensity was approximately 3 billion candle power.

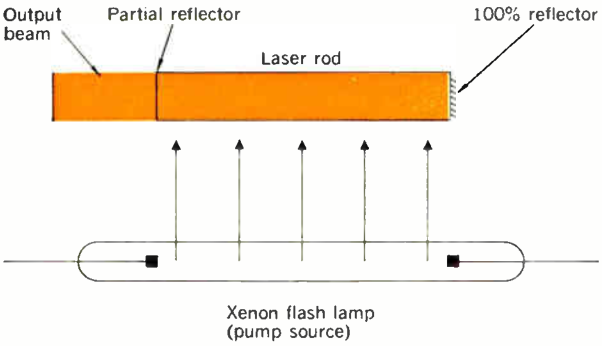


Fig. 4. Schematic drawing of laser.

The high-pressure mercury vapor lamps are now reaching brightnesses approaching that of the carbon arc. In many of the applications where the carbon arc has been successfully used, the high-pressure mercury arc is now being used. The light output across the positive crater of the mercury lamp is not uniform and therefore the light-emitting region is less extensive than that of the carbon arc. Both sources have been used in searchlights and as illumination for motion picture projectors.

To summarize, arc lamps have been used primarily as illumination sources; as energy sources the major application has been to heating or melting of materials.

The laser. The laser is a light-producing device that emits a highly collimated beam of coherent radiation. Optically, such a source of light can be described as a point source at infinity and therefore can be imaged to extremely small spot sizes. Solid-state or crystal lasers have achieved energies of over 1000 joules for pulse durations of a few milliseconds. Radiation production by this unusual means is accomplished by the release of photons of light from atoms of various materials that have been optically pumped or excited to their metastable energy level. Fig. 4 shows an optical cavity formed between two reflecting plates, with the active medium between these reflectors. Once the atoms of the material

Fig. 5. Air-cooled ruby laser with optical system for drilling very small holes. Reflecting microscope objective images radiation on material centered by rear viewing microscope. Microscope also inspects hole before it is removed.

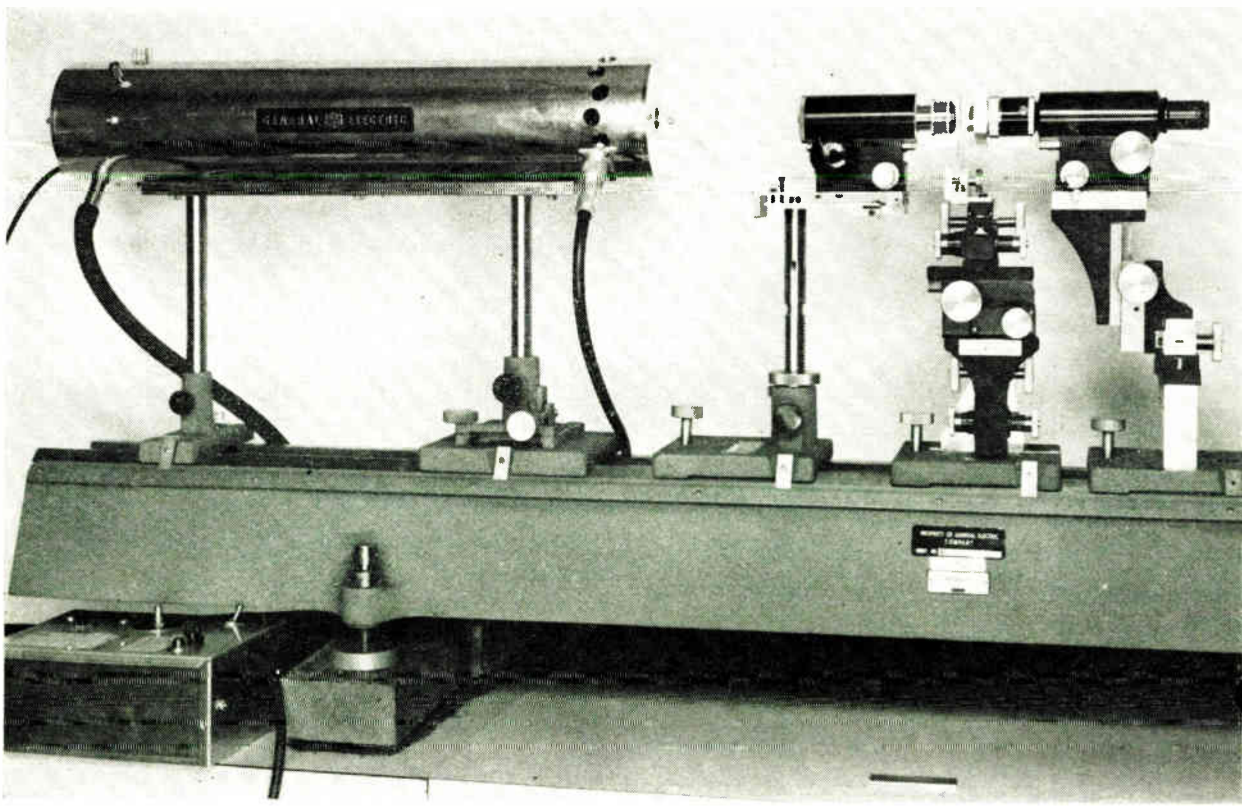
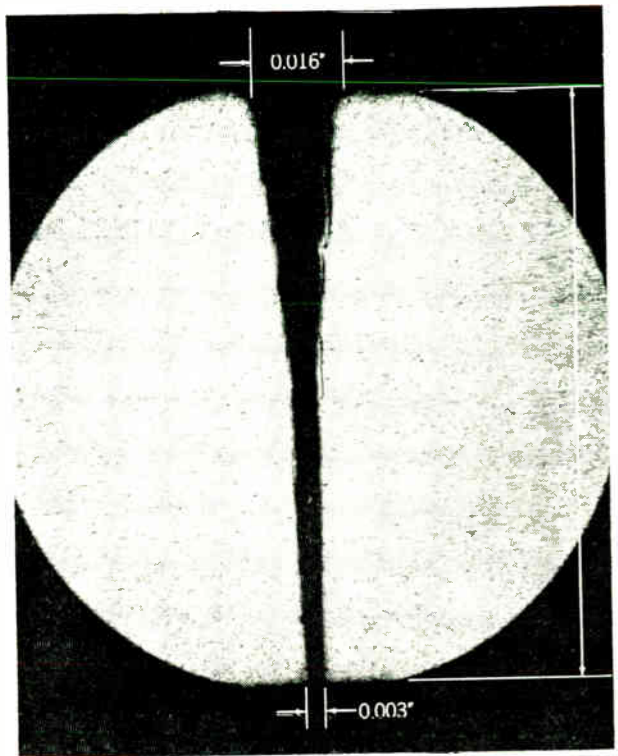
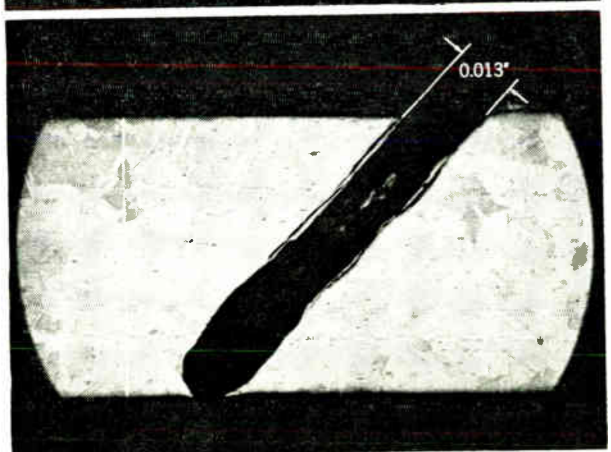


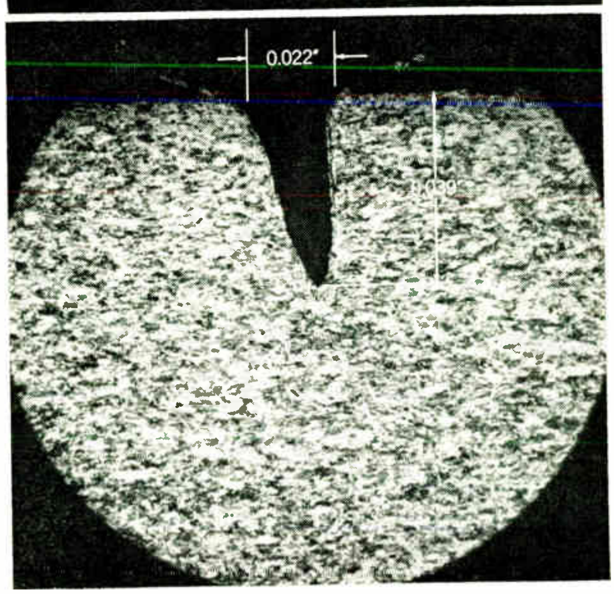
Fig. 6. Laser vaporized hole in 0.125-inch-thick stainless steel.

Fig. 7. Laser vaporized hole at an angle through 0.062-inch-thick stainless steel.

Fig. 8. Laser vaporized hole in copper. Thermal conductivity prevented deep penetration.







have been excited to their metastable level by the pumping action of a very intense light source, new photons of light are released as the energy of the atom drops to the ground state. These newly created photons of light radiate in all directions, but many of them begin traveling down the optical axis of the active medium. In doing so, they collide with other atoms which are still in the excited state, causing stimulated emission of additional photons of light, which join the original photon in both phase and direction. This action continues as the photons travel down the optical axis, until the photon avalanche reaches the output end of the laser rod. Here a portion of the radiation passes through the partially reflecting end and the remainder is reflected back into the rod, where additional amplification occurs. The nonoutput end of the rod has a fully reflecting surface and redirects the light back to the output end. As the light passes through the output reflector, a coherent, highly collimated beam of light is formed.

Laser machining. Inasmuch as energy from the laser can be imaged to a very small spot diameter, it can be used to generate extremely high temperatures for vaporizing holes in metals. Figure 5 shows a laser for use in vaporizing very small holes in different metals mounted on an optical bench with an energy imaging lens and a mechanical positioning system. For extremely small holes ranging from two to five microns in diameter, a reflecting microscope objective lens is used for imaging the laser radiation. For larger holes in the order of one to ten mils, a simple single element lens is employed.

Since the ability to form a hole in a material by the use of a laser beam depends on absorbing enough light radiation in the material surface to bring its temperature up to the vaporization point, the hardness and certain other metallurgical properties of the metal become unimportant. Figure 6 shows a hole in a piece of 304 stainless steel 125 mils thick. The hole was produced by the action of 17 pulses of a 2.2-joule 1-ms laser beam.[4] The laser radiation was imaged on the surface of the stainless steel. It is interesting to note that a hole so formed has fairly parallel walls and a depth-to-width ratio of approximately 25. The entrance diameter is 16 mils and the exit is 3 mils. Because the hole was formed by a multiple number of laser flashes, the cutting action of each pulse drilled the hole successively deeper into the metal. The light radiation was absorbed at the bottom of the hole each time by multiple reflections from the walls of the hole already formed. It should be noted also that the metallurgical structure of the material surrounding the hole has not been affected by the heating action required to form the hole. This is because the duration of each laser pulse is so short.

Holes may also be drilled at angles (Fig. 7). Here the laser beam was imaged at a 45-degree angle to the surface of the material and a hole was formed through the stainless steel at this angle.[4]

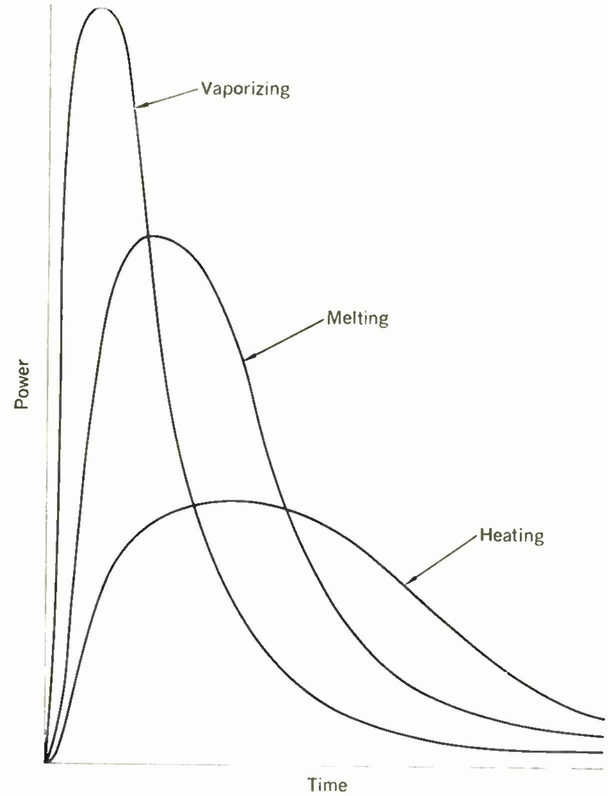


Fig. 9. Constant energy relationships for vaporizing, melting, and heating materials.


Fig. 10. Laser weld of two pieces of stainless steel.

The thermal conductivity of the material has a sizable bearing on the ease with which a hole can be formed. For instance, copper with a high conductivity is very difficult to penetrate by laser drilling action. The heat produced by the absorbed energy is conducted into the metal rapidly and the metal does not have a chance to reach its vaporization point before a large amount of the laser beam energy has been dissipated in the metal. Figure 8 shows a hole in a piece of copper. The attempt to drill through this 125-mil section of copper met failure after 20 pulses of the laser had only penetrated less than half of the distance. The laser beam employed for this experiment was adjusted to four joules per pulse.[4]

The laser has already been used for drilling very small holes in metals that are ordinarily difficult to machine. For instance, electron gun apertures are being formed in metals such as tungsten and molybdenum by laser-beam drilling. Holes as small as 2.5 microns in diameter have been so produced which is close to the limitation of 1 micron imposed by the wavelength of the ruby laser radiation—0.7 micron. (High-energy particles, such as nuclear fission fragments, have produced holes in certain materials of even finer dimensions—25 Å in diameter and 12 microns deep.) Other applications such as in dies for drawing synthetic fibers or even wire-drawing dies are under consideration. Since the laser beam can also be imaged, with cylindrical optics to a line rather than a circular spot, it is possible to form slots rather than circular holes in materials. As soon as lasers become more efficient and the pulse repetition rate for beam energies of 5 to 10 joules per pulse can be increased, they will become an important tool in specialized hole drilling or metal cutting.

Laser welding. The rate at which energy is delivered to a metal determines whether the metal is merely heated, becomes molten, or is actually vaporized. For drilling holes in a metal, it is necessary to vaporize. For welding, however, the energy should be imparted at such a rate as to bring the metal parts up to their molten temperature. In general, this is accomplished by extending the pulse duration time of the laser beam. Figure 9 shows the relationship of energy to the pulse duration time for typical heating, welding, and vaporizing. Values of energy and time for such a family of curves will vary with the thermal characteristics of the metals involved.

Laser welding to date has found certain laboratory application in the area of micro or macro welding. Fairly low-energy beams of the order of 1 to 5 joules are required for this action, with pulse durations extending out as long as 15 ms. Figure 10 shows a laser weld of two pieces of stainless steel. Here the laser pulse was approximately 15 ms long with an energy of 10 joules. The weld so produced was 10 mils wide by 10 mils deep. The metallurgical characteristics of the boundary between the parent metal and the weld show that good fusion has occurred.

Laser welding has not yet been widely exploited but it certainly holds promise, particularly in the production of high-temperature-resistant microcircuit systems. Here, again, improvement in efficiency and in pulse repetition rate are needed.

The characteristics of the laser as a machine tool are:

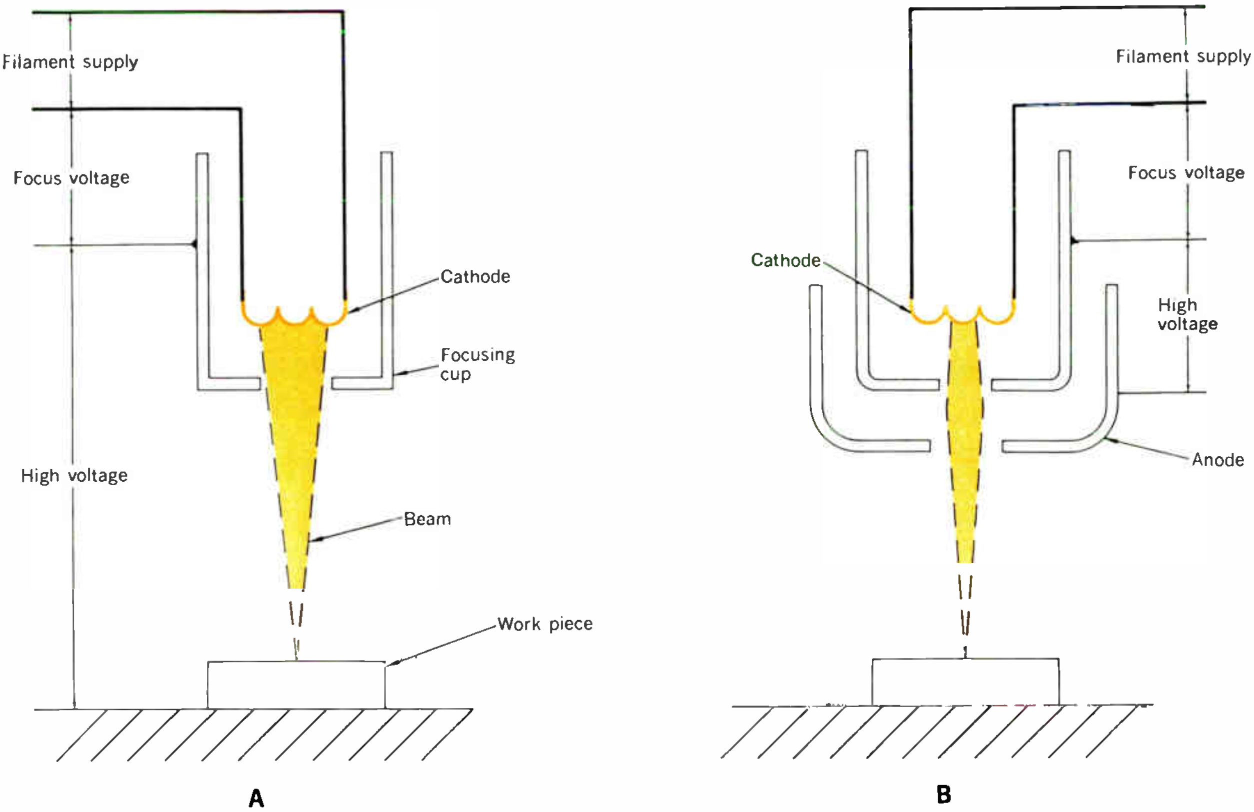


Fig. 11. (A) Workpiece-accelerated electron beam source; (B) self-accelerated electron beam source.

- Does not require vacuum.
- X rays are not generated as a by-product of the photon action striking the material.
- The efficiency of the laser is quite low (approximately one per cent of the energy supplied is converted to laser beam output).
- Holes can be formed in metals regardless of their hardness.

The electron beam

Of all the elementary particles that could be considered, electrons are the most useful and versatile for the formation of high-energy beams. They may be extracted from almost any metal or gas, since they are a prime constituent of matter. Their high ratio of charge to mass (over 1800 times that of hydrogen ions) facilitates beam extraction and manipulation by electric and magnetic fields of moderate strength. As early as 1907 a patent for an electron beam furnace was granted to Dr. M. Pirani for his development of X-ray equipment. The furnace was used to melt refractory metals in the laboratory. In 1954 the French Atomic Energy Commission made the first use of an electron beam to perform welding operations. An electron gun, similar to the type used in X-ray tubes, supplied an electron beam of 25-kV energy to the workpiece, which was rotated by a motor in a glass vacuum chamber. Soon afterward, the first United States electron beam welder was developed by the General Electric Company's Hanford Atomic Products Operation for welding metals used in nuclear reactors.[5]

Electron beam technology. Electron beam sources, or guns, may be classified according to the type of cathode employed and by the arrangement of electric and magnetic fields used to accelerate and focus the electrons. Hot-cathode guns make use of the thermionic emission from a hot filament or plate while cold cathodes utilize secondary emission, photo emission, or field emission as a source of electrons. The following is a description of these two cathode types:

Hot-cathode sources. Figure 11(A) illustrates, schematically, a workpiece-accelerated electron-beam source. In this device the electron beam is formed and controlled by applying a positive potential of a few hundred volts to draw out electrons emitted by the hot cathode. Kinetic energy is imparted by a high accelerating potential applied between the grounded work piece and the control electrode. Such an arrangement of an aperture with a weak electric field on one side and a strong one on the other constitutes an electrostatic lens capable of forming a convergent beam. By proper adjustment of working distance and electrode potentials, the electron beam can be concentrated to a small spot on the workpiece for melting or welding operations. This simple arrangement, however, lacks the flexibility and precise beam control needed for the more difficult applications.

In the device shown in Fig. 11(B), a separate accelerating electrode forms an integral part of the gun assembly. High voltage applied to this third electrode brings the electrons to full energy as they emerge from the second aperture. Since it is no longer a part of the focusing system, the workpiece can now be moved independently without affecting the electron beam adjustments. A magnetic focusing lens may be added as in Fig. 1(B) for additional focusing.

As illustrated, these devices, like optical systems, form an image of the source at the work piece. The energy concentration in the image is an inverse function of the image size and is proportional to the source "brightness"; i.e., electron emission density. For further reducing the image size, stronger lenses may be used but, as in light optics, this results in a short working distance and a reduction in the depth of focus. Thus the extent of the beam power concentration depends on the emission density and on the reduction afforded by the electron optical system.

These shortcomings of filamentary cathodes are partly overcome by forming the cathode into a segment of a sphere and heating it indirectly by electron bombardment from a separate hot filament. Electrons emitted from the concave cathode surface are concentrated into a convergent beam by an electric field normal to it. Because of its large effective emitting area and its favorable focusing geometry, this type of cathode is capable of high total intensities, and when combined with a suitable electron optical system it can produce intense concentrated beams; see the schematic of Fig. 12. As previously pointed out, working depth is sacrificed as the beam convergence is increased to obtain a smaller focal spot.

Cold emission sources. While thermionic emission from hot cathodes is the most commonly used form of electron source, other forms of emitters are becoming available. These new sources make use of field emission from a sharp metal point or secondary emission from a cold cathode by ion impact. Each of the so-called cold emitters has peculiar advantages for special classes of applications.

Field emitters[6] make use of the very intense electric fields available by applying a high negative potential to a sharp metal point to pull electrons from a cold metal such as tungsten. First discovered by R. W. Wood of Johns Hopkins University in 1897 and later explained by R. H. Fowler and L. W. Nordheim of Cambridge University, this effect results from the quantum tunneling of electrons through a surface potential barrier narrowed by an electric field of several million volts per centimeter. Current densities of 10 million amperes per square centimeter have been drawn from finely sharpened tungsten points in high vacuum. Because the emitting area is so small, densities of only a few milliamperes may be drawn continuously from a single point. Larger currents are, of course, possible by intermittent pulsing or by operating multiple points in parallel. Field emitters must be operated only in a high vacuum to avoid erosion damage by positive ion bombardment.

A principal application of the field emission source has been in flash radiography, which exploits the extremely small source size and high speed of response of a field-emitting point to take high-speed X-ray pictures with high resolution. Also for radiation effects studies, where high dose rates are required, a pulsed field emission X-ray tube can produce 10^7 to 10^9 rads per second.

While field-emission technology is still in an early stage of development, advances in the design of the associated-beam optical systems may well lead to improved X-ray machines and electron microscopes, which exploit the small source size and high brightness of field emitters to obtain much higher resolution than is now possible.

Another class of electron beam emitter has been referred to as a plasma electron beam source.[7] This type has a cylindrical or spherical cavity containing an ionized plasma from which electrons are extracted through a small beam aperture. In one form of plasma electron beam, the cathode is a hollow cylinder with perforated walls. It is supported near the center of a gas-filled enclosure at the end of a coaxially shielded stem. When the gas pressure is adjusted to about 0.01 torr in argon, or up to 0.1 torr in the lighter gases, and high voltage is applied between the grounded shield and the cathode, a unique type of gas discharge occurs. By proper adjustment of voltage and pressure the beam mode of discharge can be induced

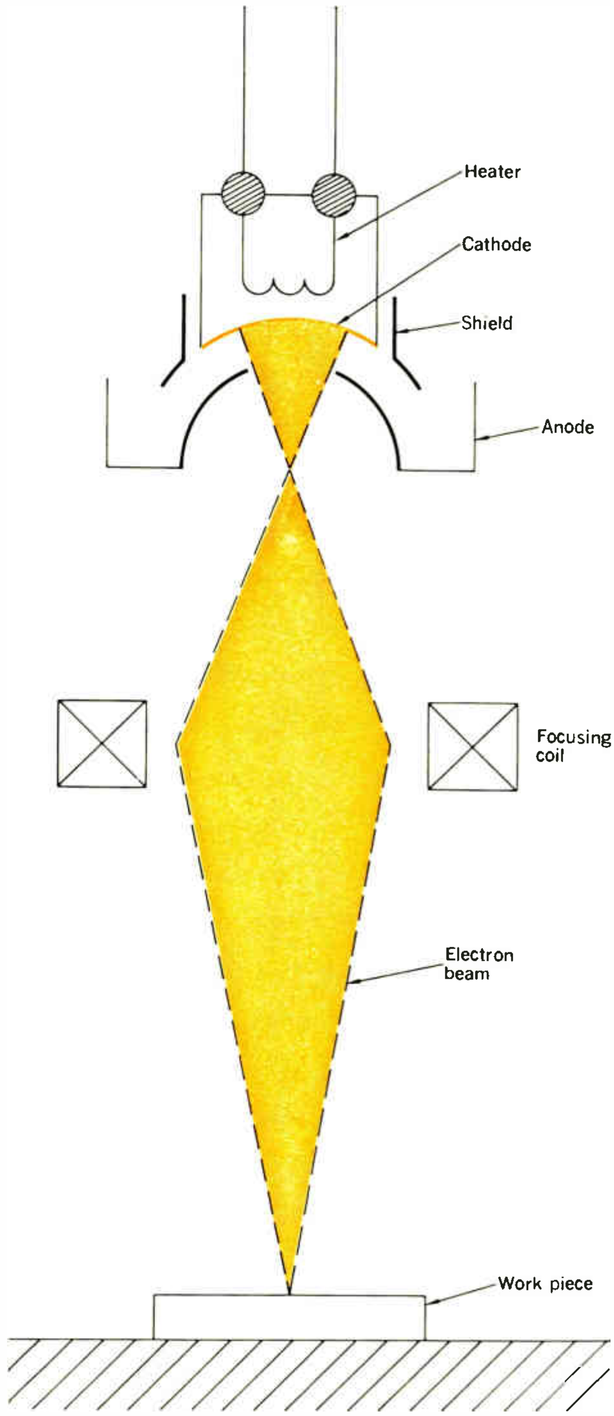


Fig. 12. High-intensity Pierce-type electron gun with indirectly heated cathode.

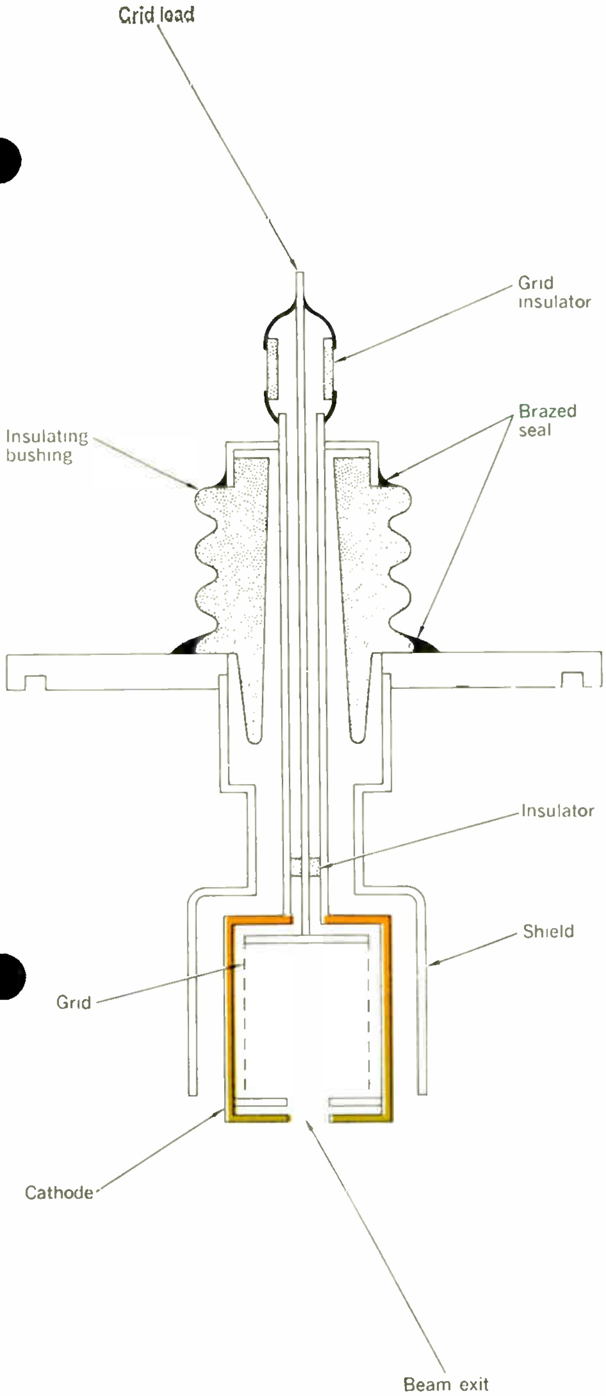


Fig. 13. Cross-sectional diagram of a shielded plasma electron beam source with a control grid.

(see title illustration). Even though the walls of this one-inch stainless steel cylinder are relatively cold, a 100-mA 20-kV electron beam issues from its ¼-inch aperture. The beam path is marked by the luminosity of gas atoms excited by occasional collisions with electrons in the beam. No external focusing fields are required.

The plasma electron beam cathode operates when positive ions are formed by a relatively small fraction of the electrons from the cathode; in their flight to the chamber walls, they collide with atoms of the ambient gas. The ions, accelerated by the strong electric field near the cathode, enter the cavity and, by collision with the inner walls, eject electrons. By this process an ionized body of plasma is formed inside the cathode. Surrounded by a space charge sheath of low luminosity, the internal plasma maintains itself at a potential several hundred volts positive with respect to the cathode. Electrons ejected from the internal walls by ion bombardment and by photoemission are accelerated across the sheath and are trapped inside the plasma where they make ionizing collisions, further intensifying the plasma. With most of their energy spent, electrons emerge from the plasma through a vortex-like constriction of its boundary, which extends through the cathode aperture. Here they are focused by the electrostatic lens action of the aperture and accelerated by the electric field in the external cathode dark space.

Since the gas density is low, only a few of the electrons in the beam collide with gas atoms, and therefore scattering of the beam is negligible. In fact, the positive ions formed in the beam path by these relatively rare collisions serve to neutralize the negative space charge and prevent spreading of the beam electrons by mutual repulsion. This well-known gas focusing phenomenon[8] thus accounts for the long, slender, self-focused beams that characterize the plasma electron beam source.

While this simple source has yielded beam currents approaching one ampere (for a three-inch size) it converts less than half of the input power into beam power. This low efficiency is due largely to the radial loss of electrons from the cathode: they are accelerated toward the chamber walls where their energy is dissipated in ionizing the gas and in heating the walls.

In a newly developed form of the plasma electron beam source[9] (Fig. 13), radial losses are suppressed by a grounded coaxial shield surrounding the cathode.* If the shield is designed to direct positive ions into the beam aperture, the wall perforations may be eliminated without impairing the energy input to maintain the internal plasma. The simple, rugged construction is well adapted to liquid cooling for high beam-power outputs. Beam conversion efficiencies as high as 80 per cent have been achieved in some models of the shielded cathode with beam-power outputs running to over 20 kW.

Continuous control of the beam power may be exercised by varying either the ambient gas pressure or the cathode voltage. An internal control grid has also been used for pulsing the beam current with a rise time of 4 to 6 μs, depending on the type of gas used. Such a grid consists of an open-mesh cylinder of fine molybdenum wire supported from an insulated stem coaxial with the cathode stem (Fig. 13). By applying a few volts of negative bias to the grid, electron emission from the cathode walls is suppressed and the beam current falls to a low value. Thus a few tenths of a watt of grid input power can control more than a kilowatt of beam power.

Though plasma electron beam sources are still in the developmental stage, they nevertheless offer these advantages: simplicity and ruggedness; operation in any gas, including air; long life; high power output; high beam conversion efficiency; high source brightness.

Problems of gas pressure regulation and associated beam stability are being overcome by automatic servo-

* In accordance with Paschen's law, a short gap can support high potentials at low gas pressures owing to the long electron-atom mean free path, which eliminates current build-up by ionizing collisions.

controlled throttle valves and by negative feedback to the control grid.

Electron beam applications. Excluding the important role played by electron beams in cathode-ray tubes, X-ray tubes, microscopy, and beam power tubes, special attention is here directed to their use in the processing, forming, and joining of materials. Some of the most promising new engineering materials are those that are most difficult to refine, shape, and join. High-power electron beam devices overcome these difficulties in a unique way: in the past few years electron beams have been applied to a variety of metallurgical processes: they can fuse practically any metal, alloy, or ceramic; they can weld, vaporize, grain refine, stress relieve, outgas, cut, metalize, vacuum braze, and grow crystals.

Some of the more important characteristics of electron beam devices that suit them to a growing number of applications are as follows:

1. High power density. By focusing to very small areas, beam power densities measured in megawatts per square centimeter are possible with accelerating voltages ranging from 10 to 100 kV or more. With the use of multiple electron guns, special beam configurations can be obtained.
2. Surface absorption. The electrons are stopped and their energy is absorbed in a thin layer of the workpiece, thus facilitating evaporation and precision machining, especially in thin materials. Also, by defocusing, selected areas may be heat treated or brazed joints may be made.
3. Control characteristics. Beam intensity, shape, and position are controllable instantly and precisely.
4. Chemical purity. Electron beam devices may be used in the nonreactive environment of a vacuum or inert gas. This serves to reduce the contamination that might occur in high-temperature refining, shaping, and joining processes.

Welding. Since the first disclosure of the electron beam welding process in 1957 by J. A. Stohr of the French Atomic Energy Commission, its development has progressed rapidly and much has been learned about the underlying phenomena affecting it.[10] In this welding process, the kinetic energy of focused electrons is converted into heat at the weld point to fuse and coalesce joints up to two or more inches in thickness. It is known that electrons in the energy range 10 to 150 kV used in the welding process can normally penetrate metals to depths of only a few thousandths of an inch. Yet when the power density of a focused beam exceeds a certain threshold value, heat is generated at such a rapid rate that the metal is melted and vaporized before thermal conduction can broaden the weld zone as with other heat sources.

If the beam is focused on a solid piece of metal it quickly drills a small hole by first vaporizing a small crater after which the vapor jet leaving the crater becomes ionized. As previously explained, positive ions in the beam path then help to concentrate the electron beam by neutralizing its negative space charge. Since the electrons have very little momentum, the drilling action is solely the result of the action of the high-velocity vapor jet. If the work piece is traversed beneath the beam, the hole moves laterally and the molten metal closes in behind it. The very narrow melt zone results in a minimum of shrinkage, and therefore less locked-in stress.

Since electron beam welding is normally done in a vacuum or in a low-pressure inert gas, many of the advantages of vacuum melting accrue. Such welds are clean and free from gas occlusions and are often stronger than the parent metal. Electron beam welding is particularly well suited for joining easily oxidized metals such as aluminum, titanium, beryllium, zirconium, and many vacuum-processed alloys.

Because fusion occurs over such a narrow zone, less energy is needed to produce a weld than with conventional methods. As little as 5 per cent of the energy input required for other types of fusion welding is consumed in making an electron beam weld.[11] This characteristic, together with the high temperatures in the focal spot zone, ideally suits the electron beam process for welding refractory metals such as tungsten, tantalum, and molybdenum. Experimental welds have been made between ceramics and refractory metals.

The photograph of a metallographic section of 0.8-inch-thick titanium alloy (Fig. 14) illustrates the characteristics of the electron beam welding process. This butt weld was made with a 150 kV 35-mA beam focussed at a point 0.6 inch below the top surface. The traverse speed was 10 inches per minute. Depth-to-width ratios as great as 20 to 1 have been reported for some types of electron beam welds.

Electron beam welding systems typically consist of an electron gun, a high-voltage dc power supply, a vacuum tank and exhaust pumps, and mechanical fixturing and traverse mechanisms for positioning of the workpiece. Also included are observation ports for an optical system with suitable dark filters for viewing the weld zone. The size of the vacuum tank and its associated pumps will, of course, be determined by the type of workpieces to be welded. A system for heavy industrial welding is shown in Fig. 15, where the arrangement includes mechanisms for moving both the electron gun and the workpiece.

Complete commercial equipments are now supplied by several firms in the United States and in Europe. Both high-voltage (50 to 150 kV) and low-voltage (10 to 50 kV) equipments are available, and beam power ratings range up to 10 kW or more. The low-current electron guns used in the high-voltage equipment can produce long, slender beams with very small focal spots and can reach into fairly deep recesses, while the low-voltage gun with higher beam current has a shorter, more convergent beam, a consequence of its relatively large cathode surface and associated beam extraction system. This disadvantage is largely offset by the compactness of gun design, made possible by the use of lower voltages and by the absence of penetrating X rays. High beam power density is a prime requirement for nearly all types of electron beam welded joints, and high total beam power is an additional requisite where deep welds must be made in materials such as aluminum and copper with their high thermal conductivity.

Electron beam welding is developing most rapidly in the aerospace and aircraft industries, where exotic alloys and refractory metals are being applied and where high strength and reliability are mandatory. It is a valuable tool for repairing or modifying expensive and intricate machine parts, and is finding its place in the electronic industry for welding tiny joints and contacts and attaching lead wires. Though it will never completely replace other forms of welding, its use will no doubt continue to spread because of its unique adaptability to special fabrication requirements.

Vacuum metallurgy. Many vacuum metallurgical processes involve the melting or vaporizing of refractory metals in a noncontaminating environment such as a vacuum or an inert gas. This requires a clean, concentrated, and precisely controlled heat source. Electron beams fill these requirements and have proved their worth in a variety of metallurgical equipments. By utilizing electron guns designed to supply high beam power at the relatively low voltage of 15 to 30 kV, the hazards of penetrating X rays are mainly avoided and the dc power supply equipment becomes less costly. Pierce-type hot-cathode guns or the plasma electron beam source are both well adapted to furnace applications.

An electron beam furnace for melting, refining, or vapor coating may consist of a vacuum tank with an electron gun at the top and a water-cooled crucible and hearth at the bottom. In operation the electron beam is directed at the crucible and may supply 20 kW, or more, of beam power directly to the melt, thus minimizing heat losses. Multiple guns may be used for higher power inputs. Because the plasma electron beam source is compact and rugged and not adversely affected by contaminants from the melt, it promises to become a valuable beam power source for such applications. Laboratory tests have demonstrated that high-quality melts of such metals as niobium and molybdenum can be made in a low-pressure argon atmosphere.[12]

Drilling and machining. By virtue of the high power concentration in a sharply focused electron beam it can readily cut through materials by vaporizing a very narrow strip in its path. Also it can drill tiny holes to a considerable depth. Holes of 5-mil diameter have been produced in 8 seconds in stainless steel $\frac{1}{2}$ inch thick. Smaller holes, of 2-mil diameter, can be formed in 0.060-inch-thick stainless steel at a rate of 100 per second. And holes of 1-mil diameter have been drilled by a newly developed hot-cathode electron gun at the Lawrence Radiation Laboratory.[13]

In electron beam machining, short bursts of power are used to evaporate successive thin layers of material. Thus precise holes or channels can be machined, because the beam pulse can vaporize the surface layer before the surrounding material that is not in the beam path can reach its melting point. Both direction and intensity of the beam can be programmed to cut the desired shape. In this way very precise machining of brittle refractory materials can be accomplished. For example, cruciform-shaped holes can be cut in metal disks to make extrusion dies for synthetic textile fibers.

In the forming of electronic microcircuitry, thin metallic films deposited on the surface of an insulator or semiconductor are selectively etched or cut through to form circuit paths. A finely focused electron beam can be used to selectively remove portions of the film by vaporizing it. For this purpose, a relatively low-energy beam (5 to 30 kV) may be employed. By this means, cuts 10 to 20 microns wide can be made in a thin metal coating. These cuts have clean straight edges and may be spaced 10 to 20 microns apart if desired. The work is viewed under a microscope and the beam may be manipulated mechanically or scanned across the surface by electrostatic or electromagnetic deflectors. Selective etching in depth is possible by varying the beam energy and intensity. In multilayer films, extremely thin layers a few hundred angstroms thick can be removed from selected spots.

Not only is the electron beam useful for micromachining, but very fine and closely spaced welded or soldered connections can be made in microcircuit assemblies. In many of these applications the electron beam is pulsed to provide a predetermined energy input for a given cutting or welding operation.

The few examples cited illustrate the growing field of electron beam applications in the electronics industry. These range from thin-film microcircuitry elements to miniature relays and the larger electron tube assemblies.

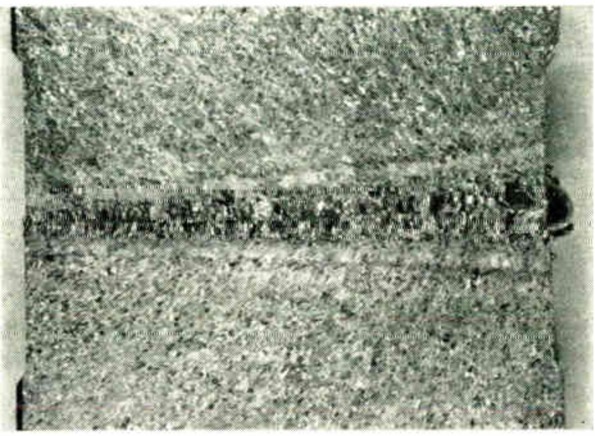
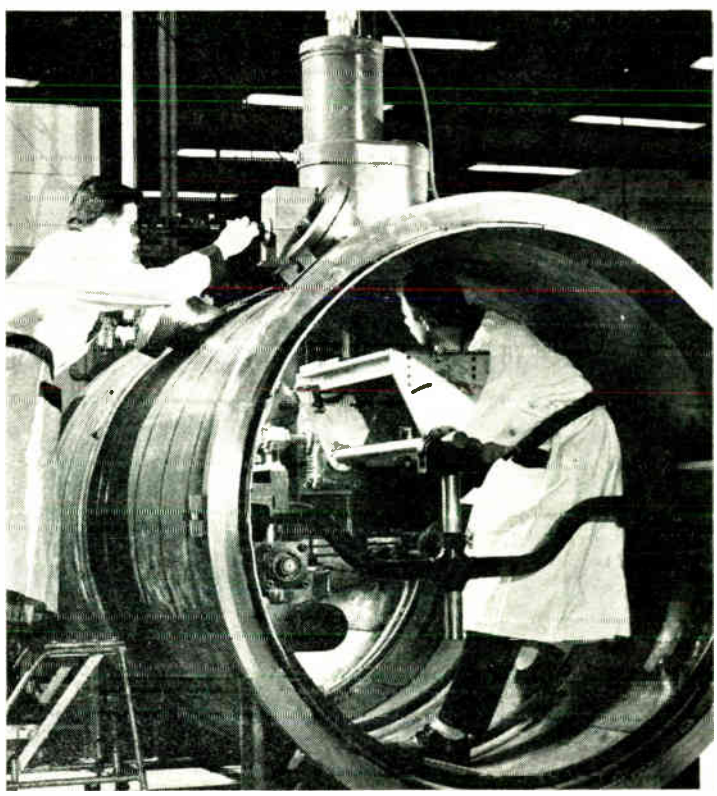


Fig. 14. Photograph showing an electron beam weld made in 0.8-inch-thick titanium alloy.

Fig. 15. Electron beam welding system with work positioning fixtures, telescope, and 5-by-9-foot vacuum chamber.



Many of these applications have been described[14] and new ones are emerging as the unique capabilities of controlled electron beams become better known.

Other uses of electron beams. While most of the electron beam applications described are based on the conversion of beam energy into heat, other equally important applications should not be overlooked. One of these is conversion of electron beam energy into electromagnetic radiation when the beam is accelerated by electric or magnetic fields, for example, by the magnetron, the klystron, and the traveling-wave tube. These devices convert electron beam energy into electromagnetic radiation of relatively high frequency, and have achieved importance in radar, radio communication, and RF heating devices.

For the generation and amplification of still higher frequencies, in the millimeter wavelength region, the interactions between electron beams and plasmas are being explored. In brief, this new approach consists of the projection of a suitably modulated electron beam into a plasma immersed in a magnetic field. By suitable adjustments of the plasma density, the magnetic field strength, and the modulation frequency, an amplified signal is obtained. Signals of extremely high frequency have been amplified in this way and there is theoretical evidence that similar devices can be made to generate controlled microwave radiation.

Still other fields for the electron beam include chemical synthesis polymerization and the sterilizing of food and drugs. Some of the new applications show great promise for future growth, provided that rugged, reliable electron beam sources, capable of operating in the presence of contaminating gases, can be developed. This need may be met by new types of sintered-oxide hot-cathode sources or by cold hollow-cathode electron beam sources.

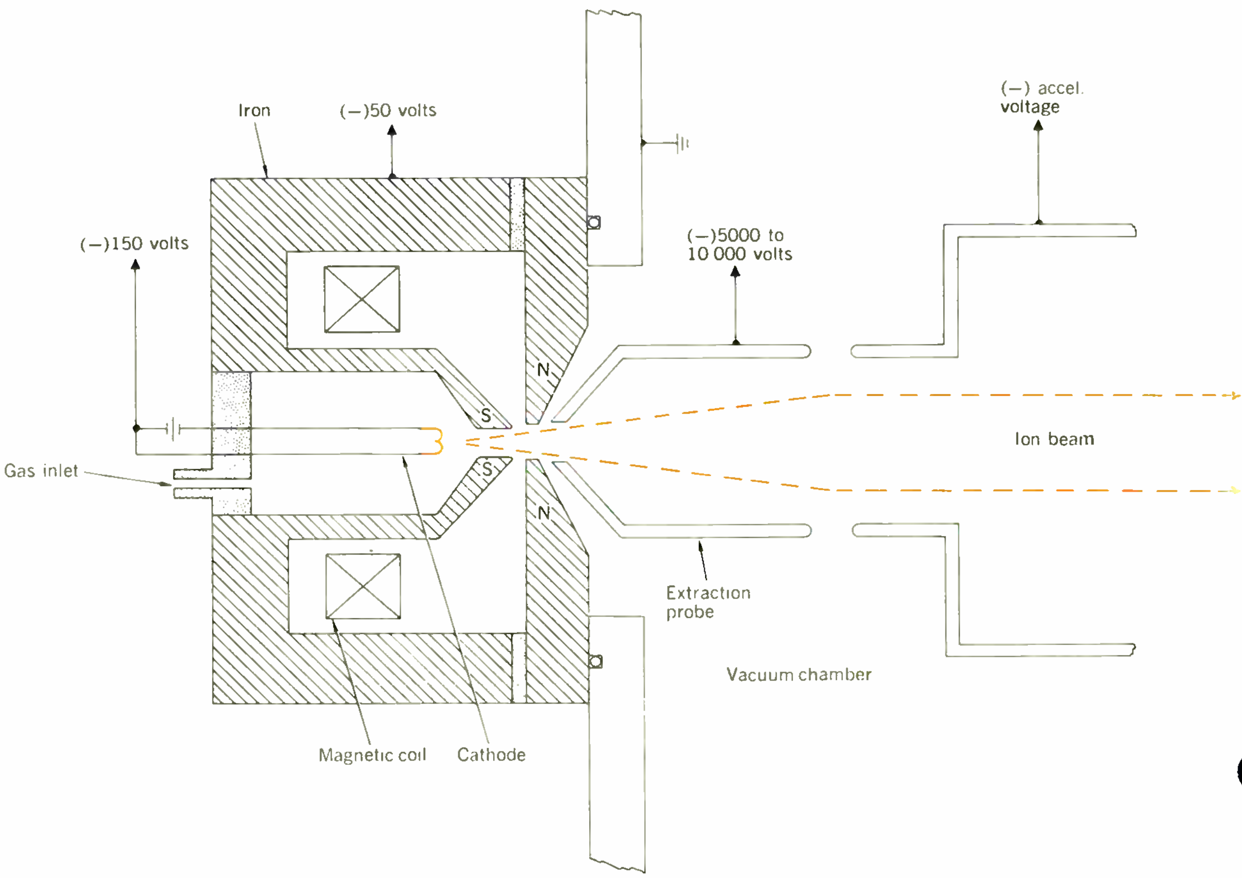
The ion beam

Positive gaseous ions are most commonly formed by removing one or more electrons from an atom or a molecule. Relatively few gases have the property of attaching electrons to form negative ions. Notable among the electronegative gases are O_2, CO_2, and SF_6. While ionized gases at relatively low pressures are the most familiar sources of ions, they may also be obtained by emission from hot metal surfaces by "contact ionization."

Because of their relatively large masses, ions attain much lower terminal velocities than do electrons when subjected to the same accelerating voltage. Also, it may be shown that ions of the same energy follow the same paths in electrostatic lens systems regardless of their masses. Magnetic lenses, however, deflect heavier particles less than the lighter ones and therefore are useful only for focusing homogeneous beams of ions. In strong magnetic fields, ions tend to move in tight circular or helical paths and may be concentrated by the action of a strongly convergent magnetic field combined with an electric field.

Ion sources. Most ion beam generators consist of a source of ions, an extracting probe or electrode, an electrostatic focusing lens and an accelerating electric field to impart the desired energy to the beam. A considerable variety of ion sources have been developed for use with

Fig. 16. High-intensity ion source.



high-energy particle accelerators employed in the study of nuclear reactions.[15] One of the most efficient of these is the so-called duoplasmatron developed by M. von Ardenne. In this ion source, electrons from a hot filament are accelerated toward a hollow iron pole tip by a potential difference of about 100 volts. As indicated schematically in Fig. 16, gas admitted to the cathode chamber at a few microns' pressure is ionized by electron impact in the narrow channel at the apex of the pole piece. The 1000-gauss magnetic field in the extraction channel concentrates the discharge and provides a high density of ions. Positive ions are drawn out of this region by the negatively charged hollow probe and focused into a beam by the electrostatic lens action of the accelerating gap at the exit end of the probe. Ion current densities in the extraction channel may approach 100 amperes per square centimeter and beam currents may be as high as half an ampere. Beam power is limited only by the voltage that can be applied to single or multistage accelerating gaps.

In all gaseous ion sources an efflux of neutral gas accompanies the extracted ion beam. This increases with the size of the exit aperture and with the gas pressure. To prevent scattering of the ion beam by collisions with neutral-gas molecules, continuous evacuation is necessary. Differential pumping is usually accomplished by passing the focused ion beam through as small an aperture as possible and pumping ahead of the aperture.

Ion beam applications. The following are some of the more significant applications of ion-beam technology:

Magnetic beam analyzer. When the parent gas is molecular, it is dissociated in the ion source and the component atoms are ionized. The emergent beam is then composed of a mixture of ions of different mass. Hydrogen, for example, yields H^+, H_2^+, and H_3^+ ions. In gases of higher atomic number, multiple ionization occurs when more than one electron is removed. The relative yield of the various component ion species depends on the gas pressure, voltage, and electron current in the ion source. If ions of a single species are desired, the beam may be separated into various components by passing it through a magnetic analyzer in which a uniform magnetic field deflects the ions in inverse proportion to their respective momenta. Thus, the individual components of a heterogeneous beam may be sorted into a mass spectrum and collected separately.

One of the earliest applications of ion beams was in the use of the magnetic mass spectrometer, which employed the sorting principle in the analysis of gases. The early instrument made possible the separation and identification of most of the natural isotopes of the lighter elements. Modern mass spectrometers have reached such a degree of sophistication that relative particle masses can now be measured with a precision of approximately 10 parts per million. This single application of ion beam technology has provided an analytical tool of inestimable value to modern industry and to research institutions.

High-energy effects. In historical sequence the next significant application of ion beams came when they were accelerated to sufficiently high energies to activate nuclear reactions. In 1931 two British physicists, J. D. Cockroft and E. J. Walton, bombarded lithium with a beam of atomic hydrogen ions (protons) of 150 kV energy and produced the nuclear transformation

$$_3\mathrm{Li}^7 + {_1\mathrm{H}^1} \rightarrow {_2\mathrm{He}^4} + {_2\mathrm{He}^4} + Q$$

(where Q = released energy) in which lithium atoms of mass 7 and atomic number 3 combine with a hydrogen atom to produce two helium atoms having a total shared kinetic energy of over 17 million volts. In this reaction, energy equal to over 100 times the initial energy of the proton is released at the expense of a net loss of 0.018 63 atomic mass units. Because only a fraction of a per cent of the ions striking the target participate in the reaction, it is not yet a useful, large-scale source of atomic energy.

Physicists were quick to sense the great possibilities of high-energy ion beams as a tool to explore the nucleus. This led to the invention of new types of ion and electron accelerators capable of imparting many millions of electron volts of energy to an ion beam. Soon hundreds of new nuclear transformations had been produced, new elements were synthesized, and intense neutron and gamma-ray fluxes were produced. This naturally lead to significant advances in the design of ion sources and in the entire technology of ion-beam formation and control.

It is appropriate here to point out the hazardous nature of the penetrating radiations which accompany most nuclear reactions and which, like the X rays elicited by electron bombardment, arise when the ion energy exceeds a threshold level. For the heavier target elements this threshold usually exceeds a million volts, particularly when ions heavier than lithium constitute the beam. For this reason, intense beams of the heavier ions may be used to bombard most targets with no danger from nuclear radiation or even from X rays, which normally arise when electrons of energy of over 15 kV strike a target.

Surface effects. Ions, when incident to a solid surface, can transfer large fractions of their momentum to individual surface atoms. This ability follows from the laws of classical mechanics governing collision processes which show that momentum transfer is at maximum when the colliding bodies are of nearly equal mass. From this qualitative statement it follows that electrons, because of their low mass, effect negligible displacement of surface atoms while ions of the same energy are very much more effective because their masses more nearly match those of the target atoms. This accounts for the phenomenon known as "sputtering," in which ions of sufficient velocity remove atoms from the surface of a solid by impact. Experiments on sputtering from oriented single crystals of a metal have shown that sputtering is not to be regarded as evaporation of atoms due to surface heating but rather as a direct result of individual momentum transfer between impacting ions and surface atoms.

The effectiveness of incident ions in removing surface atoms is measured by the so-called sputtering ratio, which is the number of atoms removed per incident ion. This ratio is a function of the mass and velocity of the incident ion and of the lattice-binding energy in the solid.

Removal of surface layers by ion bombardment is useful for cleaning or etching surfaces without destructive heating. For example, it has been shown that copper oxide layers can be completely removed from a surface by repeated cycles of bombardment with argon ions followed by heating. Heating removes the trapped ambient gas and beam atoms driven into the surface. It has also been demonstrated that impacting nitrogen ions not only can clean steel surfaces but, by being driven into the crystal lattices, can surface-harden the steel. This might well be adapted to the hardening of selected areas by exposure to a controlled ion beam. The metallographic applications of ion beam etching are quite well known and need not be described in detail.

Space propulsion. For the propulsion of future space vehicles on interplanetary missions, chemical rocket engines are inadequate because of the large mass loss that occurs in delivering a given impulse. To conserve the propellant it must be ejected at high velocities, and its effectiveness is measured in terms of its specific impulse (defined as the impulse per unit weight of propellant exhausted). Since impulse is the product of force by time, the unit of specific impulse has the dimension of time and is customarily expressed in seconds. It may be shown that the increment of velocity given to the vehicle by the expenditure of a given mass of propellant is proportional to the specific impulse.

For chemical propellants the specific impulse is about 200 to 400 seconds. This may be increased by a factor of 2 or 3 by directly heating hydrogen in a nuclear reactor. To obtain the optimum exhaust velocity required for extended deep-space missions, specific impulses as high as 5000 to 10000 are needed.[16] These numbers are arrived at by a compromise between power consumption and propellant load and they increase with the distance covered by the mission.

Ion beams provide a solution to the problem of achieving high specific impulses. In the simplest form of ion engine, an ion source supplies the propellant, which is then accelerated electrostatically; the ejection of the high-velocity ion beam supplies the mass flow to exert a sustained thrust. Even though the thrust may be measured in ounces, the net increment of velocity imparted to the vehicle by the expenditure of a small mass of propellant over a long period is larger than can be achieved by the use of much larger masses of chemical propellants.

In the choice of fuel for an ion engine one of the principal requirements is that the ion source must deliver ions of sufficient mass to yield the maximum thrust for a minimum power expenditure. Furthermore, in a good propellant the energy expended to ionize it should be as low as possible—which means that it should have a low ionization potential. The alkali metals fulfill this requirement; they also exhibit the property of "contact ionization" when they make contact with hot tungsten or tantalum. These metals, for example, having higher work functions than the ionization potential of cesium, can extract the most weakly bound electron from the cesium atom or contact. Thus atoms of cesium momentarily adsorbed on hot tungsten or tantalum evaporate as cesium ions. Cesium vapor may be ionized by passing it through a porous plug of tungsten held at red heat. The ions are then extracted and formed into a beam by suitable arrangements of charged electrodes.

To maintain neutrality of charge on the space vehicle and in the ion-engine exhaust stream, a flow of negatively charged particles electrically equivalent to the positively charged ion beam is necessary. This is accomplished by ejecting electrons or negative ions to mix downstream with the exhaust.

Extensive research and tests by the National Aeronautics and Space Administration on prototype ion beam engines have established the feasibility of this propulsion principle. Full-scale tests on space vehicles must await the development of compact and reliable nuclear-powered electric generating plants.

Future trends in ion-beam applications. Considering the relatively recent emergence of high-intensity ion sources and the possibilities for their further improvement, it seems safe to predict that even greater strides will be made in ion beam technology in the coming decade than have been made in the last two. Not only will there be progress along the lines described but in the use of ion beams for the development and processing of materials, in chemistry, and in the development of electric power from nuclear fusion processes.

Summary

The past ten years have seen tremendous growth in the use of energy beams for processing, cutting, shaping, and welding materials. Energy beams have been increasingly applied to problems of refractory materials or materials that are difficult to machine and that can not be worked by the more conventional techniques. Each beam form exhibits its own peculiar advantages and disadvantages and each is finding its own unique applications. Highly localized action is the outstanding characteristic of energy beams; it enables them to cut or weld with a minimum disturbance of metallurgical properties. This advantage over conventional techniques in many cases justifies the use of the more complex and expensive energy beam methods. As electron beam, laser beam, and ion beam equipments are improved, their applications as working tools will multiply even more rapidly in the next decade than in the past one.

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Growth of energy consumption and national income throughout the world

This study of actually recorded trends of energy consumption in every country in the world serves to provide a springboard for long-term projections

Fremont Felix *Engineering Consultant**

Introduction

Energy universally and increasingly has become a major subject of public interest; its indispensability to the orderly functioning and steady growth of a nation's economy has come to be regarded as axiomatic. In practically every country, short- and long-term expectations of energy requirements are the object of continuing statistical study and reappraisal at the highest government and industry levels.

Providing, as it does, a world-wide basis of reference and a complete record of median trends, the study presented here should permit the comparison of both the past performance and the pattern of future growth of any country with the indications derived from the combined record of all countries of the world. This has been made possible by the completeness of the United Nations statistical reports,[1-3] without which this study could never have been undertaken.

Status in 1961, and growth from 1956 to 1961. This study—including the "statistical" record shown in Table I and the analyses illustrated in Figs. 1 through 14—presents basically three categories of results: (1) consumption, national income, etc., for 1961, (2) annual compound rates of growth from 1956 to 1961, and (3) increase in 1961. The latter two categories are explained in Appendix I.

Country listing. The 153 countries of the world, the records of which are included in this study, are listed in Table I. The ten major geographic areas are identified by capital letters. In some cases, either economic or other associations of countries or specific countries that it was deemed preferable to tabulate separately have been individually listed within these ten major areas, and have also been identified by capital letters.

For charting purposes, specifically in Figs. 1 through 4, a number was assigned to each country and is coupled to the area or other grouping designation, thus permitting different aspects of the performance of the same country to be compared, specifically, against "median" performance.

Attention is drawn to the all-inclusiveness of the country listing; no countries are missing from their respective groups or areas, and the totals for each of the groups, as well as for the ten geographic areas, add up to the exact total for the world. All country points in Figs. 1 through 4 have been given equal weight in arriving at the median curves, drawn as explained in Appendix II.

Population. For each of the country groupings and geographic areas, the population for 1961 is shown in column (1) of Table I. The compound per cent annual population increase is shown in column (2). The population growth for all countries is shown in Fig. 4, as explained in Appendix III.

Electric energy

Statistical record. Columns (3) and (7) of Table I list, in condensed form, "area" data for electric energy. These provide a convenient set of references in relation

* 3 Bis Hameau Beranger, Paris 16, France.

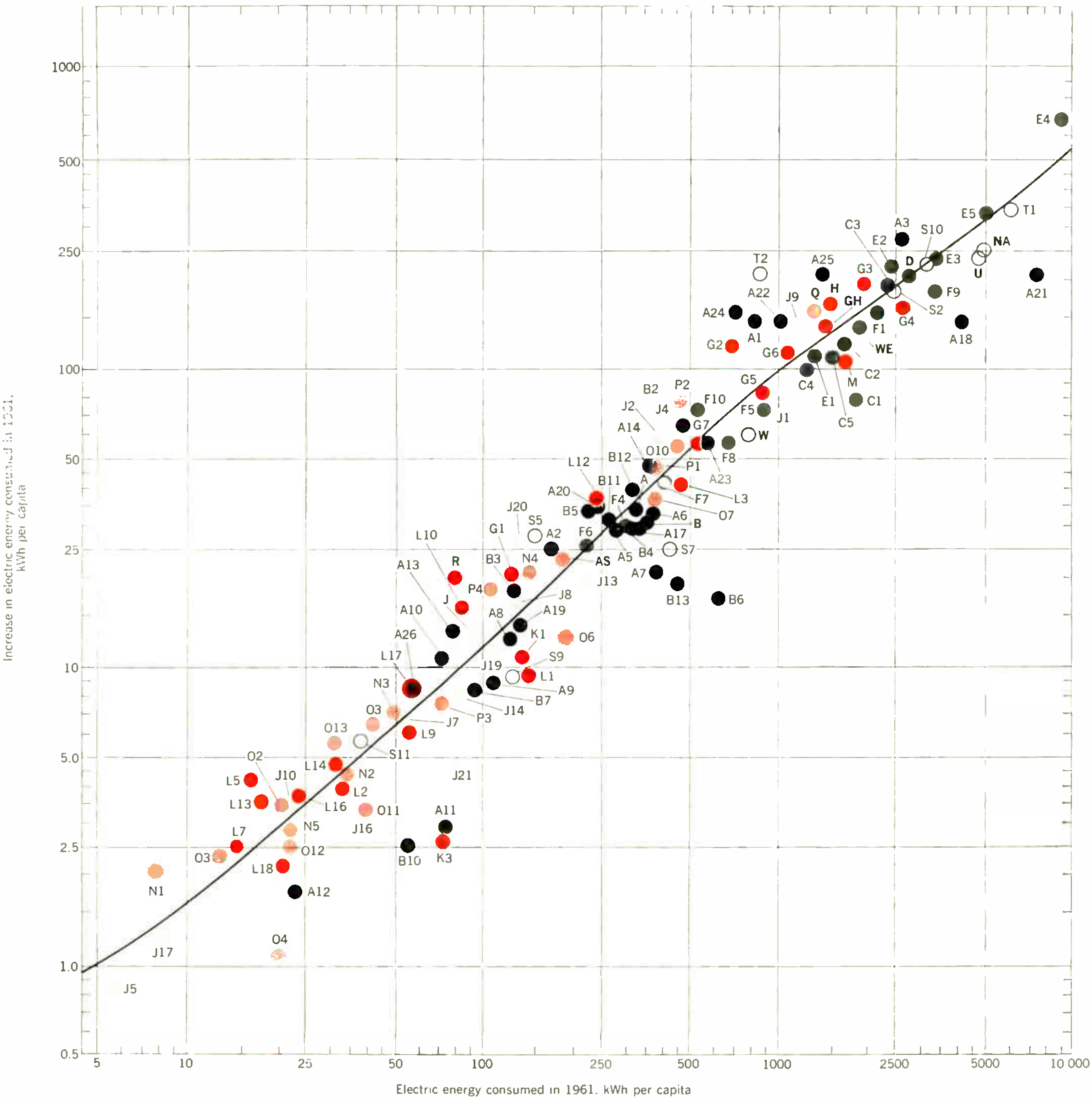


Fig. 1. Per capita increase in consumption of electric energy in 1961 for all countries of the world.

Fig. 2 (right). Per capita increase in consumption of total energy in 1961 for all countries of the world.

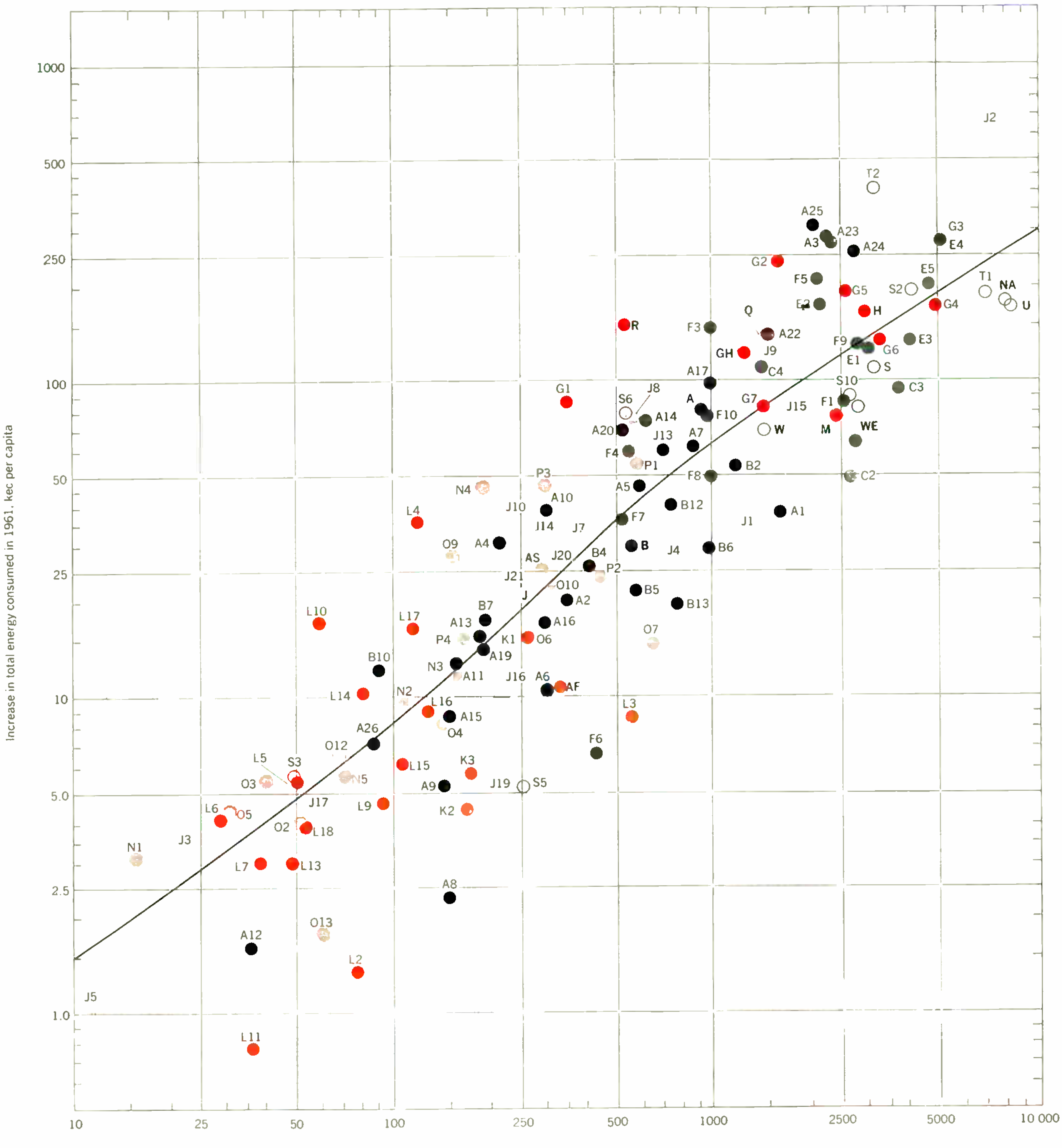
A Caribbean America:
Bahama Islands
Barbados / Bermuda
British Honduras
Colombia / Costa Rica
Cuba
Dominican Republic
El Salvador
Guadeloupe
Guatemala / Haiti
Honduras / Jamaica
Leeward Islands
Martinique / Mexico
Netherlands Antilles
Nicaragua
Panama Republic
Canal Zone
Puerto Rico
Trinidad and Tobago
Venezuela
Virgin Islands
Windward Islands

B South America (other than A):
Antarctic Fisheries
Argentina
Bolivia / Brazil
British Guiana
Chile / Ecuador
Falkland Islands
French Guiana
Paraguay / Peru
Surinam / Uruguay

Western Europe (WE):
C European Common Market
Belgium–Luxembourg
France
Germany, Federal Republic of
Italy / Netherlands
D United Kingdom
E Scandinavian countries
Denmark / Finland
Iceland
Norway / Sweden
F Other Western European countries
Austria
Faeroe Islands
Gibraltar
Greece
Ireland
Malta–Gozo
Portugal / Spain
Switzerland
Yugoslavia

Comecon countries, Europe (GH):
G European Comecon (less U.S.S.R.)
Albania / Bulgaria
Czechoslovakia
Germany, East
Hungary / Poland
Romania
H U.S.S.R.

J Middle East:
Aden
Bahrein Islands
British Somaliland
Cyprus / Ethiopia
French Somaliland
Iran / Iraq
Israel / Jordan
Kuwait
Kuwait Neutral Zone
Lebanon
Libya/Qatar
Saudi Arabia



Sudan / Somalia
Turkey
U.A.R. (Egypt)
Syria / Yemen
Africa (AF):
K Northern Africa
Algeria
Morocco
Tunisia
L Africa (other than **K** and **M**)
Congo
British East Africa
Rhodesia and Nyasaland
Cameroun
Equatorial Africa
Togo
West Central Africa
Gambia / Ghana
Liberia
Madagascar
Mauritius
Nigeria, Federation of
Angola / Cape Verde
Mozambique
Reunion
Sierra Leone
Zanzibar-Pemba
M South Africa
Asia (AS):
N Southwest and South Asia
Afghanistan
Ceylon
India/Macao
Pakistan
O Southeast Asia
Brunei / Burma
Cambodia
Indonesia/Laos
Malaya, Federation of
Singapore
Western New Guinea
North Borneo
Ryukyu Islands
Sarawak
Thailand
Vietnam, Republic of
P Eastern Asia (other than **Q**)
China (Taiwan)
Hong Kong
Korea, Republic of
Philippines
Q Japan
R Comecon countries, Asia
China (mainland)
North Korea
North Vietnam
Mongolia
S **Oceania:**
American Samoa
Australia
Australian New Guinea
British Solomon Islands
Fiji Islands
French Oceania
Guam / Hawaii
New Caledonia
New Zealand
Papua / Wake Island
West Samoa
North America (NA):
T Canada and Greenland
Canada
Greenland
San Pierre and Miquelon
U United States
W **World**

to which the data plotted in Figs. 1 through 5 can be appraised, and permit a convenient comparison of statistical results with world-wide trends.

Columns (5) and (6), respectively, present a ready parallel between the percentage contribution of the various geographical areas, etc., to the total world consumption of electric energy, and to the total addition to the world consumption of electric energy.

Increase in kWh per capita. In Fig. 1, the 1961 kWh per capita increases were plotted, for all countries, against 1961 kWh per capita. The use of a logarithmic scale of ordinates permits uniform readability over a 1000-to-1 range, and it will be noted that the plotting of the median curve, as explained in Appendix II, provides a satisfying interpretation of the relationship analyzed.

Per cent increase in kWh consumption. The "country" curve in Fig. 5 was obtained simply by dividing the ordinates of the median curve in Fig. 1 by the abscissas.

It is interesting to note that the standard rate of growth of 7 per cent per year, which is generally associated with the expansion of electric energy (doubling in ten years), should be regarded as particularly applicable only for countries consuming between 2000 and 5000 kWh per capita. Higher rates of growth are maintained during the early stages of a country's industrial development.

Five-year time spans. Appendix IV explains how the series of five-year time spans, indicated by the row of black circles at the top of Fig. 5, were posted. These circles were used to emphasize that these five-year time spans are not necessarily consecutive, although, in the following section, justification will be given for so regarding them.

Historical growth of electric energy consumption. From the same time-span calculations, explained in Appendix IV, a continuous curve was drawn, as plotted in Fig. 6, in order to make it convenient to obtain an approximate value of the increase in kWh per capita that may be achieved over any number of years, starting from any desired year taken as a reference point.

Because the logarithmic scale used in Fig. 6 does not immediately reveal the true rate-of-growth process, the kWh per capita curve in Fig. 6 was replotted, in Fig. 7, with arithmetic ordinates. From Fig. 7, the beginning of the lower half of the "S" curve, which is associated with long-time growth patterns, can be visualized. It should be noted, however, that the 100-year span on the

I. Summary of survey by geographical areas

	Population		Electric Energy					Nuclear
			1961					
Geographical Area and Code	1961 Total × 10^6	1956–61 Annual Increase, %	kWh per Capita	kWh × 10^9	% of World Total	% of World Increase	1956–61 % Annual Increase	1961 % of Total
A Caribbean America	91.3	3.1	331	31.2	1.3	1.6	10.5	
B South America (other than **A**)	126.9	3.5	358	45.4	1.8	2.0	8.7	
C European Common Market	170.6	1.6	1770	301.2	12.3	12.5	7.7	
D United Kingdom	52.8	0.6	2750	145.9	5.9	5.9	7.6	0.36
E Scandinavian countries	20.4	0.8	4320	88.1	3.6	3.5	7.6	
F Other Western European countries	85.4	0.8	856	73.1	3.0	2.8	7.6	
Western Europe **(WE)**	329.2	0.8	1848	608.3	24.8	24.6	7.7	0.14
G European Comecon (less U.S.S.R.)	99.1	0.8	1260	124.5	5.1	5.9	9.2	
H U.S.S.R.	218.0	1.7	1500	327.5	13.3	17.9	11.3	
Comecon Countries, Europe **(GH)**	317.1	1.6	1430	452.0	18.4	23.8	10.8	
J Middle East	141.9	2.5	95	13.5	0.6	0.8	13.6	
K Northern Africa	27.5	2.5	102	2.8	0.1	0.1	5.0	
L Africa (other than **K** and **M**)	153.6	2.6	65	10.0	0.4	0.5	11.3	
M South Africa	16.2	2.5	1605	26.1	1.1	1.0	6.9	
Africa **(AF)**	197.3	2.5	198	39.1	1.6	1.6	7.9	
N Southwest and South Asia	561.5	2.2	45	25.1	1.0	1.5	13.5	
O Southeast Asia	178.2	3.2	37	6.3	0.3	0.3	8.6	
P Eastern Asia (other than **Q**)	68.3	3.2	157	10.7	0.5	0.7	13.6	
Q Japan	94.0	0.9	1360	128.3	5.2	7.4	12.1	
Asia **(AS)**	902.0	2.7	189	170.4	7.0	9.9	12.2	
R Comecon countries, Asia	772.0	3.7	82	63.8	2.6	5.5	24.1	
S Oceania	15.8	1.2	2120	33.5	1.4	1.2	6.4	
T Canada and Greenland	18.3	2.6	6020	110.8	4.0	3.6	5.9	
U United States	183.7	1.8	4820	884.0	36.5	25.5	5.1	0.05
North America **(NA)**	202.0	1.9	4920	994.8	40.5	29.0	5.2	0.05
World **(W)**	3095.0	2.55	792	2452.0	100.0	100.0	7.7	0.04
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

abscissa scale still comes short of the time when the upper half of the S would begin to approach its asymptote, the calculation of which is explained in the last part of this study.

Although, as pointed out at the beginning of this article, the data from which Fig. 5 (and consequently Fig. 6) was prepared were obtained solely from increases achieved during the period from 1956 to 1961, for countries at various stages of electric energy consumption, it is noteworthy that the historical growth of several countries has, with departures above and below it, followed the general course of the kWh per capita curve in Fig. 6 (or Fig. 7). For the United States, this has been illustrated by showing, starting with 1902, the actual curve of per capita consumption. Through a striking coincidence, the 84-year abscissa for the U.S. point for 1962 comes very close to corresponding to the beginning of electric power generation, with the placing in operation by Edison of the Pearl Street Station.

Obviously, this "past history" implication is valid, over the full extent of the kWh per capita curve in Fig. 6, only for countries now consuming approximately 5000 kWh per capita, since several of the highly industrialized countries of today started to use substantial quantities of electric power in the early 1880s.

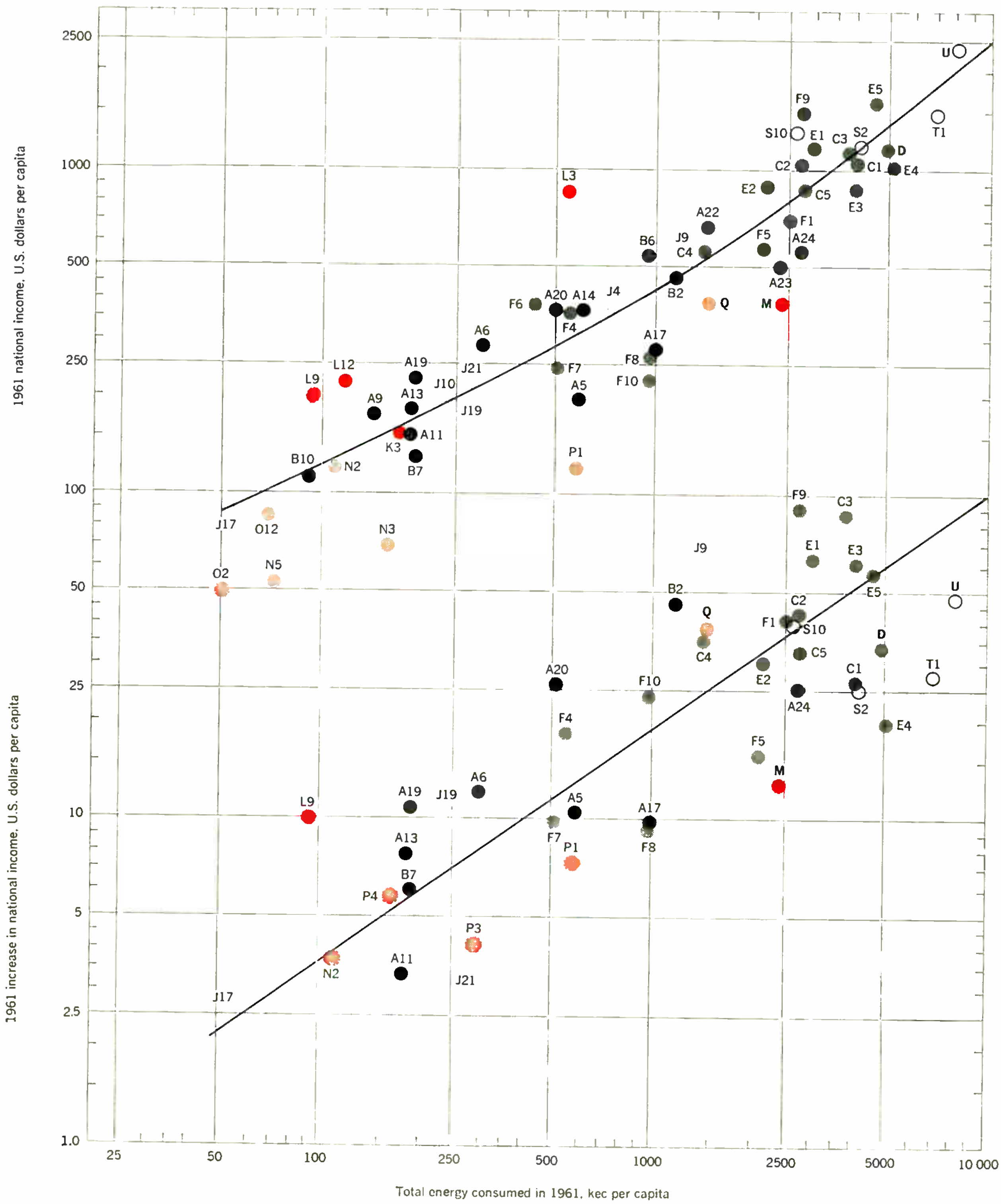
However, the kWh per capita curve in Fig. 6 is of "future history" value for all countries and it is from that point of view that a second scale of abscissas was added, reading "Years to reach 10 000 kWh per capita." This target was selected because it constitutes the limit, provided by Norway, of actual data used in this study, and therefore involves no projection or extrapolation.

The historical aspects of Figs. 6 and 7, as well as their possible use as a forecasting tool are commented upon in detail in Appendix V, which also explains how the "Population of country" and "Total kWh" curves in Fig. 6 were explained.

Appendix V also explains the example, illustrated in Fig. 8, of a 20-year forecast for a country conforming to the world median, which had in 1962 the same kWh per capita and total kWh consumption of electricity as the United States.

Fuel consumption for electric generation. Figure 9, which records the historical trend of the amount of fuel consumption per kWh in steam plants, both in the United States[4] and in Western Europe,[5] has been included here

Hydroelectricity		Solid Fuels		Liquid Fuels		Natural Gas		Total Energy					
								1961					
1961 % of Total	1956–61 % Annual Increase	1961 % of Total	1956–61 % Annual Increase	1961 % of Total	1956–61 % Annual Increase	1961 % of Total	1956–61 % Annual Increase	kec per Capita	kec $\times 10^9$	% of World Total	% of World Increase	1956–61 % Ann. Increase	Code
4.3	5.3	4.7	2.8	70.2	7.8	20.8	15.1	937	85.6	1.9	3.05	8.7	**A**
13.3	7.7	9.4	−1.6	70.2	4.7	7.1	27.5	565	71.4	1.6	1.7	5.4	**B**
7.7	6.4	58.6	−1.6	30.4	13.3	3.3	16.8	2800	476.7	10.6	6.6	3.0	**C**
0.5	11.4	73.7	−2.1	25.4	13.4	0.04	27.0	4970	263.2	5.8	1.0	0.6	**D**
37.7	8.2	16.3	−4.4	46.0	7.2	0.0	—	3870	78.8	1.7	1.7	5.1	**E**
21.0	7.4	47.0	−1.1	29.1	9.1	2.9	18.2	1142	97.5	2.2	1.4	3.1	**F**
9.6	7.3	58.1	−1.8	30.2	12.1	2.0	17.0	2785	916.2	20.3	10.7	2.4	**WE**
0.8	8.1	86.8	4.3	7.0	11.0	5.4	10.5	3200	316.9	7.0	7.2	5.1	**G**
3.4	15.3	58.8	1.0	25.8	10.6	12.0	37.6	2990	651.6	14.5	16.5	5.6	**H**
2.5	14.5	69.0	2.3	19.7	10.7	9.8	29.2	3058	968.5	21.4	23.8	5.6	**GH**
2.4	45.0	12.5	0.8	79.5	8.6	5.6	30.0	284	39.2	0.9	1.4	8.5	**J**
8.5	0.0	12.8	−5.2	72.9	6.0	5.8	10.5	204	5.6	0.1	0.1	4.6	**K**
18.8	26.0	31.3	−4.5	49.5	6.2	0.4	—	93	14.2	0.3	0.2	3.8	**L**
0.0	—	88.6	3.1	11.4	4.4	0.0	—	2414	43.5	1.0	0.7	3.2	**M**
5.0	17.5	68.7	2.5	25.5	5.6	0.8	76.0	321	63.3	1.4	1.0	3.4	**AF**
5.2	16.4	74.5	7.2	18.6	8.0	1.7	27.0	137	76.9	1.7	2.5	7.7	**N**
2.4	2.5	4.6	−5.7	76.8	9.4	16.2	4.6	122	21.7	0.5	0.6	6.0	**O**
7.6	9.1	52.9	14.1	39.3	8.7	0.2	4.6	293	20.0	0.4	0.9	11.3	**P**
17.8	4.4	46.6	4.6	34.7	24.4	0.9	39.4	1470	138.7	3.1	5.3	9.5	**Q**
12.0	6.0	51.8	6.2	33.8	15.3	2.4	11.8	285	257.3	5.7	9.3	8.9	**AS**
1.8	19.4	95.1	29.3	3.1	22.0	0.0	—	533	412.2	9.1	30.9	28.8	**R**
7.9	5.6	56.7	3.2	35.4	3.0	0.0	—	3190	50.5	1.1	0.8	3.5	**S**
29.4	5.6	14.2	−10.2	42.1	3.7	14.3	22.8	7030	128.5	2.8	1.7	2.7	**T**
3.8	3.9	23.1	−2.5	39.5	2.3	33.6	6.5	8270	1517.8	33.7	15.7	2.1	**U**
5.9	4.6	22.4	−3.5	39.7	2.5	32.0	6.7	8150	1646.3	36.4	17.4	2.2	**NA**
6.0	8.2	48.2	2.8	30.9	6.4	14.9	9.4	1459	4510.5	100.0	100.0	4.9	**W**
(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	



A Caribbean America:
Bahama Islands
Barbados / Bermuda
British Honduras
Columbia / Costa Rica
Cuba
Dominican Republic
El Salvador
Guadeloupe
Guatemala / Haiti
Honduras / Jamaica
Leeward Islands
Martinique / Mexico
Netherlands Antilles
Nicaragua
Panama Republic
Canal Zone
Puerto Rico
Trinidad and Tobago
Venezuela
Virgin Islands
Windward Islands
B South America (other than A)
Antarctic Fisheries
Argentina
Bolivia / Brazil
British Guiana
Chile / Ecuador
Falkland Islands
French Guiana
Paraguay / Peru
Surinam / Uruguay
Western Europe (WE):
C European Common Market
Belgium–Luxembourg
France
Germany, Federal Republic of
Italy / Netherlands
D United Kingdom
E Scandinavian countries
Denmark / Finland
Iceland
Norway / Sweden
F Other Western European countries
Austria
Faeroe Islands
Gibraltar
Greece
Ireland
Malta–Gozo
Portugal / Spain
Switzerland
Yugoslavia
Comecon countries, Europe(GH):
G European Comecon (less U.S.S.R.)
Albania / Bulgaria
Czechoslovakia
Germany, East
Hungary / Poland
Romania
H U.S.S.R.
J Middle East:
Aden
Bahrein Islands
British Somaliland
Cyprus / Ethiopia
French Somaliland
Iran / Iraq
Israel / Jordan
Kuwait
Kuwait Neutral Zone
Lebanon
Libya/Qatar
Saudi Arabia

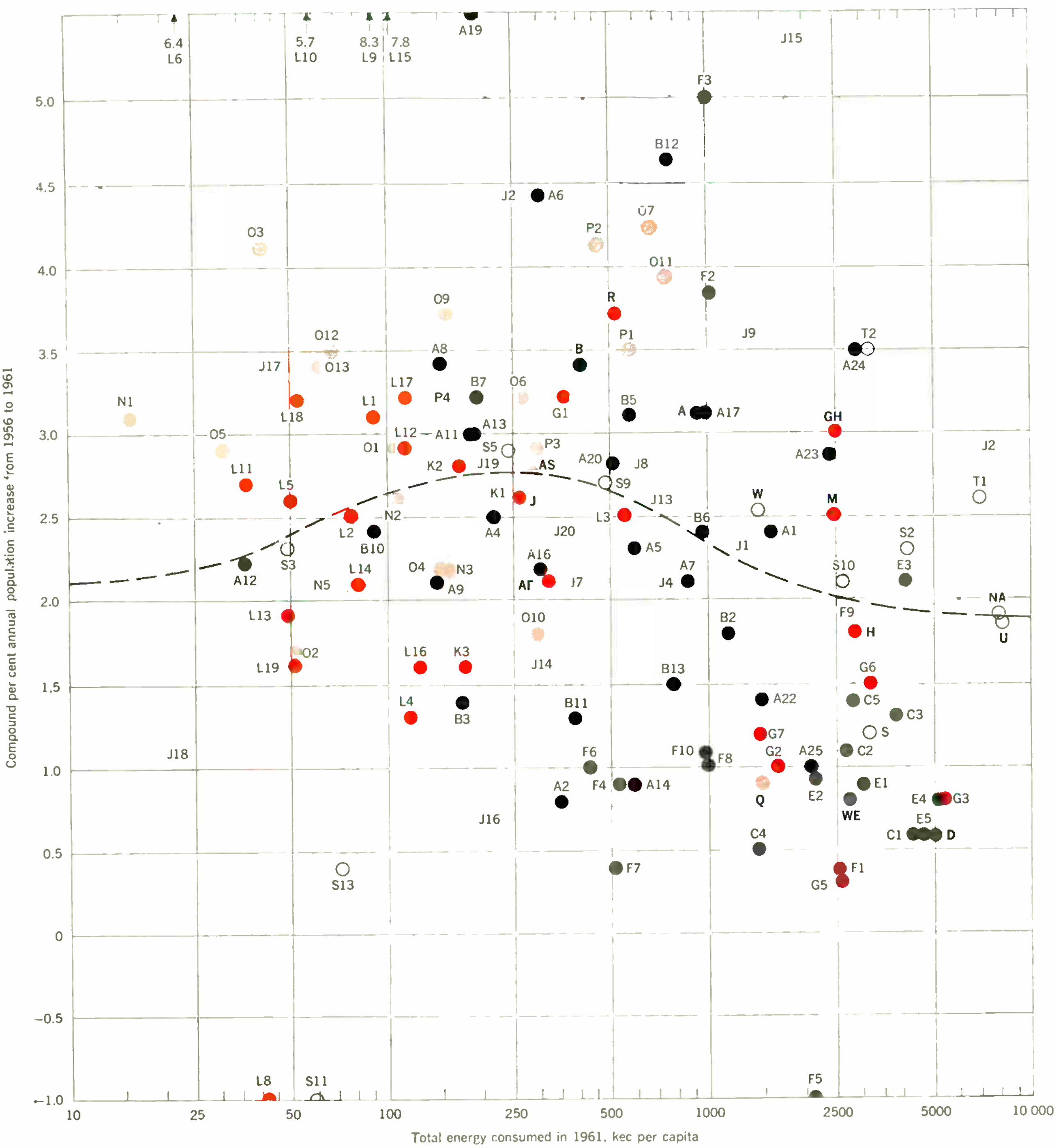


Fig. 3 (left). National income per capita and increase in national income per capita in 1961, for countries of the world. Fig. 4 (above). Annual growth of population for all countries of the world, 1956–1961.

Sudan / Somalia
Turkey
U.A.R. (Egypt)
Syria/Yemen
Africa (AF):
K Northern Africa
Algeria
Morocco
Tunisia
L Africa (other than **K** and **M**)
Congo
British East Africa
Rhodesia and Nyasaland
Cameroun
Equatorial Africa
Togo
West Central Africa
Gambia/Ghana
Liberia
Madagascar
Mauritius
Nigeria, Federation of
Angola/Cape Verde
Mozambique
Reunion
Sierra Leone
Zanzibar–Pemba
M South Africa
Asia (AS):
N Southwest and South Asia
Afghanistan
Ceylon
India/Macao
Pakistan
O Southeast Asia
Brunei/Burma
Cambodia
Indonesia/Laos
Malaya, Federation of
Singapore
Western New Guinea
North Borneo
Ryukyu Islands
Sarawak
Thailand
Vietnam, Republic of
P Eastern Asia (other than **Q**)
China (Taiwan)
Hong Kong
Korea, Republic of
Philippines
Q Japan
R Comecon countries, Asia
China (mainland)
North Korea
Nort Vietnam
Mongolia
S Oceania:
American Samoa
Australia
Australian New Guinea
British Solomon Islands
Fiji Islands
French Oceania
Guam/Hawaii
New Caledonia
New Zealand
Papua / Wake Island
West Samoa
North America (NA):
T Canada and Greenland
Canada
Greenland
San Pierre and Miquelon
U United States
W World

because it explains the use, in the "Total energy" section of this study, of 0.4 kilogram of coal per kWh, as a coefficient of equivalence. This average rate of fuel consumption per kWh generated in thermal electric power plants has been decreasing, for the past five years, at an average yearly rate of approximately 1.75 per cent for the United States and 3 per cent for Western Europe.

Total energy

Statistical record. Columns (17) through (21) in Table I list "area" data pertaining to the consumption of total energy, in the same sequence as columns (3) through (7) tabulate corresponding data on electric energy.

The coefficients of equivalence used to integrate the contribution from different fuels, on the basis of their calorific value, into equivalent kilograms of coal (kec) are listed in Appendix VI.

Corresponding data pertaining to the contribution, and the respective growth, of hydroelectricity, solid fuels, liquid fuels, and natural gas, are similarly tabulated in columns (9) through (16).

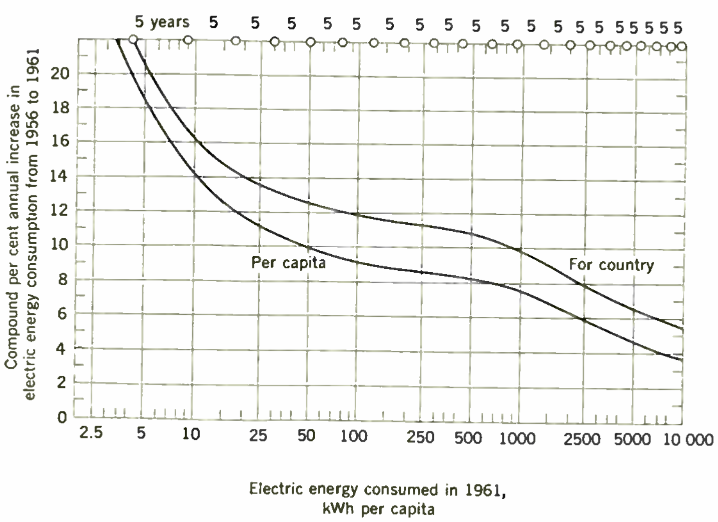
The energy from nuclear fuels, as used for the generation of electric power, is entered only in column (8) as a per cent of the total. Because of the recent entry of this form of energy, no growth rate has been shown.

Inadequacies in the integration of different sources of energy. When the total use of energy and the rate of growth of its consumption are compared on a world-wide basis, numerous inadequacies that might be corrected in a study of more limited scope have to be accepted if it is desired to include all the countries of the world.

Some of these sources of discrepancies are as follows:

1. Omission of wood and wastes, because of the lack of reliable information, which has prevented their inclusion in the United Nations Statistical Papers. However, the use of peat, wherever important, has been included.
2. Substantial differences in the use of energy for space-heating purposes, arising from widely varying climatic conditions.
3. Inclusion, in energy totals, of coke used in metallurgy as a reducing agent.

Fig. 5. Per cent increase in consumption of electric energy.



4. Varying economic efficiencies of different primary sources of energy, which this study equates solely on the basis of calorific equivalence.

For these and other reasons, such as fluctuations in the economy, the correlations arrived at can be regarded only as approximations, but this should not detract from the validity of the trends indicated by "median" curves drawn through more than 150 country points.

Increase in kec per capita. In Fig. 2, the 1961 increases in kec per capita consumption have been plotted, on a logarithmic scale of ordinates, against 1961 kec per capita.

Per cent increase in kec consumption. The "country" curve in Fig. 10 was obtained by dividing the ordinates of the median curve in Fig. 2 by the absicissas. The "per capita" curve was obtained by subtracting from the country curve the rate of population increase read from Fig. 4.

As was noted in Fig. 5 for the rate of growth of consumption of electric energy, the rate of growth of consumption of total energy is many times higher for developing countries than for highly industrialized economies. In fact, there is a close parallelism (evident in Fig. 14, in a subsequent part of this study) between the rates of growth of electric energy and of total energy.

Table I provides, in this respect, as in practically every other aspect of this study, a noteworthy confirmation of the median trends evidenced in the various curves. For instance, comparisons of the data in columns (7) and (21) for the various geographical areas show individual differences, which are generally within 1 per cent of the approximately 4 per cent difference between the electric energy and total energy rate-of-growth curves in Fig. 14.

Thus, because of its faster rate of growth, electric energy will always displace, percentagewise, some of the other forms of energy. These, however, will continue to grow so that the ever-increasing requirements for total energy will be met.

Ratio of electric energy to total energy. Because of the wide differences throughout the 153 countries analyzed in the availability of exploitable hydroelectric resources in relation to economical indigenous fuels, the ratio between the consumption of electric energy (converted to its coal equivalent) and of total energy varies considerably, even for countries at comparable levels of industrialization.

However, it was found convenient and desirable to have the use of a median relationship (as explained in Appendix VII) between kec per capita and kWh per capita for all countries of the world. This relationship is illustrated in Fig. 13, which shows that the use of electric energy, expressed in per cent of total energy, is higher for the higher values of kec per capita associated with the higher levels of industrialization.

This greater percentage use of electric energy, as the level of industrialization increases, parallels the trend toward electric energy taking on a greater percentage share of the total, as times goes on.

Five-year time spans. Similarly to the five-year circles shown in Fig. 5 for electric energy, five-year time spans have been shown at the top of Fig. 10 for total energy.

Because of the approximately 4 per cent lower rate of growth for total energy than for electric energy, five-year spans will result in a much smaller increase for total energy than for electric energy. Figure 7 shows both an electric energy–time curve and a total energy–time curve, corresponding to the five-year circles in Figs. 5 and 10, respectively. These curves show the much longer time span associated with the growth of total energy than that associated with electric energy.

Historical growth of total energy consumption. By the use of the five-year time spans calculated in Fig. 10, a continuous curve (Fig. 11) was drawn to permit a convenient charting of the growth in kec per capita, starting from any reference year.

Because the records for total energy consumption lack the homogeneity of those for the consumption of electric energy, and because the displacement of wood by coal—and, subsequently, by fuel oil—makes it more difficult to arrive at a truly meaningful historical record, the available data for the United States fit the kec per capita growth curve in Fig. 11 only for the past 25 years.

It is interesting to note, also, that, although this Fig. 11 was obtained solely from 1955–1961 data, it projects back to approximately the year 1800 corresponding to the beginning of U.S. industrial expansion stimulated by Watt's improvements to the steam engine.

This striking coincidence—together with the quarter of a century corroboration for growth of total energy consumption, plus the parallelism noted between the growth rates of total energy and electric energy—does, however, justify the expectation that the kec per capita growth curves can be used as a forecasting tool for any country in the world, starting from its own "reference year," as explained in Appendix V for electric energy, and in Appendix VIII for total energy.

Appendix VIII also comments on the total energy 20-year forecast and the corresponding ratio of electric energy and total energy, also included in Fig. 8 for a country having in 1962 the same total and per capita energy consumption as the United States.

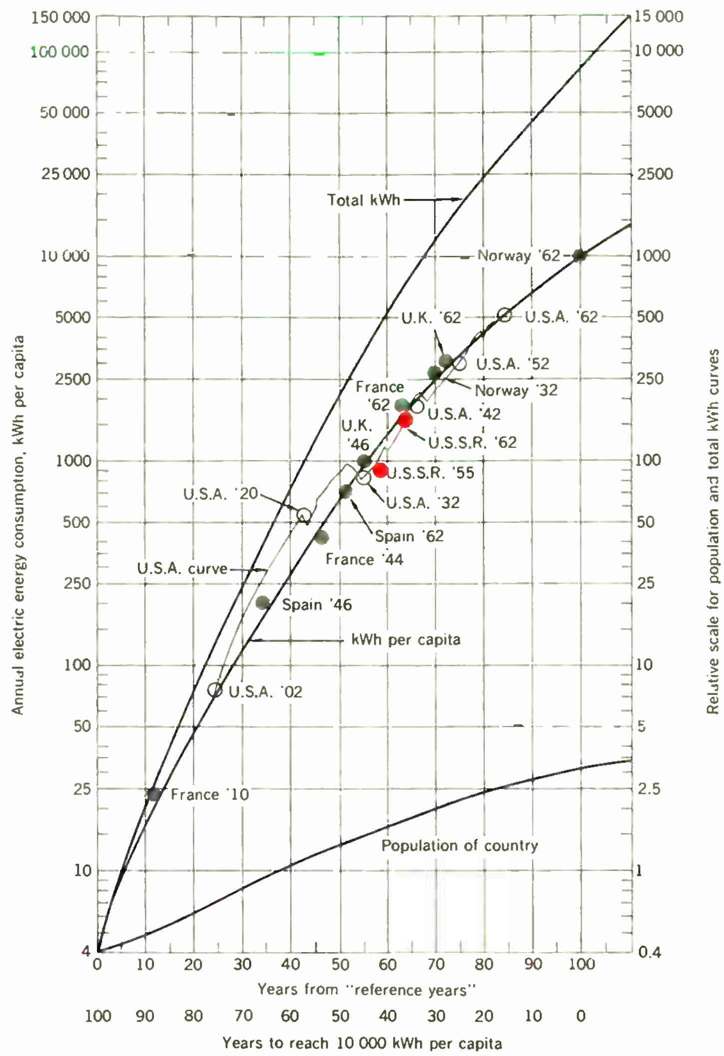


Fig. 6 Growth of per capita electric energy consumption.

National income

National income data at current prices were obtained from the United Nations "Yearbook of National Accounts Statistics, 1962,"[6] which gives this information for about 75 countries out of the 150 for which energy data were available.

Appendix IX explains how the cost-of-living index given for 67 countries in the International Monetary Fund Statistics[7] were used to convert these "current prices" data to "constant prices." Appendix X explains how the national income figures in local currencies were converted into U.S. dollars.

National income per capita and increase in national income per capita. Whereas in the first two sections of this study, kWh and kec per capita increases were respectively plotted agaist kWh and kec per capita, it was found desirable to plot both national income per capita increases and national income proper against kec per capita, as shown in Fig. 3.

Correlation between total energy and national income. Although the consumption of energy per se is not necessarily productive of additional country wealth, the achievement of higher levels of national income does require the application of industrial, transportation, and other mechanisms to production uses, plus the services identified with these—all of which are energy consuming. The relative closeness of the pattern of country points in the upper curve of Fig. 3, showing national income per capita against energy per capita, supports the validity of this correlation.

Energy contents per unit of national income. The kec per U.S. dollar ordinates of Fig. 12 were obtained by dividing the abscissas of Fig. 3 by the ordinates of the median curve for national income; this curve brings out the large increase in amount of total energy per unit of national income as the level of industrialization rises.

Although there is a wide variation in the cost of energy throughout the world, it is convenient, for order-of-magnitude purposes, and fairly accurate on a worldwide basis, to assume a cost of energy of one cent per kilogram of coal, corresponding, for other familiar units, to the costs listed in Appendix XI.

On this basis, the scale of ordinates of Fig. 12 can be read directly as the approximate percentage value of the energy contents per unit of national income, ranging from 0.5 per cent to 4 per cent.

It can be quickly established from Table I that there is nearly the same amount of energy consumed in the world (2250 billion kec) in areas having an average use of energy per capita below 2950 kec as there is in areas having a higher use of energy. Thus, 2950 kec per capita may be regarded as a world median; and, since this corresponds, in Fig. 12, to 3.33 kec per dollar, it can be said that there was an average of 3.33 cents of total energy consumed for each dollar of national income.

Conversely, the lack of energy resources, when needed for productive uses, might hinder the production of national wealth amounting to 100/3.33 = 30 times the value of the energy deficiency.

It is noteworthy that every successive year brings a gradual reduction in contents of energy per unit of national income. For the United States, this decrease has been at the rate of approximately 1 per cent per year, and is attributable to the combined effects of

1. Continuous technical improvements, resulting in higher "thermal" efficiency, not only in power plants—as shown in Fig. 9—but also in most energy-consuming devices.

2. Higher "economic" efficiency, resulting from ever-increasing load densities and market concentrations.

3. Greater emphasis on the contribution of services, which, relatively speaking, are more important sources of national income than they are consumers of energy.

Increase in national income per capita. In Fig. 3, the greater dispersion apparent in the plotting of the 1961 increase in national income than in the plotting of the 1961 national income, for the same countries, can be readily understood: economic or political fluctuations may appreciably influence the increase in national income achieved in a given period even though the national income total itself may be affected by only a small percentage.

Nevertheless, a satisfying median curve was obtained for the national income increase, and from the ratio between this and the national income median, the percentage yearly growth of national income (Fig. 14) was obtained. This curve shows that beyond 4000 kec per capita it is difficult to maintain as high annual rates of growth as between 1000 and 4000 kec per capita; but, as will be noted from the curve of increases, the more advanced economies can still add to their national incomes substantially higher yearly absolute amounts than the less industrialized countries.

Contents of electric energy per unit of national income. Corresponding to Fig. 12, Fig. 13 was prepared by using the correlation explained in Appendix VII. Since the correlation indicates that, on a world-wide basis, 2950 kec per capita corresponds to 1650 kWh per capita, for which point Fig. 13 reads 1.83 kWh per dollar, this can be regarded as an extremely approximate, but convenient, indication of the electric energy contents per dollar of national income.

On the basis of an energy equivalent of 0.4 kec per kWh, the total energy contents per dollar of national income can be approximated, on a world-wide basis, as

3.33 kec of total energy
= 0.73 kec (equivalent to 1.83 kWh) of electric energy
+ 2.60 kec of nonelectricity-producing forms of energy

Thus, electricity's share of total energy production (world average 22 per cent) must always be combined with nonelectricity-producing forms of energy (world average 78 per cent), in order to arrive at a total more closely representative of the amount of national income that all forms of energy generate.

Annual increase or decrease of the ratio of total energy to national income. Figure 14 also includes a curve that plots

Fig. 7. Growth curves showing per capita total energy and electric energy consumption.

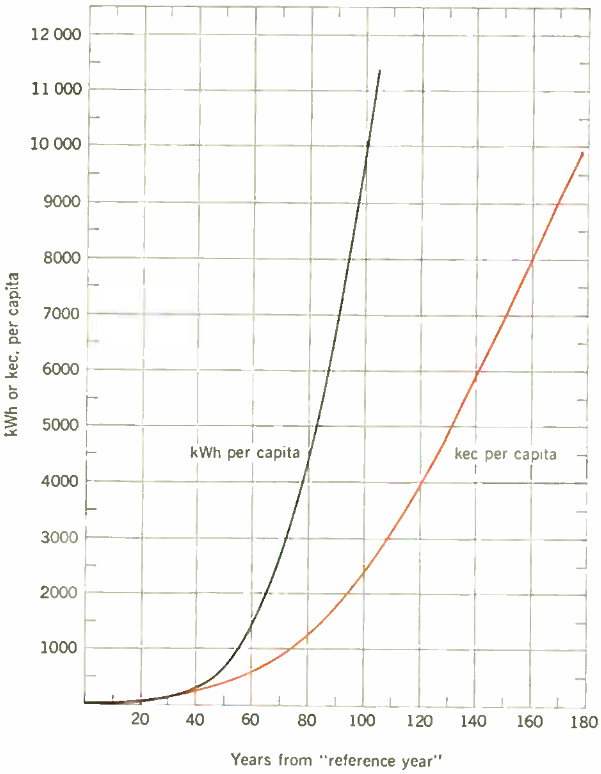
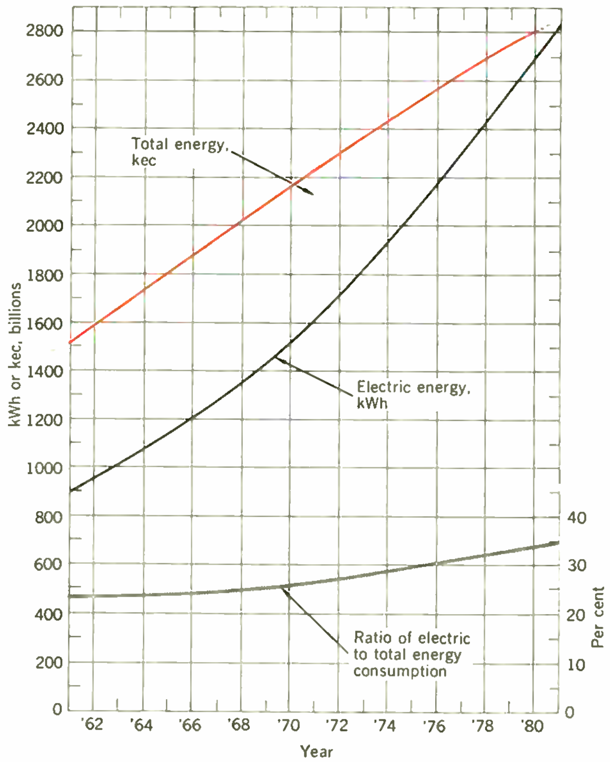


Fig. 8. Twenty-year forecast for a country with the same aggregate and per capita energy consumption as the U.S.A.



the rate of increase or decrease of the ratio of total energy to national income, as explained in Appendix XII. From this curve, it can be seen that at approximately 2950 kec per capita—which, coincidentally, happens to be the median value previously commented upon and which is also close to the average use of energy throughout Europe—the annual per cent increase in national income and the annual per cent increase in consumption of total energy are exactly the same.

At this level of industrialization, the percentage gain in a country's national income exactly matches the percentage increase in total energy consumption. For countries at higher levels of industrialization, an increase in national income is associated with a relatively smaller increase in the consumption of total energy. This is easily understandable, in view of the greater efficiency and productivity and of the greater concentration on more costly forms of national product for the more highly industrialized economies.

The foregoing statement is not inconsistent with the increased contents of energy per unit of national income, when countries of different levels of industrialization are simultaneously compared. It simply means that a country consuming, say, 8000 kec per capita can expect to achieve, in a subsequent year, 104 per cent of its income for the previous year by consuming 103 per cent of the total energy used in the previous year. Thus, from one year to the next the contents of total energy per unit of national income will have decreased by 1 per cent, as mentioned previously. Conversely, a country consuming 400 kec per capita, and which also aims to achieve 104 per cent national income in the subsequent year, should expect a consumption of total energy amounting to 108 per cent of that in the current year. A country consuming 100 kec per capita and which expects to achieve 103 per cent national income in the subsequent year, should plan on its energy requirements becoming 108.5 per cent in fulfillment of productive uses.

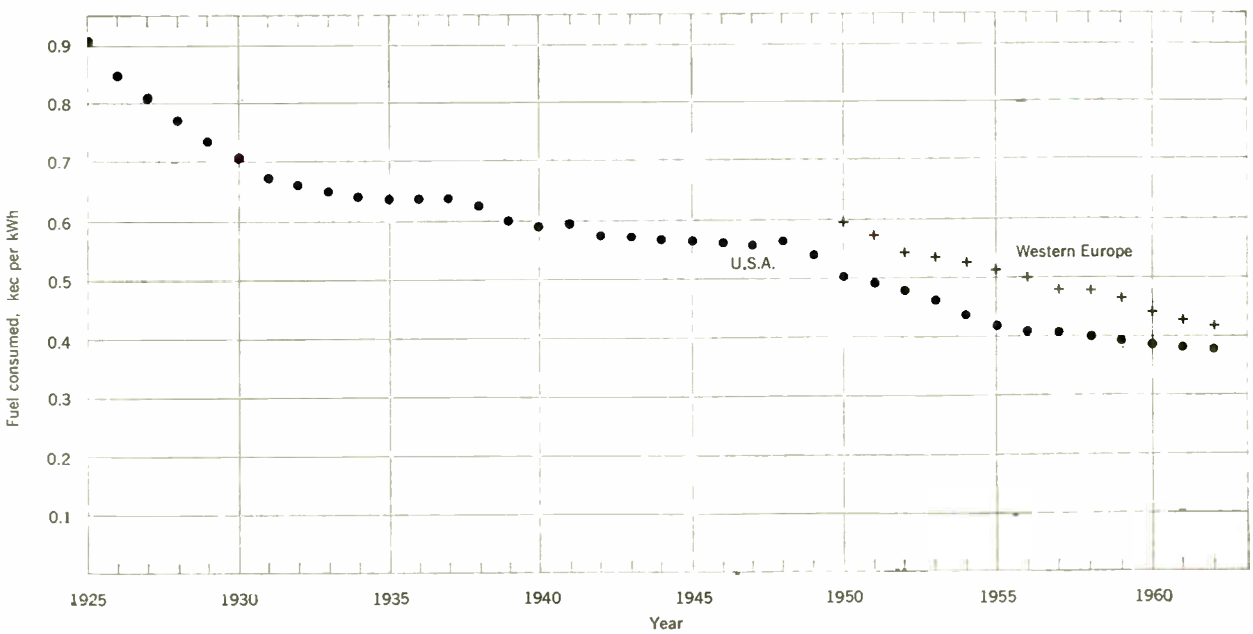
In the three examples just given, which span practically the entire range of economic development throughout the world, the correlation between the use of total energy and the associated generation of national income remains very close (as expressed by the relatively narrow range from 105.5 per cent to 99 per cent). It should be noted, however, that although at the higher end of the scale provision for additional energy consumption need only amount to three quarters of the expected relative gain in national income, at the 100-kec point the provision for additional energy consumption should be relatively three times as high as the desired gain in national income.

Annual increase of the ratio of electric energy to national income. It is apparent from Fig. 4, although a curve has not been plotted to illustrate it, that the requirements for increased consumption of electric energy are notably higher in terms of desired gains in national income. Even at 10 000 kec per capita, (7300 kWh per capita) electric energy consumption increases four thirds as fast as the desired gain in national income, and, at 100 kec per capita (40 kWh per capita), additional electric energy generation must be provided for at a rate 4.5 times that of the desired gain in national income.

The implications are particularly significant for developing countries. It is usual to refer to the lower base from which developing countries start, which is regarded as justifying the higher rates of growth that have been pointed out. Actually, these high rates of increase in energy consumption for developing countries will have to be maintained for very extended periods of time, as emphasized by the number of five-year time spans in Figs. 5 and 10.

For instance, if a developing country initiates an electrical development program at this time, when consuming 4 kWh per capita, for 50 years it will have to maintain rates of increase of electric energy consumption averaging 14 per cent, that is, twice the rate of growth regarded as normal for an industrialized country.

Fig. 9. Fuel consumption in steam plant electric generation.



Interrelationship and growth patterns. It is hoped that the extent of the coverage submitted in the tabulated record, by geographic areas, in combination with the analyses presented under the three headings, "Electric Energy," "Total Energy," and "National Income," will serve as an "all-purpose" guide and valid tool for the appraisal and approximate forecasting of energy for any country in the world.

The following are among the more significant conclusions reached:

1. Electric energy data can be validly referred to kWh per capita and total energy and national income data to kec per capita in comparisons of countries in various stages of industrial development.

2. Both electric and total energy growth requirements are about four times as high, on a percentage basis, for a developing country as they are for the most highly industrialized ones.

3. Except for the most highly industrialized countries, the difference between the rates of growth of total energy and electric energy throughout the world is relatively constant, staying within plus or minus two percentage points of an average 4 per cent.

4. On the sole basis of 1956–1961 world data, it has been possible to arrive at growth patterns that verify historical records for the United States and, at the same time, offer the promise of permitting valid forecasting.

5. Although, on a percentage basis, the analyzed growth rates indicate declines at the highest levels of industrialization, the absolute growths analyzed, whether of electric energy, total energy, or national income, show considerable dynamism. The full extent of the considerable growth that may yet be achieved in the future is estimated in the next part of this study.

Long-range extensions of world curves of consumption growth

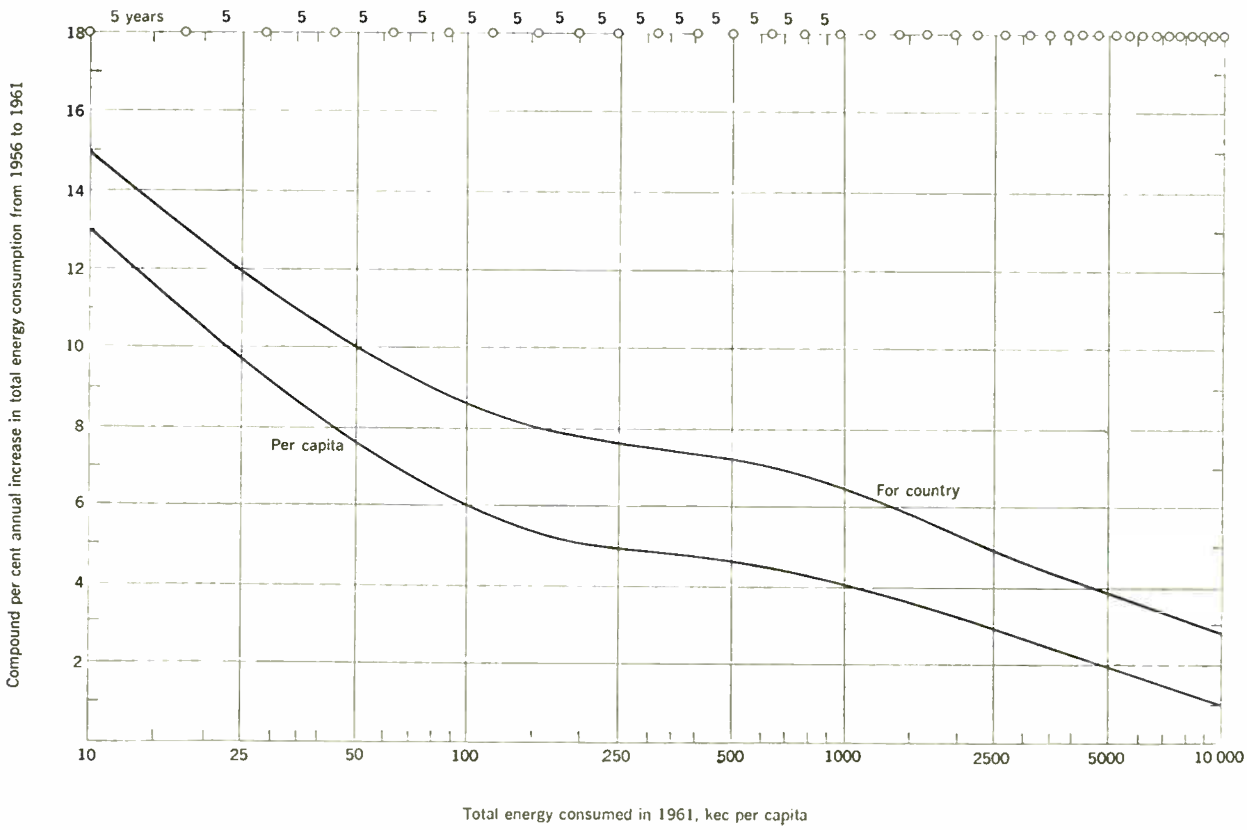
The availability of the strongly documented world curves of Figs. 6 and 11, as well as Fig. 7, has provided a particularly auspicious basis for the application of the Gompertz equation, which provides a favorably regarded form of long-term projection of the trend it recognizes in the data which it is made to fit.

Since only three equidistant points are required to establish the equation of a Gompertz curve, it is best to make a number of different trial selections before deciding on the one which is most representative.

Projections of kWh/capita and kec/capita. Inasmuch as the world curve shown in Fig. 6 was derived from country data extending up to 10 000 kWh/capita, which corresponds to the United States' expected use in 1978, this year was used as the third of the three equidistant points. After several trials, 25-year spacing was found to result in the lowest ordinate for the asymptote—that is, 79 800 kWh per capita—whereas either shorter or longer spans of years between points resulted in considerably higher ultimate levels, which appeared unrealistic.

The same years thus established (1928, 1953, and 1978 on the U.S. scale of Fig. 6) were directly used on the U.S. scale of Fig. 11. This resulted, for the kec per capita Gompertz curve, in an asymptote ordinate of 29 260 kec per capita which, as mentioned at the end of this study, appeared consistent with what might be expected

Fig. 10. Per cent increase in consumption of total energy, 1956–1961.



to be the electric energy generation requirements' share of total energy in the year 2300.

Fig. 15 shows these two curves plotted over a 500-year scale, which starts in 1800 with the beginning of the Industrial Revolution. Generation of electricity on an industrial scale does not appear until 1882.

For the year 1963, it is of interest to note that, on a per capita basis, 5560 kWh represents 7 per cent of "saturation level," for electrical generation and that 8600 kec represents 29 per cent of saturation for total energy consumption.

The years identified as "inflection years" are the next significant dates on the Gompertz curves. Up to these years, and although the yearly percentage of growth steadily decreases, each year sees a greater increase in either kWh per capita or kec per capita than the increase achieved in the previous year.

Subsequent to these inflection years—respectively 2020 and 1988—each year marks a smaller increase in kWh per capita or in kec per capita than was achieved in the previous year. It will be noted, however, that not only is the expansion still most vigorous, but also that there is, in both curves, 1.73 times as much growth still left on the "up" side, up to asymptote level, as that already reached at the inflection years.

Population growth. Because the population growth is subject to factors that do not readily lend themselves to long-term projection, it was deemed preferable not to attempt to fit it into a Gompertz equation. Instead it is treated separately, on the basis of assumptions that

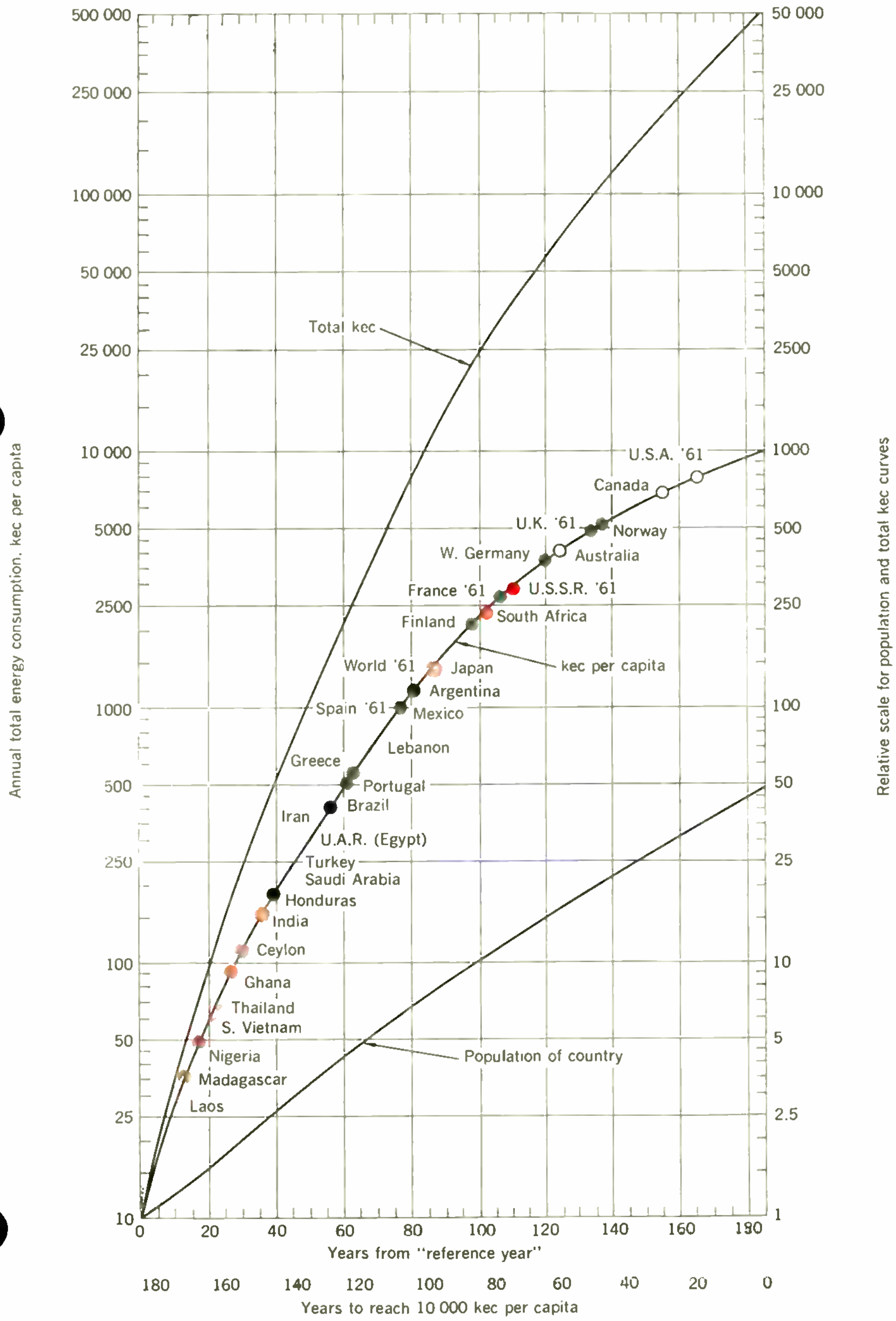


Fig. 11. Growth of total energy consumption.

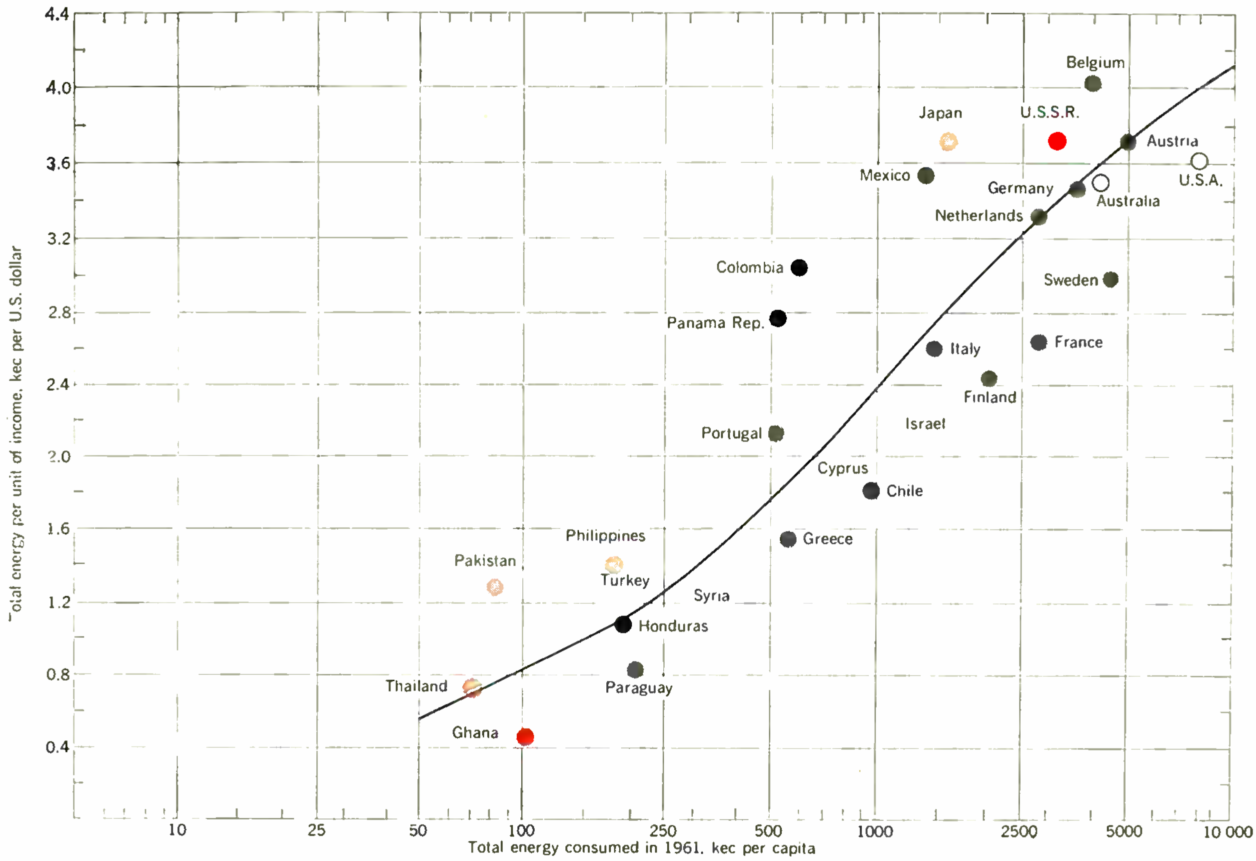
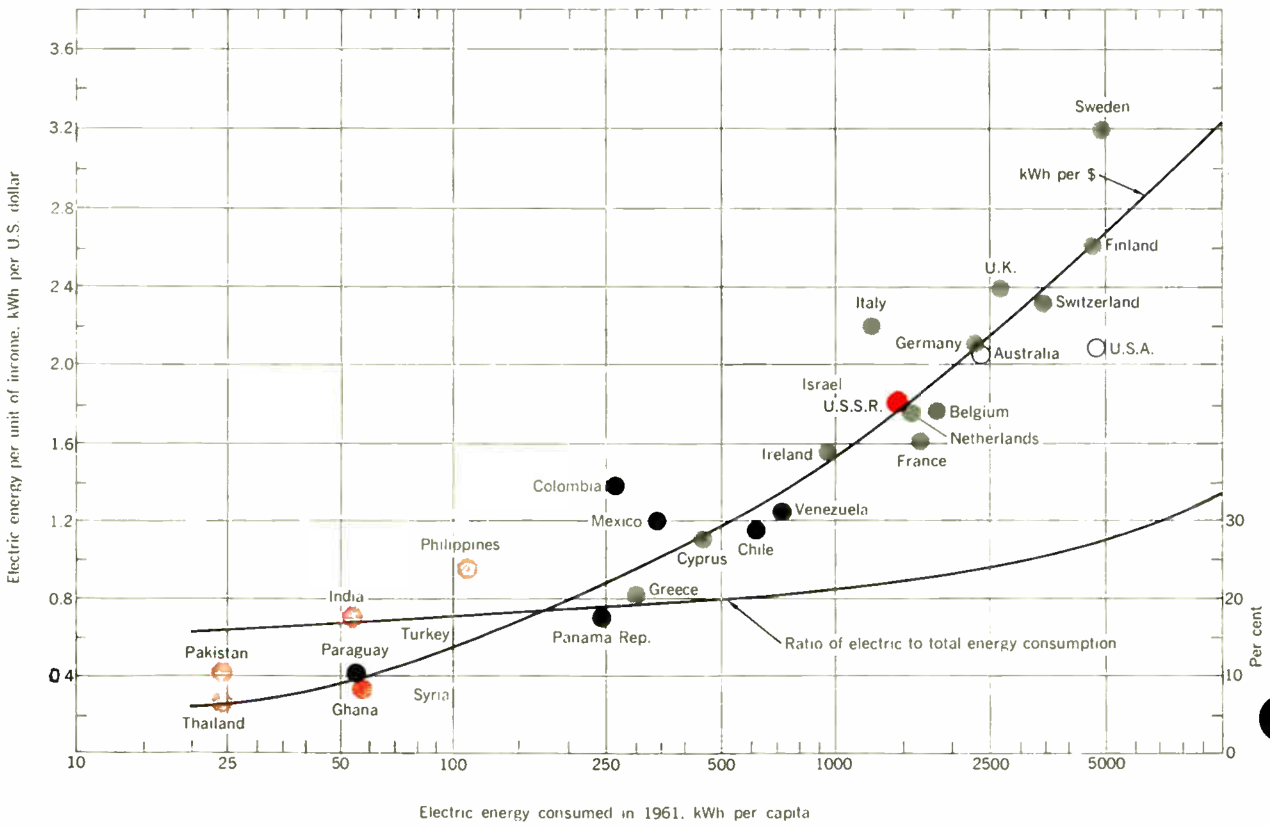


Fig. 12. Total energy per unit of national income for various countries.

Fig. 13. Amount of electric energy per unit of national income for various countries and ratio of electric energy to total energy.



practically duplicate those made in the U.S. Atomic Energy Commission's Report to the President on Civilian Nuclear Power, Appendix III, which for the year 1980 gave results consistent with the Series III Estimate of the Bureau of the Census projection.

Starting in 1962 with the current rate of population increase (1.75 per cent per year), and assuming that this will decrease each passing year by 1 per cent of its value in the previous year, the following results are obtained:

Year	1962	1980	2000	2050	2100	2150	2200	2250	2300
Rate of increase, per cent	1.75	1.44	1.22	0.75	0.45	0.28	0.17	0.11	0.06
Population, millions	189.9	247.8	322.4	525	706.3	845.9	943.1	1009.2	1054.9

Generation of kWh and consumption of kec. Multiplying the kWh per capita and kec per capita Gompertz curve values by the population figures gives the kWh generation and kec consumption curves, also shown in Fig. 15, for which the inflection years are, respectively, 2070 and 2090.

From this forecast of kWh generation, it would appear that, for a hundred years to come, the yearly increase in the total kWh generated in the United States should be greater each year than the increase recorded in the preceding year. Thus, the year 2070, in which a population of 602 million will consume 51 500 kWh per capita, resulting in a total generation of 31 trillion kWh, will record the largest one-year increase over the previous year's total.

The totals at which the consumptions of electric energy and of total energy will level off, beyond the year 2300, respectively at about 83 trillion kWh and 29 trillion kec, are of interest since they indicate that in 1963 electricity generation was only at 1.25 per cent of saturation and total energy consumption at 5 per cent of saturation.

Comparison between U.S. and U.S.S.R. energy consumption. The above considerations have illustrated some aspects of interest of these long-range forecasts. Another aspect is brought out, in Fig. 16, which, out of the 500 years spanned by Fig. 15, charts the same data, for the period from 1900 to 2080 with the addition of the corresponding projections for the U.S.S.R. obtained by the use of the same Gompertz curves, except plotted to the U.S.S.R.—instead of to the U.S.—scale of years. (These scales are 22 years apart for kWh per capita and 55 years apart for kec per capita.)

The same rate of population increase was assumed for the U.S.S.R. as for the United States, maintaining the present 1.17-to-1 ratio between the two populations, and it will be noted that in 2080 this ratio offsets the inverse ratio between kWh per capita for the two countries.

It will be noted, however, that the consumption of total energy which, as shown earlier in this study, may be regarded as an indication of national income always stays, for the U.S.S.R., well below the U.S. curve.

Forecast evaluation. Although none of them have included as distant projections as present here, there have been a number of independent forecasts of total energy consumption for the United States. The figures in Table II, which include only those published in 1961 or later, were extracted from Table 2 of Appendix III to the "1962 Report to the President on Civilian Nuclear Power" by the U.S. Atomic Energy Commission. The totals in the original table are all in quadrillion (10^{15}) Btu, and these have been converted into trillion kec on the basis (given in Appendix VIII of this study) that 1 trillion kec = 27.8 quadrillion Btu. Table II shows that the energy forecast presented in this study could be characterized as "conservative lower middle."

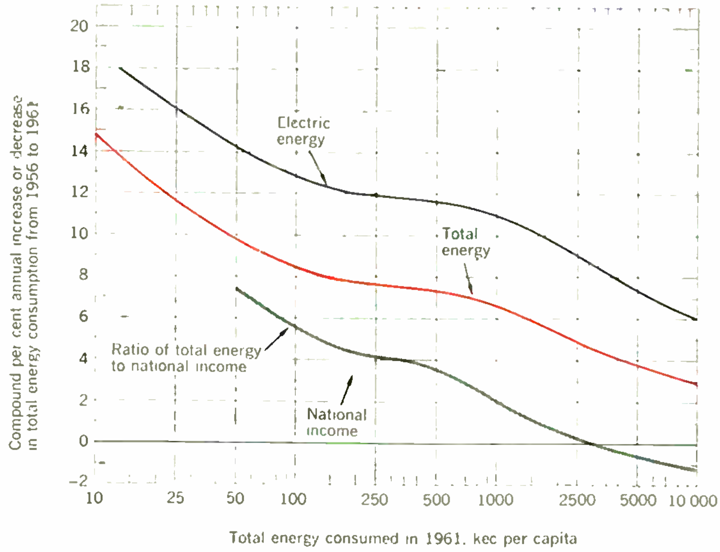


Fig. 14. Per cent change in national income per capita, in electric and total energy per capita, and in ratio of total energy to national income, 1956–1961.

The electricity forecast, the validity of which, as shown in Fig. 8, has been substantiated by comparison with other forecasts extending up to the year 2000, may be regarded as deriving long-range support, in relation to the total energy forecast, from the following. Assuming that, when both asymptote levels are approached—respectively 29 260 kec per capita and 79 800 kWh per capita—50 per cent of the total energy requirements are consumed by the generation of electricity, this would correspond to 14 630/79 800 = 0.183 kec per kWh, as compared with approximately 0.3 kec per kWh for the most efficient of today's stations and is, therefore, within the realm of probability.

Appendix I Annual compound rates of growth, and increases achieved in 1961

Throughout this study, annual compound rates of growth have been arrived at by subtracting unity from the fifth root of the ratio between 1961 and 1956 figures, thus arriving at an equivalent constant annual rate of increase which, after five years, would compound itself into the actually recorded five-year growth. Increases in 1961 were obtained by multiplying 1961 consumption or income by this annual compound rate of growth.

Appendix II Median curves

With all country points regarded as equally significant, the median curves in Figs. 1 through 4 were drawn on the basis that they should leave as many points above them as below them.

To arrive at the best "median" plotting, each of the charts was divided into ten vertical sections, respectively corresponding to the areas between the following successive abscissas: 5, 10, 25, 50, 100, 250, 500, 1000, 2500, 5000, and 10 000.

Within each of these ten sections, segments of median curves were drawn so that there would be as many country points above as below them, and the final median curve was obtained by preserving, in a single, continuous line, the maximum compatibility with the separately obtained segments. Thanks to the large number of points available, this operation could be carried out with comparative confidence.

It will be noted that the degree of correlation is not equal throughout the three divisions of this study—understandably so, since electric energy data are much more consistent than total energy data, which, in turn, are more closely correlated than national income data. However, the use of median curves provides, in all cases, a valid indication of trend.

Appendix III Plot of population growth by countries

Figures 5 and 10 show, in addition to the rate of electric and total energy consumption growth attained for a country as a whole, the increase in the consumption per capita. Per capita growth rates were obtained by subtracting from the country consumption growth rates the population growth rates read from the median curve shown in Fig. 4.

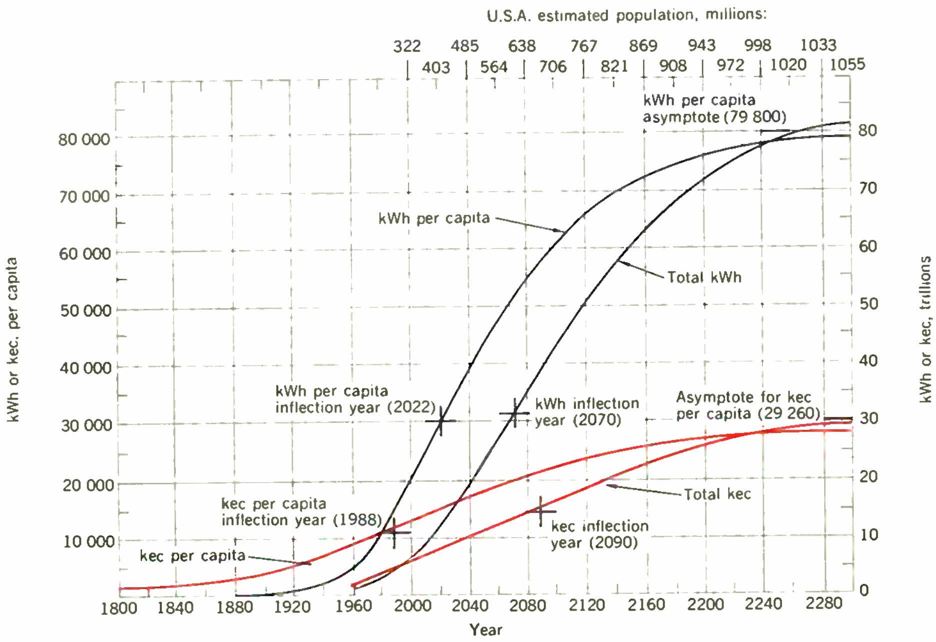


Fig. 15. Recorded and forecast growth of energy consumption in the United States. Per capita kWh and kec curves were based on Gompertz curves derived from world curves of Figs. 6, 7, and 11. Total kWh and kec curves were plotted on the basis of estimated population figures shown at the top of the graph.

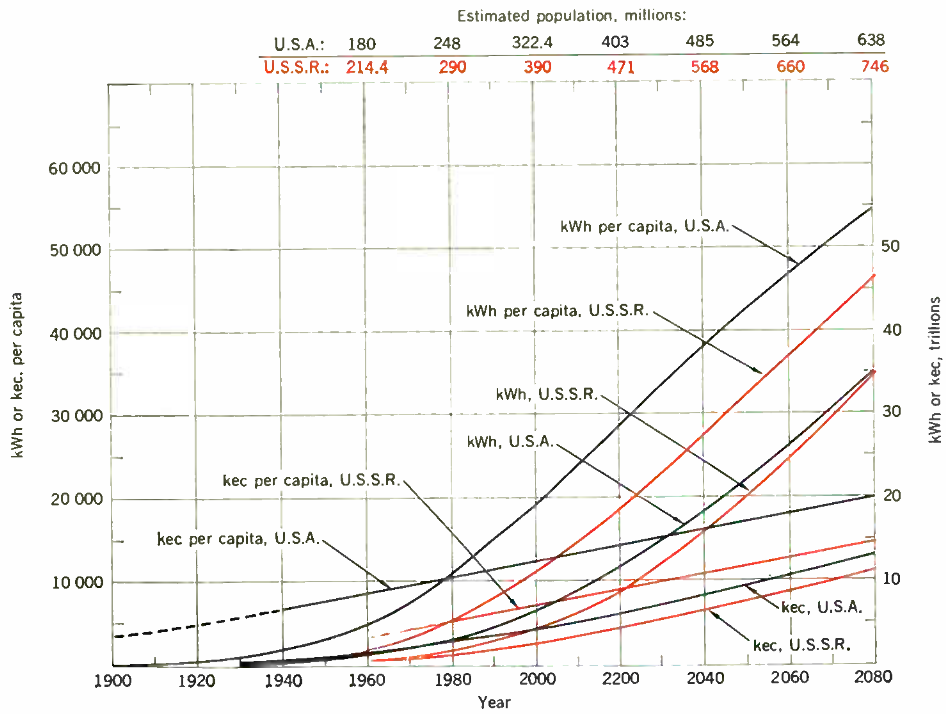


Fig. 16. Recorded and forecast growth of energy consumption in both the United States and the Soviet Union, plotted on the same basis as were the curves of Fig. 15.

Appendix IV Step-by-step calculations of growth versus time

In the per capita curve in Fig. 5, for 4 kWh per capita on the abscissa scale, the corresponding annual growth rate is 20 per cent.

If it is assumed that after five years the growth rate will have decreased to 14 per cent, the average growth rate during these five years will have been 17 per cent.

With a uniform growth of 17 per cent, the initial 4 kWh per capita figure after five years would be multiplied by 2.19, thus becoming 8.76. For 8.76 kWh per capita, the per cent annual growth curve reads 14.5 per cent, which would make the five-year average 17.25 per cent.

Five-year growth at 17.25 per cent results in a multiplying factor of 2.215, and a more exact value of the kWh per capita after five years would therefore be 8.85, for which the annual growth rate cannot be read any more closely than 14.5 per cent. Therefore, a country which, at "0" year, was consuming 4 kWh per capita, will be consuming 8.85 kWh per capita in the year "5."

By repeating this process, the circles at the top of Fig. 5 as well as the kWh per capita curves in Figs. 6 and 7 were obtained.

Appendix V Use of 1956–1961 world data for forecasting purposes

In Fig. 6 the per capita consumption values in 1962, for a small number of countries selected at random, are identified with the "reference year" on the abscissa scale for which the actual kWh per capita figure matched the curve.

In this way, for instance, in 1962 the following "reference years" were identified: Norway, 100; U.S.A., 84; U.K., 71; U.S.S.R., 62, etc.

Going as far back for each country as was permitted by available records, the points for these same countries were plotted for the corresponding "year" along the abscissa scale and, as can be readily seen, a satisfactory correlation was obtained. Thus, this curve is found to be useful for approximate forecasting within the span encompassed by present data—that is, up to exactly 10 000 kWh per capita.

This kind of forecasting is generally concerned with the total generation required, which means that the expected intervening increase in population has to be taken into account. Therefore, a separate curve, labeled "Population of country," in Fig. 6 was obtained by the same method as explained in Appendix IV, and based, this time, on the median population growth curve in Fig. 14. This curve is the basis for the left-hand curve of Fig. 7, which for convenience is shown as starting at "0" year from the same ordinate 4 that is the kWh per capita figure at this arbitrary "0" year.

The population increase over any span of years can thus be approximately estimated by the ratio of the corresponding ordinates, starting, of course, from the "reference year" under consideration.

By multiplying the population value by the kWh per capita, the curve of Fig. 6 labeled "Total kWh" was obtained, thus making it possible, when specific estimates on population growth were not available, to obtain directly an estimate of the expected increase in kWh generation.

As an example, assuming that an estimate is desired of the kWh generation for the United States in 1980:

1962 for the U.S.A. corresponds to 84 as "reference year," for which the "Total kWh" curve reads 34 000.

1980 for the U.S.A. corresponds to 102 as "reference year," for which the "Total kWh" curve reads 96 500.

Thus, the electrical generation for the year 1980 would be (96 500)/(34 000) = 2.84 times the electrical generation for 1962. Since the 1962 figure was 944 billion kWh, for 1980 this would forecast 2680 billion kWh.

Of course, the method of proration just explained can also be used to draw up a complete curve of growth forecast over any desired span of years. This was done for illustration purposes in Fig. 8, which plots an estimate of the growth of kWh consumption of a country which, like the United States, had in 1962 a kWh per capita consumption of 5098, and a total generation (including Alaska and Hawaii and industrial self-generation) of 943.677 billion kWh.

Appendix VI Coefficients of equivalence and energy contents for various fuels and electricity

	Coal Equivalent, metric tons	Energy Content, 10^3 kilocalories
1 metric ton, anthracite or bituminous coal	1	7 000
1 metric ton, coke of anthracite or bituminous coal	0.9	6 300
1 metric ton, lignite	0.3 to 0.6	2 100 to 4 200
1 metric ton, crude petroleum	1.3	9 100
1 metric ton, gasoline, kerosene, or fuel oil	1.5	10 500
1000 cubic meters, natural gas	1.33	9 310
1000 cubic meters, manufactured and coke oven gases	0.6	4 200
1000 kWh electric energy, hydro	0.375	2 625
1000 kWh electric energy, thermal or nuclear	0.4	2 800

The above energy contents are consistent with	Btu
1 pound of coal	12 600
1 pound of liquid fuel	18 000
1000 cubic feet of natural gas	1 050 000

Appendix VII Median relationship between kWh per capita and kec per capita

By comparing with Fig. 1 a similar chart (not included), in which the same kWh per capita increases were plotted against kec per capita, it was possible to arrive at the following median relationship for all countries of the world:

kec/capita	10	20	50	100	200	500	1000	2000	5000	10 000
kWh/capita	4.5	9.0	20	40.5	91	240	500	1040	3100	7 300

This has been used throughout this study in all cases where it was considered desirable.

Appendix VIII Forecast of growth of total energy consumption

An estimated curve of future total energy requirements, derived as explained in Appendix V, is shown in Fig. 8 for a country which, like the United States, had in 1961 an 8270-kec per capita consumption and a total energy consumption of 1517 billion kec. For purposes of converting the totals plotted in billion kec into quadrillion Btu, a unit often used for these projections:

II. Estimates of total energy consumption

Source	Year of Publication	Yearly Consumption of Total Energy, trillion kec 1980	2000	2050	2150	2300
Philip Sporn	1961	2.81	3.78	—	—	—
Electrical World	1961	2.92	4.50	—	—	—
Resources for the Future	1961	2.97	4.96	—	—	—
AEC Report to the President, Appendix III	1962	2.95	4.86	12.5	—	—
Draft Report to the Committee on Natural Resources of the National Academy of Sciences	1962	2.41	3.64	9.9	—	—
Characterization of forecast by Department of Interior staff:						
Upper middle		2.95	5.76	27.0	—	—
Lower middle		2.63	4.44	15.8	—	—
Lower, ultraconservative		1.94	2.34	3.78	—	—
Fremont Felix (this study)	1964	2.53	3.98	9.07	20.65	29.30

1000 billion kec = 27.8 quadrillion Btu

In Fig. 8, the ratio between electric energy and total energy has also been shown. In calculating this ratio, an approximately 10 per cent reduction over a span of 20 years has been assumed for the ratio of equivalence between kWh and kec, starting in 1961, at 0.38 (as read, for the United States, from Fig. 9).

Appendix IX Correction for constant prices

The correction of national income figures at current prices to corresponding figures at constant prices was obtained as shown in the following example:

It is assumed that a country had, in terms of local currency, an income of 100 in 1956 and 147 in 1961, which would correspond to an annual increase, at current prices, of 8 per cent.

It is also assumed that the cost of living index[7] was 100 in 1956 and 116 in 1961, which would correspond to an annual increase of 3 per cent in the cost of living.

The annual multiplier of national income at constant prices was obtained as $(147/116)^{1/5} = 1.0495$, which means a 4.95 per cent annual increase in national income at constant prices.

It is to be noted that intervening currency depreciations or appreciations, in terms of the U.S. dollar, for instance, are immaterial to this calculation.

Appendix X Conversion of national income figures to U.S. dollars

Converting local currency 1961 national income into U.S. dollars was obtained by using the rate of exchange given in the United Nations Statistical Bulletin.[3]

The national income per capita increases in 1961, which are shown in the lower curve of Fig. 3, were obtained merely by multiplying the annual rate of increase of national income by the 1961 national income per capita in U.S. dollars.

Appendix XI Cost of energy assumed for approximation purposes

1¢ per kilogram of coal
corresponds to:
36¢ per million Btu
37¢ per thousand cubic feet of natural gas
$14.30 per 10 million kilocalories
$ 2.16 per barrel of fuel oil
$10.00 per metric ton of coal

As a matter of interest, in 1960 the average cost of all the fuel consumed for electric generation, in steam plants in the United States, was 0.7 cent per kec.

Appendix XII Increase or decrease of the ratio of total energy to national income

The "ratio" curve in Fig. 14 was obtained as the ratio of the ordinates of the curves of increase of total energy and of national income, increased in each case by 100. Unity was subtracted from the ratio thus obtained.

Appendix XIII Contribution of various energy sources and ratio of electric to total energy

The data in columns (9) through (16) show the percentage contribution of the various sources to the total energy summarized in columns (17) and (18).

The actual consumption of nuclear fuels, hydro or imported electricity, solid fuels, liquid fuels, or natural gas can be immediately obtained, in kec, by multiplying the total in column (18) by the corresponding percentage in columns (8), (9), (11), (13), or (15), and these kec figures can, in turn, be converted into kWh, tons of oil, cubic feet of natural gas, etc., by using the coefficients of equivalence in Appendix VI.

The share of total energy, used in (or equivalent to) the generation of electric energy, as a per cent of the total, can be obtained immediately for any of the areas or groups of countries tabulated, as 0.4 times kWh per kec, using either the per capita figures in columns (3) and (17) or the aggregate figures in columns (4) and (18).

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This article is based on a paper presented at the American Power Conference, Chicago, Ill., April 14–16, 1964. The full text of the paper will be published in the *Proceedings* of the conference later in 1964.

The author expresses his appreciation to the United Nations for the completeness of the Statistical Papers,[1-2] which include every country in the world, to the Edison Electric Institute's Department of Economics and Statistics for their thoroughness in recording the progress of electricity generation in the United States, and to the other organizations listed in the references for their dedicated work. The expert mathematical assistance rendered by Leo B. Shohan, senior economist for the National Industrial Conference Board's Division of Economic Research, in the application of the Gompertz equations is also acknowledged.

Matrix functions and applications

Part V—Similarity reductions by rational or orthogonal matrices

This last part of a five-part series includes two methods for transforming a matrix into its companion matrix by rational operations. A method is also given for orthogonally transforming a real symmetric matrix to triple diagonal form and then computing its eigenvalues without using the coefficients of the characteristic polynomial

J. S. Frame *Michigan State University*

5.1 Computations related to the companion matrix

Associated with any polynomial $D(\lambda)$ of degree n, having leading coefficient $d_0 = 1$ (*monic* polynomial) is an $n \times n$ matrix C of particularly simple form whose minimal and characteristic polynomial is $D(\lambda)$. This so-called *companion matrix* of $D(\lambda)$ is

$$C = \begin{bmatrix} 0 & 0 & \dots & 0 & -d_n \\ 1 & 0 & & 0 & -d_{n-1} \\ 0 & 1 & & 0 & -d_{n-2} \\ \cdot & \cdot & & \cdot & \\ \cdot & \cdot & & \cdot & \\ \cdot & \cdot & & \cdot & \\ 0 & 0 & & 1 & -d_1 \end{bmatrix} \tag{5.1.1}$$

Its characteristic equation is easily shown to be

$$|\lambda U - C| = D(\lambda) = \sum_{k=0}^{n} d_{n-k}\lambda^k \qquad d_0 = 1 \tag{5.1.2}$$

The n matrices $U, C, C^2, \dots C^{n-1}$ are linearly independent, since their first columns $C^{j-1}U_{\cdot 1}$ (for $j = 1, 2, \dots n$) are the n columns of the $n \times n$ unit matrix U. Since C cannot satisfy an equation of degree less than n, its minimal and characteristic polynomial are both $D(\lambda)$.

The matrix C (or its transpose C^T) is also called the companion matrix for any $n \times n$ matrix A whose minimal polynomial is $D(\lambda)$. Such a matrix will be shown below to be similar to C.

Some important properties of C are related to the Vandermonde matrix V and the "left upper triangular" symmetric coefficient matrix W of Eq. (4.1.11) in Part IV, both derived from $D(\lambda)$.

We find that the matrix CW is symmetric:

$$CW = \begin{bmatrix} -d_n & 0 & 0 & \dots & 0 & 0 \\ 0 & d_{n-1} & d_{n-2} & \dots & d_1 & 1 \\ 0 & d_{n-2} & d_{n-3} & \dots & 1 & 0 \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ 0 & d_1 & 1 & \dots & 0 & 0 \\ 0 & 1 & 0 & & 0 & 0 \end{bmatrix} = (CW)^T = WC^T \tag{5.1.3}$$

Hence W transforms C into C^T.

Theorem 5.1.1 Right modal matrices for the companion matrix C of (5.1.1) and its transpose C^T are WV^T and V^T, respectively.

Proof: If Λ is the Jordan upper triangular spectral matrix for C, and V the corresponding Vandermonde matrix, a direct computation shows that

$$VC = \Lambda^T V \tag{5.1.4}$$

Hence from (5.1.4) and (5.1.3) we have

$$C^T V^T = V^T \Lambda \tag{5.1.5}$$

$$C(WV^T) = WC^T V^T = (WV^T)\Lambda \tag{5.1.6}$$

The modal columns for C, which are columns of WV^T and rows of VW, may be easily computed from $D(\lambda)$ by reading in reversed order the rows that are computed by the synthetic division algorithm as explained in Part IV, Eq. (4.3.39).

Example 1 A polynomial $D(\lambda)$ with a triple root and a simple root will illustrate the computation of WV^T by synthetic division.

$$\begin{aligned} D(\lambda) &= \lambda^4 - 2\lambda^3 - 36\lambda^2 + 162\lambda - 189 \\ &= (\lambda - 3)^3(\lambda + 7) \end{aligned} \tag{5.1.7}$$

For $\lambda_1 = 3$

$$\begin{array}{rrrrrl}
1 & -2 & -36 & 162 & -189 & |\underline{3} \\
 & 3 & 3 & -99 & 189 & \\
\hline
\mathbf{1} & 1 & -33 & \mathbf{63} & 0 & = D(3) \\
 & 3 & 12 & -63 & & \\
\hline
\mathbf{1} & 4 & \mathbf{-21} & 0 & = D'(3) & \\
 & 3 & 21 & & & \\
\hline
\mathbf{1} & \mathbf{7} & 0 & = D''(3)/2! & &
\end{array}$$

For $\lambda_1 = -7$

$$\begin{array}{rrrrrl}
1 & -2 & -36 & 162 & -189 & |\underline{-7} \\
 & -7 & 63 & -189 & 189 & \\
\hline
\mathbf{1} & \mathbf{-9} & \mathbf{27} & \mathbf{-27} & 0 & = D(-7)
\end{array} \tag{5.1.8}$$

Computed rows, indicated by boldface type, are written in reversed order to obtain the rows of VW or the columns of the modal matrix WV^T for C.

$$WV^T = \begin{bmatrix} 63 & -21 & 7 & -27 \\ -33 & 4 & 1 & 27 \\ 1 & 1 & 0 & -9 \\ 1 & 0 & 0 & 1 \end{bmatrix} \text{(modal matrix for } C) \tag{5.1.9}$$

We verify that

$$\underset{\substack{\text{Companion} \\ \text{matrix}}}{\underset{C}{\begin{bmatrix} 0 & 0 & 0 & 189 \\ 1 & 0 & 0 & -162 \\ 0 & 1 & 0 & 36 \\ 0 & 0 & 1 & 2 \end{bmatrix}}} (WV^T) = (WV^T) \underset{\substack{\text{Jordan} \\ \text{matrix}}}{\underset{\Lambda}{\begin{bmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -7 \end{bmatrix}}} \tag{5.1.10}$$

The *vector iterate* (Krylov's) method and the *pivotal similarity* (Danilevskii's) method are two different techniques for finding the characteristic polynomial $D(\lambda)$ and modal matrix S of an $n \times n$ matrix A through its companion matrix C, without computing the conjoint matrix $B(\lambda)$. These computations both require fewer multiplications (about $2.5n^3$) in the *nonderogatory* case (minimal equation equals characteristic equation) than the conjoint method (about n^4), but both are more difficult to program, since they require special treatment when certain computed quantities happen to vanish. Especially complicated is the so-called *derogatory* case, when the minimal polynomial of A is of lower degree than the characteristic polynomial. Then A is not similar to an $n \times n$ companion matrix C, but to a direct sum of such matrices of lower dimensions. These methods give insight into the structure of matrices, but may be unstable when used with large-order matrices in machine computation. Even with multiple precision arithmetic, small round-off errors in the coefficients of the characteristic polynomial may lead to large errors in the eigenvalues and eigenvectors, as pointed out by Wilkinson and others. The examples below, with integral coefficients, show hand computations in which no round-off errors are involved.

The vector iterate method. Given an $n \times n$ matrix A we select an arbitrary column vector $X_0 \neq 0$ (such as $U_{\cdot 1}$) and define the iterates X_k of X_0 by repeated left multiplications by A.

$$X_k = AX_{k-1} = A^k X_0 \quad \text{for } k = 0, 1, 2, \ldots \tag{5.1.11}$$

We compute the $n \times n$ matrix X and the augmented $n \times (n+1)$ matrix X_+, defined by

$$X = (X_0, X_1, \ldots, X_{n-1}) \tag{5.1.12}$$

$$X_+ = (X_0, X_1, \ldots, X_{n-1}, X_n) = (X, X_n) = (X_0, AX)$$

Let W_0 be the $n \times 1$ column vector with jth entry d_{n+1-j}. Then by the Hamilton-Cayley theorem,

$$X_+ \begin{bmatrix} W_0 \\ 1 \end{bmatrix} = XW_0 + X_n = \sum_{k=0}^{n} X_k d_{n-k}$$

$$= \left(\sum_{k=0}^{n} A^k d_{n-k} \right) X_0 = D(A) X_0 = 0$$

$$W_0 = \begin{bmatrix} d_n \\ d_{n-1} \\ \cdot \\ \cdot \\ d_1 \end{bmatrix} \tag{5.1.13}$$

If the vectors $X_0, X_1, \ldots, X_{n-1}$ are linearly independent, then X^{-1} exists, and we can compute $W_0 = -X^{-1}X_n$ by reducing X_+ to row echelon form, as described in Part I.

$$X_+ \rightarrow X^{-1}X_+ = X^{-1}(X, X_n) = (U, -W_0)$$

$$= (X^{-1}X_0, X^{-1}AX) = (U_{\cdot 1}, C) \tag{5.1.14}$$

This echelon reduction yields the companion matrix $C = X^{-1}AX$ similar to A. Also the matrix $S = X(WV^T)$ is a right modal matrix for A since, by (5.1.14) and (5.1.3),

$$A(XWV^T) = XCWV^T = (XWV^T)\Lambda \tag{5.1.15}$$

About $n^3 - n^2$ multiplications are required to compute X_+, $n^3/2$ to reduce this to $(U_{\cdot 1}, C)$, n^2 to compute WV^T by synthetic division, and n^3 to find $X(WV^T)$; or a total of about $2.5n^3$ multiplications when X is nonsingular.

Example 2 Transform the following nonderogatory 4×4 matrix A with triple eigenvalue and find its characteristic polynomial and eigenvectors by the vector iterate method.

$$A = \begin{bmatrix} 2 & 6 & 8 & -3 \\ 7 & -6 & 8 & -4 \\ 4 & -8 & 0 & -2 \\ -4 & 10 & -2 & -1 \end{bmatrix}$$

$$X_+ = \begin{bmatrix} 1 & 2 & 90 & -194 & 446 \\ 0 & 7 & 20 & -42 & 118 \\ 0 & 4 & -40 & 84 & -164 \\ 0 & -4 & 58 & -138 & 326 \end{bmatrix} \tag{5.1.16}$$

We choose $X_0 = U_{\cdot 1}$, write $X_1 = A_{\cdot 1}$, and compute $X_2 = AX$, $X_3 = AX_2$, $X_4 = AX_3$. Clearing successive columns of X_+ by row operations, we then have

$$X_+ \rightarrow \begin{bmatrix} 1 & 0 & 110 & -236 & 528 \\ 0 & 1 & -10 & 21 & -41 \\ 0 & 0 & 1 & -3 & 9 \\ 0 & 0 & 90 & -189 & 405 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 94 & -462 \\ 0 & 1 & 0 & -9 & 49 \\ 0 & 0 & 1 & -3 & 9 \\ 0 & 0 & 0 & 81 & -405 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 8 \\ 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & -5 \end{bmatrix} \tag{5.1.17}$$

The characteristic polynomial is read from the last column of (5.1.17).

$$D(\lambda) = \lambda^4 + 5\lambda^3 + 6\lambda^2 - 4\lambda - 8 \tag{5.1.18}$$

Synthetic division yields the eigenvalues of C (or of A), and the modal matrix WV^T for C. (Multiple precision may be required if this method is to be used in machine computation, and other methods for finding eigenvalues may be preferred.)

$$\begin{array}{rrrrrl}
1 & 5 & 6 & -4 & -8 & \quad \underline{\mid -2} \\
 & -2 & -6 & 0 & 8 & \\ \hline
\mathbf{1} & \mathbf{3} & \mathbf{0} & \mathbf{-4} & 0 & = D(-2) \\
 & -2 & -2 & 4 & & \\ \hline
\mathbf{1} & \mathbf{1} & \mathbf{-2} & 0 & = & D'(-2) \\
 & -2 & 2 & & & \\ \hline
\mathbf{1} & \mathbf{-1} & 0 & = & D''(-2)/2! &
\end{array}$$

$$\begin{array}{rrrrrl}
1 & 5 & 6 & -4 & -8 & \quad \underline{\mid 1} \\
 & 1 & 6 & 12 & 8 & \\ \hline
\mathbf{1} & \mathbf{6} & \mathbf{12} & \mathbf{8} & 0 & = D(1)
\end{array}$$

$$\Lambda = \begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = S^{-1}AS \quad (5.1.19)$$

Columns of WV^T are read from these rows, reversed.

$$\underset{X}{\begin{bmatrix} 1 & 2 & 90 & -194 \\ 0 & 7 & 20 & -42 \\ 0 & 4 & -40 & 84 \\ 0 & -4 & 58 & -138 \end{bmatrix}} \underset{(WV^T)}{\begin{bmatrix} \mathbf{-4} & \mathbf{-2} & \mathbf{-1} & \mathbf{8} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{12} \\ \mathbf{3} & \mathbf{1} & \mathbf{0} & \mathbf{6} \\ \mathbf{1} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}}$$

$$= \underset{S}{\begin{bmatrix} 72 & 90 & 1 & 378 \\ 18 & 27 & 7 & 162 \\ -36 & -36 & 4 & -108 \\ 36 & 54 & -4 & 162 \end{bmatrix}} \quad (5.1.20)$$

The first and last columns of S are eigenvectors for the eigenvalues -2 and 1, respectively. Constituent matrices A_j are not obtained directly, but may be obtained by transforming the constituent matrices Λ_j of Λ by S, as follows: $A_j = S\Lambda_jS^{-1}$.

If only the first r of the vectors $X_0, X_1, \ldots$ are linearly independent, this procedure must be modified. This situation will become evident when the echelon reduction of X_+ yields a matrix LX_+ of rank $r < n$, having $n - r$ rows of 0's at the bottom. Deletion of the first column from the upper left $r \times (r + 1)$ matrix leaves an $r \times r$ companion matrix, but iterates of additional vectors may be needed to obtain the n linearly independent columns of the required matrix similar to A that is quasi-upper triangular with diagonal blocks in companion matrix form, each corresponding to a factor of $D(\lambda)$ which divides the minimal polynomial.

Example 3 Transform the following matrix A by the vector iterate method.

$$A = \begin{bmatrix} 2 & 3 & 2 & -3 \\ 3 & 2 & -3 & 2 \\ 3 & -2 & 1 & 2 \\ -2 & 3 & 2 & 1 \end{bmatrix}$$

$$X_+ = \left[\begin{array}{rrrr|rr} 1 & 2 & 25 & 18 & 0 & 3 \\ 0 & 3 & -1 & 94 & 1 & 2 \\ 0 & 3 & -1 & 94 & 0 & -2 \\ 0 & -2 & 9 & -46 & 0 & 3 \end{array}\right] \quad (5.1.21)$$

Here we take $X_0 = U_{\cdot 1}$, $X_1 = AU_{\cdot 1} = A_{\cdot 1}$, $X_k = AX_{k-1}$, but find that A^4X_0 is linearly dependent on its predecessors. A new independent vector such as $U_{\cdot 2}$ must be placed in column 5 to obtain an augmented matrix X_+ of maximum rank 4. Its iterate $AU_{\cdot 2}$ is then placed in column 6. Columns 1, 2, 3, and 5 of X_+ form a nonsingular X, and columns 2, 3, 4, and 6 form AX. Row operations on X_+ that reduce it to echelon form will reduce AX to $X^{-1}AX$

$$X_+ \rightarrow \left[\begin{array}{rrrr|rr} 1 & 0 & 9 & -78 & 0 & 1 \\ 0 & 1 & 8 & 48 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 25 & 50 & 0 & 5 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{rrrr|r|r} 1 & 0 & 0 & -96 & 0 & -0.8 \\ 0 & 1 & 0 & 32 & 0 & -0.6 \\ 0 & 0 & 1 & 2 & 0 & 0.2 \\ \hline 0 & 0 & 0 & 0 & 1 & 4 \end{array}\right] = X^{-1}X_+ \quad (5.1.22)$$

$$X = \begin{bmatrix} 1 & 2 & 25 & 0 \\ 0 & 3 & -1 & 1 \\ 0 & 3 & -1 & 0 \\ 0 & -2 & 9 & 0 \end{bmatrix} \quad X^{-1}AX = \left[\begin{array}{rrr|r} 0 & 0 & -96 & -0.8 \\ 1 & 0 & 32 & -0.6 \\ 0 & 1 & 2 & 0.2 \\ \hline 0 & 0 & 0 & 4 \end{array}\right]$$

To transform this into a direct sum of companion matrices an additional transformation is needed by a matrix T whose last column is orthogonal to the rows of $X^{-1}AX - 4U$. Procedures for finding T are related to the methods of Theorem 4.4.3 in Part IV.

$$T = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix} \quad XT = \begin{bmatrix} 1 & 2 & 25 & -3 \\ 0 & 3 & -1 & 2 \\ 0 & 3 & -1 & -3 \\ 0 & -2 & 9 & 2 \end{bmatrix}$$

$$C = (XT)^{-1}A(XT) = \left[\begin{array}{rrr|r} 0 & 0 & -96 & 0 \\ 1 & 0 & 32 & 0 \\ 0 & 1 & 2 & 0 \\ \hline 0 & 0 & 0 & 4 \end{array}\right] \quad (5.1.23)$$

The characteristic equation and minimal equation of C are

$$\begin{aligned} D(\lambda) &= (\lambda^3 - 2\lambda^2 - 32\lambda + 96)(\lambda - 4) \\ &= (\lambda - 4)^3(\lambda + 6) \\ m(\lambda) &= (\lambda^3 - 2\lambda^2 - 32\lambda + 96) \\ &= (\lambda - 4)^2(\lambda + 6) \end{aligned}$$

The modal matrix M for C is a direct sum of modal matrices for the diagonal blocks, and the matrix $S = XTM$ is a modal matrix for A.

$$M = \left[\begin{array}{rrr|r} -24 & 6 & 16 & 0 \\ 2 & 1 & -8 & 0 \\ 1 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \quad (5.1.24)$$

$$S = XTM = \begin{bmatrix} 5 & 8 & 25 & -3 \\ 5 & 3 & -25 & 2 \\ 5 & 3 & -25 & -3 \\ 5 & -2 & 25 & 2 \end{bmatrix}$$

The pivotal similarity method (Danilevskii). In the Danilevskii method, a given $n \times n$ matrix $A = A^{(1)}$ is transformed in $n-1$ stages into a succession of similar matrices $A^{(k)}$, for $k = 2, 3, \ldots n$, such that $A^{(n)}$ is a subdirect sum of companion matrices, and $A^{(k)}$ agrees with $A^{(n)}$ in its first $k-1$ columns. Finally, $A^{(n)}$ is transformed into a direct sum C of companion matrices.

If all subdiagonal entries in column $k-1$ of $A^{(k-1)}$ are 0, we omit stage k. Otherwise we select a nonzero subdiagonal entry p_k in column $k-1$ of $A^{(k-1)}$ (using the numerically largest entry for rounded-off machine computation; any one for exact computation) and call it the kth *pivot*. Stage k has three parts:

1. If p_k is in row $j > k$, interchange rows j and k of $A^{(k-1)}$ and then interchange columns j and k by use of a transposition matrix T_{kj} to get

$$M_k = T_{kj}A^{(k-1)}T_{kj} \qquad (5.1.25)$$

where column $k-1$ of M_k is a vector V_k with pivot p_k as kth entry.

2. Let P_k be a matrix obtained from the unit matrix U by replacing column k by V_k. Then $|P_k| = p_k$ and $P_k^{-1}M_k$ will have $U_{\cdot k}$ in column $k-1$. Multiply M_k by P_k^{-1} on the left by the "pivotal operation" to be described below.

3. Multiply $P_k^{-1}M_k$ on the right by P_k by simply replacing its kth column by $(P_k^{-1}M_k)V_k$. Define

$$A^{(k)} = P_k^{-1}M_kP_k$$

In the "pivotal operation" row k is first divided by the pivot p_k. Then multiples of this row are subtracted from the other rows to clear to zero the remaining nonzero entries of column $k-1$ that contained the pivot. For $i \neq k$, entry m_{ij} is replaced by

$$\begin{vmatrix} p_k & m_{kj} \\ m_{i,k-1} & m_{ij} \end{vmatrix} \div p_k \qquad (5.1.26)$$

At the end it may be necessary to perform further transformations by a matrix T to clear the nondiagonal blocks.

Example 4 Transform the matrix A of Example 3 by the pivotal similarity method.

Solution:

$$A = A^{(1)} = \begin{bmatrix} 2 & 3 & 2 & -3 \\ 3 & 2 & -3 & 2 \\ 3 & -2 & 1 & 2 \\ -2 & 3 & 2 & 1 \end{bmatrix} = M_2$$

$$V_2 = \begin{bmatrix} 2 \\ 3 \\ 3 \\ -2 \end{bmatrix} \qquad P_2 = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{bmatrix} \qquad (5.1.27)$$

The pivot p_k is the 21-entry 3. No transposition needed.

$$P_2^{-1}M_2 = \begin{bmatrix} 0 & 5/3 & 12/3 & -13/3 \\ 1 & 2/3 & -3/3 & 2/3 \\ 0 & -4 & 4 & 0 \\ 0 & 13/3 & 0 & 7/3 \end{bmatrix}$$

$$P_2^{-1}M_2V_2 = \begin{bmatrix} 77/3 \\ -7/3 \\ 0 \\ 25/3 \end{bmatrix} \qquad (5.1.28)$$

Insert the new second column to get $A^{(2)}$. Take $P_3 = {}^{25}/_3$ as new pivot, and interchange rows 3 and 4, then columns 3 and 4, to obtain

$$M_3 = T_{34}A^{(2)}T_{34} = \begin{bmatrix} 0 & 77/3 & -13/3 & 4 \\ 1 & -1/3 & 2/3 & -1 \\ 0 & 25/3 & 7/3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 1 & 0 & 77/3 & 0 \\ 0 & 1 & -1/3 & 0 \\ 0 & 0 & 25/3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_3^{-1}M_3 = \begin{bmatrix} 0 & 0 & -288/25 & 4 \\ 1 & 0 & 19/25 & -1 \\ 0 & 1 & 7/25 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

$$P_3^{-1}M_3V_3 = \begin{bmatrix} -96 \\ 32 \\ 2 \\ 0 \end{bmatrix} \qquad (5.1.29)$$

$$A^{(3)} = \begin{bmatrix} 0 & 0 & -96 & 4 \\ 1 & 0 & 32 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} = A^{(4)}$$

Stage 3 is omitted since subdiagonal entries of column three are 0. Check that all the similar matrices $A^{(k)}$ and M_k have the same trace 6. Finally, find T with last column orthogonal to $A^{(4)} - 4U$. Then

$$\underset{T^{-1}}{\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \underset{A^{(4)}}{\begin{bmatrix} 0 & 0 & -96 & 4 \\ 1 & 0 & 32 & -1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}} \underset{T}{\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$$= \begin{bmatrix} 0 & 0 & -96 & 0 \\ 1 & 0 & 32 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \qquad (5.1.30)$$

$$= C$$

The modal matrix M for C was given in (5.1.24). Here a modal matrix for A would be S, where

$$S = P_2T_{34}P_3TM = \begin{bmatrix} 5 & 8 & 25 & 1 \\ 5 & 3 & -25 & 0 \\ 5 & 3 & -25 & 1 \\ 5 & -2 & 25 & 0 \end{bmatrix} \qquad (5.1.31)$$

Advantages of the pivotal similarity method over the vector iterate method are: (1) About $2n^3$ multiplications are required for obtaining all eigenvectors instead of $2.5n^3$. (2) The possibility of linear dependence because of poor choices of initial vectors X_0 is avoided.

However, neither method appears to be as simple and straightforward as the adjoint method of Theorem 4.2.2 (Part IV) for calculating constituent matrices.

5.2 Tridiagonalization of symmetric matrices by reflection

An $n \times n$ tridiagonal matrix $T^{(n)}$ is one whose entries t_{ij} are 0 for $|i-j| > 1$. Givens[1] has developed an elegant method for computing the eigenvalues of a real symmetric tridiagonal matrix, without computing the coefficients in the characteristic polynomial, that has advantages of greater stability for machine computation. A variation of this method will be presented in which the eigenvectors can be obtained simultaneously if certain products of entries are not too small to be used as divisors. A slight modification of Householder's method[2] will first be described and illustrated, whereby an arbitrary real symmetric $n \times n$ matrix is transformed into a tridiagonal matrix by a succession of $n-1$ simple orthogonal reflection matrices S_k that are also symmetric.

Starting with $A = A^{(n)}$, we define $A^{(k)}$ for $k < n$ by

$$A^{(k)} = S_k A^{(k+1)} S_k \qquad S_k = S_k^{-1} \tag{5.2.1}$$

where $A^{(k+1)}$ and S_k have the partitioned form

$$A^{(k+1)} = \left[\begin{array}{c|c|c} B_k & C_k & 0 \\ \hline C_k^T & \multicolumn{2}{c}{T^{(n-k)}} \\ 0 & & \end{array}\right]$$

$$T^{(n-k)} = \begin{bmatrix} b_{k+1} & c_{k+1} & 0 & \cdot & 0 \\ c_{k+1} & b_{k+2} & c_{k+2} & \cdot & 0 \\ 0 & c_{k+2} & \cdot & \cdot & 0 \\ \cdot & & & b_{n-1} & c_{n-1} \\ 0 & 0 & 0 & c_{n-1} & b_n \end{bmatrix} \tag{5.2.2}$$

$$S_k = S_k^{-1} = S_k^T = \left[\begin{array}{c|c} U_k - 2r_kV_kV_k^T & 0 \\ \hline 0 & U_{n-k} \end{array}\right]$$

$$r_k^{-1} = V_k^T V_k \tag{5.2.3}$$

The reflection matrix S_k, which is both orthogonal and symmetric, depends on a vector V_k that is easily derived from the column vector C_k in $A^{(k+1)}$, or its transpose C_k^T. Let

$$C_k^T = (c_{k1}, c_{k2}, \dots, c_{kk})$$

$$c_k^2 = C_k^T C_k = \sum_{j=1}^{k} c_{kj}^2 \tag{5.2.4}$$

Choose the sign of c_k opposite to that of c_{kk} and define

$$V_k^T = (c_{k1}, c_{k2}, \dots, c_{k,k-1}, c_{kk} - c_k) \tag{5.2.5}$$

Then

$$V_k^TV_k = C_k^TC_k - 2c_{kk}c_k + c_k^2 = 2c_k(c_k - c_{kk}) = \frac{1}{r_k}$$

$$V_k^TC_k = C_k^TC_k - c_kc_{kk} = \frac{1}{2r_k} \tag{5.2.6}$$

$$(U_k - 2r_kV_kV_k^T)C_k = C_k - V_k = (0, 0, \dots, 0, c_k)^T \tag{5.2.7}$$

Hence by (5.2.1), (5.2.3), and (5.2.7), the matrix $A^{(k)}$ is obtained from $A^{(k+1)}$ by replacing C_k^T and C_k by the vector $(0, 0, \dots, c_k)$ and its transpose, and B_k by

$$\tilde{B}_k = (U_k - 2r_kV_kV_k^T)B_k(U_k - 2r_kV_kV_k^T) = \left(\begin{array}{c|c} B_{k-1} & C_{k-1} \\ \hline C_{k-1}^T & b_k \end{array}\right) \tag{5.2.8}$$

To compute the matrix $\tilde{B}_k$, we may first evaluate X_k, p_k, and Y_k, where

$$X_k = 2r_kB_kV_k \qquad p_k = r_kV_k^TX_k \qquad Y_k = X_k - p_kV_k \tag{5.2.9}$$

$$\tilde{B}_k = (U_k - 2r_kV_kV_k^T)(B_k - X_kV_k^T)$$
$$= B_k - X_kV_k^T - V_kX_k^T + 2p_kV_kV_k^T$$

$$\tilde{B}_k = B_k - Y_kV_k^T - V_kY_k^T \tag{5.2.10}$$

Example 5 The computation is illustrated by the following 4×4 symmetric matrix $A = A^{(4)}$

$$A^{(4)} = \begin{bmatrix} 4 & 3 & 1 & 6 \\ 3 & -6 & -3 & -2 \\ 1 & -3 & 3 & 3 \\ 6 & -2 & 3 & 5 \end{bmatrix} = \left[\begin{array}{c|c} B_3 & C_3 \\ \hline C_3^T & b_4 \end{array}\right] \tag{5.2.11}$$

$$C_3^T = (6, -2, 3)$$
$$c_3 = -\sqrt{(6)^2 + (-2)^2 + 3^2} = -7$$

$$V_3^T = (6, -2, 10) \qquad V_3^TV_3 = 140 = \frac{1}{r_3} \tag{5.2.12}$$

$$X_3^T = 2r_3V_3^TB_3 = (0.40, 0.00, 0.60)$$
$$p_3 = r_3V_3^TX_3 = 0.06$$

$$Y_3^T = X_3^T - p_3V_3^T = (0.04, 0.12, 0.00) \tag{5.2.13}$$

$$V_3Y_3^T = \begin{bmatrix} 0.24 & 0.72 & 0 \\ -0.08 & -0.24 & 0 \\ 0.40 & 1.20 & 0 \end{bmatrix}$$

$$A^{(3)} = \left[\begin{array}{cc|c|c} 3.52 & 2.36 & 0.60 & 0 \\ 2.36 & -5.52 & -4.20 & 0 \\ \hline 0.60 & -4.20 & 3.00 & -7 \\ \hline 0 & 0 & -7 & 5 \end{array}\right] \tag{5.2.14}$$

$$C_2^T = (0.60, -4.20)$$

$$c_2 = \sqrt{0.36 + 17.64} = 3\sqrt{2}$$

$$V_2^T = [0.60, (-4.2 - 3\sqrt{2})]$$

$$V_2^TV_2 = 36(1 + 0.7\sqrt{2}) = \frac{1}{r_2} \tag{5.2.15}$$

$$p_2 = \frac{(188 - 135\sqrt{2})}{18}$$

$$Y_2^T = [-0.40, (2.80 - 2\sqrt{2})]$$

$$V_2Y_2^T = \begin{bmatrix} -0.24 & 1.68 - 1.2\sqrt{2} \\ 1.68 + 1.2\sqrt{2} & 0.24 \end{bmatrix} \tag{5.2.16}$$

$$T = A^{(2)} = \begin{bmatrix} 4 & -1 & 0 & 0 \\ -1 & -6 & 3\sqrt{2} & 0 \\ 0 & 3\sqrt{2} & 3 & -7 \\ 0 & 0 & -7 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} b_1 & c_1 & 0 & 0 \\ c_1 & b_2 & c_2 & 0 \\ 0 & c_2 & b_3 & c_3 \\ 0 & 0 & c_3 & b_4 \end{bmatrix} \tag{5.2.17}$$

The matrix T is a real tridiagonal matrix orthosimilar to A.

Eigenvalues and eigenvectors of a real symmetric tridiagonal matrix. Let T be an $n \times n$ real symmetric tridiagonal matrix with diagonal entries $t_{ii} = b_i$ and off diagonal entries $t_{i,i+1} = t_{i+1,i} = c_i$. We may assume that all the c_i are *not zero* since if one or more were zero we could partition T into a direct sum of tridiagonal blocks and apply the theory to each block separately.

Since T is hermitian, all its eigenvalues λ_j are real. If X_j is a corresponding eigenvector, it satisfies the equations

$$(T - \lambda_jU)X_j = 0 \tag{5.2.18}$$

On dropping subscripts j, these equations take the form

$$
\begin{aligned}
0 &= (b_1 - \lambda)x_1 + c_1x_2 \\
0 &= c_1x_1 + (b_2 - \lambda)x_2 + c_2x_3 \qquad (5.2.19)\\
0 &= c_2x_2 + (b_3 - \lambda)x_3 + c_3x_4 \\
&\cdots \\
0 &= c_{n-1}x_{n-1} + (b_n - \lambda)x_n
\end{aligned}
$$

The following multiples of the ratios x_k/x_1 are values at an eigenvalue λ_j of certain monic polynomials $P_k(\lambda)$ of degree k. The last of these, $P_n(\lambda) = D(\lambda)$ is the characteristic polynomial. We compute its value at λ without evaluating its coefficients explicitly.

$$
\begin{aligned}
P_0(\lambda) &= 1 &&= 1 \\
P_1(\lambda) &= \frac{c_1x_2}{x_1} &&= \lambda - b_1 \\
P_2(\lambda) &= \frac{c_1c_2x_3}{x_1} &&= (\lambda - b_2)P_1(\lambda) - c_1{}^2P_0(\lambda) \\
P_k(\lambda) &= \frac{c_1c_2\ldots c_kx_{k+1}}{x_1} &&= (\lambda - b_k)P_{k-1}(\lambda) - c_{k-1}{}^2P_{k-2}(\lambda) \\
P_n(\lambda) &= D(\lambda) &&= (\lambda - b_n)P_{n-1}(\lambda) - c_{n-1}{}^2P_{n-2}(\lambda)
\end{aligned} \qquad (5.2.20)
$$

Equations (5.2.19) are equivalent to (5.2.20) plus $P_n(\lambda) = 0$. By evaluating successive polynomials $P_k(\lambda)$ at a value λ_j for which $P_n(\lambda_j) = 0$, we compute the components

$$\frac{x_k}{x_1} = \frac{P_{k-1}(\lambda_j)}{c_1c_2\ldots c_{k-1}} \qquad (5.2.21)$$

of the corresponding eigenvector X_j at the same time, provided that the denominators $c_1c_2 \ldots c_{k-1}$ are not so small that the indicated division is unstable.

These polynomials have the important property that at any root of $P_{k-1}(\lambda)$ the polynomials $P_k(\lambda)$ and $P_{k-2}(\lambda)$ have opposite signs. Hence if $\nu(\lambda)$ denotes the number of variations of sign (sign changes) in the sequence $P_0 = 1$, $P_1(\lambda)\ldots P_n(\lambda)$ then $\nu(\lambda)$ changes only when λ passes through a root of $P_n(\lambda)$ itself. The polynomials $P_k(\lambda)$, for $k = 0, 1, \ldots, n$, are said to form a *Sturm sequence*.

Theorem 5.2.1 The variation count $\nu(\lambda)$ in the sequence $P_k(\lambda)$ is equal to the number of roots of $P_n(\lambda) = 0$ that exceed λ.

Proof: Each polynomial P_k has leading coefficient 1. Hence for large negative λ the signs alternate, whereas for large positive λ they are all positive. Hence

$$\nu(-\infty) = n \qquad \nu(+\infty) = 0 \qquad (5.2.22)$$

As λ increases, no variation change occurs except a loss of one variation in passing each root. Q.E.D.

Note that when $c_k \neq 0$, the eigenvalues of T are all *real and distinct*.

To locate the eigenvalues of T, we first seek some convenient upper and lower bounds expressed in terms of the easily computed quantities

$$S_1 = \text{tr } T = \Sigma\lambda_j = \Sigma b_i = n\bar{\lambda} \qquad (5.2.23)$$

$$S_2 = \text{tr } T^2 = \Sigma\lambda_j{}^2 = \Sigma b_i{}^2 + 2\Sigma c_i{}^2 = n(\sigma^2 + \bar{\lambda}^2) \qquad (5.2.24)$$

where $\bar{\lambda}$ and σ denote the mean and standard deviation of the n real eigenvalues.

Theorem 5.2.2 The eigenvalues λ_j of the real symmetric tridiagonal matrix T satisfy the inequalities

$$\left|\lambda_j - \frac{S_1}{n}\right|^2 \leq \left(1 - \frac{1}{n}\right)\left(S_2 - \frac{S_1{}^2}{n}\right) = M^2 \qquad (5.2.25)$$

where S_k is the trace of T^k.

Proof: Since all eigenvalues λ_j of T are real

$$
\begin{aligned}
&\left|\lambda_j - \frac{S_1}{n} \pm \frac{M}{n-1}\right|^2 \leq \sum_{k=1}^{n}\left(\lambda_k - \frac{S_1}{n} \pm \frac{M}{n-1}\right)^2 \\
&\leq S_2 - \frac{2S_1{}^2}{n} + \frac{S_1{}^2}{n^2} + \frac{nM^2}{(n-1)^2} = \left(\frac{nM}{n-1}\right)^2 \qquad (5.2.26)\\
&\left|\lambda_j - \frac{S_1}{n} \pm \frac{M}{n-1}\right| \leq M + \frac{M}{n-1}
\end{aligned}
$$

Hence

$$\left|\lambda_j - \frac{S_1}{n}\right| \leq M \qquad (5.2.27)$$

Q.E.D.

To compute the eigenvectors and eigenvalues of T

1. Find lower and upper bounds, such as

$$\frac{S_1}{n} \pm \frac{[(n-1)(nS_2 - S_1{}^2)]^{1/2}}{n} \qquad (5.2.28)$$

A less exact but more easily computed pair of bounds for eigenvalues λ_j of T is $-L$ and L, where

$$L = \max_i \sum_j |t_{ij}| \qquad (5.2.29)$$

2. Using (5.2.20), compute the sequence $P_k(\lambda)$, $k = 0, 1, \ldots, n$, for several values of λ between the upper and lower bound, so that each change in variation count $\nu(\lambda)$ is located between consecutive integers.
3. Obtain as close an approximation as desired for each eigenvalue λ_j and check that the sum is $S_1 = \text{tr } T$.
4. After finding each eigenvalue to the desired accuracy, we may compute the corresponding eigenvector components from (5.2.21) if the denominators involved do not introduce large round-off errors. Otherwise a more stable method may be required.

Example 6 Find the eigenvalues and eigenvectors of the matrix T of (5.2.17).

Solution: First find S_1, S_2, and the bounds.

$$S_1 = \Sigma b_i = 4 - 6 + 3 + 5 = 6 \qquad \bar{\lambda} = \frac{6}{4} = 1.5$$

$$S_2 = \Sigma b_i{}^2 + 2\Sigma c_i{}^2 = 86 + 2(1 + 18 + 49) = 222$$

$$M^2 = \frac{3(888 - 36)}{4^2} = \frac{639}{4} = 159.75 = (12.64)^2$$

$$-11.14 = \bar{\lambda} - M < \lambda_j < \bar{\lambda} + M = 14.14 \qquad (5.2.30)$$

Next compute Sturm sequences $P_k(\lambda)$ for some values $\lambda = -8, -2, 5$, and 11 spaced through the interval and find that the variation counts are 3, 3, 1, and 1, respectively. Check that the counts at $\lambda = -9, -1, 4$, and 12 are respectively 4, 2, 2, and 0, and conclude that

$$
\begin{aligned}
-9 < \lambda_1 < -8 \qquad -2 < \lambda_2 < -1 \\
4 < \lambda_3 < 5 \qquad 11 < \lambda_4 < 12
\end{aligned} \qquad (5.2.31)
$$

Successive polynomials $P_k(\lambda)$ and the related components x_{kj} of the eigenvector X_j are

$$P_0 = 1 = x_{1j}$$

$$P_1 = \lambda - 4 = -x_{2j}$$

$$P_2 = (\lambda + 6)P_1 - 1 = -3\sqrt{2}x_{3j}$$

$$P_3 = (\lambda - 3)P_2 - 18P_1 = 21\sqrt{2}x_{1j}$$

$$P_4 = (\lambda - 5)P_3 - 49P_2 = D(\lambda) \qquad (5.2.32)$$

A few of these computations and the variation count $\nu(\lambda)$ will illustrate the procedure for closing in on a root. For machine computation, the cruder bounds −16 and 16 might be chosen instead, and the test values for λ would be found by successive bisection of intervals in which the variation count changes.

λ	P_0	$P_1(\lambda)$	$P_2(\lambda)$	$P_3(\lambda)$	$P_4(\lambda) = D(\lambda)$	$\nu(\lambda)$
−8.0	1	−12.0	23	−37	−646	3
−9.0	1	−13.0	38	−222	1246	4
−8.4	1	−12.4	28.76	−104.66	−6.74	3
−8.404	1	−12.404	28.8192	−105.3823	0.4033	4
−2.0	1	−6.0	−25.0	233	−406.0	3
−1.0	1	−5.0	−26.0	194	110.0	2
−1.23	1	−5.23	−25.9471	203.896	1.1358	2
5.0	1	1.0	10.0	2.0	−490.0	1
4.0	1	0.0	− 1.0	− 1.0	50.0	2
4.102	1	0.102	0.0304	− 1.803	0.1289	2
11.0	1	7.0	118.0	818.0	−874.0	1
12.0	1	8.0	143.0	1143.0	994.0	0
11.53	1	7.53	131.0009	981.8977	− 7.2523	1

(5.2.33)

Successive approximations of the same type, assisted by interpolating in the column of values of $D(\lambda)$, lead to the following values for λ_j.

Eigenvalues λ_j

$$\begin{aligned} \lambda_1 &= -8.403\,774\,776\,0 \\ \lambda_2 &= -1.232\,324\,345\,5 \\ \lambda_3 &= 4.102\,259\,883\,8 \\ \lambda_4 &= 11.533\,837\,937\,7 \\ \text{Sum} &= 2.000\,000\,000\,0 \end{aligned} \qquad (5.2.34)$$

These eigenvalues λ_j of T are also eigenvalues of A. Corresponding eigenvectors of T in (5.2.17) are computed from (5.2.32).

Eigenvectors X_j^T

$$\begin{array}{llll} (1, & 12.403\,774\,48, & -6.791\,967\,02, & -3.547\,043\,00) \\ (1, & 5.232\,323\,35, & 6.115\,537\,16, & 6.868\,828\,26) \\ (1, & -0.102\,259\,88, & -0.007\,791\,36, & -0.060\,751\,98) \\ (1, & -7.533\,837\,94, & -30.899\,881\,2, & 33.104\,458\,1) \end{array} \qquad (5.2.35)$$

The mutually orthogonal eigenvectors X_j could be reduced to unit vectors to form an orthogonal modal matrix X for T. Constituent idempotent matrices for T are $X_j^T X_j/(X_j X_j^T)$.

A simple left multiplication of X by the matrices S_k used in transforming A in (5.2.11) to T in (5.2.17) gives an orthogonal modal matrix S_3S_2X for A.

Conclusion

Through use of examples amenable to hand computation, this series of articles has illustrated a few of the many methods for calculating eigenvalues and eigenvectors of matrices. A thorough analysis of the effect of round-off error in machine computations is important, but beyond the scope of this study. The interested reader should consult recent papers by Wilkinson, Lanczos, Givens, Householder, and others.

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How a computer system can learn

The process by which a computer can learn is demonstrated by asking it to solve increasingly more difficult versions of the Tower of Hanoi puzzle. Ultimately, the system may learn how to generalize in a particular problem domain

Aiko Hormann *System Development Corporation*

Present uses of computers, valuable as they may be, are far from the ultimate in what might be accomplished. One of the reasons is that the solution of even well-defined problems, for which goals and rules are precisely known, can be extremely difficult to program (e.g., chess-playing programs). But intellectual capacities of machines might be extended by means of an adaptive system to handle increasingly complex and varied tasks.

A possible approach might be to preprogram a system. With this technique, a large background of information and capabilities—to the limits of human ability in both programming techniques and knowledge of the problem domain—is injected into the system. The system is then left on its own to learn the rest of the techniques required for problem solution, provided that efficient learning capabilities are also in the system.

Some problems that are not well defined, such as those which arise in socioeconomic, industrial, and military problem situations, might be handled effectively and economically by a good adaptive system that can accept repeated reformulations and refinements of given problem definitions and problem-solving strategies and methods.

The research effort to be described is concerned with the fundamental question of how such an adaptive, intelligent system can be constructed. The writer believes that achieving a machine intelligence of high caliber, in both diversity and brilliance, may be illusory but that a great deal can be done now to raise the present level of machine intelligence and to find new ways to use computers.

Although some of the basic questions man has been asking about himself—How do men learn to generalize?, What separates a good problem solver from a poor one?—remain unanswered, there are useful results from these investigations that have yielded deeper insight into the problem of machine intelligence. The design of our research machine, called Gaku after the Japanese word denoting learning, has been influenced by these results. However, some nonhuman features of the machine have also been incorporated to explore many possibilities and to discover fruitful techniques that are not necessarily a deliberate imitation of those used by humans.

The features considered and the techniques employed have been tried on a highly experimental basis. We are not sure of their workability or desirability, separately or together. As a result, many trials and modifications are anticipated.

Before the practical application of an adaptive system to solving important problems can be considered, simpler, well-defined, and completely solved problems must be investigated for an easier evaluation of the machine's attempts at solution. This has been done with Gaku.

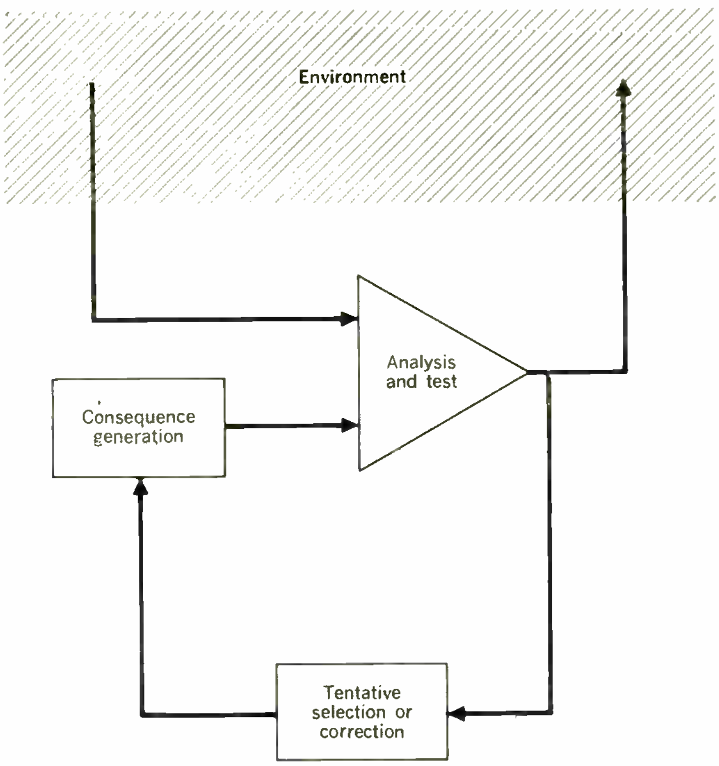


Fig. 1. Three-phase cycle characteristic of how humans solve problems can be illustrated as shown. Note the feedback loop.

Behavior of Gaku

In a typical problem-solving situation, a human learner has very little information about *how* to solve a given problem even though the problem may be completely and precisely defined. Often at first his behavior looks like random trial and error. As the problem unfolds, however, his behavior becomes more selective, directed, and organized. Previously inadequate actions are corrected or adjusted by the use of new information that gradually becomes available as a consequence of previous actions. The solution strategy is often discovered, as it were, in the course of the action itself.

These characteristics of humans solving problems may be abstracted into a cycling process as shown in Fig. 1. The cycle passes through three phases: analysis and test, tentative selection or correction, and consequence generation. A feedback loop is formed when the analysis and test phase of the cycle receives the consequence of a proposed course of action from the consequence generation phase. Upon re-entering the analysis and test phase, reformulation or reanalysis of the given task is done by comparing the consequences received and the description of the given task. A selection of a new course of action or a modification of the previously proposed act is then effected in the tentative selection or correction phase. The three phases are passed repeatedly until either a success or a failure is finally determined in the analysis and test phase of the cycle.

The complete system has four mechanisms, which are coordinated by a fifth called a mechanism coordinator, as illustrated in Fig. 2. Each of the four mechanisms uses the cycling process just described, but each works in a different stage of problem-solving activity.

The programming mechanism. The purpose of the programming mechanism is internal programming—the construction of programs by manipulation of basic operations and prestructured programs. Specific objects manipulated by the programming mechanism are sometimes basic operations, sometimes previously generated subroutines (modified or unmodified), and sometimes a mixture of both, depending on the system's past experience and the particular task requested of the mechanism.

The problem-oriented mechanism. Sequences of unit actions that are defined by a given task environment are manipulated and generated by the problem-oriented mechanism. It accomplishes this action either directly or through the programming mechanism. Since the problem-oriented mechanism constructs and actually carries out the required sequence of legal moves in solving the given problem, the mechanism directly determines the behavior of the system.

Some useful methods and techniques are provided for

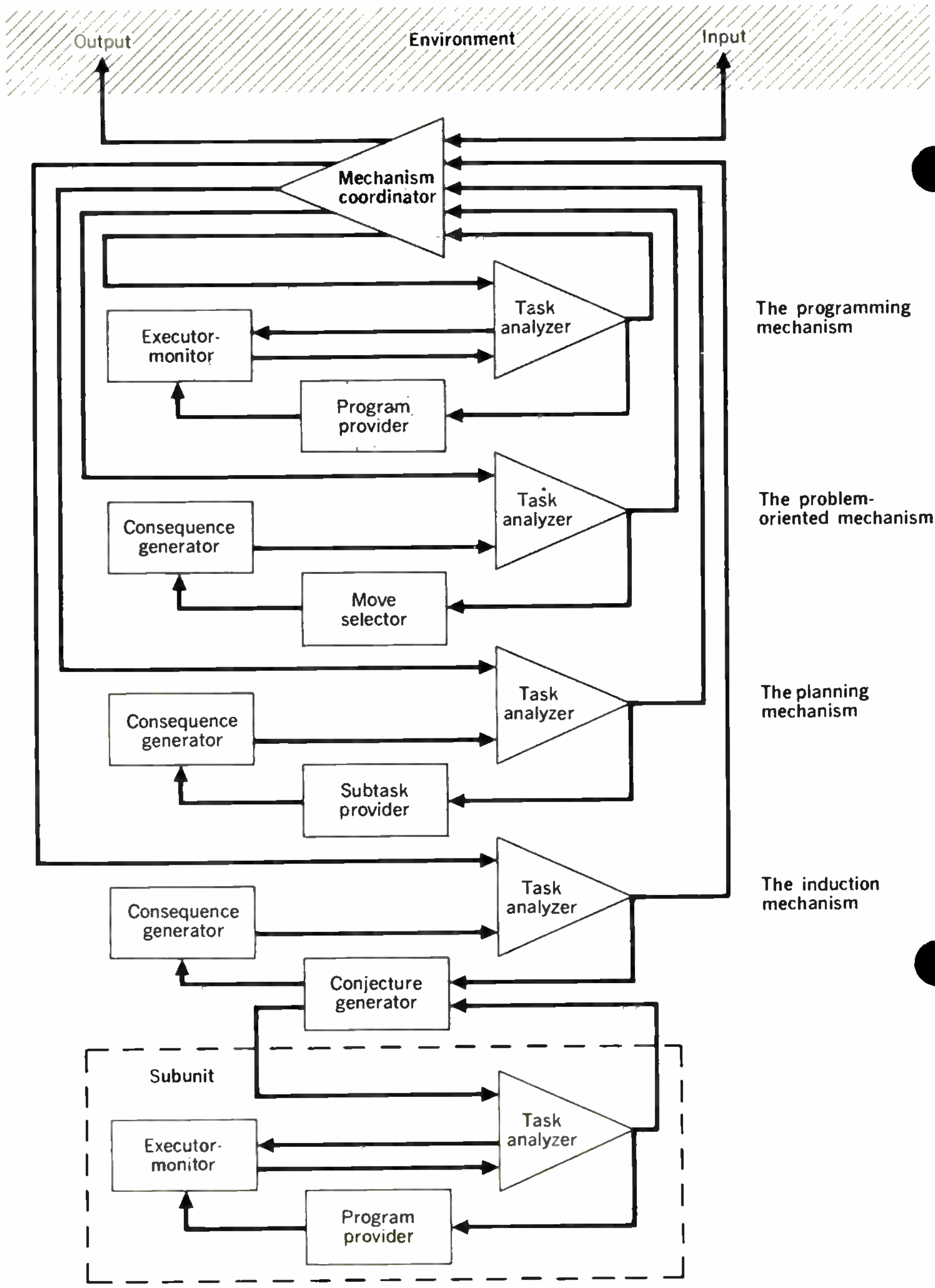


Fig. 2. Four mechanisms used in Gaku are coordinated by a mechanism coordinator. Each mechanism contains the cycling process shown in Fig. 1.

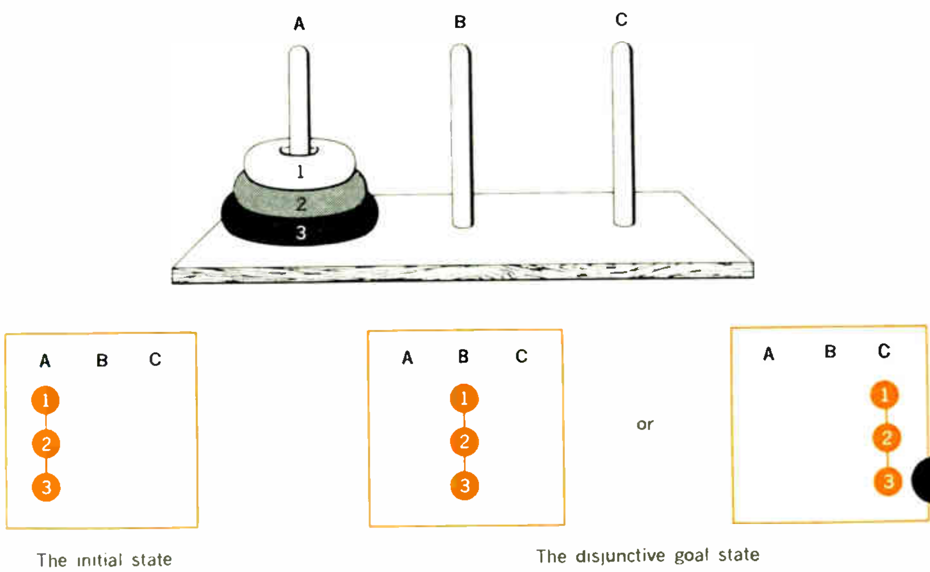


Fig. 3. The three-disk version of the Tower of Hanoi puzzle. The upper portion shows the actual puzzle. The lower portion shows how the computer looks at the puzzle with counters instead of disks, and cells for pegs.

enabling the mechanism to search for a usable sequence of unit actions more efficiently than if it were to search simply by exhaustive or random trial and error. These heuristic methods and techniques are of a step-by-step nature. For complex problems, however, such step-by-step heuristics will fail unless there is also a mechanism for analyzing problem structure and placing guideposts on the road to the goal. Because of the piecemeal manner of attacking problems, the problem-oriented mechanism tends to be hindered within a narrow and restricted framework, a situation that is remedied with the assistance of the planning mechanism.

The planning mechanism. A larger view of a given task, often at an abstract level, is taken by the planning mechanism. After surveying the task as a whole, the planning mechanism subdivides the task into a hierarchy of subtasks each of which is presumably easier to perform than the original. This hierarchy of subtasks comprises a rough sketch of a possible course of action that guides the problem-oriented mechanism.

Specific objects manipulated by the planning mechanism are called "state" descriptions—i.e., descriptions of a set of conditions that represent stepping stones or intermediate "nodes" to the final solution. Finding a solution can be thought of as establishing or selecting a valid path—one made up of legal or allowed steps—between successive intermediate states. The individual states are handled by the mechanism coordinator by instructing the problem-oriented mechanism to solve only one portion of the problem at a time.

The induction mechanism. The induction mechanism takes a still larger view of a given task. It does so by surveying the system's past experience with various problems, and applies this experience to related problems that have not been encountered previously. In this manner the induction mechanism influences the problem-oriented mechanism to make efficient use of its past experience.

The mechanisms that have been described represent an implicit means for telling Gaku how to solve problems. Although an often heard expression, "A machine can do only what it is told to do," is true, the triviality implied by "only" is far from realistic. There are many ways to tell a machine what to do. For example, Gaku is not told how exact steps of a solution procedure are carried out for each new problem. Instead, it is given the four mechanisms described, with general rules for making decisions. The objective is for the total system to be able to execute a discriminating search, through a large number of possible combinations, to find combinations which will serve as solutions to the given problem. Such a procedure is more effective than is a random or exhaustive search.

The Tower of Hanoi puzzle

The role played by each mechanism and other features of Gaku become more meaningful when presented in a particular problem-solving context.

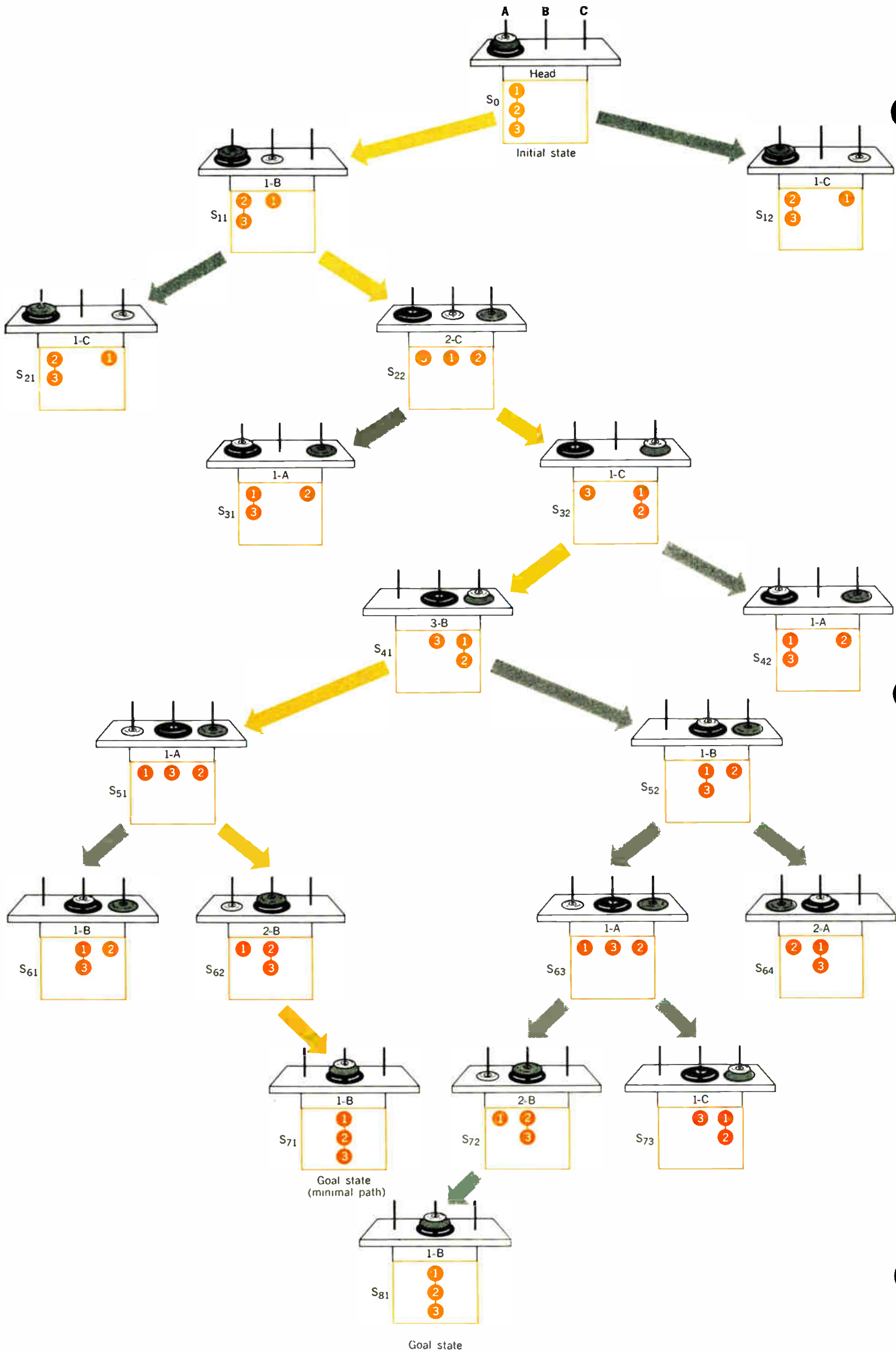
The Tower of Hanoi puzzle was invented by a French mathematician and sold as a toy in 1833. It was originally described as a simplified version of a mythical Tower of Brahman in a temple in the Indian city of Benares. The tower was said to consist of 64 disks of gold originally stacked on one peg that were being transferred by the temple priests, according to a precise ritual, to one of two other pegs. Once the transfer was complete, according to the myth, the temple would crumble into dust and the world would vanish in a clap of thunder. Assuming the priests worked night and day, moving one disk every second, and provided they knew the shortest sequence of moves, it would take about 585 billion years to finish the job.

The Tower of Hanoi puzzle is one of many puzzles that can be described as board-and-counter games for which the required solution is a sequence—often specified to be the shortest sequence possible—of legal moves that start with the given initial state and reach a desired state. In this case, the problem, as displayed in Fig. 3, is to transfer the tower of disks from one peg to either of two empty pegs in the fewest possible moves. Only one disk may be moved at a time and no disk may ever be placed on top of a smaller disk.

The Tower of Hanoi puzzle was chosen as the first testing vehicle for Gaku's performance for several reasons. The solution is relatively simple but not trivial. For example, when human subjects are given the task of finding the shortest sequence of moves for eight disks, the time required for solution ranges from minutes to days. For some it is unsolvable. Also, evaluation of performance is easier as the solution is known to the experimenter. Finally, the puzzle can be varied by altering the number of disks and pegs, thus allowing a training sequence from the simple to the more difficult within the same class of tasks. The puzzle has the important property that the methods for simple cases, with suitable abstraction, do provide some help in solving harder cases in a fairly nontrivial way.

Puzzle-solving procedure. The mechanism coordinator receives Gaku's first task, the three-disk version of the Tower of Hanoi as shown in Fig. 3. In addition, conditions for legal moves tell Gaku that if the exploration should go beyond 50 nodes or 10 levels, whichever is reached first, in the tree of intermediate states and attempted moves, that fact should be reported to the trainer who will then feed back appropriate suggestions.

The mechanism coordinator's first task is to activate routines for abstraction and characterization of the given task. To achieve relative simplicity in this informal description, many programming details and special terms are avoided. They are replaced by more general, perhaps more vague, terms that are employed on an intuitive basis. The abstraction routine compares the initial state and the goal state one part at a time and records its findings. For example, as far as Gaku is concerned, the disks are counters numbered from 1 to 3 with number 1 the smallest and number 3 the largest. The pegs are represented by cells A, B, and C. As part of the abstraction routine, such items are considered as the number of counters (numbered disks) involved, positions of the counters, and the contents of cells (pegs) in the initial state that match those in the goal state, both with and without taking into account the order of occurrence of the counters. The resulting abstracts are then processed by the characterization routine that helps Gaku create subgoals and preference criteria in selecting moves. Significant among them are *preferred moves* that take counters out of cell A or disks off peg A (since the ultimate goal is to get all counters into cell B or C or all disks onto peg B or C); those moves that take counters into cell A are labeled undesirable, since this is a step backward.



A
B
C
Head
S_0
Initial state
1-B
S_{11}
1-C
S_{12}
1-C
S_{21}
2-C
S_{22}
1-A
S_{31}
1-C
S_{32}
3-B
S_{41}
1-A
S_{42}
1-A
S_{51}
1-B
S_{52}
1-B
S_{61}
2-B
S_{62}
1-A
S_{63}
2-A
S_{64}
1-B
S_{71}
Goal state
(minimal path)
2-B
S_{72}
1-C
S_{73}
1-B
S_{81}
Goal state

If Gaku had had previous experience with similar puzzles, their characteristics in the past-experience record would indicate possible assistance toward solution by the planning mechanism and the induction mechanism. For this first task, however, there is no record of past experience. As a result, the mechanism coordinator activates the problem-oriented mechanism. The task analyzer portion of the problem-oriented mechanism, having received the information from the mechanism coordinator, creates the first node of a move-tree (see S_0 in Fig. 4) that contains the initial state. The move selector then finds two moves called 1-B and 1-C. Move 1-B means move counter 1 to cell B (move the white disk to peg B) and move 1-C means move counter 1 to cell C (move the white disk to peg C), leading to nodes S_{11} and S_{12}, respectively, as shown in Fig. 4. Since both moves are equally preferred (they take a counter out of cell A, which is equivalent to taking a disk off peg A), the task analyzer chooses one of them randomly, stores it as the new current state, and creates a new node corresponding to S_{11} in the move-tree.

The next move chosen is 2-C because it again meets the preference criterion of taking a counter out of cell A. The other legal move, 1-C, does not meet this preference criterion and, therefore, is rejected. As a result, S_{22} is designated as the next current state. At state S_{22}, neither of the two legal moves 1-A or 1-C is preferred since neither takes a counter out of cell A. Consulting with the preference criteria, the move selector discovers that moving counter 3 either to cell B or cell C (moving the black disk either to peg B or peg C) is desirable. The move selector then checks to see if this can be done by giving the suggested moves 3-B and 3-C. The consequence generator, in an attempt to make the suggested moves, checks the legality conditions and finds that counter 3 can be moved only to an empty cell (black disk to an empty peg). The current state S_{22} in Fig. 4 shows clearly that neither move 3-B nor 3-C can be made.

Removal of this impediment is now attempted by a recursive use of the problem-oriented mechanism. The consequence generator, which is itself a part of the problem-oriented mechanism, makes a request that either B or C be made empty without disturbing counter 3 (black disk) in A, and calls the problem-oriented mechanism. Thus, the entire cycle of the mechanism becomes involved again at one lower level.

The task analyzer automatically takes care of this second-level entrance by a push-down cell, peculiar to the list-processing technique. This action is necessary so that the second entrance to the mechanism, before exit is made from the first involvement of the mechanism, does not destroy the information needed for exit from the first one.

At the lower level, preference criteria, generated by the request to make B or C empty, tell the move selector that those moves taking counters out of B or C are preferred. The move selector generates 1-A and 1-C as legal moves from which 1-C is selected because 1-A is listed as undesirable. The consequence generator produces this as the new current state S_{32}. Since the requested condition has been satisfied, the signal "task accomplished" is sent out and exit from the entire mechanism is made.

When the control is returned to the consequence generator that initiated the request, the current state S_{32} now has B empty and the move 3-B can be made to produce the new state S_{41}.

At state S_{41}, move 1-B is preferred to 1-A because the latter is listed as undesirable (the move does not keep cell A empty). This move leads to the state S_{52} and more exploration of the move-tree is necessary until the goal state S_{81} is reached. Since the path leading to S_{71}, as shown by colored rules or branches in Fig. 4, is a minimal path, Gaku is instructed to find a sequence shorter than eight steps. More backtracking and exploration of the move-tree occurs until finally the goal state S_{71} is reached in seven steps as required.

In recording its experience with the three-disk case, Gaku notes its characteristics and preference criteria used successfully in choosing moves. An unsuccessful criterion, which previously influenced the problem-oriented mechanism not to move any disks back to peg A, after A becomes empty, receives a negative value. The mechanism coordinator records the sequence of successful moves for the three-disk case along with its abstraction, which says, in essence, that "moving disk V from A to B was accomplished," where V is a grouping parameter that combines the three disks into a single symbol.

Extensions of the puzzle

Next, Gaku is given the four-disk case of the Tower of Hanoi puzzle as shown in Fig. 5. This time the mechanism coordinator can utilize its past experience. Gaku exhibits its early stage of learning and shows how a little information on past experience can be used to best advantage by coordinating the functions of the problem-oriented mechanism and the induction mechanism.

To produce a new list of characteristics, the mechanism coordinator activates the abstraction and characterization routines as before. This new list is then compared item by item with the old one. The comparison discloses that the new task is identical to the old one except that the new task involves one more disk. The old record on the three-disk case is brought back to the working memory. With this much preparation, the mechanism coordinator activates the planning and induction mechanisms to aid the performance of the problem-oriented mechanism. These two mechanisms work simultaneously and independently of each other, each reporting its suggestions to the mechanism coordinator.

With the information in the working memory provided by the mechanism coordinator, the task analyzer of the planning mechanism uses the abstracted form of the accomplished task to suggest the context in which the subtask provider is to work. As shown in Fig. 6, the subtask provider finds two legal moves, V-B and V-C, from the initial state, but chooses V-B, a move already recorded as being solved. This choice is given to the consequence generator, which then provides the new current state, S_{11}.

It is illegal for the problem-oriented mechanism to move more than one counter at a time because the mech-

Fig. 4. Part of a move-tree for the three-disk version of the Tower of Hanoi puzzle. The smallest disk, which is white, corresponds to counter 1; the gray disk corresponds to counter 2; and the black disk to counter 3. The shortest sequence of moves from the initial state to the goal state is shown by the colored rules.

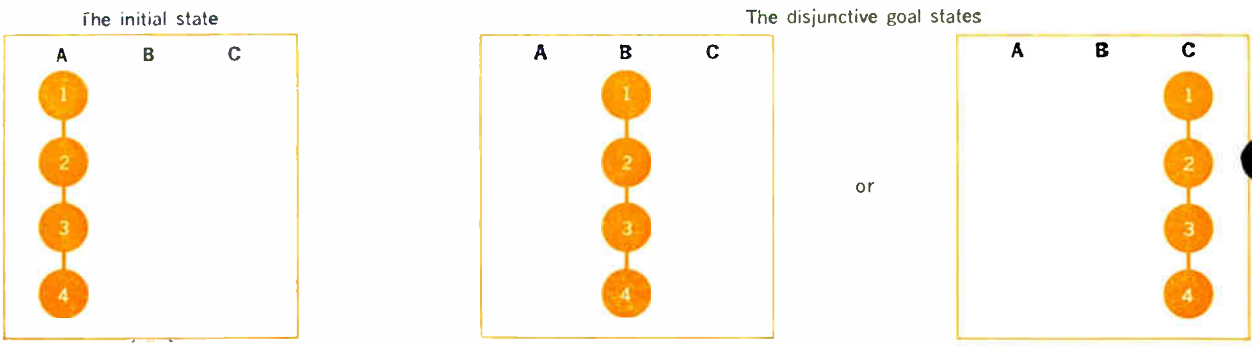


Fig. 5. The four-disk version of the Tower of Hanoi puzzle.

Fig. 6. Subtasks of the four-disk version of the Tower of Hanoi puzzle.

anism deals only with unit actions defined by the task. However, in the planning mechanism, the main objective is to get an overall picture of the entire problem. Therefore, the three disks are moved together as if they were one, a technique analogous to using a previously proved theorem in theorem-proving steps. In other words, the current proof is reached in one step while the utilized theorem itself requires many steps.

With move 4-C chosen as being in the preferred list, state S_{21} is formed and stored as the third item. The fourth item is the goal state. This list of items or states is processed by the mechanism coordinator, taking a pair of states at a time, to form a subtask. The subtasks (M_1, M_2, and M_3 in Fig. 6) thus created are given to the problem-oriented mechanism for the actual performance that will be acceptable by the environment.

What if the four-disk case is given a unique goal state S_{32} instead of two possible states, and S_{31} is arrived at unsuccessfully? The mechanism then has to explore more intermediate states to finally reach S_{32}, possibly ending up with an exhaustive search. However, planning of this kind is relatively cheap. It takes only a few examinations at this abstracted level to find the path exhaustively. In contrast, if the individual moves are considered at the level of detail necessary in the problem-oriented mechanism, an exhaustive search for the four-disk case will take 65 534 examinations of intermediate states.

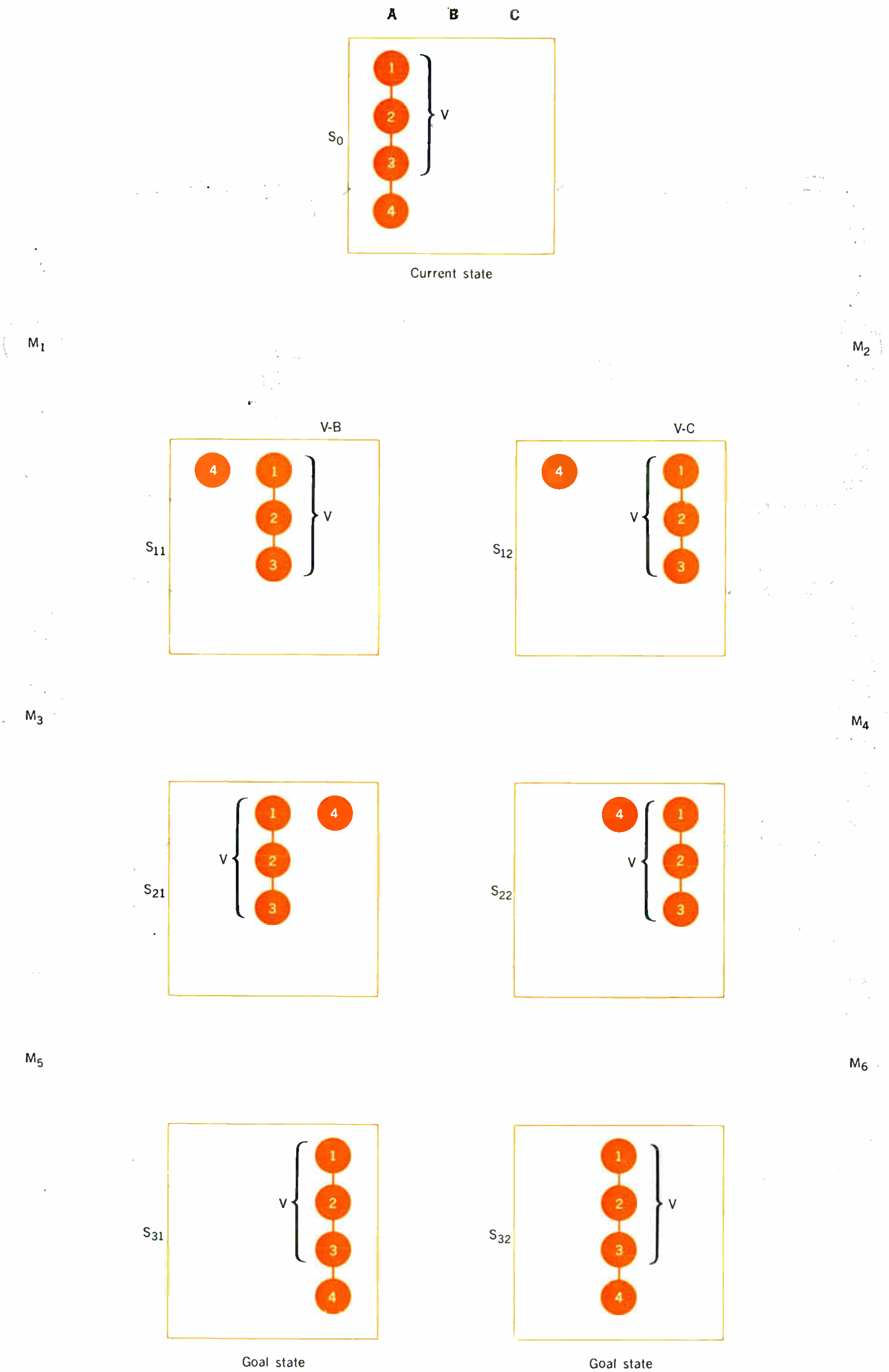
In the meantime, the induction mechanism attempts to conjecture a general pattern of successful moves from the three-disk case in order to suggest a sequence of moves for the four-disk case. The task analyzer of the induction mechanism analyzes a given sequence of successful moves to determine a pattern category to be tried out, and passes the information on to the conjecture generator. It then produces programs that represent the conjectured pattern with the aid of its own subunit (see the induction mechanism in Fig. 2). The suggested moves are generated by the produced programs under the supervision of the consequence generator and are given to the mechanism coordinator, which at this point is able to use the total information obtained from the planning and induction mechanisms to guide the problem-oriented mechanism.

For the given sequence of successful moves in this example, the task analyzer makes the decision that the "cyclic" pattern category is to be tried first. The conjecture generator then activates its subunit, which is very much like the programming mechanism but has more limited domain and range for its input and output. The first input to this subunit is the list of counters 1 2 1 3 1 2 1 with the cyclic signal that causes the task analyzer of the subunit to look for and find the first recurrent position of the first item on the list—in this case, the third item.

The task analyzer now takes the first two items, 1 and 2, as defining a cycle phase, and asks the program provider to construct a program that will generate 12 repeatedly as 1 2 1 2 . . . The constructed program is executed by the executive-monitor and the results are checked by the task analyzer, one by one, against the given list until a point of mismatch, the fourth item, is detected. A new cycle phase is found to be 1 2 1 3 and a program is produced to generate 1 2 1 3 repeatedly. The conjecture generator, which activated the subunit, now receives a success signal and the generated program.

The same procedure is used for the list of cell names B C C B A B B, and the repeated pattern finally accepted turns out to be B C C B A B. The conjecture generator combines these two programs so that together they will produce a sequence of counter-cell pairs as suggested moves, and outputs the resulting program to the consequence generator.

The consequence generator produces a sequence of moves by executing the generator program. Since the total number of moves for the four-disk case is currently unknown, the mechanism uses "20" from the given limit of effort this time with contents 100 and 20 and generates 20 moves that are then output to the mechanism coordinator.


A
B
C
S_0
V
1
2
3
4
Current state
M_1
M_2
V-B
V-C
S_{11}
S_{12}
M_3
M_4
S_{21}
S_{22}
M_5
M_6
S_{31}
S_{32}
Goal state
Goal state

The mechanism coordinator uses the total information obtained from both mechanisms to guide the problem-oriented mechanism that attempts to solve the four-disk case. Actions of the problem-oriented mechanism are more straightforward this time because, instead of examining all the legal moves at each state, it uses the suggested move as the first choice trial. There is, however, some exploration of the move-tree because the suggested sequence contains some wrong moves.

After the complete sequence of moves is found, the correct and suggested moves are compared and unmatched elements noted in order to determine whether the pattern-generator programs can be modified or whether a new pattern must be tried. A comparison of the suggested and correct moves is:

Suggested moves	1 2 1 3 1 2 1 3 1 2 1 3 1 2 1
	B C C B A B B C C B A B B C C
Correct moves	1 2 1 3 1 2 1 4 1 2 1 3 1 2 1
	B C C B A B B C C A A C B C C

Boldface items indicate where the mismatch occurred.

The mechanism coordinator, after the comparison, informs the induction mechanism of its decision that the modification of the programs must be tried.

The conjecture generator of the induction mechanism then modifies previously constructed programs by parameterization; i.e., it replaces unmatched symbols with parameters.

When the five-disk case is presented, the same procedure of making suggested moves and comparing them against the successful moves is followed and, again, if a mismatch is found, a subsequent modification is made by parameterization.

When the new programs are used to suggest moves for the six-disk case, all of them prove to be correct. In fact, the parameterized program, which has now been constructed, will solve all n-disk cases as long as the initial state has n disks in A and the possible goal states are given. Of course, Gaku will never know the facts unless told by the trainer. As Gaku becomes more experienced with the puzzle, and the conjecture program is used successfully, the performance value of the conjecture increases so that the mechanism coordinator will direct a straightforward use of the program.

Primary and secondary learning

Research on Gaku has been concerned mainly with primary learning—learning by firsthand experience through adaptation to new situations. Future research, part of which has already been started, will include secondary learning—learning from outside sources other than from a person's own experience, e.g., for humans, reading books and listening to teachers, parents, and friends.

When the human analog is considered, secondary learning constitutes a large part of an average person's learning. Unless a man were brought up as a wolf-child, he could hardly avoid learning through verbal communication. Indeed, through his verbalization ability, man's learning can extend beyond the limitations of time and space. As an illustration of a situation in which a man is deprived of secondary learning, an average person will make very slow progress in learning to play chess solely from play experience, devoid of learning from chess books and from communicating with his fellow players.

It is even speculated that some generalization processes are learned through secondary means. Ability to generalize, to make the best possible use of experience, is essential if a man or a brain artifact is to manifest more than trivial intelligence. However, much of the generalization ability can be acquired in classrooms and through textbooks, where general principles are taught along with examples and where some generalizing processes, or acts of generalization, are demonstrated or re-enacted. Sometimes questions are asked in such a way as to encourage the students' own generalization attempts. Learning is often by imitation. Gaku will be given a similar opportunity for secondary learning. It can imitate the processes of generalization for specific cases as shown by its teacher. At a later stage, it could apply the same or similar processes to comparable situations, provided that such similarity concepts were also taught or developed. Attempts to generalize generalization processes may then become possible in an internal bootstrap manner.

Gaku will be given an opportunity for both primary and secondary learning. There will be a mixture of what might be called lectures (secondary learning), exercise problems (primary learning), and hints of graded sequence (secondary learning evoking primary learning) on each subject area, as well as a demonstration of particular problem-solving techniques and generalization processes. At the outset, it is assumed that no false information is given to Gaku, unlike the situation in which humans receive false information that is both intentional and unintentional. Therefore, secondary learning in the system will have more direct effects on Gaku's behavior.

The purpose of integrating into Gaku the ability to undergo secondary learning is threefold. First, it is an attempt to promote Gaku's rapid intellectual growth, as contrasted to the situation where only primary learning is possible. Second, the investigation is expected to yield deeper insight into the problem of generalization and into the effects of primary and secondary learning that are made mutually dependent within the system.

Third, inasmuch as secondary learning depends largely on the ability to communicate, a study of effects of communication languages and internal representation schemes may lead to easier and more efficient communication between men and computers than exists today. With improved communication and more intelligent machines, there could be many interesting and practical applications of dividing intellectual labor between computers and men. The present dividing line for a natural or optimum division—that of letting a computer do more tedius routine work and a man the higher-level thinking—might be shifted.

For the present stage of Gaku, the man–machine relation pertaining to the division of intellectual labor is that the man is Gaku's teacher rather than its colleague. After educating systems like Gaku to a much higher level of intellectual sophistication, men can demand that machines participate to a greater extent in the man–machine partnership in solving more difficult problems.

Conclusions

Aside from the intrinsic difficulties of problem solving, there are many conceptual and technical difficulties that must be overcome before an intelligent learning system can be realized that is really powerful in interesting and practical ways. Among them are two types of communication difficulties: the first is a difficulty of communicating with the computer from the designer and programmer's point of view during the conceptualization and implementation stages of the system; the second involves communication problems between the implemented system and its user or tutor, whichever role a human takes in man–machine interaction. The two are related, of course.

In the first case, the designer's concepts of the inner workings of an intelligent learning system, i.e., of learning processes of mechanisms, memory organization, problem-solving techniques and methods, etc., must be first interpreted in terms of concepts and techniques applicable to computer programming. They must be converted into sufficiently complete and precise specifications which, in turn, must be converted into an internal representation scheme, ultimately reaching to the most concrete level, namely, the machine code. During this conversion from the designer's conceptual scheme of the system to its internal representation in the computer, a great deal of generality and flexibility may be lost if the particular choice schemes is inadequate.

The second type of difficulty is that man communicates with the system with far greater restrictions than he does with his fellow workers. In normal adult communication, understanding of a complex concept expressed in words depends on the listener's background. Additional explanation or paraphrasing may be necessary in more words, but these words themselves are usually rich in concepts and people need not descend to the most concrete level of the verbal communication. What is assumed here is some commonness in the background information, pieces of which are evoked by certain associations with words and objects. People seem to form complex associations and acquire a large body of background information gradually over a long period of time.

Human communication with computers is restricted because existing computer systems do not have a background of information and general information-handling capabilities in any way comparable to that of an average adult. Therefore, it will be desirable to "inject" into the system's memory a large body of organized information about the environment and about the system itself, provided the system also possesses efficient information-handling capabilities. Internal representation of these must be associated with terms and expressions in a language that is flexible and understandable to those who are not necessarily familiar with the system.

In addition to the language restriction or the lack of background knowledge, the lack of adaptability of existing systems constitutes a major impediment to their usefulness to man. It is very difficult to get computers to handle problems that are possible of solution but poorly defined. If the man is allowed to expand or modify (or both) a set of initial assumptions about the problem definition and solution strategies, and if terms and processes that are specifically introduced in the problem context are remembered so that they need not be introduced afresh each time, then the system can "grow" intellectually with the man-user. As the system gains experience, the extent and kind of communication between the system and its user will change radically. This phenomenon can be observed in two people who have worked together on a project. They begin to assume a great deal of common knowledge in their communication, and stop going back to basic definitions when they discuss their problems. An outsider to the project, hearing such conversations, will be baffled.

Of course, this "letting the system grow" technique has its pitfalls. It is generally true that the efficiency of an information-handling system is inversely proportional to the amount and complexity of information involved. However, a system capable of learning should also *learn* to generalize.

It is in the success or failure of attempts to solve this problem that the future of Gaku lies. The *products* of generalization, that is, general rules or principles of unification, can be easily used and evaluated *once they are discovered.* The *processes* of generalization are another matter; only dimly do we understand the ways in which intelligence functions to synthesize large blocks of information and establish useful connections between them. At this point, research on the Gaku system is concentrated on the attempt to discern these processes. As new insights gained in these areas are incorporated into the body of the system, they should increase the capabilities of the system to achieve further insights. Though "general" generalization processes are complex and elusive, enabling a system to generalize in *a particular* problem domain appears possible given the present state of the art of artificial intelligence.

This article is based on an article in the April 1964 issue of *SDC Magazine*, a publication of the System Development Corporation. A more detailed description of Gaku's performance, including an additional example in the area of secondary learning, is scheduled for publication in *Behavioral Science* early next year.

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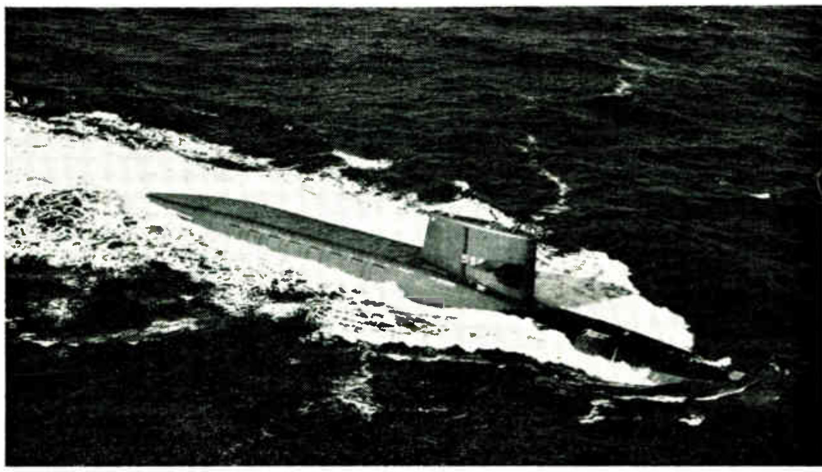
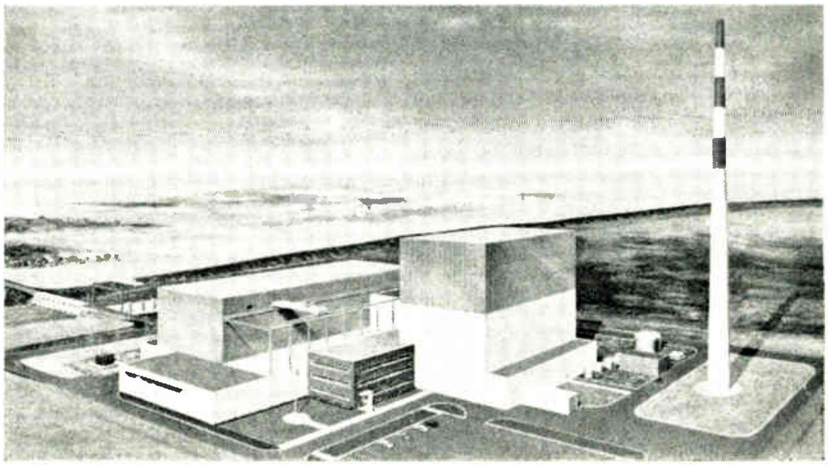
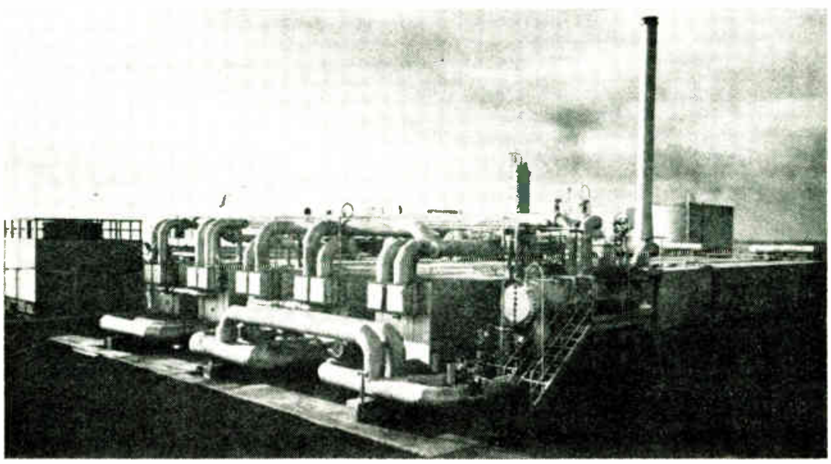




Nuclear power today and tomorrow

U.S. nuclear power plant capacity is now 1.2 GWe. By year 2000, 45 per cent of electric power will be nuclear. Great expansion is also anticipated in ship propulsion, space, and desalting of sea water

Frank Schwoerer, Jr., Warren F. Witzig Nuclear Utility Services, Inc.



In less than 25 years, the nuclear chain reaction has progressed from the laboratory to the bomb to extravagant hopes for peaceful utilization, and to disillusionment when the hopes were not quickly realized. We now arrive at today's position in which nuclear power is the preferred power source in many situations and enjoys excellent prospects for expanded utilization in the future. For electric power generation, nuclear power is already economically competitive in certain wide areas of the world and may soon become a necessity in conserving natural resources and limiting air pollution. Below water and in outer space, nuclear power is the means of performing missions otherwise impossible. These examples highlight the truly remarkable progress that has been made in taming the atom.

About fifteen intermediate and large-scale nuclear reactors for electric power generation have been completed and operated successfully in the United States and Western Europe; approximately 15 more reactors have been contracted for and are in various stages of construction.[1,2] In certain parts of the United States, Europe, and Southeast Asia, it is evident that a high percentage of all future large-scale electric plants will be nuclear powered. This record of accomplishment has been based on steady improvements in the economics of nuclear power together with an outstanding record of safety.

As spectacular as the growth of nuclear power in the generation of electricity, has been the growth of the nuclear U.S. Navy. Today, approximately 40 nuclear ships are in commission, including attack and ballistic-missile submarines, an aircraft carrier, a cruiser, and a frigate (or large destroyer).[1] In naval craft, and particularly in the submarine, nuclear power has provided completely new weapons systems capabilities, exemplified by the polar cruise of the *Nautilus*, the around-the-world submerged cruise of the *Triton*, and the everyday missions of the fleet ballistic-missile submarines.

To a lesser degree than in electric power generation and naval ship propulsion, nuclear power has also been applied to portable power plants for the military services, power plants for earth satellites and rocket propulsion, and the power plant for the nuclear ship *Savannah*.

In general, it can be said that nuclear power has come into substantial use through either one or the other of two conditions: (1) a promise of clear economic advantage, or (2) a capability beyond that possible with other energy sources. These factors are also the keys to future usages of nuclear power. During the next ten years, nuclear power probably will be utilized still more extensively in central station power plants, naval ship propulsion, and space. It is intriguing to speculate whether the reactor designs will exhibit more or less variety; the present trend is toward standardization of types. There will be emphasis in central station reactors on fuel cycles that better conserve fissionable material.

Title page illustrations:

USS George Washington, Polaris-missile submarine.

Projected Oyster Creek nuclear station.

Multistage flash distillation water plant, San Diego, Calif. This is but one of many desalination processes.

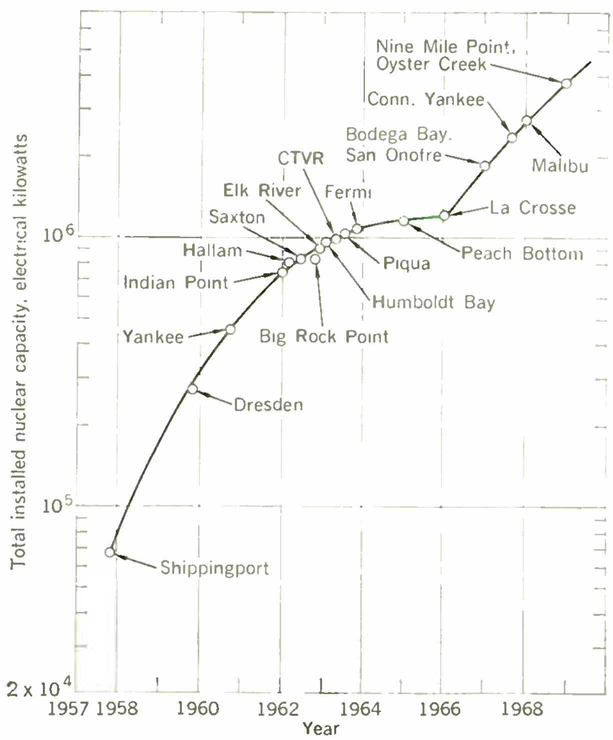
Also over the next ten years, the applications of nuclear power will probably be broadened to include the desalting of sea water on a large scale. These and other prospects are further discussed in the following sections.

Electric power generation

The growth in the installed capacity of nuclear-powered central station electric generating plants is shown in Fig. 1. Today, the total capacity of nuclear plants in the United States is approximately 1.2 GWe (1200 MWe). The operating reactors have demonstrated outstanding safety, high reliability, excellent load-following performance, and good economics. For example, the operating costs (as distinguished from the total power costs) of Consolidated Edison's Indian Point Reactor (5.4 mills/kWh) are today equal to the average operating costs for the Consolidated Edison system.[3]

This fine record of achievement has stimulated the projected construction, over the next five years, of six large plants with a combined generating capacity of about 2.6 GWe. Expected shortly is the announcement of other new plants, all to be constructed by private utilities with minimum governmental assistance. Three of the announced new reactor plants (Bodega Bay, San Onofre, Malibu) will be situated in California; for these, projected power generation costs will run about 5 to 7 mills/kWh. The other three plants (Connecticut Yankee, Niagara-Mohawk, and Oyster Creek) will be in the northeastern area of the United States, where the projected power generation costs run as low as 4 mills/kWh for the Oyster Creek Plant[4] (Table I). With power costs as low as this, plainly competitive with fossil fuel sources, it is not surprising that in Southern California and in Northeastern

Fig. 1. U.S. civilian power reactors. Total installed capacity of nuclear plants is approximately 1.2 GWe.



United States, approximately 25 per cent of all the announced new central station plants are to be nuclear.[5]

In looking beyond the new central station plants that have been publicly announced, the appendixes to the AEC's 1962 Report to the President[6] are a guide. An installed nuclear capacity in the United States of 16 GWe by 1975, 40 GWe by 1980, and 734 GWe by the year 2000 are predicted (Fig. 2). As percentages of the total electric generating capacity of the country, the nuclear capabilities for the years 1975, 1980, and 2000 will be about 4, 17, and 45 per cent, respectively.

Nuclear power in foreign countries. In Western Europe, to even a greater degree than in the United States, nuclear power is competing favorably with fossil fuels. PWR (pressurized water-cooled reactors) and BWR (boiling-water reactors), designed in the United States, have been installed or ordered for installation in Belgium, Italy, France, Switzerland, and Spain. A lively controversy is under way in Great Britain as to whether that country's future nuclear program should continue with reactors of the gas-cooled type (of which the total installed capacity is now approximately 500 MWe), or should switch to water reactors. It is of significance that the question of nuclear reactors as opposed to coal has not even been mentioned; all new large-scale power installations in Great Britain are expected to be nuclear. According to a recent Euratom projection, the total installed nuclear capacity in Western European countries will be 40 GWe by 1980.[7]

Recently, countries in Southeast Asia, specifically India and Pakistan, have announced intentions to purchase nuclear reactors for new electricity generating stations. Because of the high fossil fuel costs in Southeast Asia (about 50–75 cents per 1 million Btu,[8] or about 1½ to 2 times higher than in New England, California, or Western Europe), nuclear power is certain to be a major factor in the growth of the economies of the countries in this area. It is conceivable that toward the end of the next ten years, the rate of installation of nuclear reactor plants in Southeast Asia will exceed the rate in either the United States or Western Europe, at least on a percentage basis.

Reactor types. All of the six large central-station reactors recently announced for construction in the United States are of light water-cooled and moderated design. So, also, with the possible exception of advanced gas-cooled reactors in Great Britain and France, are reactors for foreign markets. Of the new water reactors, the total number to be constructed is split about evenly between BWR and PWR types. Compared with the diversity of re-

I. Projected plant revenue requirements, mills/kWh*

	Years of Operation					
	1–5	6–10	11–15	16–20	21–25	26–30
Plant fixed charges	1.67	1.57	1.46	1.45	1.66	1.84
Working capital	0.20	0.35	0.39	0.42	0.48	0.54
Fuel cycle	1.64	1.28	1.23	1.23	1.23	1.23
Operation and maintenance	0.51	0.51	0.51	0.54	0.67	0.80
Totals	4.02	3.71	3.59	3.64	4.04	4.41

*Oyster Creek Nuclear Station.

Heat generated by nuclear fissions in the reactor core produces a steam-water mixture (typically at 1000 psig and about 5 per cent quality). The steam and water phases are separated, and saturated steam is supplied to the turbine while saturated water is recirculated to the core inlet. In large reactors (over 60 MWe) pumped recirculation is employed but in a small reactor natural recirculation is practical. The number of recirculation loops varies with reactor size; typically, four would be used in a 500-MWe reactor. The net thermal efficiency of the reactor plant is about 32 per cent.

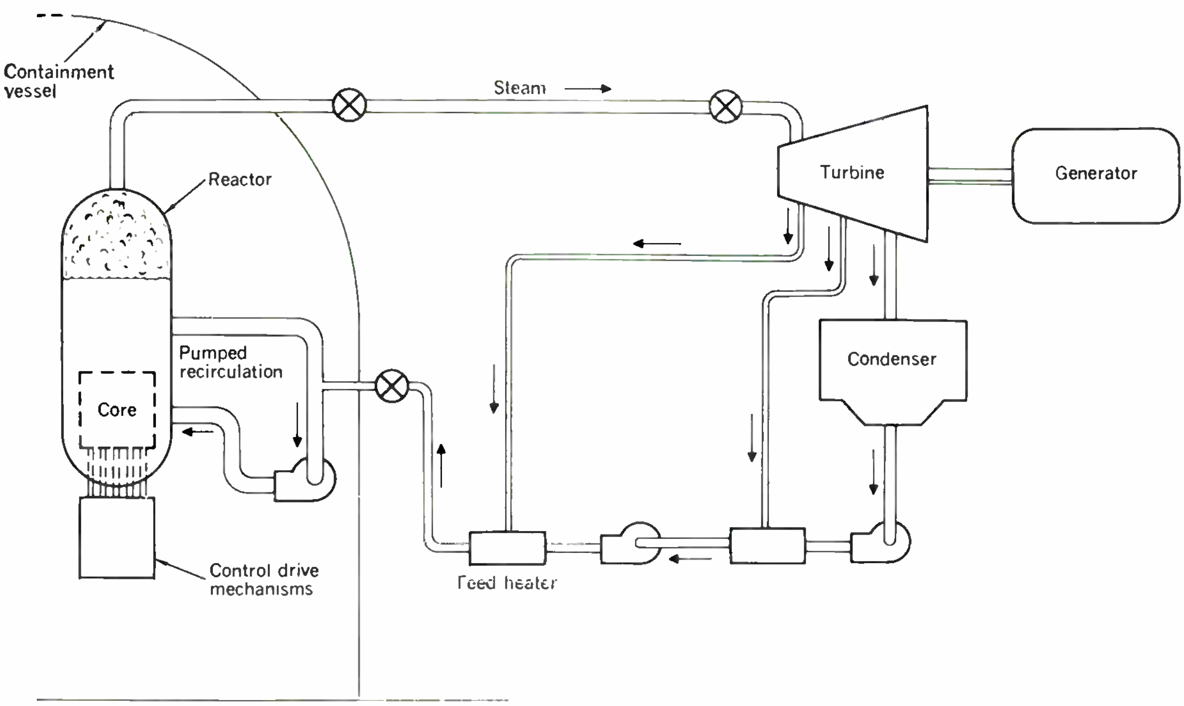
Although primary coolant is supplied directlv to the turbine, only a tiny fraction of the radioactivity in the coolant is carried with the steam phase. Double shaft seals prevent its release from the turbine casing. A reactor contaminant vessel and quick-closing valves in the steam line protect against release of radioactivity in the remote eventuality of a primary system rupture.

Rod-type fuel elements of about ½-inch diameter, spaced on approximately ⅝-inch centers, are used. The fuel is UO_2, enriched to between 2 and 2½ per cent of U^{235}, and the fuel is clad with Zircaloy or stainless steel. A typical core power density is 40 thermal kilowatts per liter. Neutron-absorbing controls rods are used to control the chain reaction.

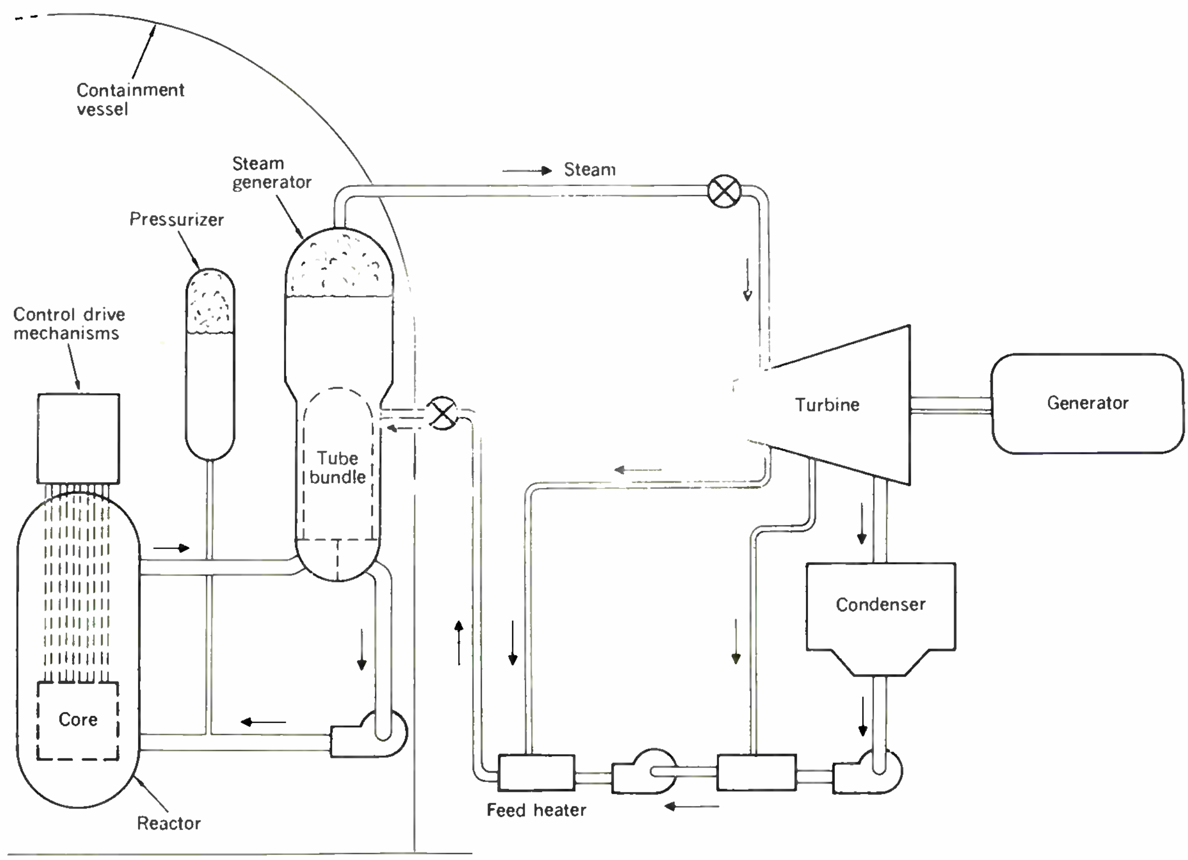
Heat generated by nuclear fissions in the reactor core is transported by primary system water (typically at 2000 psig and 590°F) to the steam generator. In the steam generator, the primary water is cooled (typically to 540°F) by the transfer of heat across the tube bundle to the secondary system. Primary water is then recirculated to the reactor core. Saturated steam (typically at 700 psig) is produced on the secondary side of the steam generator and is supplied to the turbine. The net thermal efficiency of the reactor plant is about 30 per cent.

The reactor core consists of an array of rod-type fuel elements, of about ⅜-inch diameter and consisting of uranium dioxide (UO_2) contained within Zircaloy or stainless steel cladding. The fuel elements are oriented parallel to the flow direction and are spaced on a square lattice with about ½-inch between centers. The UO_2 fuel is enriched to between 2½ and 3 per cent of U^{235}. Typically, the average power density of the core is 60–70 thermal kilowatts per liter. Control rods of neutron absorbing material (such as boron or silver, cadmium and indium) sometimes supplemented by boron in solution in the primary water, are positioned by drive mechanisms to control the chain reaction. The illustration shows one coolant loop consisting of piping, the steam generator, and circulating pump. A large reactor will have several such loops (typically four in a 500-MWe reactor). A single pressurizer controls primary system pressure and accommodates small volume changes of the primary coolant that result from changes in temperature and pressure. Primary system is sealed during operation within the reactor plant containment vessel.

Boiling water reactor (BWR)



Pressurized water reactor (PWR)

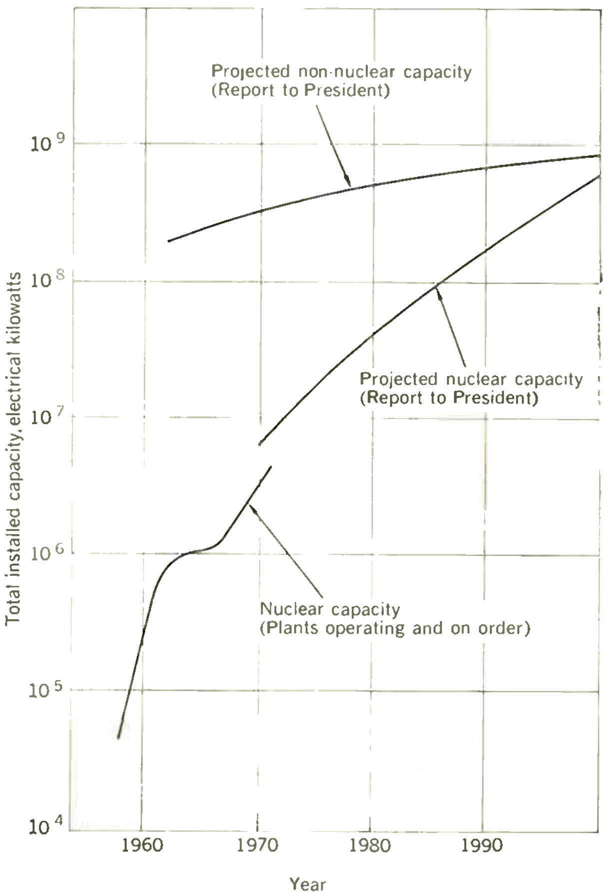


actors completed from 1962 to 1965, which include, in addition to light-water reactors, two reactors with nuclear superheat (Pathfinder and Bonus), a sodium-graphite reactor (Hallam), a heavy water-cooled and moderated reactor (CVTR), an organic-cooled reactor (Piqua), a fast breeder reactor (Fermi), and a high-temperature gas-cooled reactor (Peach Bottom), the present trend appears to be toward a virtual monopoly of the power generation field by BWR and PWR types. Whether this trend continues will depend in large measure upon the success of recent Atomic Energy Commission invitations to utilities to participate in the construction of large-scale reactors of alternate types.[9]

Five reactor concepts are included in those reactors in which the AEC has solicited utility interest. The concepts are: (1) the spectral-shift controlled reactor; (2) the sodium graphite reactor; (3) the high-temperature gas-cooled reactor; (4) the heavy-water pressurized reactor; and (5) the seed blanket reactor. While each of these concepts offers one or more advantages (for example in neutron economy, fuel inventory, or thermal efficiency over the more highly developed BWR and PWR types), it is not evident that any one of the new concepts provides a sufficiently great overall advantage to come into wide use in the future.

Improved fuel utilization. A more certain element in the future of central-station reactors is that the long-range need to conserve fissile material resources (as emphasized in the President's report[6]) will result in future reactors

Fig. 2. Projected electric utility generating capacity.



being designed to approach breeding. Breeding is defined as the process of producing more fissile material (by neutron captures in fertile material) than is consumed. Figure 3 illustrates the two theoretically possible breeding cycles. Because fissions of plutonium in a fast reactor result in a larger number of excess neutrons than fissions of U-233 in a thermal reactor, the breeding gain or net production of fissile material is considerably larger in fast, plutonium-fueled reactors. In fact, only the fast reactors are capable of making appreciably more fissile fuel than they consume, and thermal breeding, if practical at all, will be barely capable of replenishing fissile fuel consumed. Thus, in a growing nuclear industry, fast breeders must be counted upon to supply fissile fuel for new reactors coming into operation, if our natural resources of U-235 are not to be rapidly depleted.

Unfortunately, the development of fast breeder reactors is hampered by severe engineering problems related mainly to reactivity control and materials. It is not likely that reactors of this type will enjoy an appreciable usage in the next 10 to 20 years. Therefore, as an interim measure, thermal reactors with improved conversion ratios approaching a breeding ratio of one will come into use through development of the thorium fuel cycle. Thorium can be employed as the fertile material in standard BWR and PWR designs, as it was in the first Indian Point core. To date, U-235 plus thorium-fueled reactors have not shown an economic advantage over reactors utilizing a slightly enriched uranium. However, considering the increased fuel depletions that have been shown to be feasible by advancements in materials technology, the prospects for advantageous use of thorium have improved.

Private ownership of nuclear fuel. The United States Government at present holds title to all enriched nuclear fuel. Under the present law, fuel is leased to approved users, at an interest rate of 4¾ per cent per year. In many cases—for example, the first fuel loading in a new reactor plant—this interest (use charge) is waived. These practices create an artificial situation wherein there is an inadequate charge for increased enrichment of the fuel, and consequently, relatively small incentive for good fuel economy.

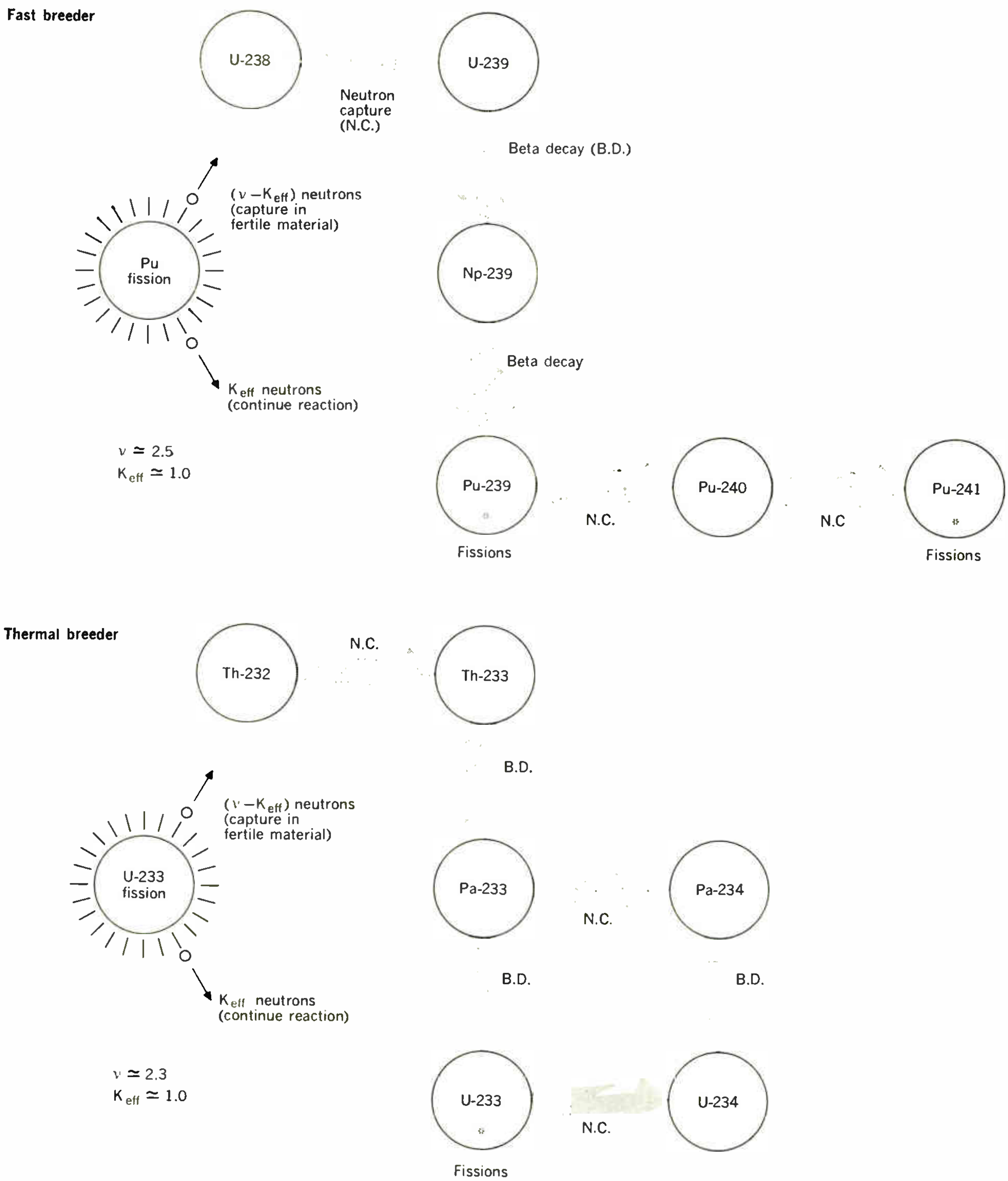
For many years, the possibility of introducing private ownership of nuclear fuels has been discussed. In March 1963, the AEC submitted legislative proposals to the Congress to provide a transition to private ownership of enriched fuels by June 30, 1973. Under the terms of the proposed legislation, private ownership would be permitted immediately, and made mandatory after the specified date. Contracts for toll enrichment of privately owned fuel in Commission facilities and government purchase of plutonium and U-238 from spent fuel would also be authorized.

The reaction of the nuclear industry, as represented by the Atomic Industrial Forum, is favorable toward private ownership. In fact, industry has proposed that the transition be accelerated by making private ownership mandatory on Jan. 1, 1971.[10] Thus, it is evident that utilities building reactor plants today will be contending with private ownership of fuel. The effect on fuel-cycle economies of Zircaloy-clad BWR and PWR reactors now being designed will be minor. An increase in the interest rate on fuel inventory from the present 4¾ per cent per year to a maximum probable commercial rate of 12 per cent per year would increase the fuel-cycle cost only 0.15 to 0.2 mill/kWh. However, if in conjunction with the transition to private ownership the base price of uranium were allowed to drop to the current world market price (about $6 per pound for U_3O_8 as compared to $8 per pound in the United States), the two factors would offset each other and there would be essentially no net change in fuel cost.

The transition to private ownership of nuclear fuels is not expected to affect the rate of growth of nuclear power. Increased emphasis on neutron economy, to reduce necessary enrichments, can be foreseen, as well as increased power density to reduce the in-core residence time. Development programs, such as the high-power density program sponsored by Consumers' Power Company and General Electric in the Big Rock Point reactor, are making significant progress toward proving the capability of present reactor designs to operate satisfactorily at 25 to 50 per cent higher power.

Nuclear safety. From the outset, the civilian nuclear

Fig. 3. Two theoretically possible breeding cycles.



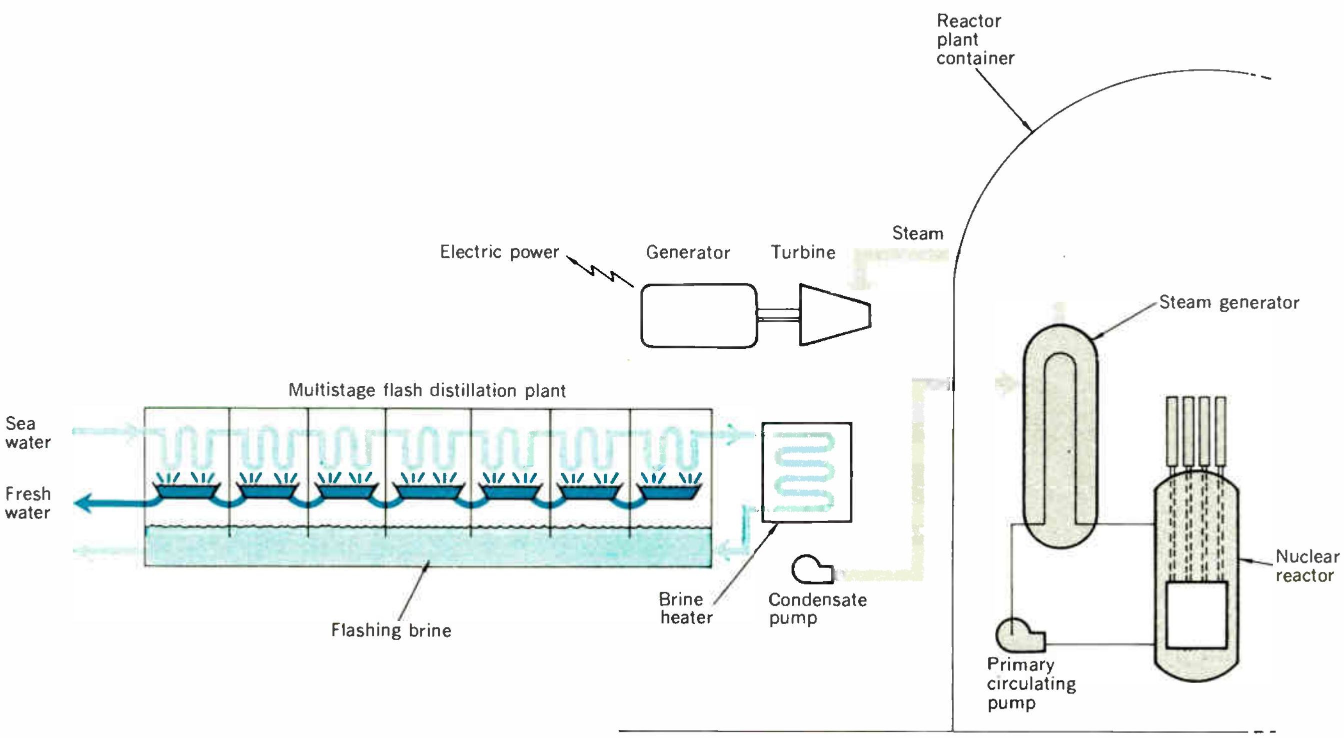
power program has placed overriding emphasis on safety, and most particularly on protecting the general public from any possible release of radioactivity. Safe design and operation have been assured through a combination of many elements, including the inherent self-regulation of the nuclear reactor itself, fast-response instrumentation and automatic shutdown provisions, elaborate engineered safeguards, extensive operator training programs, rigorous licensing provisions, and continuous monitoring of operation by health physicists and licensing authorities. These multiple precautions are directed toward two basic objectives: (1) prevention of reactor accidents, or, in the remote possibility of an accident, localization of the effects to within the reactor plant; and (2) safe handling of radioactive wastes. The effectiveness of the nuclear safety program is evidenced by the fact that no incidents involving either the operating crews or the general public have occurred from operation of any commercial reactor. In fact, it is becoming recognized that fossil fuel wastes (sulfur dioxide, nitrous oxide, and fly ash) could be more hazardous to the public health than the carefully handled and contained nuclear wastes.[11]

In a nuclear reactor plant, the radioactive fission products confined within multiple barriers are: (1) the fuel material that contains over 99 per cent of the fission products within its crystalline or molecular lattice, and is corrosion resistant in the reactor coolant environment in the event of a cladding defect; (2) the fuel element cladding, which is highly corrosion resistant and carefully inspected to assure its integrity; (3) the reactor pressure vessel and coolant piping; and (4) the reactor plant containment structure. Instrumentation, automatic "fail-safe" shutdown mechanisms, and the self-limiting characteristics of reactor excursions protect against radioactivity release from nuclear accident. The inherent safety of UO_2-fueled reactors was dramatically demonstrated in the recent SPERT-1 reactor destruction test.[12] The total energy release was limited to one half to two thirds of that anticipated. The reactor remained operable after the test. Radioactivity levels, both on-site and off-site, also remained low.

The reactor containment structure is a pressure vessel, completely enclosing the reactor and conservatively designed to contain the complete release of substantially all radioactive materials, even in the highly improbable event of a double-ended rupture of a main coolant pipe and subsequent melting of the fuel elements. The containment structure is periodically leak-tested as a requirement of the operating license. Typical permissible leakage rates are 0.1 to 1.0 per cent per day, with an internal pressure corresponding to the maximum credible accident.

Reactor siting. Additional protection of the general public is afforded by AEC regulations pertaining to siting of nuclear power plants. Basically, it is required that reactors be located away from areas of high population. This requirement is certain to become the most difficult to meet as the number of installed reactors increases. In California today, for example, suitable reactor sites are so few that Southern California Edison obtained permission to construct its San Onofre plant on the Marine Corps' Camp Pendleton preserve. The New York metropolitan area is another place where reactor siting has posed difficulty. As a consequence of this situation, in recognition of the inherent safety of commercial reactors, and with the incorporation of additional engineered safeguards, Consolidated Edison of New York proposed to construct a large reactor plant at Ravenswood on the east bank of the Harlem River in New York

Fig. 4. Typical nuclear-powered plant for water desalting.



City. Licensing action on the plant has been obviated by a subsequent utility decision to bring hydroelectric power from Canada to meet immediate needs.

Inevitably, other situations similar to Ravenswood will develop. It also appears certain to many authorities in the nuclear field[13,14] that recognition of the inherent safety of commercial reactor plants will result in favorable licensing decisions on close-in sites. The major obstacle appears to be education of the general public, a large segment of which still likens a power reactor to a bomb. It is probable that, at least initially, reactors in or near population centers will incorporate additional engineered safeguards beyond those common today. Examples are double containment shells with intermediate bleed-off and collection of any leakage, and concrete shielding of the containment shell itself. The cost of these additional measures would be relatively small (0.1–0.15 mill/kWh), and will not deter nuclear power growth.

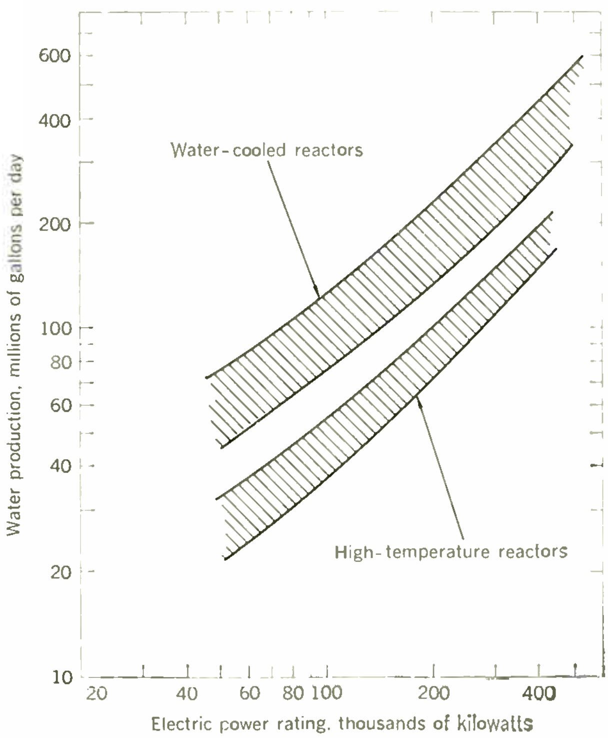
Handling of radioactive wastes. Still another challenging aspect of the future commercial reactor program is safe disposal of radioactive wastes. Nuclear wastes include spent fuel, containing highly radioactive fission products, contaminated liquids, and radioactive gases. Though satisfactory techniques have been developed to process the wastes from existing plants, the safe disposal of wastes will assume increased importance as the number of nuclear power plants increases. Major development programs are currently under way, for example, at the Oak Ridge National Laboratory, to devise new and improved techniques of waste disposal.

Almost all of the radioactive products of the fission reaction are contained within the reactor fuel elements. When these elements have reached the end of their useful life, they are removed from the reactor and transported in heavily shielded casks to a fuel-reprocessing center, such as the one to be operated commercially by Nuclear Fuels Services, Inc. The radioactive elements are separated by chemical processing and then stored in liquid form in underground tanks for a sufficiently long period (30 years or more) to allow for a substantial natural decay of radioactivity. For many years, spent fuel elements from the AEC's plutonium reactors at Hanford and Savannah River have been processed in a similar manner. The present storage methods appear both feasible and economically acceptable for at least the next 20 years, until current research and development programs devise more effective long-term disposal means.

In 1960, after 15 years of operation, the quantity of wastes stored in underground tanks was about 70 million gallons and since 1960 the amount of stored wastes has been increasing by about 5 million gallons per year.[15] By comparison, cumulative amounts of high-level wastes from commercial reactors are projected to be less than 10 million gallons by 1980, and about 100 million gallons by the year 2000.[15] Thus, commercial reactors will not be the dominant source of radioactive wastes for at least 30 to 40 years.

A small fraction of the radioactive products from the fission reaction must be handled at the reactor site. These include mainly activation products in the reactor coolant; for example, Co-60, which results from neutron capture in cobalt in structural materials used in the reactor. Hydrogen and radioactive gases, predominantly xenon and kryton, also exist in the coolant. These radioactive products are disposed of on the site by a variety of techniques, all comparatively standard in the chemical industry. Essentially, all of the contained radioactive material is separated, collected, and disposed of before gases and liquids are discharged at the reactor site. Through careful and continually monitored disposal means, the effect on the public of the very small amount of radioactive material that escapes at the reactor site in maintained at a level less than one thousandth of that absorbed by the average person from his luminous-dial alarm clock and television set, and less than one hundred thousandth of what one might receive from natural radiation and medical X rays during his lifetime.[15]

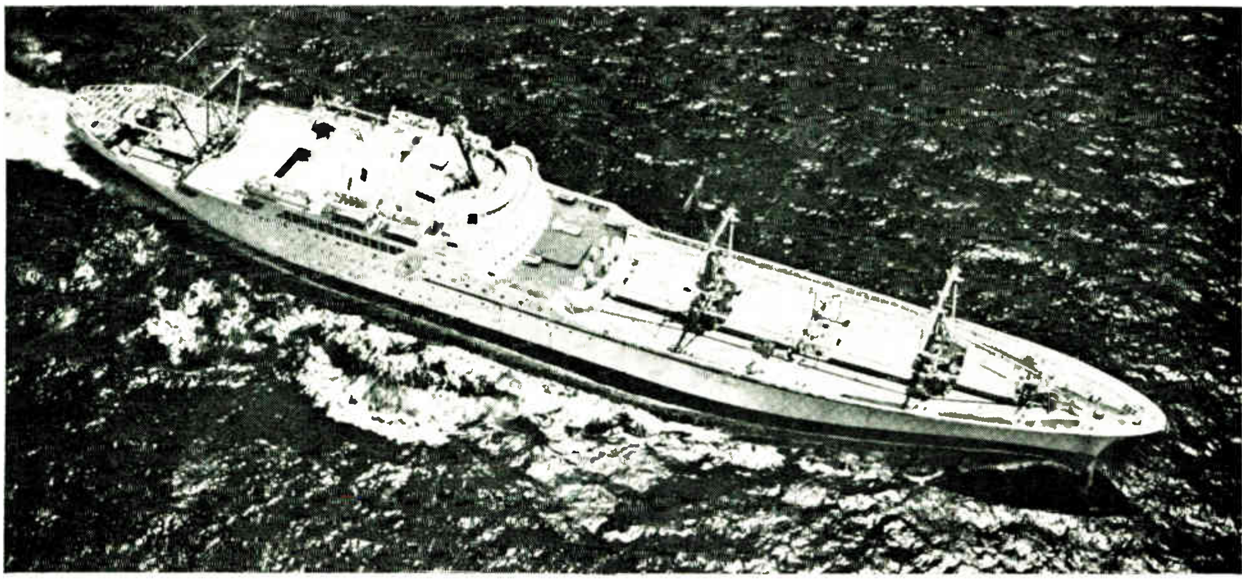
Fig. 5. Dual-purpose reactor plant for water desalination and electric power generation has typical outputs as shown.



Process heat

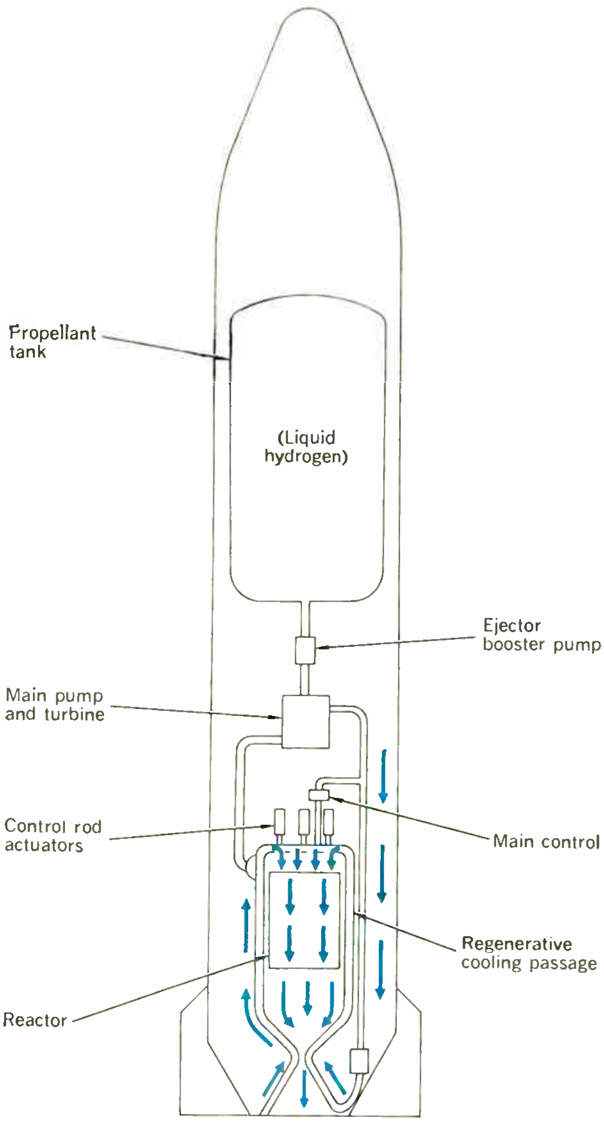
For many years it has been speculated that the application of nuclear heat to processing industries constitutes at least as large a potential market as application to electric power generation. Factors inhibiting this application of nuclear power have been that: (1) most requirements for process heat are in small blocks of power where nuclear power is economically least attractive, and (2) the temperatures required in many applications are higher than can be provided by the most highly developed reactor types—BWR and PWR.

A new and seemingly ideal process heat application of nuclear power is in the desalting of large quantities of water. A number of economic studies have been performed recently, and these show that the lowest water costs are realized when the reactor is of the dual-purpose type, producing both electric power and water. The result is a fortunate one, since in those areas of the world having the greatest requirements for fresh water, the availability of water is bound to cause a growth in the economy which will in turn create demands for electric power. Depending



View of NS Savannah, nuclear-propelled merchantman.

Fig. 6. Schematic of nuclear-powered booster rocket.



on the plant size, capital interest rate, and value of electric power generated, water costs ranging from about 20 to 40 cents per thousand gallons are presently considered to be achievable.[16] These costs are lower than water costs from a desalting plant utilizing fossil fuel at 35 cents per million Btu for the heat source. Thus, it can be expected that in locations where natural fresh water supplies are inadequate, nuclear desalting plants will be installed. Since the reactor technology follows directly from power reactors, and since various desalting processes are currently being demonstrated in intermediate-size prototypes,[17] very little additional developmental work is required, and consequently substantial usage of nuclear power in the water desalting application should come into being by 1975.

A typical nuclear-powered desalting plant is shown in Fig. 4. Multistage flash distillation is but one of many desalting processes that may be employed; it has the advantages of having been demonstrated and of being readily scaled up to large product capacity.[16]

An interesting consequence of dual-purpose reactor plant for electric power generation and water desalination is that the plant can be adapted to seasonal and daily variations in electric power demand. Since water may be stored inexpensively, the periods of low electric demand can be utilized to produce water. The result is a potentially increased plant load factor and correspondingly lower power and water costs. Typical amounts of water and power production are shown in Fig. 5.

Ship propulsion

Naval ships. As indicated, the growth of the nuclear U.S. Navy has been possible because nuclear power results in tremendously improved and even completely new weapons system capabilities. Thus, economics has not been a predominant factor. Nevertheless, the need to operate within limited budgets forces cost consciousness upon the nuclear Navy. Prices for nuclear reactors and associated equipment have been reduced many times since the construction of *Nautilus*, the first nuclear submarine. For submarines, nuclear propulsion has been an outstanding success[18] and is now well established. Over the next ten years, at least 50 new submarines will be put into service[1] and it can be expected that through evolution of design, performance will continue to improve; however, no fundamentally new developments are foreseen in this area.

The outlook for naval surface ships is less clear. The recent and controversial decision to utilize oil fuel rather than nuclear power in the newest aircraft carrier poses a major threat to the future of nuclear power in surface ships. However, the comparative economics in this case were close[18] and it seems likely that, as nuclear costs decrease with further development, the U.S. Navy will follow the previously expressed intent to use nuclear power in all major surface ships.

Merchant ships. Application of nuclear power to merchant ship propulsion has been significantly less attractive economically to date than application to central station power generation, or to naval surface ship propulsion. This is a natural consequence of the lower power ratings of the merchant ship propulsion plants and concern over the frequency with which merchant ships must berth close to large population centers. After being in the doldrums for many years, recent developments, including operation of the *NS Savannah*, and the development of compact, low-cost reactor designs by United States manufacturers, have led to a revival of interest in a nuclear-powered merchant marine.[19] An AEC "white paper" on the subject is now in preparation.

The conditions under which nuclear power for merchant ship propulsion is economically most competitive are: (1) large ship size, (2) high availability, which generally means long voyages and short turn-around times in port, and (3) high speed. These requirements result directly from the higher capital cost of nuclear power and its relatively low fuel cost compared to conventional oil-fired power plants. In the past, it has appeared that the three given conditions would be best realized in a large long-range tanker or a high-speed passenger liner in Pacific service. A new application is now receiving attention: the creation of a U.S. merchant marine of high-speed cargo ships (with a speed of about 30 knots as compared with 15 to 18 knots for most existing shipping). It has been pointed out that high speed tends to generate demand for service.[20] Therefore, it is argued, a fleet of high-speed nuclear cargo ships would regain a substantial share of the total shipping volume for carriers. Since federal subsidy of the merchant marine is required, whatever the power source, the additional federal subsidy for nuclear power is not considered detrimental to the plan. In fact, the additional construction subsidy would be compensated for by the improved competitive position such ships could create.

Other Applications

Portable reactors. The future for small, portable reactors for electric power generation is very much in doubt. A recent AEC "white paper" concluded that, on strictly economic grounds, there is no incentive to utilize nuclear power to meet electrical demands at advanced bases, except in such remote areas as McMurdo Sound or possibly Greenland, where operation costs of oil-fired plants are high. Existing small reactors, on the whole, have performed well. The basic difficulty is that portable reactors are at the opposite end of the size spectrum from central station reactors, and nuclear power is economically less competitive in small sizes.

Power for hardened sites. The most promising application for small reactors of the "portable" type appears to be as power supplies for underground "hardened" sites. Here the application is much like that for the submarine. In the event of a nuclear attack on a sensitive control center or missile base, it is desirable to be able to operate for some reasonable time with complete physical isolation from the external environment. Here the reactor, with its long lifetime for a small fuel charge, its excellent load-following capabilities, and mechanical durability, is superior to both conventional and exotic energy sources.

In the event of a major national program to build and equip shelters and to plan for postnuclear-attack recovery, a very large market for nuclear reactors of small size (100 to 2000 kWe) would develop.

At the lower end of the small-size range, direct conversion from heat to electric power is competitive with small turbine–generator sets. Reactors in the smaller sizes are thus likely to be developed in conjunction with thermoelectric energy converters.

Space applications. Closely analogous to the isolated hardened site is the spacecraft, earth satellite, or space probe. Here again nuclear power provided a unique capability and, not surprisingly, both small reactors and isotopic power sources are being developed in relatively large numbers for this application.

Nuclear power is also of interest for space propulsion. Here the applications can be divided into two general types: (1) high thrust for short duration for booster rockets to escape the earth's gravitational field and (2) low thrust for long duration for orbital maneuvers and travel to distant planets. The booster rocket, shown schematically in Fig. 6, has the advantage of appreciably higher specific impulse than chemical rockets, and as a consequence could perform a given space mission with lower initial lift-off thrust. As an example,[21] for a moon-landing expedition, a vehicle with first-stage chemical and second-stage nuclear rockets would require a lift-off thrust of only 1.5 to 2.25 million pounds as compared to about 6 million pounds for an all-chemical multistage rocket. Under the NERVA program, a nuclear rocket is being developed for missions into deep space.

For smaller thrusts over longer periods, the use of ions accelerated by a nuclear reactor as the power source is of interest. Application of an ion engine to a space mission would appear logically to follow the successful operation of the nuclear booster rocket. Development work on the ion engine is proceeding concurrently with work on the nuclear booster.

Plowshare. Another proposed use of nuclear power is exemplified in the Plowshare project, which is concerned with use of underground explosions of nuclear devices to accomplish peaceful purposes. Recently, publicity has been given to a plan to excavate a sea-level canal across lower Mexico utilizing underground detonation of a number of nuclear devices. Plans to dig harbors by nuclear explosions have also been advanced.

Many problems are still to be solved, but tests indicate that underground nuclear explosions produce a very small yield of radioactivity to the environment. If this indication can be substantiated, not only will large civil engineering projects be accomplished at astonishingly low cost, but the Plowshare concept will offer possibilities of great potential for performing various chemical processes on a truly massive scale.

Conclusions

Nuclear power is firmly established today in three applications: electric power generation, naval submarine propulsion, and space auxiliary power. The number of nuclear power plants in these applications will more than double in the next ten years, and the total power generation capacity will increase by a factor of almost ten. In the field of electric power generation, costs can be expected to drop below 3.5 mills/kWh by 1980, as the fuel-cycle cost drops to about 1 mill/kWh, the capital costs drop to $100/kWe, and operating crews are reduced through use of automatic controls from about 60 to approximately 10 persons.

In other fields of application, the use of nuclear power will also grow. Nuclear power appears today to be on the threshold of expanded use for propulsion of naval surface ships and the merchant marine, despite the recent decision against use of nuclear power in the newest aircraft carrier. Factors contributing to the future use of nuclear power in ship propulsion will be reduced costs and the development of compact, lightweight reactor plants. New concepts of propulsion will be introduced into the merchant marine to take full advantage of nuclear power. These developments could range from automatic cargo handling to nuclear-propelled tractor vehicles towing cargo-carrying barges or, possibly, submersible tankers.

Also, several new areas of application, where nuclear power promises pronounced advantages in performance and cost, include (1) booster rockets for deep space missions, (2) desalting of sea water, (3) power packages for hardened sites, and (4) underground explosions of nuclear devices. It appears that use of nuclear power in these applications will be limited only by how rapidly the requirements themselves develop.

With the increase in nuclear power plants, there has been a tendency for a few reactor types to dominate the individual fields of application. This is in contrast to a rather wide variety of types considered in former years. For example, all naval reactors are currently of the pressurized water type. Further, new central station reactor sales in the United States and abroad have recently been dominated by the BWR and PWR types. This tendency to pair off applications with specific reactor types is abetted by the realities of the business process. A manufacturer who sells several reactors of one type can prorate his design costs over several units, offer performance warranties based on smaller margins, and thus sell at lower prices. This situation is making it increasingly difficult for the so-called advanced reactor types to compete economically. And as the popular reactor types are further developed and the coal industry tries to meet the challenge of lower costs, the newer concepts will have even greater difficulty in meeting competition.

The present situation in which reactor types are identified with a specific manufacturer is unfortunate because the question of choice in reactor concepts has become too closely linked with maintaining competition in the industry. A healthier competitive situation would exist if there were several suppliers for each basic type.

Despite the significant accomplishments of the AEC and the nuclear industry to date, which include early prospects for private ownership of nuclear fuels and commercial processing of spent fuel, further development of nuclear power is still very much dependent on government aid. The following are the particularly important areas still requiring federal government participation: (1) materials development and irradiations tests, (2) heat transfer studies, (3) basic nuclear physics, (4) purchase of bred fuel, and (5) improved methods of radioactive waste disposal.

In retrospect, it seems remarkable that nuclear power has come so far in less than 25 years. But, lest the technical problems be solved only to have the growth of nuclear power held back by uninformed public opinion, it is important for the nuclear industry to begin a program of public education. Experience to date, and a series of specific tests have shown that nuclear power is inherently safe. That the public be convinced of this is most important because the growth of metropolitan areas will result in a growing need for reactor plants in or near population centers.

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IEEE Reports for 1963

Introduction

Clarence H. Linder, President

It is my pleasure to present to the membership the Reports of the Secretary and Treasurer, which record the activities and vital statistics of IEEE in 1963. These documents cover in some detail the end results of a year's experience of those who have shared in the management of IEEE: members of the Boards, Committees, and Staff on whom have fallen the responsibility for decisions and execution of Institute policy. On behalf of the members I wish to thank them for their dedicated service to IEEE.

Secretary Pratt's report records accomplishments of which we can all be proud. The reorganization and merger of all IEEE Regions and Sections was completed during the year, well in advance of the forecasts of the Merger Committee. At the year's end there were 181 Sections, 38 Subsections, 346 Group Chapters, and 260 Student Branches. The IEEE Groups had a combined membership of 96,417. Thirty thousand editorial pages were needed to publish 2700 papers, 900 letters to the editor, and innumerable news items on IEEE activities.

Treasurer Clark's report shows that the Institute operated at a substantial deficit in 1963. A deficit had been foreseen by the Merger Committee and the Directors of IRE and AIEE prior to the merger. At that time it was anticipated that the reduction in dues for former AIEE members and the starting costs of the merger would result in a deficit of about $260,000 in 1963. The actual deficit is higher than this, owing primarily to a reduction in advertising income. A smaller operating deficit, also anticipated at the time of merger, is in prospect for 1964. However, owing to certain nonoperating expenditures mentioned below, the total deficit for 1964 will exceed that for 1963. By 1965, it is planned that IEEE will be operating on a balanced budget, provided that dues and advertising income meet expectations.

The IEEE Board has undertaken this year an investment of funds for improved equipment and staff facilities. During the deliberations of the Merger Committee, it was realized that the record-keeping, accounting, and mail-label systems of IRE and AIEE were not adequate for the larger and more complex operations of IEEE. To assure that these essential functions can be performed efficiently and accurately, an entirely new computer system was designed, with the assistance of Price Waterhouse & Company, and let out for bid in July 1963. The contract for this system was awarded in December, and the equipment is scheduled for delivery late this year.

The IEEE Computer Center will be located in the Exhibition Hall on the first floor of the United Engineering Center in New York. This location at our 47th Street Headquarters is well suited for the purpose, particularly since it is hoped that the computer can be shared with our sister engineering societies in future years.

To prepare the new space for the staff and the computer installation will require a capital investment of several hundred thousand dollars. The expenditure of these substantial sums has been authorized only after careful deliberation by the Board. Your Directors view them as an investment which will be repaid, within the next decade, by reduced costs in handling our voluminous records and mailing operations. The master member data file, for example, will contain over 50 million characters, and mailing labels are required for over $2^1/_2$ million copies of IEEE publications each year.

The Board has also considered the obligation of IEEE to assist in raising funds for the completion of the United Engineering Center and has decided to make a contribution of $175,000 to the Supplementary Fund Raising Campaign of UET (United Engineering Trustees) from IEEE treasury funds. This contribution is considered appropriate in view of the broadened base of IEEE's involvement in the United Engineering Center.

Your Board of Directors has under advisement a matter of financial importance in another category. On February 20, 1964, IEEE was notified by the Internal Revenue Service (after a number of years of consideration of the matter by the Service) that IEEE's annual exhibition, held as an integral part of its annual convention, involved the operation by it of a business not substantially related to IEEE's tax-exempt purposes and that the net income therefrom, if any, is subject to federal tax. The Service requested IEEE to file tax returns with respect to that income for the years 1954 to date.

IEEE, through its legal counsel, intends to contest imposition of the proposed tax by appropriate administrative and legal action. The maximum amount of alleged tax liability, which, together with interest thereon, could be material, is not yet known since the matter is now being studied by IEEE's legal counsel and auditors, and IEEE has not received any statement of proposed adjustments of its tax liability from the Internal Revenue Service.

Your Board of Directors considers it important that all IEEE members know that the possibility of taxation of convention-exhibition income was discussed at length by AIEE and IRE officers in the merger negotiations and, consequently, the recent action by the Internal Revenue Service was not unanticipated.

Prudent stewardship over many years by the Boards of Directors of AIEE and IRE has provided an accumulation of surplus funds, which now stand the IEEE in good stead. As can be seen from the auditor's report, the capital resources of the IEEE are sufficient, by a substantial margin, to cover the expenditures, both authorized and potential, referred to above. Your Board of Directors will exercise great care to assure that the present sound financial condition of the Institute is maintained. The membership will be promptly informed, through the pages of *IEEE Spectrum*, when the tax question mentioned here is finally settled.

Report of the Secretary—1963

Haraden Pratt

To the Board of Directors,
The Institute of Electrical and
Electronics Engineers, Inc.

Gentlemen:

Herewith is presented the Report of the Secretary for the year 1963, the first of operations under the merger arrangement that created the IEEE.

Being a formative year in the attacking of problems incident to the merger, some solutions, such as the policy regarding publications, have been adopted and others are in progress but not finalized. In particular, the coordination of office operating routines and management organization have been difficult due to the quite different practices prevalent in the two former societies and some embarrassment ensued both for members and for management, due to errors that were made in the struggle to achieve unification. However, by the close of the year, most of these difficulties came under control.

It should be mentioned that new pension and insurance programs were adopted and made effective during the year.

Growth and other statistics in the Report indicate that the new society is moving forward vigorously and that the problems of combination of old Sections into IEEE Sections have been worked out.

Respectfully submitted,
Haraden Pratt
Secretary

March 7, 1964

Section A—Membership

Table I gives the distribution of the membership by grade and by percentage and Table II gives the geographical distribution. Of a total of 3094 life members, 316 are contributors to the Life Member Fund, and 2778 are noncontributors.

Section B—Section and Subsection Activities

At the end of 1963, the number of IEEE members residing in 103 countries around the globe was 154,509. Of these, all but 3567 lived within the territory of an IEEE Section or Subsection. The distribution of Sections and Subsections is indicated in Table III.

Table I. IEEE Membership by Grade, by Percentage, December 31, 1963

Grade	Number of Members	Percentage
Honorary (H)	5	
Fellow (F)	2,589	1.7
Senior Member (SM)	26,274	17.0
Member (M)	82,750	53.5
Associate (A)	14,215	9.2
Student (S)	28,676	18.6
Totals	154,509	100.0

Table II. Geographical IEEE Membership Distribution, December 31, 1963

Region	Number of Members	Percentage
1	39,461	25.5
2	26,032	16.9
3	13,217	8.6
4	18,067	11.7
5	13,048	8.4
6	30,586	19.8
Subtotal (U.S.)	140,411	90.9
U.S. possessions	210	0.1
U. S. overseas military	908	0.6
	1,118	0.7
7 (Canada)	6,383	4.1
8 (Europe)	3,473	2.3
9 (elsewhere)	3,124	2.0
Subtotal	12,980	8.4
Grand total	154,509	100.0

Table III. Distribution of Sections and Subsections

	Sections	Subsections
United States	150	40
Canada	16	2
Abroad	15	
	181	42

A major activity of many Sections and the larger Subsections is the publication of a local monthly Bulletin to fulfill the need for announcing to the Section member the increasing activities of the Section, including (1) Section Meetings, (2) Professional Technical Group Chapter Meetings, and (3) Technical Discussion Group Meetings. Fifty-four of the Sections and Subsections now issue these monthly publications.

The number of meetings reported to Headquarters during the year was 1545 by Sections, 484 by Subsections, 789 by Professional Technical Group Chapters, and 460 by Technical Discussion Groups, for a total of 3278 local meetings.

Section C—Report of Technical Activities (PTG and TOC)

1. General

Continuation of the former IRE Professional Groups as the IEEE Professional Technical Groups, and the former AIEE and IRE Technical Committee structures as the IEEE Technical Operations Committee, was maintained through the first year of IEEE operations. During that year, because of overlapping fields of interest and the basic differences in operational methods of Groups and Committees, many problems of mutual concern came under discussion.

1.1 Ten areas of activity were outlined to facilitate discussions among Groups and Committees.

1.2 Professional Technical Groups and Technical Discussion Groups, operating on the Section level, conducted 789 meetings and 460 lecture courses.

1.3 Cooperation between the Groups and Committees developed in such areas as Standards and participation of Groups in Committee activities.

1.4 General meetings involving both PTG and TOC participation were held, notably the Winter General Meeting and the International Convention in New York, the Summer General Meeting at Toronto, Wescon at San Francisco, Pacific Annual Meeting in Spokane, and the National Electronics Conference at Chicago.

1.5 Trends have developed looking toward a more unified approach for possible consolidation of technical papers generated by the PTGs and Committees.

2. Professional Technical Group (PTG) Operations

2.1 The number of Groups has grown from 29 to 32.

2.2 Twenty special conferences, symposia, and other meetings were sponsored within IEEE by the PTGs. In addition there were ten jointly sponsored meetings with other engineering societies.

2.3 Three new Groups, 6 mergers involving committees and 11 sets of merger negotiations were initiated.

Accomplished to date were the following mergers:

February—PTG on Broadcasting with TOC Broadcasting Comm.

May—PTG on Reliability with TOC Subcommittee on Reliability and Quality Control

June—PTG on Electron Devices with TOC Solid State Devices Comm.

August—PTG on Industrial Electronics with TOC Recording & Controlling Instruments Comm.

PTG on Instrumentation & Measurement with TOC Elec. & HF Instruments Comm., Fundamental Electrical Standards Comm., Indicating & Integrating Instruments Comm.

October—PTG on Nuclear Science with TOC Nuclear Instrumentation Comm. and Nucleonics Comm.

New PTGs developed from AIEE ITGs and TOC Committees:

PTG-Aerospace from ITG on Aerospace and TOC Comm. on:

Aerospace & Energy Conversion; Aerospace & Support Systems; Aerospace Instrumentation; and Flight Vehicle Systems

PTG-Power from Power Division TOC

PTG-Electrical Insulation from TOC Electrical Insulation Comm.

Potential mergers:

PTG-Aerospace with PTG-ANE
PTG-AC with ITG-AC, TOC Feedback Control Systems Comm. and Industrial Control Comm.
PTG-BME with TOC ETMB Comm.
PTG-CP with TOC Electronic Transformers Comm.
PTG-CP and PTG-PEP
PTG-CS with TOC Communication Division
PTG-EC with TOC Computing Devices Comm.
PTG-ED with TOC Electron Tubes Comm.
PTG-IECI with TOC T.C. 10 Standards Comm. on Industrial Electronics
PTG-NS with TOC Nuclear Techniques Standards Comm.

A potential new group is that of the TOC Science and Electronics Division. (Four committees of this division are interested in forming a Group, the name and scope of which have not yet been proposed.)

2.4 At the year's end there were 96,356 Group members and 833 affiliates. The number of Group Chapters has grown to 346 in 67 Sections.

Thirty of the Groups published *Transactions* and 22 of these published "Newsletters." It is estimated that 35,000 IEEE members belong, on the average, to two Groups. These Group members received the publications and Conference notices and other special material generated by the respective Groups. Apart from the *Transactions*, which are mailed by the printer, approximately 1000 Group mailings were handled by IEEE Headquarters.

3. TOC Organization

The Technical Operations Committee (TOC) organization has undergone substantial change during the past year.

Discontinuance of several TOC Subcommittees, where the need for function had dissipated, has reduced their number from nine to four: Information Retrieval, Special Technical Conferences, TOC Manual, and West Coast. Following presentation of its latest progress report, in the fall of 1963, it was decided that the Ad Hoc Committee on Processing, Quality and Distribution of Papers should continue in existence until the end of the year. The joint PTG/TOC Ad Hoc Committee to Recommend Future Meetings Pattern was established and added to the roster.

One of the two TOC Institute Technical Groups, the ITG on Aero-Space, with the four Technical Committees involved, moved into the PTG organization; the other, the ITG on Automatic Control, remained an integral part of TOC through 1963, with PTGAC merger negotiations pending finalization. TOC units merging with PTGs or forming new PTGs, interdivisional transfers of Technical Committees, TC and divisional consolidations, etc., have brought about a reorganized Divisional-Technical Committee structure.

The four present divisions, (Power, Industry and General Applications, Science and Electronics, and Communications), supervise and coordinate the activities of 59 Technical Committees and 252 Subcommittees. The Technical Committees perform the following functions:

a. Procure, review, and classify for publication, technical material: During 1963 the Communication Division, Instrumentation Division, and Science & Electronics Division together published 696 transactions pages in the bimonthly *Communication and Electronics;* General Applications Division and Industry Division together published 372 transactions pages in the bimonthly *Applications and Industry;* and Power Division published 1148 transactions pages in the bimonthly, *Power Apparatus and Systems;* a total of 2216 transactions pages.

b. Organize and present this technical material at IEEE meetings, symposia, and special technical conferences.

c. Through the Subcommittees, generate Standards material for coordination by the Standards Committee.

The Standards Committee is the most active of the four TOC General Committees (which include New Technical Activities, Research, and Safety). Twenty-two voting members constitute the Standards Committee, which approves the content of Standards publications, establishes the rules and procedures for the various functions of the Committee, and forms the nucleus of a far-reaching organization. Because the Committee must keep in touch with the thinking and activities of the Standards-generating units within the IEEE (the Technical Committees, Technical Committees—Standards, and recently the Professional Technical Groups), and other organizations (ASA, CSA, EEI, EIA, ISA, NBS, NEMA, SMPTE, and UL), a very important auxiliary group consisting of several hundred appointees that serve as liaison representatives to and from these activities, was established very early in the year.

Communication between the liaison representatives of the IEEE-based activities and those of the other organizations is fostered extensively. The Committee devoted major portions of six meetings during 1963 to the establishment of procedures for the most effective method of operation. To accomplish this task, ten *ad hoc* committees were appointed to study key subjects and present recommendations to the Standards Committee for adoption and implementation.

The Technical Committees—Standards referred to in the foregoing grouping of Standards-generating units within the IEEE are 25 Technical Committees carried over from the IRE structure and are unique to the Standards functions in that they concentrate their attention on definitions, symbols, and methods of measurement as compared with the many-faceted operations of other entities of the IEEE that also submit proposals for Standards Publications to the Standards Committee, i.e., the Technical Committees from the AIEE structure and the Professional Technical Groups evolved from the Professional Groups of IRE. Twenty-four Standards were completed during 1963 and six are in process.

3.1 Twenty-one special conferences, symposia, and other meetings were sponsored by the Committees. In addition, there were five jointly sponsored meetings with other engineering societies.

4. The Joint Technical Advisory Committee (JTAC)

The Joint Technical Advisory Committee held a total of four meetings from July 1 to December 31, 1962 (the first half of its fiscal year 1962–1963), while under the sponsorship of IRE and EIA. During the calendar year 1963, under the sponsorship of IEEE and EIA, JTAC held five meetings. The Fifteenth Anniversary dinner was held in May 1963 at the Waldorf-Astoria Hotel, New York City.

Seriously concerned with the many increasing demands on the spectrum, and in the light of the many rapid technical changes which have taken place in the past decade, JTAC decided to revise its 1952 publication *Radio Spectrum Conservation*. With the full support and encouragement of its sponsors, JTAC set up a Subcommittee (62.1) for this purpose under the chairmanship of Philip F. Siling. Work is progressing rapidly and the new edition will be published this year.

The FCC, in November 1962, advised the JTAC that it planned to continue and expand its study of man-made noise and inquired as to JTAC's plans in this area. JTAC has endeavored since 1961 to stimulate the interest of engineers in the increasing technical and economic problems of electromagnetic compatibility. JTAC members and the secretary have conferred with representatives of federal agencies and industry to determine the extent of interest and various activities in interference control and reduction. These activities included two classified presentations, one by the Chief Signal Officer at the Pentagon and the other by the Tri-Service Electromagnetic Compatibility Center at Annapolis. In November 1963, JTAC established a Subcommittee (63.1), under the chairmanship of Richard P. Gifford, to explore the neglected area and to encourage educational and remedial studies in the EMC/RFI area. Jerome B. Wiesner, Acting Special Assistant to the President for Telecommunications, in December 1963, indicated interest in this area and suggested that JTAC consider becoming a focal point for professional study of the needed technical programs and objectives. The resultant data will define the seriousness of electromagnetic interference as a national problem.

On July 24, 1963, the FCC adopted Docket 15130 in the Matter of Reliability and Related Design Parameters of Microwave Radio Relay Communication Systems and Resultant Impact upon Spectrum Utilization. In this docket the FCC referred to an earlier JTAC report submitted by request in which JTAC recommended FCC adoption of a policy permitting the use of frequency diversity in all cases except where local interference would result. JTAC also recommended that a carrier frequency separation of at least 5 to 10 per cent should be maintained to obtain full benefits. The docket further stated that, although JTAC provided the Commission with pertinent analysis and commentary on the relative merits of diversity techniques, the Commission still lacked sufficient data to permit a considered judgment on this method of achieving reliability, at the cost of spectrum space. In November 1963, JTAC established Subcommittee 63.2 under the chairmanship of William H. Radford to submit a report on FCC Docket 15130.

5. The International Radio Consultative Committee (CCIR)

The Executive Committee of the U.S. National CCIR held two meetings at which reports were submitted by the Chairman of the U.S. Study Groups. The Executive Committee approved the United States responses to the studies assigned at the IXth Plenary Assembly prior to submission at the Xth Plenary Assembly which was held in January 1963 in Geneva, Switzerland. At the Xth Plenary Assembly, the United States also submitted proposed new studies. The Extraordinary Administrative Radio Conference to Allocate Frequency Bands for Space Radiocommunication Purposes was held in October 1963 in Geneva. There was individual IEEE representation at both meetings. Lists of material received from the Executive Committee and from Geneva at IEEE during 1963 were distributed quarterly to the chairmen of the IEEE Professional Technical Groups and the Technical Committees, and to the members of JTAC.

6. The International Scientific Radio Union

The U.S. National Committee of the International Scientific Radio Union (URSI), together with the Boeing Scientific Research Laboratories and the University of Washington, jointly sponsored the Symposium on Signal Statistics, which was held December 6 and 7 prior to the URSI-IEEE 1963 Fall Meeting. The 1963 URSI General Assembly was held in Tokyo. Approximately 800 attended the Assembly, at which propagation problems formed the principal topic of discussion. There was individual IEEE representation at the Assembly in Tokyo.

7. IEEE Intersociety Relations Committee

The IEEE Intersociety Relations Committee met four times during the year. The members for 1963 were: Warren H. Chase, Chairman; E. Finley Carter; Lloyd V. Berkner; A.A. Johnson; T. M. Linville; Frederick B. Llewellyn; Ronald L. McFarland; Charles F. Savage, Jr.; and J. W. Simpson.

The Committee's scope and responsibilities are described in IEEE Bylaw 409.8. At its initial meeting in March 1963, the Committee recognized that IEEE is a nonnational society. The Intersociety Relations Committee then moved that it recommend to the Executive Committee that its scope be amended by the following addition to the first paragraph:

> "In accordance with the Constitution and Bylaws which recognize IEEE as a society nonnational in scope, the proceedings of this committee shall be organized, with respect to intersociety problems, country by country. Accordingly, this committee may find it desirable from time to time to organize subcommittees to deal with problems of one country or another."

The Intersociety Relations Committee originally existed as an AIEE committee. It was stimulated and its activities and responsibilities were greatly increased in 1963 as the result of the AIEE-IRE merger into IEEE. In every case where IRE or AIEE had appointed an official representative to any other organization, the individual's availability and competence for the particular assignment were carefully reconsidered by the ISRC. This required a large amount of evaluation of personnel in order to accomplish the most effective liaison with other organizations.

At the inception of the IEEE, many long-term appointments were in effect as the result of previous action by the respective appointing agencies of IRE and AIEE. These appointments run until their respective termination dates. The IEEE, on recommendation of the ISRC, appointed representatives to new terms in 20 organizations during 1963.

Section D—Editorial Activities

General. The year 1963 was a year of maintaining the *status quo* for the present while making major policy decisions for the future. On the recommendation of the Ad Hoc Committee on Editorial Policy, formed in the fall of 1962, the Board of Directors authorized (1) the establishment of a new publication, called *IEEE Spectrum*, to go to all members except Student members, (2) the continuance of the *Proceedings of the IEEE* on a paid subscription basis, and (3) the discontinuance of *Electrical Engineering*. *Spectrum* was to be general in nature and educational in intent, while *Proceedings* was to be a research-oriented journal covering all fields served by the IEEE. These changes took effect January 1, 1964.

Meanwhile, in 1963, the former publications of the AIEE and the IRE were continued in good health, without interruption and without significant change in quantity or character. As a result, the IEEE published more than 2700 papers and 900 letters to the editor, for a total of 30,120 editorial pages during the year.

Proceedings of the IEEE. The year was highlighted by the appearance in the *Proceedings* of three special issues: on quantum electronics; new energy sources; and an international issue. The number of papers submitted during the year and their disposition remained about the same as in the past, specifically, 287 papers received of which 26 per cent were accepted, 34 per cent were referred to the *Transactions* for publication consideration, and 40 per cent were rejected. The number of letters in the "Correspondence" section increased from 400 the previous year to 506.

As a result the *Proceedings* carried 2037 editorial pages and 1541 pages of advertising and filler during the year, for a total of 3578 pages.

Electrical Engineering: Although advertising decreased substantially, the editorial content of *Electrical Engineering* was maintained at about the same level as in previous years. Of the 1378 pages published, 997 were devoted to technical and editorial matter.

Transactions. The title *Transactions*, having been used prominently by both predecessor societies, now encompasses a major share of the publication activity of the IEEE. The *Transactions* of the 30 PTGs accounted for 106 issues totaling 10,736 pages during the year, while the three bimonthly *Transactions* of the TOC produced 18 issues and 3088 pages. In both cases the page totals showed an increase over the preceding year.

In keeping with prior practice, the bimonthlies will be available as annual bound volumes in the spring of 1964.

Student Journal. In September 1962, the *AIEE EE Digest* and the *IRE Student Quarterly* were combined into *Student Journal and EE Digest* as a step toward the merger of the two organizations. Consolidation of the two publications into one was completed in January 1963 with the issuance of the *IEEE Student Journal*, a bimonthly magazine which is sent free to all Student Members. In 1963 this publication contributed 280 pages of technical and other career information to its 27,000 readers.

Special Publications and Preprints. As in past years, the International Convention, held in New York City in March, produced a ten-part *Convention Record* totaling 1988 pages. In addition, ten special and technical conference publications were issued during the year totaling 1502 pages. Finally, a total of 624 papers, amounting to 9452 pages, were individually preprinted in photo-offset form for 13 meetings and conferences.

Section E—Major Meetings and Conferences

The IEEE, its Regions, Sections, Technical Committees, and PTGs, sponsored or cosponsored 87 major technical meetings and conferences during 1963. The high point of the year was the annual International Convention in March, which attracted 71,337 members and visitors to New York for a four-day program of technical papers and exhibits.

Section F—Student Branches

Six Student Branches were established during 1963, raising the total to 260, seventeen of which are in Canada and six in Latin America and Europe. The new Student Branches and the IEEE Sections in which they are located are as follows: Universidad de Concepcion (Chile), Ohio Technical Institute (Columbus), Universita de Padova (Italy), Spring Garden Institute (Philadelphia), The Community College and Technical Institute of Temple University (Philadelphia), and Union Technical Institute (North Jersey).

Report of the Treasurer

W. R. Clark

Our financial statements, as certified by Price Waterhouse & Company, who have audited the books of the IEEE for 1963, are contained at the end of this report. In the Statement of Financial Position the working capital amounted to $1,306,262, with the marketable securities stated at cost, and to $2,097,000, with securities stated at market values at the end of 1963. Our total funds were $3,792,275 and $4,626,000 respectively. Since our total expenses for operations during fiscal 1963 amount to $5,708,593, we are in sound financial position.

Realignment of Securities Portfolio

In order to protect the reserves of the Institute, our financial counsel, Wood, Struthers and Winthrop, advised us to redistribute the combined reserves of the two former institutes to obtain a better balance between common stock and bonds. Accordingly, our portfolio was revised during 1963 and the investment picture as of December 31, 1963, was as shown in the following table.

Our financial counsel considers a distribution of about 40 per cent in bonds and the rest in stocks a safe distribution for an organization of our type. Some of the Restricted Funds are also invested in accordance with advice of investment counsel. These Restricted Funds (listed in the audited report) are available for the awards indicated.

PORTFOLIO AT END OF FISCAL 1963

Stock	Book Value Amount	Book Value %	Market Value Amount	Market Value %	Appreciation (or Depreciation)
Preferred	$114,121	4.6	$111,000	3.4	(3,121)
Common	939,246	37.7	1,813,061	55.3	873,815
Total Stock	1,053,367	42.3	1,924,061	58.7	870,694
Bonds $1,498,500 (face value)	1,436,184	57.7	1,355,057	41.3	(81,127)
GRAND TOTAL	$2,489,551	100.0	$3,279,118	100.0	$789,567

Operations

The nonrecurring expenses, which were not forecast, accounted for practically all of the difference between the $424,019 actual excess of expenses over income and the budgeted $322,100 deficit established by the Board of Directors. These expenses included those involved in the design and study of a new computer to replace the existing system, which is totally inadequate for the combined Institutes, an inventory adjustment to reduce a large stock of old issues of our publications, and legal expenses associated with various items resulting from the merger.

In addition, the net advertising income fell below forecast because of the unexpected change in publication policy, extra help was needed at the start of the year to promptly merge the activities of the two Societies, and the expenses for Special Technical Conferences exceeded expectations. These items, in addition to the nonrecurring items, account for the difference between the actual deficit and the $259,900 deficit forecasted just prior to the merger.

1964 Prospects

Prior to the merger, it was estimated that the second year of operations would result in a $131,250 deficit. The current operating expenses for 1964 are expected to result in a deficit of less than this amount. However, nonrecurring expenses involved in resolving the income tax question, the consolidation of Headquarters staff, the preparation for the new computer operation, and the contribution to the United Engineering Center to fulfill IEEE's obligation in completing the building, indicate the deficit for 1964 will exceed that for 1963. These expenditures will result in greater operating efficiencies in future years and a sounder financial position, and will enable the Institute to better serve its members.

Price Waterhouse & Co.

60 Broad Street
New York 10004
March 23, 1964

To the Board of Directors of

The Institute of Electrical and Electronics Engineers (Incorporated)

We have examined the statement of financial position of The Institute of Electrical and Electronics Engineers (Incorporated) as of December 31, 1963, and the statements of income and operating fund and changes in restricted funds for the year. Our examination was made in accordance with generally accepted auditing standards and accordingly included such tests of the accounting records and such other auditing procedures as we considered necessary in the circumstances.

On January 1, 1963, The Institute of Radio Engineers Inc. was merged into the American Institute of Electrical Engineers and the name of the latter organization changed to The Institute of Electrical and Electronics Engineers (Incorporated).

In our opinion, subject to the final determination of the Institute's income tax liability, if any, as referred to in Note 3, the accompanying statements present fairly the financial position of The Institute of Electrical and Electronics Engineers (Incorporated) at December 31, 1963, and the results of its operations for the year, in conformity with generally accepted accounting principles.

Price Waterhouse & Co.

THE INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS (INCORPORATED)
STATEMENT OF FINANCIAL POSITION
DECEMBER 31, 1963

Operating Fund		
Current assets:		
Cash		$ 565,111
Marketable securities, at cost, market value $3,280,000		2,489,552
Accounts receivable		130,660
Notes receivable, bearing 6% interest per annum		128,279
Prepaid expenses, etc.		300,230
Total current assets		3,613,832
Less—Current liabilities:		
Accounts and accrued expenses payable		547,106
Funds held for the use of professional groups		255,049
Deferred income:		
Dues	$ 567,715	
Subscriptions	388,606	
Convention	544,190	
Other	4,904	
		1,505,415
Total current liabilities		2,307,570
Working capital		1,306,262
Fixed assets:		
Land and buildings, at cost	1,641,455	
Office equipment and leasehold improvements, at cost less accumulated depreciation and amortization of $263,041	427,982	
		2,069,437
Operating fund balance (accompanying statement)		3,375,699
Property Fund		
Advance to United Engineering Trustees, Inc. (Note 2)		265,000
Restricted Funds		
Cash		53,858
Marketable securities, at cost, market value $141,000		97,718
Restricted fund balance (accompanying statement)		151,576
Total funds		$3,792,275

THE INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS (INCORPORATED)
STATEMENT OF INCOME AND OPERATING FUND
FOR THE YEAR ENDED DECEMBER 31, 1963

Income:	
Membership entrance fees and dues	$1,868,407
Advertising	1,397,263
Convention and technical conferences	1,106,008
Subscriptions	723,532
Miscellaneous income, which includes interest and dividends of $118,230	189,274
Total income	5,284,484
Expenses:	
Exclusive of salaries shown below:	
Section rebates	383,159
Advertising costs	438,242
Publication printing cost	1,411,660
Convention and technical conferences	902,059
Sales items	32,714

General and administrative expenses (Note 1)	1,062,174
Salaries	1,478,585
Total expenses	5,708,593
Excess of expenses over income	(424,109)
Operating fund balance, January 1, 1963	3,799,808
Operating fund balance, December 31, 1963 (Note 3)	$3,375,699

THE INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS (INCORPORATED) STATEMENT OF CHANGES IN RESTRICTED FUNDS FOR THE YEAR ENDED DECEMBER 31, 1963

Restricted fund	Fund balance January 1, 1963	Receipts from contributions and marketable securities	Disbursements for awards and related costs	Fund balance December 31, 1963
Life Member Fund	$ 51,157	$ 9,333	$5,439	$ 55,051
International Electrical Congress—St. Louis Library Fund	6,685	137	137	6,685
Edison Medal Fund	12,194	1,167*	892	12,469
Edison Endowment Fund	8,254	267	267*	8,254
Lamme Medal Fund	9,397	345	411	9,331
Mailloux Fund	1,087	42	42	1,087
Volta Memorial Fund	6,698	15,685	2,215	20,168
Kettering Award Fund	2,247	26		2,273
Browder J. Thompson Memorial Prize Award Fund	5,384	96		5,480
Harry Diamond Memorial Prize Award Fund	1,042	28		1,070
Vladimir K. Zworykin Television Award Fund	5,124	109		5,233
W. R. G. Baker Award Fund	8,228	360		8,588
William J. Morlock Award Fund	5,000	44		5,044
W. W. McDowell Award Fund	10,000	88		10,088
William D. George Memorial Fund		755		755
Total	$132,497	$28,482	$9,403	$151,576

* Includes an interfund transfer of $267.

NOTES TO FINANCIAL STATEMENTS

Note 1:

In 1963 the Institute amended the retirement plans of AIEE and IRE to create a noncontributory pension plan for its employees. The unfunded past service liability under the plan is approximately $110,000.

Note 2:

In accordance with a Founder's agreement between the Institute and the United Engineering Trustees, Inc. the Institute has agreed to permanently maintain its principal offices in the United Engineering Center, which in 1963 involved a lease payment of approximately $94,000. The $265,000 advance to United Engineering Trustees, Inc. is repayable only out of available reserve funds on dissolution of United Engineering Trustees, Inc. and carries interest at an annual rate of 4%.

Note 3:

On February 20, 1964 the Institute was notified by the Internal Revenue Service that the Institute's annual convention involved the operation by the Institute of an unrelated trade or business and that the net income therefrom, if any, is subject to federal tax. The Service has requested the Institute to file tax returns with respect to that income for the years 1951 to date.

The Institute, through its legal counsel, intends to contest imposition of the proposed tax by appropriate administrative and legal action. The maximum amount of potential tax liability, which could be material, is not yet known since the matter is presently being studied by the Institute's legal counsel and auditors and the Institute had not received any statement of proposed adjustments of its tax liability from the Internal Revenue Service.

Authors

C. Q. Lemmond (SM) received the B.E.E. degree from North Carolina State College in 1944. He joined the General Electric Company's Engineering Test Program in June 1944 and later entered the company's Creative Engineering Program. In 1948 he accepted a permanent position in the General Engineering Laboratory (now the Advanced Technology Laboratories) as a development engineer. His first assignments dealt with radiation detectors and their associated circuitry. He was later project director of electronic control equipment for the 15-MeV industrial betatron.

In 1949 he was placed in charge of the Electron and Optics Unit, where he supervised the design and development of various electron optical instruments. In 1956 the unit was combined with the Color and Optics Engineering Unit and Mr. Lemmond was named manager of the new component, called Optical Engineering. In this capacity he has been responsible for the design and development of electron and ion optical systems, including ultraviolet and infrared wavelengths. The group has engaged in the design of periscopes, borescopes, infrared sights and scanners, and many other related projects. At present the areas of investigation include optical information processing, high-density electron optical storage and retrieval of information, and high-energy-laser applications.

L. H. Stauffer (SM) received the B.S. degree in physics from Utah State University in 1927 and the doctorate in physics from the University of California in 1930. After 11 years as an assistant professor of physics at the University of Idaho, he joined the General Electric Company in 1941. He worked on magnetic amplifier circuit development in the General Engineering Laboratory until 1943, when he moved to the Carbon Products Laboratory in the Apparatus Department. Here he worked on the development and testing of carbon brushes for high-altitude aircraft use, and on the development of a high-temperature graphitizing furnace. In 1947 he assumed his present position of physicist in the Advanced Technology Laboratories, where he is concerned with the theoretical and practical aspects of physics, including high-energy particle accelerators. He has participated in the development of ion sources, analyzers, ion focusing systems, and accelerator tubes. His work has also included development of methods for measuring magnetic properties of strip steel, a study of long-range missile phenomena, and development of a grid-controlled plasma electron beam source. His recent activities have been largely in the fields of gaseous electronics and particle physics. He is now engaged in the development and application of beam-forming hollow cathodes for operation in low-pressure gas atmospheres.

Fremont Felix (SM, L) received his scholastic and engineering education in Paris, where he received the engineering degree from the Ecole Supérieure d'Electricité in 1922. He first came to the United States in 1926, and since that time has been associated with many U.S. electrical manufacturing, engineering, and construction organizations, on whose behalf he has been overseas since 1946, residing first in Istanbul, then in Milan, and now in Paris. He has been associated with the development of single-phase traction motors, steel mill and other industrial application engineering, aircraft gas-turbine application, and the introduction of U.S. enriched-uranium-concept nuclear plants to Europe.

Mr. Felix is a member of the U.S. Council of the International Chamber of Commerce and is a delegate to its Commissions on European Affairs, on International Trade Practices, and on Asian and Far Eastern Affairs. His overseas missions have taken him to numerous countries throughout the world and have contributed to the approach of which the article in this issue represents the most complete form. Earlier studies were reported in a 1955 AIEE conference paper, "National Income and the Use of Electrical Energy," and a 1963 IEEE conference paper, "Electrical Energy, Total Energy, and National Income," which has been published in India and Pakistan in English, and in translation in eight other countries.

J. S. Frame A biographical sketch of Dr. Frame appears on page 239 of the March issue.

Frank Schwoerer, Jr. is manager of engineering at Nuclear Utility Services, Inc., an independent consulting firm in Washington, D.C. He has provided technical direction for AEC-financed studies of nuclear reactors for water desalination and of dual reactors versus a single reactor for a given power-plant installation. He also participated in providing technical assistance to the U.S. Army program for both the operation of existing reactors and advanced application studies. Prior to joining NUS, he was engaged for seven years in the design and development of advanced reactor concepts at Westinghouse Electric Corporation's Bettis Atomic Power Laboratory. Principal accomplishments were a comparative evaluation of seed-blanket and slightly enriched pressurized-water reactors, the conceptual design of a self-controlling reactor employing new concepts in the interaction of the nuclear and thermal conditions in the core with the entire reactor power plant, and thermal–mechanical design of an advanced converter reactor utilizing the thorium fuel cycle. In the aircraft gas-turbine field, he has worked on the development of compressors and turbines for Westinghouse.

Mr. Schwoerer received the B.S. degree in naval architecture and marine engineering from the Webb Institute of Naval Architecture in 1944 and the M.S. in marine engineering from Massachusetts Institute of Technology in 1947. He also has done graduate work at the University of Pennsylvania.

Warren F. Witzig (SM) is senior vice president of Nuclear Utility Services, Inc., which he helped establish in 1960. He is responsible for the technical direction of the company's work on large power reactors, small military reactors, and the use of nuclear reactors and isotopes in aerospace applications. He serves on the Pathfinder Atomic Power Plant Safety Committee and the Consumers Power Company Big Rock Point Nuclear Power Plant R & D Review Committee. He has directed consulting activities covering a broad range of technical disciplines in the nuclear field. These have included most aspects of reactor design and operation, with particular emphasis on reactor core design and optimization of reactor fuel cycles. While with Westinghouse Electric Corporation he was responsible for the design of the reactors in the Skipjack and Polaris series of submarines, and served as manager of nuclear fuels irradiation research programs and manager of the Natural Circulation Reactor Project. Prior to this he performed heat transfer and high-vacuum work in the Westinghouse Research Laboratories.

Dr. Witzig received the B.S. degree in electrical engineering from Rensselaer Polytechnic Institute, and the M.S. degree in electrical engineering and the Ph.D. in physics from the University of Pittsburgh. He is a member of the American Nuclear Society, American Physical Society, Sigma Xi, Sigma Pi Sigma, Eta Kappa Nu, and the Nuclear Standards Board.

Aiko Hormann was born in Yokohama, Japan, and brought up in Tokyo. After having been selected as an exchange student to the United States, with four scholarships, she majored in mathematics at the Southeast Missouri State College, from which she received the B.S. degree. She subsequently received the M.A. degree in mathematics from the University of Missouri, following which she took two additional years of graduate study in mathematics at the University of Pittsburgh.

As a mathematician for Westinghouse Research Laboratories, she engaged in computer programming of mathematical and engineering problems, and contributed to the design and development of compiler languages and diagnostic programs for compilers. At System Development Corporation she participated in the design and development of a list-processing language, ROVER. At present she is a member of SDC's Artificial Intelligence Staff and is working on Gaku.

Mrs. Hormann is a member of the Mathematical Association of America, the Association for Computing Machinery, the American Association for the Advancement of Science, and Pi Mu Epsilon.

IEEE publications

scanning the issues
advance abstracts
translated journals
special publications

Scanning the issues

Thermionic AC Generation. Since thermionic diodes produce a low-voltage dc output, large generators necessarily entail very high total currents. One way of reducing these to manageable size is to connect a large number of small cells in series. Beyond a certain point, however, this leads to a prohibitive decrease in reliability.

An alternate method would be to connect only a small number of cells in series, convert their output to alternating current, and use a step-up transformer to produce the high-voltage moderate current demanded by the load. The dc-to-ac conversion could be achieved by means of rotating machinery but this would sacrifice one of the chief advantages of thermionic generators, the absence of moving parts. On the other hand, the development of high-power static inverters is not a simple problem.

A thermionic converter is itself an electronic tube; this suggests the possibility of using the generator as its own modulating element. In principle, if the output of a thermionic generator is modulated so as to produce a periodically varying direct current, the ac component of that output can be extracted (and stepped up) by means of a transformer.

A recent analysis of the use of magnetic fields to modulate a thermionic converter shows that an ac efficiency equal to approximately 70 per cent of the corresponding dc efficiency is obtainable.

These results apply to the generation of sinusoidal alternating current. There are indications that higher efficiencies are possible with other waveforms, that is, those approaching square waveshapes. It is felt that, even for sinusoidal alternating current, the penalty in performance and efficiency is not excessive in view of the advantages inherent in direct ac generation. (C. L. Eisen and A. Schock, "Thermionic AC Generation," *IEEE Trans. on Communication and Electronics*, May 1964.)

Tube Reliability. With the advent of the transistor, relatively little has been heard on the subject of tube reliability over the past dozen years. It is interesting to note, therefore, that the vacuum tube art has not been standing still during this period and that, in fact, the life span of the average receiving tube has been substantially increased.

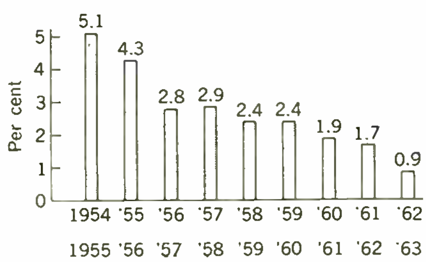
The improvement in tube reliability is clearly revealed by recently published results of an extensive life-testing program conducted over the past ten years. Since July 1954, Sylvania has tested approximately 33000 of its tubes in more than 2000 television receivers produced by all major U.S. manufacturers. In the tests, the receivers were operated at 130 volts, higher than normal line voltage, in order more readily to disclose inherent tube weaknesses. The tests were run for 1500 hours of tube operation on a cycle of 50 minutes on and 10 minutes off.

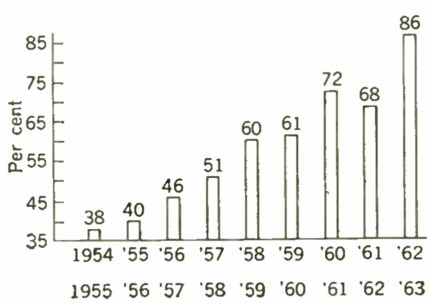
These tests show that during the period 1954–1963, average tube failure rate (at 130 volts) has been reduced from 5 per cent per 1000 hours to 0.9 per cent per 1000 hours. At the same time, the proportion of tube types having zero failures through 1500 hours at 130 volts has increased from 38 per cent to 86 per cent.

In 8000-hour tests conducted at normal line voltage (117 volts) during the last two years, the tube failure rate was found to be only 0.25 per cent per 1000 hours. This is equivalent to approximately 96 per cent TV receiver reliability through 1000 hours.

And it appears that failure-rate improvement is going to continue unabated. The failure rate under the accelerated 130-volt condition is estimated to be not more than 0.5 per cent for the one-year testing period ending this month. At normal 117-volt operation, this would reflect an estimated failure rate of not more than 0.15 per cent per 1000 hours. (G. V. Herrold, "Tube Reliability in TV Receivers," *IEEE Trans. on Broadcast and Television Receivers*, May 1964.)

Tube failure graphs. Upper illustration shows tube failure rates in per cent per 1000 hours; lower, proportion of total tube types which showed zero failure through 1500 hours. The time scale for each bar is from July of one year to July of the next.





Ring-Plane Circuits. There appears to be growing interest in a type of slow-wave structure known as ring-plane circuits. The name comes from their physical structure, which consists of circular rings or slotted pipes supported by one or more radial planes.

Ring-plane circuits have a number of interesting properties. Their large diameter can accommodate large-diameter electron beams, and they have relatively large impedances for strong beam interaction. They can easily be constructed to the required tolerances in the small sizes associated with operation at millimeter wavelengths and can easily be cooled by conduction through the supporting planes. The latter two characteristics make these circuits well suited for use in high-power, high-frequency traveling-wave tubes.

In fact, a laboratory model of a traveling-wave-tube amplifier using a ring-plane circuit recently produced a

Designed for Application

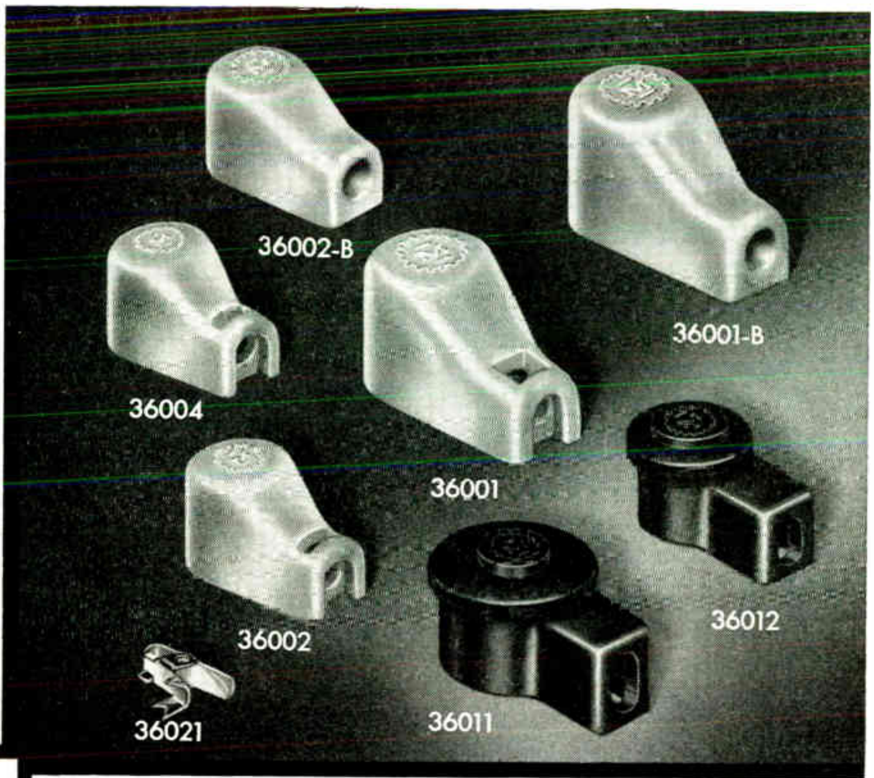
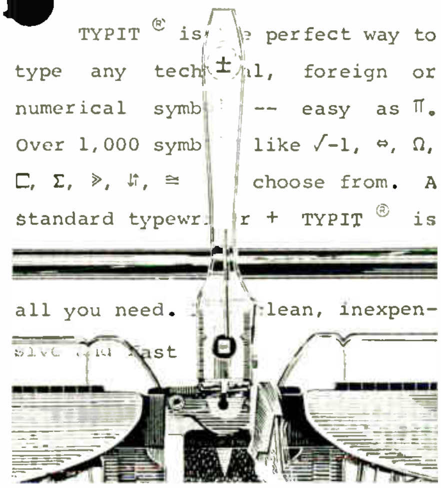


PLATE AND GRID CAPS

Illustrated are the stock military and standard ceramic Millen plate and grid caps and the snap lock caps for mobile and industrial applications requiring tighter than normal grip. Standard plate caps have phosphor bronze clips; military plate caps have beryllium copper clips.

JAMES MILLEN MFG. CO., INC.
MALDEN
MASSACHUSETTS



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Correspondence

'Artificial Intelligentsia' Supplement

As a prerequisite to the following the reader is referred to the Proposal Request and the response of "Calculated Risks, Inc.," in Louis Fein's paper (February issue, pp. 74–78). This additional "proposal" is herewith submitted since we feel that the response of Calculated Risks, Inc., indicated that they were beginning to think about problems for which we already had solutions. It should be noted that the technical statements are intended to reflect our actual work.

Paul W. Cooper
Sylvania Electronic Systems
Waltham, Mass.

Hyperquadrics Unlimited

April 29, 1964

TO: Bright Field
ATTN: SMARTIES
SUBJECT: Proposal in Response to RFB No. 123(456)-ABC-789-D

Dear Sir:

As usual we have submitted our proposal late. However, we hope that you find our two product lines, **post-Freud**[1] and **post-Socrates**[2] classification machines, sufficiently advanced to compensate for our belated response.

It has been a common practice to treat statistical pattern recognition problems under assumptions of *normality*, and various **Gauss**-type machines have been in use. In order to deal with *abnormally* distributed patterns, a number of organizations have been recently examining the problem of developing a **Dr. Freud** machine for transforming abnormal distributions into normal ones for **Gauss** to operate upon.

Recognizing that there are many more abnormal distributions than normal ones, we have developed a device, called **post-Freud,** which works with the abnormal distributions directly. We do not change abnormal ones into normal ones, but accept them as they are.

Recognizing that any decision boundary partitioning a sample space into category regions can be optimum for many probability distribution forms, we have demonstrated that the simpler boundary forms arising under assumptions of normality are optimum for many interesting abnormal distributions as well. These boundary forms, the hyperplanes, hyperspheres, and hyperquadrics, are implemented, respectively, as correlators, generalized energy detectors, and comparison of quadratic forms.

Although some of these partitioning procedures depend upon the distribution function form, others are exactly the same for the abnormal as for the normal, although the performance differs. However, in certain limiting cases of increased dimension both abnormal and normal perform the same, and to the observer the possibility of deciding whether the behavior reflects normality or abnormality vanishes. In fact, in some cases the abnormal perform better, which invites one to ponder the very meaning of normalcy.

At times the problem can be simplified by an efficient reduction in dimensionality. Yet, where necessary, the performance can be improved by adding a new dimension. Taking our cue from the Greek prefix *hyper*, meaning *higher*, adding a new dimension then brings us on to a *higher plane*.

It has been felt that in some quarters pattern recognition researchers have