

ELECTRONIC TECHNOLOGY SERIES

ELECTROSTATICS

a

RIDER

publication

\$1.35

ELECTROSTATICS

Edited by
Alexander Schure, Ph.D., Ed.D.



JOHN F. RIDER PUBLISHER, INC.

116 West 14th Street • New York 11, N. Y.

Copyright June 1958 by
JOHN F. RIDER PUBLISHER, INC.

All rights reserved. This book or parts thereof may
not be reproduced in any form or in any language
without permission of the publisher.

Library of Congress Catalog Card Number 58-10386

Printed in the United States of America

PREFACE

The study of electrostatics, which is concerned with the behavior of electrical charges at rest, is an integral part of electronic technology. The concepts covered in a discussion of electrostatic principles apply in practical and theoretical situations ranging from the action of a capacitor to an understanding of many electrodynamic problems.

The text covers the subject comprehensively and thoroughly. Using a mathematical treatment that, while simple, is central to the scientific considerations presented, it enables the student or interested technician to obtain a thorough understanding of the given topics that is quantitative as well as qualitative. The book is firmly grounded on a thorough exposition of relevant physical fundamentals. The topics, once presented, are related to practical situations, such as a novel and interesting description of the earth's electric field. Drill is given in selected problems to afford the reader additional profitable information and an opportunity to apply the principles taught.

Specific attention is given to atomic structure; conductors and insulators; the law of electric charges; electrostatic detection equipment; the triboelectric series; Coulomb's law; unit systems, including the cgs and mks; practical electric units; the nature and characteristics of the electric field; the equation of field intensity; Gauss' Law; charge distribution; oil-drop equations; dielectrics and dielectric constants; electric potential; equipotential surfaces; potential gradient; capacitance and capacitors; and applications such as the Van

PREFACE

de Graaff generator. In all, the coverage is sufficiently inclusive to yield a base for further explorations into more advanced work.

Grateful acknowledgment is made to the staff of the New York Institute of Technology for its assistance in the preparation of the manuscript of this book.

*June 1958
New York, N. Y.*

A. S.

CONTENTS

<i>Chapter</i>		<i>Page</i>
1	Matter and Electric Charges	1
2	Unit Systems	14
3	Nature and Characteristics of the Electric Field	21
4	Electric Potential, Capacitance, and Capacitors	43
5	Electrostatic Devices and Applications	57

Chapter 1

MATTER AND ELECTRIC CHARGES

1. Introduction

The study of electricity, as a human endeavor, encompasses a vast amount of widely diversified material. It reaches into communications, industrial processes of all varieties, transportation and transportation control, scientific research, and military systems of every imaginable type.

Despite its ramifications and complexities, electricity has its beginnings in a few relatively simple concepts. Like every other science, it did not begin to yield its secrets to human research until this research began to take on a semblance of *quantitativeness*. For a study to be quantitative all its basic concepts must, first of all, be expressible in *numbers* which are measured in *units* having definitions that are rigidly specific. The statement "the trip from city X to city Y is a long one" is completely uninformative because the word "long" is a relative quantity having no intrinsic numerical value. There is just as little information contained in the statement that the "current flowing from terminal X to terminal Y in an electric circuit is large." To the power engineer a "large" current may be one that can fuse a one-inch-diameter copper bus bar; a radio-tube engineer may consider a current one-millionth this size to be a large one.

The development and comprehension of electrical units of measure properly begin in *electrostatics*, the study of electric charge at

rest. It is here that the forces between electric particles can be observed experimentally and measured before motion begins, and that the fundamental laws that govern these forces can be derived and stated.

2. Atomic Structure

After the work of Rutherford, Bohr, and their contemporaries, the atom was for many years considered to contain only three particles: electrons, protons, and neutrons. Today the great atom-smashers reveal the presence of numerous other subatomic particles, which appear and disappear in atomic interactions. The existence of positrons, pi mesons, tau mesons, kappa mesons, neutrinos, and many others is no longer questionable, and new particles are being brought to light yearly. Apart from knowing that these new particles exist, however, the student of electricity need not concern himself with them. Virtually every phase of this wide subject permits description in terms of electrons and protons only. (Neutrons enter the picture more or less incidentally.)

Electrons exert forces on each other. If they are free to move, a group of them will scatter almost instantaneously in a conducting material, so that they become as far apart as possible. This is taken as evidence that a *force of repulsion* exists between them. Similarly, protons repel each other. Adjacent electrons and protons, however, display a mutual *attraction*.

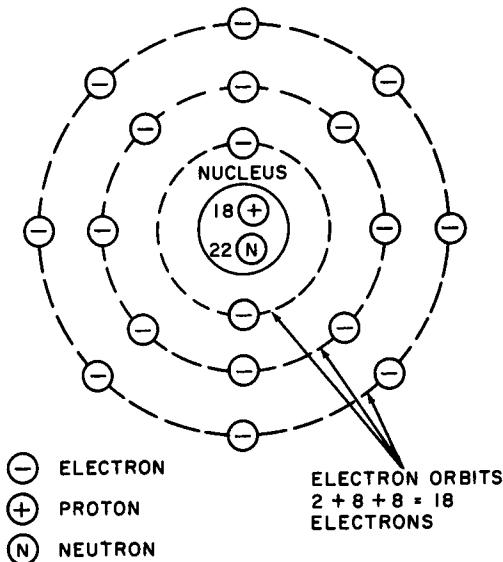
The existence of two distinct kinds of forces leads to the conclusion that there are two kinds of electrification or *charge*. On a *purely arbitrary basis*, any substance that is repelled by a glass rod that has been rubbed with silk is said to be *positively charged*. When an object is repelled by a hard-rubber rod that has been rubbed with fur, it is considered *negatively charged*. These terms have no significance other than that of being opposite; there is no inherent meaning in "negative" or "positive" aside from the fact that bodies bearing these types of electrification behave differently.

A *neutral atom* contains equal numbers of electrons and protons. The condition of neutrality implies a kind of electrical *cancellation*, in which it is considered that the strength of the proton's charge is exactly equal and opposite to the charge on the electron. On the basis of the arbitrary choice made above, a proton is designated as a positive charge and an electron as a negative charge. Although the picture of an atom as a miniature solar system in which

the protons and neutrons form the nucleus or "sun," and the electrons are compared to planets orbiting around the nucleus is no longer considered rigorous enough in atomic research, it still suffices for basic electrical study. An atom of argon pictured in this way is given in Fig. 1.

There is ample experimental and theoretical evidence now available that proves that the massive particles of the atom are all contained in the nucleus; that protons and neutrons possess approxi-

Fig. 1. An atom may be pictured as a nucleus consisting of protons and neutrons, with electrons revolving about them in fixed orbits. An atom of argon is shown.



mately the same mass; and that an electron behaves as though its mass were approximately $1/1840$ of a proton. Small electrical forces applied directly to the atom are, therefore, much more likely to move electrons rather than the more massive particles. The greater the mass of a particle, the greater is its *inertia* or tendency to resist a change in motion. Hence electrons are *mobile*, while protons and neutrons are ordinarily pictured as *static* under ordinary electrostatic forces.

Thus stresses placed on the structure of a given group of atoms are much more likely to displace electrons from their orbits rather than affect the nucleus. This idea is found to be quite valid, because electrons may move from atom to atom freely in certain materials

under the influence of electrical forces. If, for example, one electron wanders from its orbit around the nucleus of an argon atom, the condition of neutrality no longer exists and the atom now has a net *positive* charge. Such a charged atom is called an *ion* (in this case a *positive ion*). The roving electron may later attach itself to a neutral atom and establish a net *negative* charge on it, thus creating a *negative ion*.

3. Conductors and Insulators

In most substances, particularly non-metals like rubber, plastics, and ceramics, the orbiting electrons are very closely bound to the nucleus, and it is difficult to separate even one electron from the rest of the structure. An *electric current* is a movement of electrons and, since this movement is impeded in the materials under consideration, such substances are termed *nonconductors* or *insulators*. No substance is a perfect insulator, but the current that the usual insulator will conduct with the application of a given electrical force is extremely small compared to that for *conductors* under the same conditions.

All metals and some non-metals such as carbon and silicon contain many mobile free electrons. These electrons pass from atom to atom under the impetus of even small electrical forces. Because materials of this type permit an easy passage of electrons, they are called *conductors*. The reader is reminded that both terms—conductor and nonconductor—are relative. Silver is the best known conductor and, compared to it, iron is a very poor conductor indeed. Yet iron conducts electrons so much better than, say, wood, that it is still referred to as a conductor, whereas wood is called an insulator.

4. The Law of Electric Charges

For introductory study in electrostatics, it is satisfactory to consider an electric charge as a "disembodied" entity that bears either an excess or deficiency of electrons. Because an atom is electrically neutral (contains equal numbers of positive and negative particles), an *electric charge* may be any one of the following (see Fig. 2):

An electron—a single negative charge

A proton—a single positive charge

A group of electrons—equal in charge to the number of electrons present; negative charge.

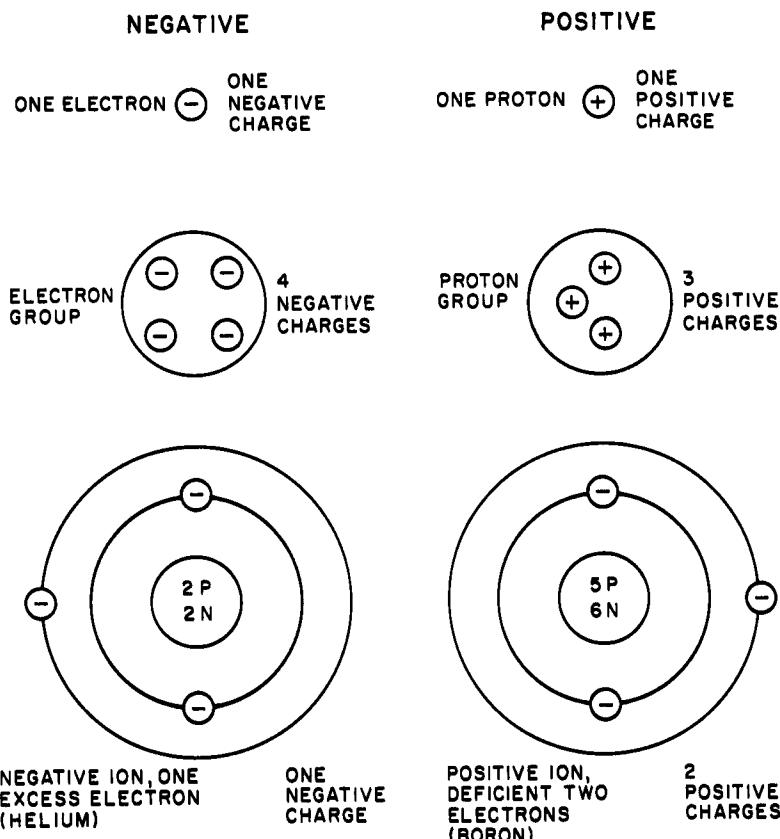


Fig. 2. Some types of electrically charged particles.

A group of protons—equal in charge to the number of protons in the group; positive charge.

A negative ion—an atom that has gained electrons, bearing a negative charge equal to the *excess electrons*.

A positive ion—an atom that has lost electrons, bearing a positive charge equal to the *deficiency of electrons*.

Mention has been made before of the effect these charges have upon each other. This may be formally stated as the *Law of Electric Charges*:

Like charges repel each other; unlike charges attract each other.

It is generally conceded that these forces of attraction and repulsion have a greater effect upon electrons than upon positive charges

of any type, due to the low mass of the electron and its greater mobility. On the other hand, even positive charges are subject to acceleration under the action of electric forces, particularly when these charges are either static or in motion in free space. (A stream of positive ions may undergo a deflection in path when acted upon by electric forces; the extent of the deflection is less marked than it would be if the same forces were applied to a stream of electrons, but it is measurable.)

The Law of Electric Charges may be extended to explain several important phenomena. Assume that a concentrated negative charge has been placed on a small metal disc with an insulated handle (a

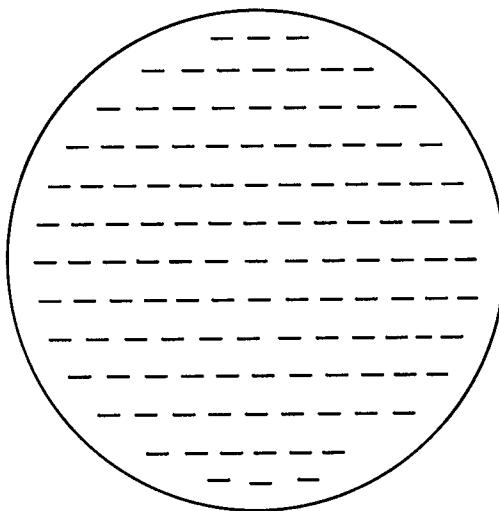


Fig. 3. A charged plate, showing even distribution of the charges on it.

so-called "proof plane") and that the disc is touched to a large aluminum plate mounted on four insulated legs. When the plate is tested with an electrostatic detector, it is found that the original charge on the proof plane has in large part been transferred to the aluminum plate, and that the charge has spread out evenly over the larger surface, as shown in Fig. 3.

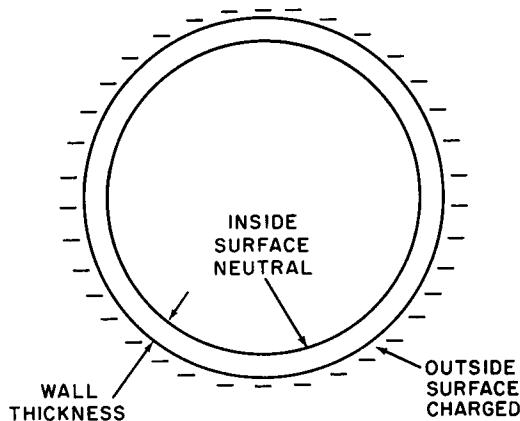
This distribution is explained by the conductivity of the aluminum and the mutual repulsion of the negative charges.

Another example deserving note is the condition of charge of a hollow sphere having a finite and measurable wall thickness. Although the sphere is made of conducting material, the entire charge

is found to be present on the *outside* surface. The inner surface is completely neutral.

A sphere in cross section may be pictured as two concentric circles, as shown in Fig. 4. The mutual repulsion of like charges compels the electrons or ions to move as far apart as possible. In

Fig. 4. An electric charge exists only on the outside surface of a charged sphere.



the case of the sphere, the outer surface represents the greatest physical separation. This phenomenon is utilized in safeguarding the operating personnel of large Van de Graaff static generators such as those used for atom-smashing; it is a startling fact that the safest area for the operators is right inside the huge sphere where millions of volts are being generated. (See Fig. 5.)

5. Electrostatic Detection Equipment

The equipment with which electrical phenomena are detected and measured rarely gives any indication of the processes that occur. Light passing through a lens is easily visible; the size of a basketball bladder changes as compressed air is pumped into it, but electricity is invisible and has no measurable weight. The terminals of a large capacitor appear the same whether it is charged to a lethal potential or not. The task set for electrostatic instruments is that of making electricity evident to our senses.

The gold-leaf electroscope (Fig. 6) is such an instrument. Essentially, the electroscope consists of a glass flask in which a metal rod is supported by passing it through the center of an insulating

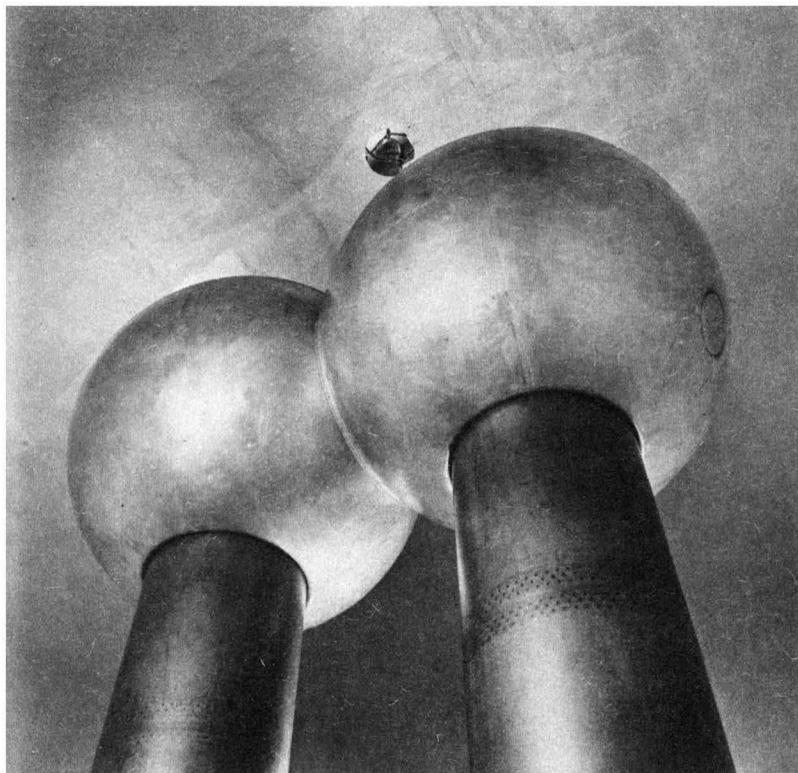


Fig. 5. Van de Graaff 2,000,000-volt generator. Massachusetts Institute of Technology.

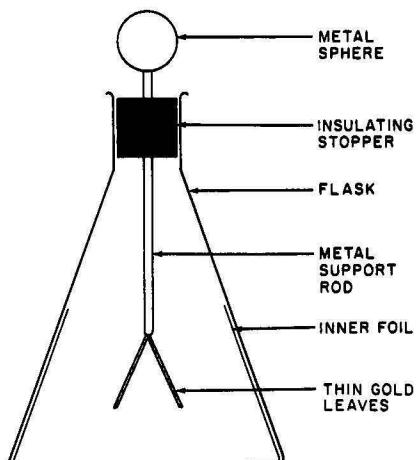


Fig. 6. Gold-leaf electroscope.

stopper. The rod is topped by a spherical conductor; suspended from the bottom are two pieces of thin gold foil, extremely small in mass and measuring about $\frac{1}{4}$ inch by 1 inch. Often, the lower section of the flask is lined inside with tin or aluminum foil.

If a proof plane bearing a positive charge (i.e., a deficiency of electrons) is touched to the top ball, the gold leaves immediately diverge. The extent of the divergence in a given instrument is a function of the magnitude of the original charge on the proof plane.

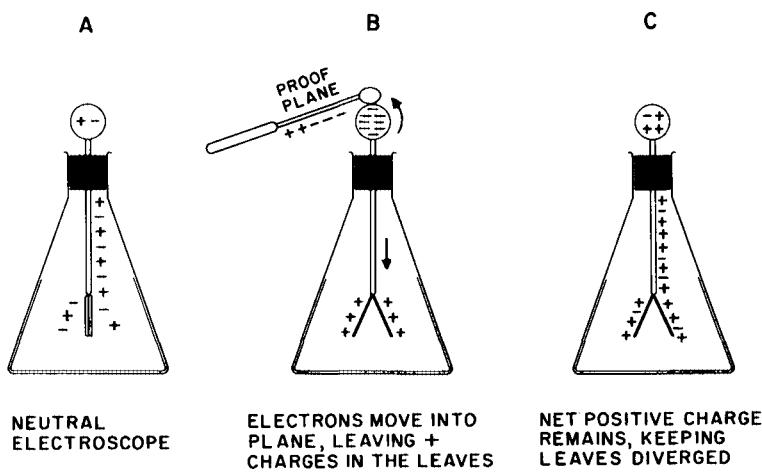


Fig. 7. Charging an electrostatic generator by contact. The charge on the leaves is the same as that of the charge applied.

In a *neutral* electroscope, the atoms, positive ions, and "free" electrons have distributed uniformly over all conducting surfaces and, hence, leave no "net" charge in the absence of an outside charge. Upon the approach and contact of the proof plane, however, the negative particles (electrons) are drawn up into the sphere. Because the plane and the sphere make contact, the electrons are further drawn into the plane and neutralize (to some degree) the positive charge on it. When the proof plane is removed, the system is left with a deficiency of electrons, and thus has a net positive charge. This sequence is shown in Fig. 7.

In its charged state, the electroscope may be used to detect the presence of charges of either sign. Should a negative body approach a positively charged electroscope, such as that of Figure 7C, residual electrons in the system will be driven down into the gold leaves,

partially neutralizing the + charge on them. The divergence of the leaves will diminish, signifying that the charge on the approaching body is negative. Upon the approach of a positively charged body,

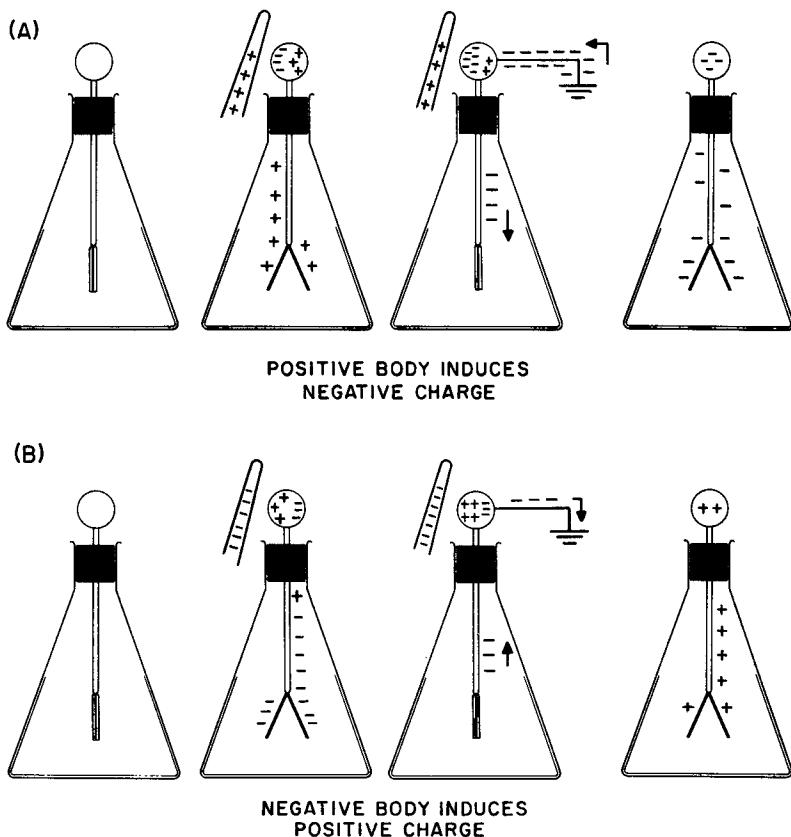


Fig. 8. Charging an electroscope by induction. The charge on the leaves is opposite to that of the charge applied.

however, the leaves will diverge still further, because additional electrons will be drawn upward away from the leaves, making the net positive charge greater than it was before.

The process shown in Fig. 7 is called *charging by contact*. It always results in the appearance of a charge on the electroscope leaves having the *same* sign as the charging body. It is also possible to place a charge of *opposite* sign on any body by using the method of *induction*. This process is shown in Fig. 8. Assume that a posi-

tively charged body, such as a rod of glass rubbed with silk, is brought near the sphere of a neutral electroscope without touching it. Electrons move into the top sphere by electrostatic attraction of opposite charges and the leaves diverge as a result of the net positive charge left on them by the withdrawal of electrons. The sphere is now touched with the finger; as this is done, the leaves will be seen to collapse. This is caused by electrons entering the system from the earth through the body and the finger. These "free" electrons neutralize the positive charge on the side of the sphere away from the rod and on the leaves, whereas the electrons on the other side of the rod are tightly "bound" by the attraction of the positively charged glass rod.

The finger is then withdrawn. This has no effect on the leaves, because the excess negative charges present in the system are still tightly bound in the sphere by the attraction of the glass rod. When the rod is removed, however, the leaves instantly diverge. The removal of the rod permits the negative charges to distribute themselves throughout the metallic portions of the electroscope. Because negative charges *entered the system* via the finger-to-earth connection, the electroscope is left with a net negative charge. Thus, when charged by induction, the electroscope receives electricity of the opposite sign from that of the inducing charge.

A similar sequence takes place when a negatively charged rubber rod is brought near the ball of the electroscope. The steps in this process are shown in Fig. 8B.

6. The Triboelectric Series

As early as 600 B.C., the Greeks observed that a piece of amber when rubbed against their clothes acquired the property of attracting small pieces of parchment and lint. Although this effect was associated only with amber for a long period of time, it was ultimately found that other substances also possessed this property.

A glass rod and a stick of sealing wax after being rubbed with the same piece of silk attract each other. On the other hand, the rod of sealing wax repels a block of sulfur that has also been rubbed with silk. It appears that the glass gives off electrons to the silk, and the silk to the sealing wax. The glass is positively charged, the sealing wax negatively charged, and they attract each other. The silk, however, gives off electrons to the sulfur as well as to the wax; therefore, the sulfur and wax repel each other. Which of two bodies

will become positively electrified by friction depends upon the particular condition of its surface, and depends upon its molecular nature. These ideas have led to the development of a table based on experiment called the *triboelectric series*. In general, if any two bodies in the table are brought into intimate contact, the one that appears first in the table takes on the positive charge.

THE TRIBOELECTRIC SERIES

- | | |
|--------------|-----------------------|
| 1. asbestos | 6. silk |
| 2. glass | 7. cotton |
| 3. mica | 8. resin, sealing wax |
| 4. wool | 9. hard rubber |
| 5. cat's fur | 10. sulfur |

This list, which originated with Michael Faraday, was based upon *rubbing contact* between substances. There is some indication in the results of current experiments that electrification comes not from friction but simply from intimate contact between materials.

7. Coulomb's Law

Newton's Law of Universal Gravitation first revealed the inverse square characteristics of gravitational force fields. The statement of the law includes the measurable fact that the force between two particles due to a field of force between them always varies inversely as the square of the distance between them. Although Newton did not realize the universality of this concept, a better appreciation of it was brought about when Coulomb, almost 100 years later, discovered that the same rule applied to static charges of electricity and magnetic poles as to masses in gravitational fields. Using an extremely delicate and precise instrument called a torsion balance, Coulomb determined that:

The force between two charged bodies that are small compared to the distance between them is directly proportional to the product of their charge magnitudes and inversely proportional to the square of the distance between them.

Expressed algebraically, Coulomb's Law reads:

$$F = \frac{qq'}{Kr^2} \quad (1)$$

where F is the force of attraction or repulsion between the charged bodies, K is a constant to be discussed shortly, and r is the distance between the centers of the two bodies whose charge magnitudes are represented by q and q' .

The constant K depends upon two factors: first, upon the units used to express the other quantities and second, upon the nature of the medium between the charged bodies.

A satisfactory comprehension of electrostatic phenomena can be realized only by achieving a thorough understanding of the meanings and relationships of units of measure in this field. The confusion experienced by students of the subject is understandable, since many authors treat units too lightly. For this reason, the underlying standards of accepted unit systems will be studied in the next chapter.

8. Review Questions

1. Why do electrons rather than protons move when acted upon by electric forces?
2. Discuss an ion in terms of atomic theory; distinguish between negative and positive ions.
3. According to current atomic theory, what is the real difference between an electrical conductor and an insulator?
4. State the Law of Electric Charges.
5. Explain qualitatively why a concentrated negative charge placed on a conducting surface spreads out so that it is uniformly distributed on the surface.
6. Explain qualitatively why there appears to be zero charge on the inside of a charged conducting sphere.
7. A neutral electroscope is charged by bringing it into contact with a negative rod. What charge will appear on the leaves of the electroscope? Explain.
8. Describe the process in which a neutral electroscope may be charged negatively by means of a positively charged proof plane.
9. What is the triboelectric series? If each of the following pairs of substances are brought into intimate contact, which one of the pair will take on a positive charge? (a) mica and sulfur, (b) cat's fur and glass, (c) cotton and asbestos, (d) hard rubber and wool.
10. State Coulomb's Law. Why is the constant K required in the equation that expresses Coulomb's Law?

Chapter 2

UNIT SYSTEMS

9. Basic Systems

Although the English system of weights and measures is more familiar to the layman, virtually all scientific work is carried on in units that stem from the metric system. In this country, mechanical and civil engineers still use the foot-pound-second (*fps*) system, but other branches of engineering have adopted metric units more or less unanimously.

In the metric system, the fundamental unit of length is the *meter*; originally intended to be one-ten-millionth of the distance from the North Pole to the equator around a great circle, the meter is now redefined as the distance between two scratches on a platinum-iridium bar located in the Bureau of Standards in the United States. Since a meter contains 39.37 inches, it is approximately a yard in length. One distinct advantage of the metric system is that it is *decimally* organized. (Fundamental units are multiplied or divided by factors of ten to form new, conveniently-handled sub- and multiple-units.) In the chart that follows, the standard prefixes are

italicized; these prefixes are invariably used to mean the same thing regardless of the units to which they are assigned.

Fundamental length unit = 1 meter		
1 <i>micrometer</i>	10^{-6} meter	(μm)
1 <i>millimeter</i>	10^{-3} meter	(mm)
1 <i>centimeter</i>	10^{-2} meter	(cm)
1 <i>kilometer</i>	10^3 meters	(km)
1 <i>megameter</i>	10^6 meters	(Mm)

If a weightless cube measuring 1 cm on each side is filled with water, the volume of the water is one *cubic centimeter* (1 cm^3 or 1 cc). This volume of water is then said to constitute the basic unit of *mass*, i.e., one *gram* (1 g or 1 gm). Mass is not to be confused with weight; mass is a constant for any given body of matter (within relativistic limits) and is determined by the number, kind, and concentration of the molecules that go into the structure of the body. It is often said that mass of a body is its material *content*. Weight, on the other hand, is a *force* produced by locating a given mass within a gravitational field. Thus weight of a given constant mass is not, in itself, constant. For example, a man who weighs 180 lb on the earth would weigh only 30 lb on the moon, although his material content does not change as a result of the trip. Thus a gram is defined as the mass of one cubic centimeter of distilled water. Subdivisions and multiples of the gram may be expressed by means of the same prefixes as described for the meter. That is, 1 microgram = 10^{-6} grams, 1 milligram = 10^{-3} grams, and 1 kilogram = 10^3 grams.

Intervals of time are always compared by the motions of bodies. Two intervals are defined as equal, when a body moving under exactly the same circumstances moves equally far in both cases. One of the simplest natural units of time is the period of rotation of the earth determined by observing successive noonday positions of the sun. The interval between two successive noons is called the *solar day*, and the basic unit of time selected for the metric system (as well as the English system, incidentally) is the *mean solar second* or the 86,400th part of a mean solar day. For the purposes of this book, it shall be acceptable to conclude that there are 60 mean solar seconds in one minute and 3600 mean solar seconds in one hour.

Many lengths measured in everyday life can be conveniently stated in centimeters (cm), masses in grams (gm), and intervals in

seconds (sec). Thus, the metric system stated in terms of these basic units is often referred to as the *cgs system* (centimeter-gram-second).

10. Force in the CGS System

A clear understanding of the meaning of *force* is possible only from a consideration of Newton's Second Law of Motion. In this famous law, Newton stated that the acceleration taken on by a body under the action of an applied force (push or pull) is directly proportional to the force and inversely proportional to the mass of the body. Algebraically stated:

$$a = \frac{Kf}{m} \quad (2)$$

in which acceleration is measured in cm/sec^2 and mass is measured in grams. K is a constant that can be made to become unity if the unit of force is correctly chosen. Solving for Kf:

$$Kf = ma \quad (3)$$

Using the cgs units for mass and acceleration, the product ma must be expressed in $\text{gm} \cdot \text{cm/sec}^2$ (mass in gm \times acceleration in cm/sec^2). This "long-winded" unit is known by a simpler name, the *dyne*. If we are willing to express *force* in *dynes*, then Equation 3 may be rewritten:

$$K \times \text{dynes} = \text{dynes} \quad (4)$$

Hence, the constant K is now equal to 1 and may be dropped from the equation, and Newton's Second Law may be stated:

$$f = ma \quad (5)$$

where f is in *dynes*, m is in grams, and acceleration is in cm/sec^2 . Under these conditions, a dyne may now be defined as the force required to impart an acceleration of one cm/sec^2 to a mass of one gm.

11. Weight Expressed in CGS Units

Weight is a special case of a force acting on a mass in a gravitational field. On the surface of the earth, a freely falling body accelerates under the action of gravity at the rate of approximately 980 cm/sec^2 . (This figure is slightly different in different geographical locations and is taken merely as a rough average.) It is customary to

assign the letter "g" to this acceleration, and, with weight symbolized as "w," the Second Law becomes:

$$W = mg \quad (6)$$

Because our units must be consistent if K is to remain 1, w must be expressed in *dynes* if mass (m) is in gm and acceleration (g) is in cm/sec².

This immediately leads to the conclusion that *one gm of mass on the earth's surface weighs 980 dynes*. This conclusion is reached when substitution of known numbers is made in Equation 6:

$$w = 1 \text{ gm} \times 980 \text{ cm/sec}^2 \quad (7)$$

or

$$w \text{ (of 1 gm mass)} = 980 \text{ gm-cm/sec}^2 = 980 \text{ dynes} \quad (8)$$

The foregoing discussion points out the fact that *any weight obtained in grams is actually that number multiplied by 980, expressed in dynes*.

The dyne is an important unit in electrostatics and will be related to Coulomb's Law shortly.

12. The MKS System of Units

The *mks* system of weights and measures differs from the *cgs* system in the relative of the units. The following table of conversions will assist you in understanding the changes that must be made:

$$1 \text{ kg} = 1000 \text{ gm}$$

$$1 \text{ m} = 100 \text{ cm}$$

Like the gram, the kilogram is a unit of mass. Weight is measured in *newtons*. We may substitute these quantities into Equation 6 as follows:

$$w = mg$$

or

$$\text{weight in newtons} = \text{mass in kilograms} \times g \text{ in meters/sec}^2$$

Since g is 980 cm/sec², g is also expressible as 9.8 m/sec². Thus, the *weight* of a 1-kg mass is:

$$w \text{ (1 kg mass)} = 9.8 \text{ kg-m/sec}^2 = 9.8 \text{ newtons} \quad (9)$$

Because many textbooks express Coulomb's Law in mks units, it is just as important to be capable of handling this law in mks units as it is to be able to state it in terms of cgs units.

13. Units for Coulomb's Law

In the statement of Coulomb's Law in Section 7 and in its equation (Equation 1), units were not assigned.

If units in the cgs system are desired, then we are constrained to measure *force in dynes*, and *distance in cm*. K is set equal to 1 for a vacuum, but the magnitude of the unit charges— q and q'— remain undefined. To keep the equation free of difficult-to-remember conversion factors, the electrostatic unit of charge magnitude is defined:

One electrostatic unit of charge (esu) is that charge magnitude that will repel another identical unit charge with a force of one dyne, when the distance between them is one centimeter and when the charges are in a vacuum.

Substituting into Equation 1, this definition results in:

$$1 \text{ dyne} = \frac{1 \text{ esu charge} \times 1 \text{ esu charge}}{K \times (1 \text{ cm})^2} \quad (10)$$

The force between two unit charges in air differs so little from that in a vacuum that K is also considered unity for air.

As has been shown, the suitable choice of units makes it possible to phrase Coulomb's Law in very simple terms. As the reader will recognize, the choice of the esu is dictated in the first place by Coulomb's Law, its selection being based on the dyne as a force unit and the centimeter as a distance unit. The esu unit of charge is a measure of *quantity*; by measurement it has been shown to be the equivalent of the charge on approximately 2.1×10^9 (over two thousand million) electrons.

Another very common unit of electrical quantity is the *coulomb*. It is a much larger unit of charge, equivalent to 3×10^9 esu or about 6.3×10^{18} electrons. If q and q' are expressed in coulombs, r in meters, and force in newtons (as required by the mks system), Coulomb's Law becomes more complex, numerically:

$$F = \frac{qq'}{(1.11 \times 10^{-10})r^2} \quad (11)$$

The quantity in the parenthesis is the value of K required to convert the basic definition of Coulomb's Law in the cgs system (in which the unit of charge is the esu, force is in dynes, and distance in cm) to a true relationship in the mks system where F is in newtons, q in coulombs, and r in meters.

14. Practical Electrical Units

Although practical electrical units (volts, ohms, amperes, coulombs, etc.) are employed more often in computations involving current electricity, they are often encountered in electrostatic problems as well.

At the basis of this system is the practical unit of *electrical quantity, the coulomb*. The coulomb, of course, measures the same factor as the electrostatic unit charge (esu) and, as stated earlier, it is a much larger unit, being 3×10^9 greater in value.

In accordance with the Law of Conservation of Energy, work must be done to move any quantity of electricity from one electric potential to another. *Work* as a physical term and *energy* are virtually synonymous (energy is the *capacity* for doing work), and it is more customary to speak of electrical energy rather than the work done.

The mechanical definition of work is the product of force applied and distance moved in the direction of the force or:

$$W = Fs \quad (12)$$

where f is force in dynes, s is distance in cm, and W is work in dyne-cm. The dyne-cm is called an *erg*. The practical unit of electrical energy is called the *joule*. One joule contains 10^7 ergs.

The ratio of electrical energy to charge appears very often in calculations and practical problems. The ratio defines the practical unit of electrical potential energy or simply "potential," the *volt*. In terms of the quantities discussed above, the volt may be defined as follows: *one volt of potential difference exists between two points if one joule of work must be done to move one coulomb of charge from one point to the other.*

Rate of flow of charge is commonly called *current intensity*. The unit of measure for current intensity is the *ampere*, which is defined as a flow of 1 coulomb of charge past a given point per second. (This

flow will under definite conditions deposit silver at the rate of .001118 grams per second from a specified solution of silver nitrate.)

$$I = \frac{Q}{t} \quad (13)$$

where I is in amperes, Q is in coulombs, and t is in seconds.

The resistance to the flow of an electric current offered by a conductor is measured in *ohms*. *One ohm is defined as that resistance through which one ampere passes* when the applied potential is one volt. (An ohm may also be legally defined as the resistance at zero degrees centigrade of a column of mercury of uniform cross section, having a length of 106.3 cm and a mass of 14.45 grams; this is also approximately the resistance of 1000 ft. of number 10 soft-drawn annealed copper wire at zero degrees centigrade.)

15. Review Questions

- Convert the following to centimeters, using exponential notation wherever necessary. (a) 30 meters, (b) 3×10^4 micrometers, (c) 8.3 kilometers, (d) 0.0006 megameters.
- Explain the difference between mass and weight. Of the two quantities, which one would be expected to vary (for a given object) if a person were to take this object from the equator to the north pole?
- State the equation for Newton's Second Law of Motion. If mass is expressed in grams, acceleration in cm/sec/sec, and K is unity, what unit is the appropriate unit for the force?
- What is the weight in dynes of a 1-gram mass? Explain how you obtain this figure.
- Prove that a newton is equal to a kg-m/sec². How many dynes are there in 1 newton?
- Define an esu in terms of two identical unit charges.
- Define the joule. How many ergs are required to make up one joule?
- Define the volt, the ohm, and the ampere.
- Relate the volt, the ohm, and the ampere in a single equation. By what name is this relationship known?
- What current in amperes flows through a 10 ohm electric broiler coil when the line voltage is 120 volts?

Chapter 3

NATURE AND CHARACTERISTICS OF THE ELECTRIC FIELD

16. The Nature and Definition of the Electric Field

Work in a physical sense is done on a body when a force is applied to it, causing it to move against a resistance in the direction of the force. When a boy pulls a sled, his hand is in contact with the rope and the rope is tied to the sled; when a carpenter swings a hammer, his hand must grasp the handle in intimate contact and the hammer does no work until its head comes in contact with the nail.

There are three distinct phenomena in which the rule of contact appears to be violated: gravitational attraction, magnetic attraction and repulsion, and electrostatic attraction and repulsion. All three follow the law of inverse squares, that is, that the force is inversely proportional to the square of the distance between the centers of the reacting bodies. But in all three reactions, contact between the body exerting the force and the body upon which the force acts is unnecessary.

Investigations of gravity, magnetism, and electrostatics have given rise to *field theories* for each of these phenomena. In the case of electrostatic forces, a condition of *stress* appears to exist in the medium surrounding an electric charge so that a force acts upon a positive or negative charge placed in this medium. This condition of stress is easily detectable when the charges are sufficiently large; when

large enough, the medium itself may rupture causing an *arc discharge*, which is usually accompanied by a characteristic electric spark and heat. The ability of charged objects to influence each other over a distance is attributed to the stress in the medium, and the region in which this stress appears is known as the *electric* or *electrostatic field*.

Electrostatic stress or distortion of the medium can be represented by *lines of force*. The direction of such a line of force is arbitrarily chosen as the path along which a unit positive charge would

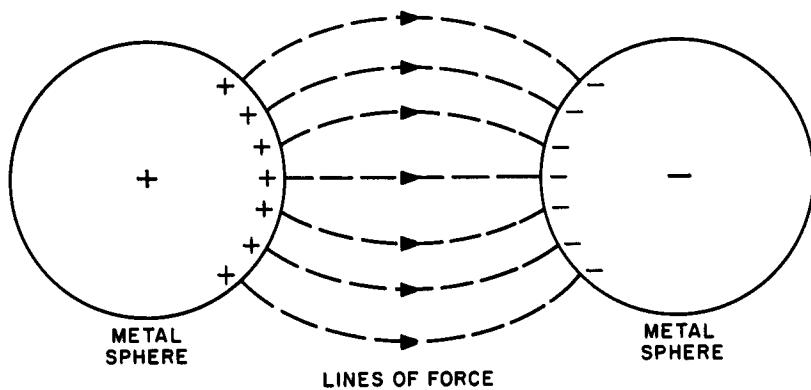


Fig. 9. An electrostatic field is considered to be made up of lines of force that pass from a positively charged body to a negatively charged one.

move if it were free to do so. A fairly complete "picture" of an electric field was first drawn up by Faraday who attributed the following characteristics to lines of force:

1. An electric field may be divided up into lines of force.
2. Each line terminates at a positive charge on one end and a negative charge on the other.
3. The lines, throughout the field, coincide with the direction of the electric stress.
4. The lines behave as though they were made of stretched elastic, always tending to contract and bring together the negative and positive charges.
5. A line of force between two conducting surfaces must always meet the conducting surface perpendicularly. This must be so from the very nature of the assumed static conditions. If a line of force entered or left a conducting surface at any other angle than *normal*,

it would have a tangential component at the surface which would cause the movement of charges within the conductor. This would constitute a continuous electric current and, since currents do not flow along the surface of the conductor in an electric field in a static system, the junction of the line of force and the surface must be a right angle. (See Fig. 9.)

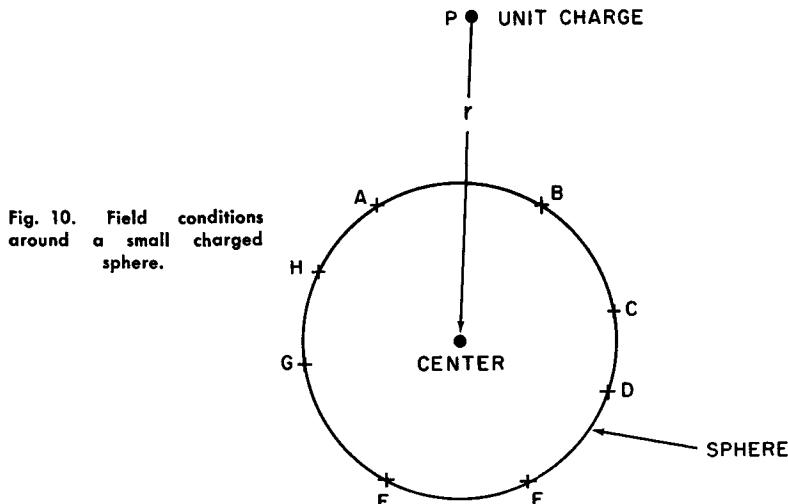
17. The Equation of Field Intensity

The intensity of an electric field is defined as the force acting on a unit charge, and is denoted by E . Thus, by definition:

$$E = \frac{F}{q} \quad (14)$$

in which (in the cgs system) F is in dynes, q is in esu, and E is in dynes per unit charge.

Equation 14 alone, however, is not sufficient for the solution of many problems. In addition, it is necessary to be able to find the



number of lines of force associated with point charges. To determine the number of lines of force that leave or enter each unit charge, we first consider the field conditions in the vicinity of a small charged sphere (Fig. 10). The surface of the sphere contains q positive charges. At a distance r from the center of the sphere and lying well

beyond the surface is a unit charge (one esu). The unit charge is close to positive charges A and B, at a greater distance from charges H, C, G and D, and at a still greater distance from F and E. Furthermore, from previous considerations (Section 4), we know that the charges are distributed uniformly around the surface of the sphere. The question may then be asked, "What is the average effect of these distributed charges on the unit charge r cm away from the sphere's center?" By averaging the distances of the uniformly distributed charges, the tentative conclusion may be reached that the net effect of the surface charges is the same as though they were all concentrated at the center of the sphere. This conclusion may be further justified when it is recalled that lines of force must intersect a charged surface perpendicularly so that a continuation of each line must pass through the center of the sphere. (Any radius of a sphere intersects the surface of the sphere at right angles to it.) Thus, insofar as the effect of the surface charges upon a unit external charge is concerned, this charge is acted upon by a force that is identical to one that would be produced by q charges at the center of the sphere.

Coulomb's Law given in Equation 1 is written $F = qq'/Kr^2$. Many secondary equations derived from Coulomb's Law contain the factor 4π . These equations are applied more frequently than the basic expression, and it is convenient to eliminate the 4π from them. This is accomplished by defining a new constant, ϵ_0 , as follows:

$$\epsilon_0 = K/4\pi$$

which yields

$$K = 4\pi\epsilon_0 \quad (16)$$

whence Coulomb's Law becomes

$$F = \frac{qq'}{4\pi\epsilon_0 r^2} \quad (17)$$

where q and q' are in coulombs, r is in meters, and F is in newtons. From the numerical value of K given in Equation 11, we can state for the mks system that:

$$\begin{aligned} \epsilon_0 &= \frac{K}{4\pi} = \frac{1.11 \times 10^{-10}}{4\pi} \\ &= 8.85 \times 10^{-12} \frac{\text{coulomb}^2}{\text{newton-meter}^2} \end{aligned} \quad (18)$$

Example: Two alpha particles, each having a charge of 3.2×10^{-19} coulombs, are separated by a distance of 10^{-13} meters. Find the force of repulsion between them in dynes.

Solution: From Equation 17

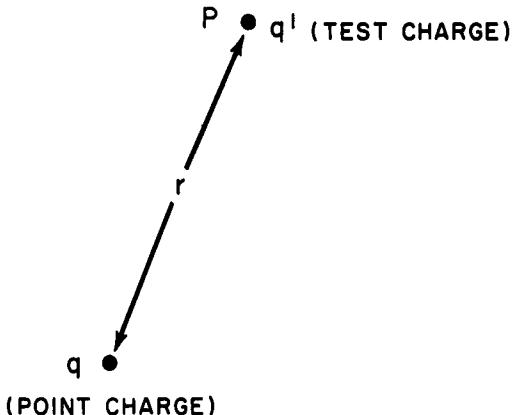
$$\begin{aligned} F &= \frac{3.2 \times 10^{-19} \times 3.2 \times 10^{-19}}{4 \times 3.14 \times 8.85 \times 10^{-12} \times (10^{-13})^2} \\ &= 9.17 \times 10^{-2} \text{ newtons} \end{aligned}$$

Because 1 newton = 10^5 dynes,

$$F = 9170 \text{ dynes}$$

Returning now to the charged sphere of Fig. 10 and its electric field, it has already been mentioned that the magnitude of the field is the force per unit charge ($E = F/q$). Force is a vector quantity, however, so that direction as well as magnitude must be assigned to the field. The direction of an electric field at a particular point in

Fig. 11. The electric field of a point charge, q , acting on a test charge, q' .



the field is arbitrarily taken as the direction of the force acting on a positive test charge placed at that point. Thus, the force on a negative charge such as an electron is considered opposite to the direction of the field.

The nature of the medium through which an electric field acts determines the value of the constant K in Coulomb's Law ($K = 4\pi\epsilon_0$), hence K is known as the *dielectric constant* of the medium. To determine electric field intensity at a point P separated from a point charge q by a distance r , consider a test charge having a charge magnitude equal to q' placed at P as in Fig. 11.

From Equation (17)

$$\mathbf{F} = \frac{qq'}{4\pi\epsilon_0 r^2}$$

but electric field intensity (from Equation 14) is

$$\mathbf{E} = \frac{\mathbf{F}}{q'}$$

so that

$$\mathbf{E} = \frac{\mathbf{q}}{4\pi\epsilon_0 r^2} \quad (19)$$

and the direction of the field is away from the point charge q if it is positive and toward it if q is negative.

Example: A point charge having a magnitude of plus 12×10^{-9} coulombs is located 6 cm away from point A. Compute the electric field intensity at

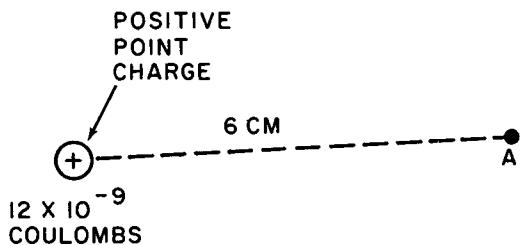


Fig. 12. Illustration for Example.

point A (Fig. 12) and state whether the field acts to the right or to the left.

Solution: The magnitude of the force vector acting on point A due to the positive charge is given by

$$\mathbf{E} = \frac{12 \times 10^{-9}}{4 \times 3.14 \times 8.85 \times 10^{-12} \times (0.06)^2}$$

$$\mathbf{E} = 3.0 \times 10^4 \text{ newtons per coulomb}$$

Because the point charge is positive, the field is directed away from it or toward the right.

We are now prepared to discuss the method used to represent an electric field by lines of force. Any number of lines might be chosen to describe a field of given intensity, but if this were done, the resulting picture would be entirely qualitative with respect to field magnitude. In order to have the lines of force represent both *magnitude*

and *direction*, they are spaced from each other in such a manner that the number of lines passing through a unit area of a surface at right angles to the field is always equal to the product of ϵ_0 and E. That is, *for a unit area of surface*

$$\text{Number of lines} = \epsilon_0 \times E \quad (20)$$

where ϵ_0 is $K/4\pi$ and E is field intensity. Applying this definition to any area A, we have:

$$\text{Number of lines} = \epsilon_0 EA \quad (21)$$

A point charge produces lines of force having an outward radial direction. If such a charge is placed at the geometric center of a sphere of any radius, each line of force will pass through the spherical surface perpendicularly, thus fulfilling the conditions set forth above. Let the radius of the sphere be r; then the field intensity at every point on the spherical surface due to the point charge q is

$$E = q/4\pi\epsilon_0 r^2$$

(Equation 19) and the total number of lines passing through the sphere is given by Equation 22, because the surface area of a sphere is $4\pi r^2$.

$$\text{Number of lines} = \epsilon_0 EA = \frac{\epsilon_0 Q}{4\pi\epsilon_0 r^2} \times 4\pi r^2$$

$$\text{Number of lines} = q \quad (22)$$

The conclusion to be drawn from this analysis is that the number of lines that pass through a sphere concentric with a point charge is exactly equal to the magnitude of the point charge. Since the radius factor cancels out, it is evident that *the same number of lines of force pass through every sphere containing a charge q at its center, regardless of the radius.*

Lines of force are continuous, each one being terminated at one end by a positive charge and at the other by a negative charge. This characteristic of continuity leads to an important generalization, originally stated by Gauss and known as Gauss' Law.

The net number of lines of force that leave a closed surface containing an electric charge is equal to the net positive charge inside the surface. If there is no net charge inside the surface, the net number of lines crossing the surface is zero.

The meaning of "net" when referring to charges, means the difference between the number of positive and the number of negative charges (or their algebraic sum). When referring to lines of force, the net number of lines crossing a surface is equal to the algebraic

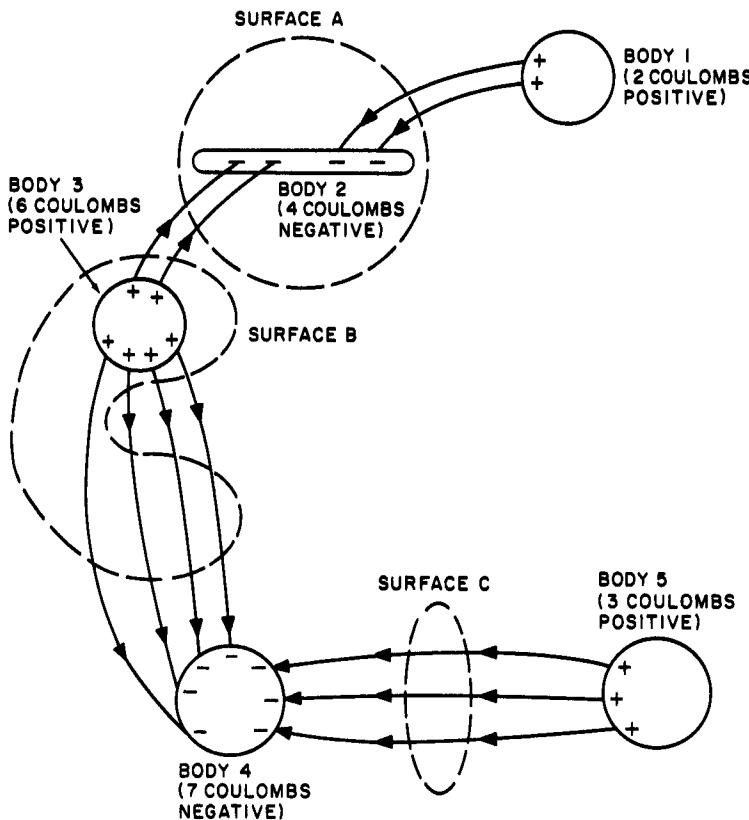


Fig. 13. Illustration for an application of Gauss' Law.

sum of the lines passing outward through the surface and the number of lines passing inward through the same surface.

An application of Gauss' Law is shown in Fig. 13. Correlation between the number of positive and negative charges, the number of lines of force, and Gauss' Law may be obtained from this figure as follows.

(1) Starting with charged body 1, two positive charges (2 coulombs) give rise to two outward-moving lines of force, which terminate on

two negative charges on body 2. Thus two lines enter or cross surface A on this side.

(2) Two lines cross the other side of the surface A boundary, coming from two of the six positive charges on body 3, again going inward. This makes a total of 4 lines entering surface A which, according to Gauss' Law, is equal to the four negative charges inside of surface A residing on body 2. This gives us an important corollary:

The net number of lines of force that enter a closed surface containing an electric charge is equal to the net negative charge inside the surface.

(3) Body 3 has 6 positive charges. Surface B is intersected at 12 points. Of these, 9 lines intersect the surface going outward, while 3 intersect going inward, making a net of 6 lines going outward. Again, this corresponds with the number of positive charges in surface B in accordance with Gauss' Law.

(4) Surface C has no charge within it at all, hence the net crossings should total to zero. As seen, three lines from body 5 enter and leave the surface leaving a net of zero. Since surface C has no charges inside it, no lines can terminate within it.

(5) Because all lines must terminate at a positive charge at one end and a negative charge at the other, there must be an equal number of positive and negative charges if the representation is to be correct. There are 11 charges of each sign in Figure 13, so that the number and direction of the lines conform with the theoretical requirements.

18. Examples of the Use of Gauss' Law

Gauss' Law may be applied to many different surfaces for the purpose of calculating the field intensity that results from various charge distributions. It may also be used to test certain basic assumptions that are taken as self-evident, such as the distribution of charge and the field is a conductor.

The Field Surrounding a Charged Sphere. A sphere of radius r_1 is shown in cross section in Figure 14. Surrounding this conducting sphere is a second sphere of nonconducting material having a radius r_2 . The inner conducting sphere carries a positive charge of q coulombs uniformly spread out on its surface. With this symmetrical charge distribution and geometry, the lines of force may be considered to be directed outward radially from the inner sphere. Because

the outer sphere is concentric with the inner sphere, all the lines of force that cross it must do so at right angles to its surface. From Equation 21 we have

$$\text{Number of lines} = \epsilon_0 EA$$

and for a sphere of radius r_1

$$\text{Number of lines} = \epsilon_0 E (4\pi r_1^2)$$

By Gauss' Law, the number of lines leaving the sphere must equal the net positive charge within it, hence:

$$q = \epsilon_0 E (4\pi r_1^2)$$

Solving for E :

$$E = \frac{q}{4\pi\epsilon_0 r_1^2}$$

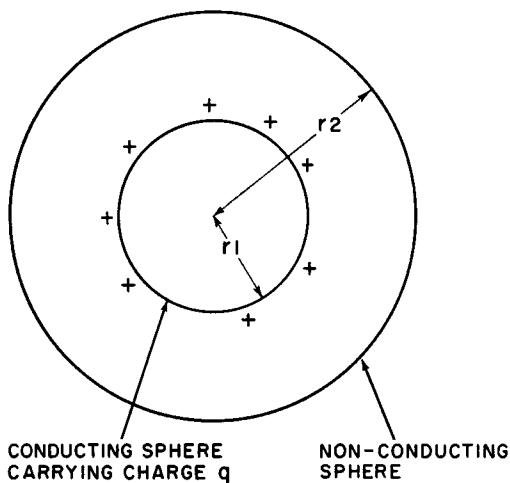


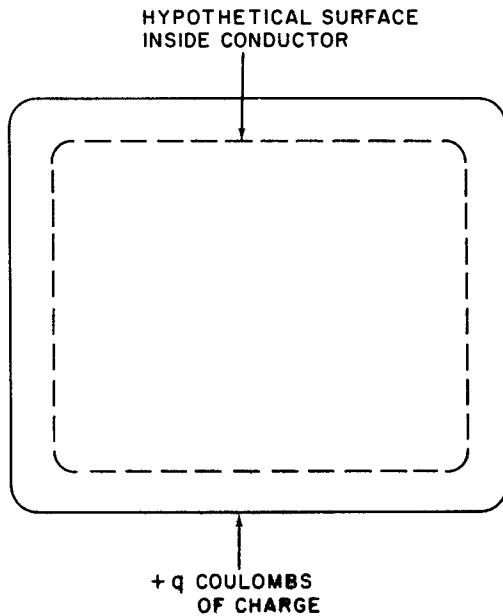
Fig. 14. Study of the field intensity around a charged conducting sphere.

It will at once be observed that this expression is identical with that previously derived for the intensity of the field at a distance r from a *point charge* (Equation 19). This means that the field intensity surrounding a charged sphere is the same as that of a point charge of the same total magnitude and that the charge on the sphere may be considered to be concentrated at its geometric center. As a second implication, the net field inside the sphere is zero.

The Field Within a Conducting Surface. We have shown in Section 4 that charges placed on a conducting surface tend to move as far apart as possible, due to mutual repulsion. Using Gauss' Law, we can now test this idea more rigorously.

Electrons in a conductor are free to move. If an electric field is present in the conductor, there will be motion of the free electrons as long as the field exists; conversely, the electronic motion will be zero if there is no electric field. As a corollary we may then conclude that we may assume the field in a conductor to be zero if the electrons are quiescent.

Fig. 15. Gauss' Law applied to a conductor.



Consider now a three-dimensional conducting solid (shown in cross section in Fig. 15). A charge of q coulombs has been placed on the conductor. From Gauss' Law, we may at once state that the net number of lines of force crossing any surface located an infinitesimal distance inside of the outer boundary of the conductor, such as the surface shown by the dashed line in Fig. 15, must be equal to the net positive charge within the surface. These charges are static. Since there is no motion of charges, the field within the conductor must be zero and the number of lines of force crossing the hypothetical sur-

face must also be zero. In accordance with Gauss' Law, however, the net charge in the conductor must be equal to the number of lines of force crossing the surface in an outward direction. Thus, *the net charge inside the conductor must also be zero*. Because a charge of q coulombs was placed on the conductor initially, all of the charges must be located between the hypothetical surface and the outer surface of the conductor. Having originally assumed that the hypotheti-

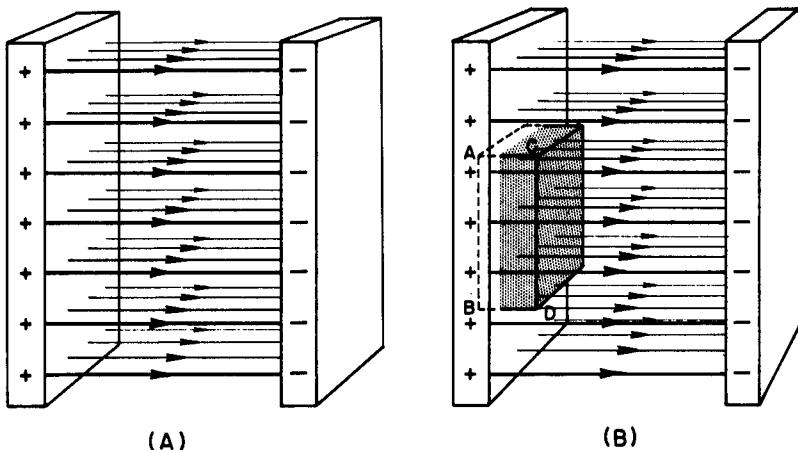


Fig. 16. Study of the field between two conducting plates.

cal surface was located an infinitesimal distance inside the conductor, we must conclude that *all the charges are present on the outside of the conductor*.

The Field Between Two Parallel Conducting Surfaces. If two parallel conducting surfaces (Fig. 16) are charged to equal magnitudes, but with opposite sign, the electric field between them consists essentially of straight lines of force uniformly spaced and parallel to each other. To use Gauss' Law for the determination of the magnitude of the electric field between the plates, let us imagine a closed surface having a rectangular cross section placed between the plates as shown in Fig. 16B. Side AB of this surface lies inside the conductor on the left, whereas side CD is outside the conductor and within the electric field.

Since there is no electric field in the conductor at the left (see the preceding example), there are no lines of force passing through

side AB of the closed surface. Hence, the net number of lines of force passing through the closed surface are those that emerge through side CD. To determine the number of lines involved in this transition, we shall again use Equation 21.

$$\text{Number of lines} = \epsilon_0 EA$$

in which E is the intensity of the electric field between the plates and A is the area of side CD of the closed surface.

The total charge inside the closed surface is equal to the area A of side AB that encases the charges on the conducting plate multiplied by the charge per unit area on the plate itself. If γ represents the charge per unit area, then:

$$q = \gamma A \quad (23)$$

Applying Gauss' Law, we have:

$$\epsilon_0 EA = \gamma A \quad (24)$$

Hence

$$E\epsilon_0 = \gamma$$

or

$$E = \frac{\gamma}{\epsilon_0} \quad (25)$$

Hence, the field intensity in newtons per coulomb between two parallel, oppositely charged plates is equal to the number of coulombs per square meter of charge on either plate divided by the constant ϵ_0 . ($\epsilon_0 = 8.85 \times 10^{-12}$ coulomb 2 /newton - meter 2)

Example: The field intensity between two parallel electrodes is 30,000 newtons per coulomb. An electron placed in this region will be acted upon by a force that moves it toward the positive plate. Calculate the magnitude of this force in dynes, assuming the charge on an electron to be approximately 1.6×10^{-19} coulomb.

Solution: Find first the force on the electron in newtons.

$$E = \frac{F}{q}$$

$$F = Eq$$

$$F = 3 \times 10^4 \times 1.6 \times 10^{-19}$$

$$F = 4.8 \times 10^{-15} \text{ newton}$$

Since there are 105 dynes in one newton,

$$F = 4.8 \times 10^{-10} \text{ dyne}$$

Example: The charge on a sphere is 10^{-3} coulomb, distributed evenly over its surface. Calculate the field intensity in newtons per coulomb at a point outside the sphere, 5 meters from the center.

Solution: Use Equation 19 .

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

$$E = \frac{10^{-3}}{4 \times 3.14 \times 8.85 \times 10^{-12} \times 25}$$

$$E = \frac{10^{-3}}{2779 \times 10^{-12}}$$

$$E = 3.6 \times 10^5 \text{ newtons per coulomb}$$

19. Distribution of Charge

Experiments conducted along the lines of Faraday's "ice-pail" investigation bring to light a number of important facts concerning

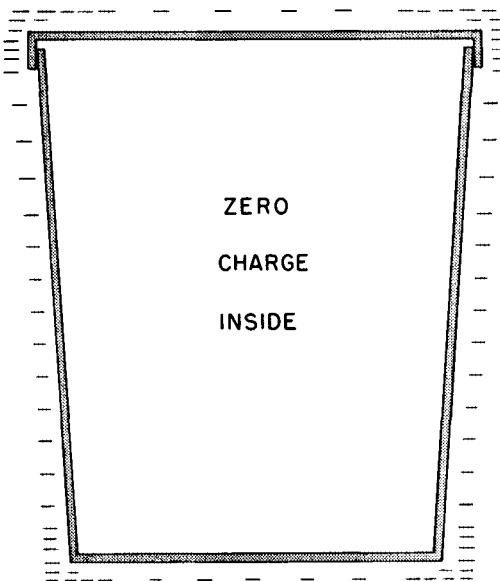


Fig. 17. A charged closed pail.

the manner in which electric charges distribute themselves on conductors of various shapes.

The distribution of an electric charge may be examined by means of a metal disc mounted on an insulating handle and called a proof plane. If the disc is placed against the surface of the charged and insulated pail shown in Fig. 17 and then removed, it takes with

it a portion of the charge, which may then be tested by means of an electroscope. The experiment proves that the greatest charge is obtained from the outer surface of the pail near its upper edge and on the outer corner of the bottom. Less is found on the middle of the cylindrical sides and none at all in the interior, if the pail has a metal cover. The absence of charge on the inside surface of a closed conductor was discussed in Section 4. The heavy concentration of charges on the corners of the pail leads to the conclusion that the greatest density of charge is found at points of sharpest curvature. *Charge density varies inversely with radius of curvature.*

When the density of charge exceeds certain limits, the air in the neighborhood of a charged surface is made slightly conducting by a

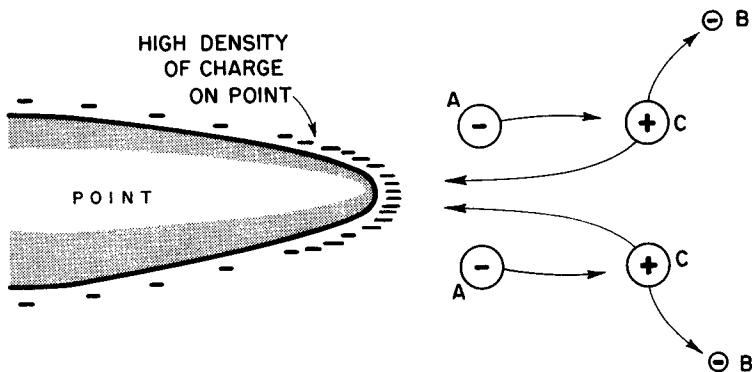


Fig. 18. The process of point discharge. (A) A negative ion is accelerated by the strong electric field near the point due to the point's high charge density. (B) Electrons dislodged from neutral gas molecules due to collisions with the accelerated ions, leaving positive ions (C), which move toward the point.

collision process involving charged gas molecules with uncharged ones. This process is called "ionization." The strong electrostatic field around a sharp point accelerates any ions already present due to the effect of cosmic rays, etc. These ions collide with neutral molecules, often dislodging additional electrons. This leaves the gas molecules positively charged, and they move either toward or away from the point, depending upon the charge on the point. In either case, there are enough charged atoms of opposite sign to be attracted to the high charge density, where they pick up an excess of charge and are immediately repelled. It is found, therefore, that charges "leak" from sharp points much more rapidly than they do from surfaces of larger radius or curvature. The process is shown in Fig. 18.

Point discharge explains the efficiency of certain types of lightning rods, which enable a residence or other building to maintain itself in a more or less neutral electrical state by discharging through the sharp points. The phenomenon also explains why points must be avoided in high voltage power supplies such as those used in television receivers. Unless all connections are rounded, energy is lost through the effects of *corona*, another name for electrical discharge through ionized air.

20. Millikan Method of Measuring Electron Charge

In 1913, Robert Millikan reported on a series of painstaking experiments which established that the electron possesses a unique, indivisible quantity of electricity. Millikan's measurements also established the value of this quantity with high precision. His method, known as the "oil drop" experiment, was carried on as follows: A fine spray of oil is blown into a chamber (Fig. 19). The droplets settle toward the bottom of the chamber with varying speeds, depending chiefly upon their size. The surface tension of the droplets causes them to take on a spherical shape and they fall through the viscous medium (the air in the chamber) at a speed determined by the equation called Stokes' Law:

$$v = \frac{w}{6\pi r Z} \quad (26)$$

where v is the final velocity of fall, w is the net weight of the particle after accounting for buoyancy, r the radius of the falling sphere, and Z is a constant that depends upon viscosity of the medium (coefficient of viscosity).

Sooner or later, one of the droplets falls through the small hole at the bottom of the chamber. As soon as one does so, the hole is closed and the single droplet finds itself in a region between two accurately spaced parallel plates across which a potential of several thousand volts is connected. The droplet is highly illuminated by the light of a carbon arc lamp, from which the heat rays are removed by a filter to prevent convection currents from affecting the experiment. An observing microscope is focused on the space between the plates and the movement of the drop is measured with respect to two pairs of cross-hairs. The time required for its passage from one to the other of the cross-hairs is precisely measured by means of a stopwatch, hence the speed of the droplet may be easily computed. This speed

is a function only of the diameter and weight of the droplet and the viscosity of the air in accordance with Stokes's Law (Equation 26).

In its passage through the chamber, each droplet takes on a small electric charge due to contact and induction effects. Assume that the droplet being observed carries a negative charge. As it approaches the lower plate, the switch is closed thus establishing an electric field in the space between the plates. Since the lower plate is negative, the droplet will begin to move upward in the field and again pass the cross-hairs of the microscope, this time at a different velocity.

At this instant, an X-ray tube at the opposite end of the plate area is momentarily flashed, causing the air in the vicinity of the

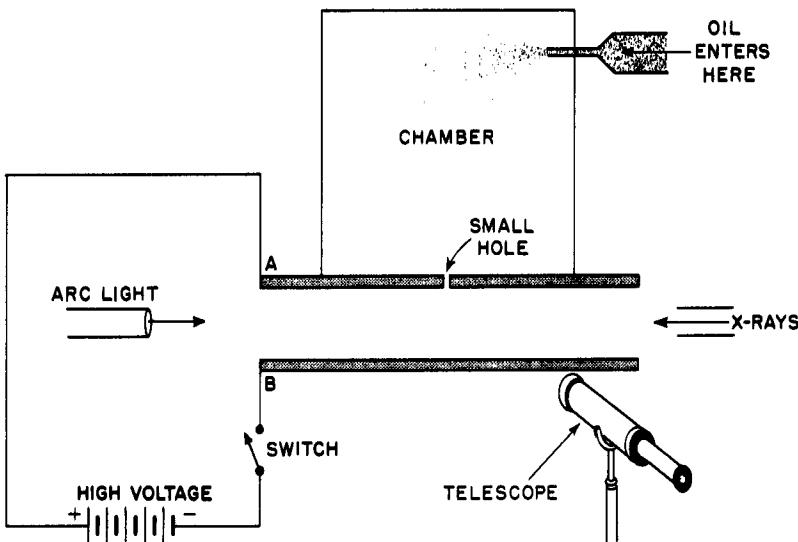


Fig. 19. The apparatus for the Millikan oil-drop experiment.

droplet to become ionized. The droplet then may *pick up or lose electrons* and thus change its net charge which, in turn, causes a modification of its velocity and possibly direction of motion. The change in motion may be the result of a change of charge amounting to one, two, or more electrons lost or gained, but in any case the *gain or loss must be equal to a whole number of electrons*.

By thus watching and measuring the velocity of a single droplet often for hours at a time, Millikan found that no matter what the size, material, or original charge of the droplet, the *change in charge*

was always a whole number multiple of 1.591×10^{-19} coulombs. Thus, the value and uniqueness of the charge of an electron were established.

21. The Oil-Drop Equations

The experimental procedure just described may be translated into appropriate equations that provide the actual value of the charge of an electron in terms of the experimental data.

Let v_0 designate the steady-state velocity of an oil droplet when the electric field is not present and v the upward velocity of the same droplet under the influence of the field. Stoke's Law may then be written for these two conditions as:

$$v_0 = \frac{w}{6\pi r Z} \quad (27)$$

$$v = \frac{Eq - w}{6\pi r Z} \quad (28)$$

As before, Z is the viscosity of the air, E is the electric field strength in the region of droplet movement, w is the weight of the droplet, r is its radius, and q the absolute charge on the droplet. The numerator of Equation 28 is the net force acting on the droplet. (It is positive when it results in upward motion against the action of gravity.) For simplicity, the effect of the buoyancy may be neglected without introducing serious error. If Equations 27 and 28 are added algebraically, the w factor drops out and the resulting equation may be solved for the charge on the droplet. Thus:

$$v + v_0 = \frac{w}{6\pi r Z} + \frac{Eq - w}{6\pi r Z} = \frac{Eq}{6\pi r Z}$$

Solving for q :

$$q = \frac{6\pi r Z (V_0 + v)}{E} \quad (29)$$

All of the quantities on the right side are directly measurable except the value of r , the radius of the droplet. The viscosity coefficient Z may be obtained by several precise methods, v_0 and v are determined by observation with the viewing microscope, and the electric field intensity E may be easily obtained from the voltage between the charged plates and their separation, as will be shown in a later paragraph. The radius r must, however, be determined indi-

rectly. The droplet is spherical due to surface tension, and the oil of which it is formed has a measurable density D. Thus, since density is defined as mass per unit volume,¹

$$w = V D g \quad (30)$$

where V is the volume of the droplet, D is the density of the oil, and g is the gravitational acceleration.

The volume of a sphere is given by

$$V = 4/3 \pi r^3 \quad (31)$$

Substituting for V in Equation 30:

$$w = 4/3 \pi r^3 D g \quad (32)$$

This value for w may be substituted in the form of Stokes' Law for the velocity v_0 in the absence of a field (Equation 27),

$$\begin{aligned} v_0 &= \frac{4/3 \pi r^3 D g}{6 \pi r Z} \\ &= \frac{2/9 r^2 D g}{Z} \end{aligned} \quad (33)$$

Solving for r^2 and taking the square root of both sides,

$$\begin{aligned} r^2 &= \frac{9}{2} \frac{v_0 Z}{D g} \\ r &= \sqrt{v_0 Z / 2 D g} \end{aligned} \quad (34)$$

Substituting this value of r into Equation 29,

$$\begin{aligned} q &= \frac{(6\pi Z)(3\sqrt{v_0/2Dg})(v_0 + v)}{E} \\ &= \frac{18\pi Z(v_0 + v)\sqrt{v_0 Z / 2 D g}}{E} \end{aligned} \quad (35)$$

This expression may be rewritten as

$$q = \frac{4}{3} \pi \left(\frac{9Z}{2} \right)^{3/2} \left(\frac{v_0}{Dg} \right)^{1/2} \frac{(v_0 + v)}{E} \quad (36)$$

Now everything on the right side of Equation 36 is experimentally determinable. It was by observing the difference in the various

¹From Equation 6, $w = mg$. The total mass of the drop is equal to the product of its mass per unit volume and its volume.

q 's calculated in this way for the same droplet that Millikan arrived at his conclusions concerning the uniqueness and value of the electronic charge, e . The best of modern determinations gives a value of e equal to 1.6018×10^{-19} coulomb, which varies only slightly from Millikan's value.

22. Dielectrics

Conductors and insulators were touched upon in Section 3. In conductors, the electrons are not tightly bound to their atomic nuclei and are able to pass freely from atom to atom. In insulators, the bond is much tighter and a substantial voltage is needed to cause electron movement. Because the difference between conductors and insulators is one of magnitude rather than mode of behavior, the term *insulator* is relative and applies to the properties of the medium with respect to current conduction. When the behavior of an insulating medium is analyzed relative to its dielectric properties, i.e. number and density of electric field lines, resistance to electric breakdown, etc., the medium is called a *dielectric*. The behavior of a given substance when used as an insulator may be very different from its behavior when applied as a dielectric.

For instance, air is one of the best insulators known; in the dry state, its conductivity is very low even under the stress of large voltages. Yet, as a dielectric it has a relatively low breakdown threshold, flashover occurring at about 75,000 volts per inch of gap. Rubber in comparison can withstand better than 400,000 volts per inch of dielectric thickness, but is not nearly as good an insulator as air, permitting leakage currents to flow under conditions at which air would stop them completely.

The ability of a substance to withstand breakdown is known as its *dielectric strength*. This characteristic is expressed in volts per unit thickness when the substance is placed between flat electrodes with rounded edges. Thus, the dielectric strength of air is 3,000 volts per mm, that of rubber is 16,000 volts per mm, and that of varnished cambric about 32,000 volts per mm. Modern ceramic and synthetic plastic dielectrics have even higher ratings.

The change in potential per unit thickness applied across a dielectric is called the *potential gradient*. For instance, 48,000 volts impressed across 30 mils of dielectric material establishes a potential gradient of 1,600 volts per mil. Potential gradient is always expressed

in terms of voltages that are well below the breakdown potential, since, after a discharge occurs, there is likely to be charring or cracking of the insulating material, with a consequent change of all its dielectric properties in the area of the breakdown.

23. The Dielectric Constant

In restating Coulomb's Law (Section 17) mention was made of the constant K as the dielectric constant of the medium. Let us now define this term. When a d-c voltage is applied to two flat electrodes separated by a gap, a charge of $+q$ is found on one and a charge of $-q$ is found on the other. Should a slab of glass or polystyrene be now inserted in the gap, without altering the applied voltage, the quantity of charge rises to $+q'$ and $-q'$. The increment in charge must, therefore, be due entirely to the presence of the new dielectric material. Thus, the dielectric constant of any insulating medium is defined as the ratio of the charge with an air (or more precisely vacuum) dielectric to the charge with the material dielectric without change of applied potential or:

$$\text{dielectric constant} = \frac{q}{q'} \quad (37)$$

The dielectric constants of several common insulators are given in the chart below.

DIELECTRIC CONSTANTS

Air	1	Water	81
Bakelite	4.5 to 5.5	Polystyrene	2.5 to 7.5
Glass	4.0 to 10.0	Porcelain	5.7 to 6.8
Paraffin	2.3	Mica	5.4 to 8.0
Rubber	2.0 to 2.5		

24 Review Questions

1. Explain why lines of force between two conducting surfaces must intersect the surfaces at right angles.
2. Why is it convenient to discard the constant K originally used in Coulomb's Law in favor of a new constant ϵ_0 ?
3. Define electric field intensity in words and in the form of an equation. What are its units in the cgs system?
4. Prove that the same number of lines of force must pass through the surface of a hollow sphere of any radius if it contains a charge q at its center.

ELECTROSTATICS

5. What is the convention regarding the direction of an electric field produced by a point charge q if the point charge is positive? if it is negative?
6. State Gauss' Law. If K in Coulomb's Law had not been replaced by ϵ_0 , how would Gauss' Law be stated?
7. Calculate the intensity of the electric field surrounding a sphere of 10 cm radius at a distance of 25 cm from the sphere if the charge on its surface is 2 coulombs.
8. Compute the field intensity between two parallel oppositely charged plates each of 100 cm^2 area. The charge on each plate is 0.5 coulomb.
9. Explain the effect of points on a charged body. Define the terms ionization and corona.
10. Describe qualitatively the method used by Millikan to obtain the charge on an electron.
11. Define *dielectric strength* and *dielectric constant*.

Chapter 4

ELECTRIC POTENTIAL, CAPACITANCE, AND CAPACITORS

25. Potential and Potential Difference

As mentioned in Section 14, the potential difference between two points may be defined as the ratio of energy to charge, or

$$\text{Potential Difference} = \frac{\text{Energy}}{\text{charge}} \quad (38)$$

It remains to determine the units that may be properly applied to this relation and to visualize the physical meaning that underlies the concept of potential difference.

The *absolute potential* of a given body is measured by the number of ergs or joules required to bring a unit charge q of like sign (so that work must be done against the repelling force) up to the body from infinity. Absolute potential is virtually impossible to determine, but for convenience the earth is assumed to be at zero potential and all other potentials are measured relative to it.

It may be shown by the integral calculus that the work done in moving any point charge q' from infinity to a distance r from another point charge q (in the mks system) is given by the equation

$$W = \frac{qq'}{4\pi\epsilon_0 r} \text{ joules} \quad (39)$$

The equivalence of work and energy makes it possible to state that the work done according to this equation is also the potential energy

given to the point charge as a result of moving it from infinity to a distance r from another point charge of similar sign. Since electrical potential is the ratio of energy to charge,

$$\text{Potential} = \frac{qq'}{4\pi\epsilon_0 r q'} \quad (40)$$

$$V = \frac{q}{4\pi\epsilon_0 r} \text{ joules per coulomb} \quad (41)$$

According to the definition of the volt given in Chapter 2, however, it is fundamentally the same unit as the joule per coulomb, hence the expression above is said to be the potential in *volts* of a point charge q with reference to some zero level taken at a great distance away from the point charge.

Determination of the potential around a charged conducting sphere will demonstrate several interesting points. We have already shown that the electric field around a charged sphere may be computed by considering that the entire charge is concentrated at the center of the sphere. The potential of such a sphere is, therefore, also given by Equation 41, in which r must be taken equal to or greater than the radius of the sphere.

Now consider such a sphere being charged in air by some electrostatic method. (Such methods will be discussed in Chap. 5.) When the electric field intensity becomes sufficiently great, the air in the vicinity of the sphere becomes conductive and the charge leaks off as fast as it is generated. The question arises as to what factor or factors determine the maximum potential to which the sphere can be charged. This question may be answered by reasoning as follows: designating the maximum electric field intensity by E_{\max} , we can determine the largest charge that the sphere can hold by solving Equation 19 for q :

$$q_{\max} = (E_{\max})(4\pi\epsilon_0 r_s^2) \quad (42)$$

in which r_s is the radius of the sphere. Substituting this value of q_{\max} into Equation 41,

$$V_{\max} = r_s E_{\max} \quad (43)$$

Experimental evidence shows that air becomes conducting when the field intensity approaches 3×10^6 volts per meter, hence the maximum potential in volts (the *voltage*) that can be built up on a con-

ducting sphere of given radius regardless of the charging method is:

$$V_{\max} = 3 \times 10^6 (r_s) \text{ volts} \quad (44)$$

where r_s is measured in meters. This development provides a supplementary explanation of the phenomenon of discharge from points discussed in Chap. 3. Since the maximum voltage is a function of the radius of the charged body, a surface that has an extremely small radius of curvature cannot sustain a high potential without causing ionization of the air and consequent discharge. In addition to this, it also illustrates the reason for the use of extremely large spheres in million-volt Van de Graaff generators.

Example: What is the maximum potential to which a "point" having a radius of .01 mm can be raised?

Solution:

$$V_{\max} = 3 \times 10^6 \times 10^{-5} = 30 \text{ volts}$$

Example: A Van de Graaff generator has a sphere of approximately 2-meter radius. What is the maximum potential to which it can be charged?

Solution:

$$V_{\max} = 3 \times 10^6 \times 2 = 6,000,000 \text{ volts.}$$

Physicists, using the mks system, measure field strength in newtons per coulomb, rather than in volts per meter. These units are, however, exactly equivalent. A volt is defined as a joule/coulomb, but a joule is a newton-meter.

Substituting these units in Equation 19,

$$\text{Field strength } E = \frac{\text{volts}}{\text{meter}} = \frac{\frac{\text{joules}}{\text{coulomb}}}{\text{meter}} = \frac{\text{joules}}{\frac{\text{coulomb}}{\text{meter}}} = \frac{\text{newton-meters}}{\text{coulomb}}$$

The distance unit cancels leaving

$$E = \text{newtons/coulomb} \quad (45)$$

26. Equipotential Surfaces

If two points between charged plates, such as A and B in Fig. 20, are given a difference of potential, work is required to move a charge up the potential from A to B. If the charge is moved *parallel* to the plates, however, *work is not required* because the *motion is occurring in a direction in which there is no component of field force due to the electric field*. If there is no resisting force component, work is not

ELECTROSTATICS

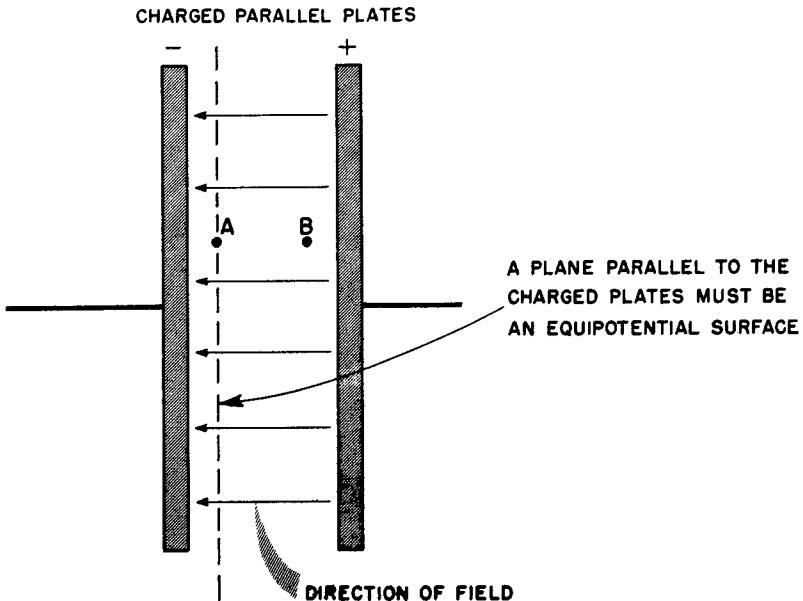


Fig. 20. Study of an equipotential surface.

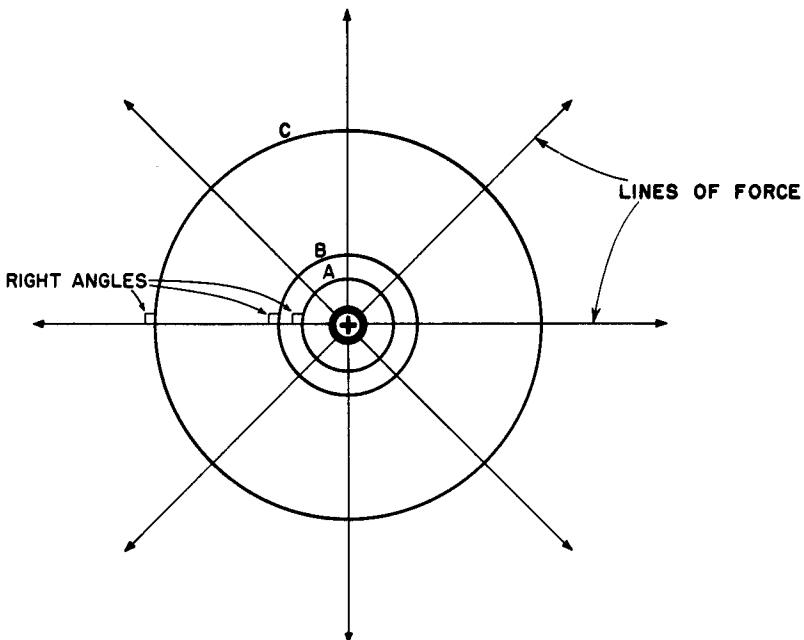


Fig. 21. The electric field around a charged sphere passing through equipotential surfaces.

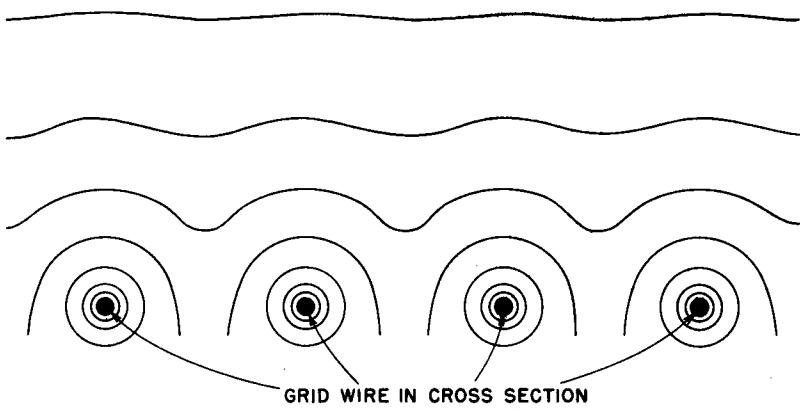


Fig. 22. Equipotential surfaces around the grid of a vacuum tube.

necessary to maintain uniform motion. For this reason, all points in a plane passing through point A and parallel to the plates have the same potential as point A. This follows from the fact that there can be a potential *difference* only if work is required to move the charge. A surface on which all points have the same potential is called an *equipotential surface*.

This concept has several important implications. The surface of any conductor with a static charge must be an equipotential surface. If a potential difference existed for an instant between points on the conducting surface, there would be an immediate movement of charge from the points of high potential to those of low potential and this movement would continue as long as the potential difference existed. As we shall see later, the plates of a capacitor, irrespective of their shape or position, must also be equipotential surfaces.

A second important idea is that *electrostatic lines of force must pass through equipotential surfaces at right angles*. If this were not the case, components of the field *along* the surface would exist, and movement of charges would take place. A difference of potential would thus exist along the surface, contrary to the definition. As may be observed, this is true of the equipotential surface in Fig. 14. Again, consider a charged sphere like that of Fig. 21. The charged sphere is the innermost circle bearing the plus sign. Since the field of the sphere is *radial* in nature, each force line must pass through con-

centric spheres of larger diameter perpendicularly. Hence, surfaces A, B, and C are equipotential surfaces. It should be noted that a similar condition exists for coaxial cylinders.

With more general distribution of electric field sources, equipotential surfaces take on more diversified shapes. Such surfaces, as they might appear around the control grid of a vacuum tube, are shown in Fig. 22.

In drawing a series of equipotential surfaces, it is conventional to indicate them as surfaces having uniform voltage increments. (Each surface is shown separated from the adjacent one by an equal potential difference.) When this is done, the surfaces are increasingly fur-

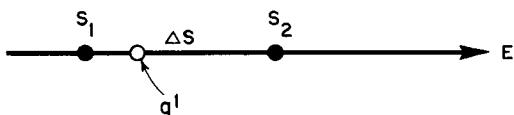


Fig. 23. The force acting on a test charge due to an electric field.

ther apart, as the distance from a single charged source increases. This is due to the greater divergence of the radial lines from a central charged body as the distance grows.

27. Potential Gradient

An important term often encountered in electrostatics is *potential gradient*. To study its significance, consider the force acting on a test charge q' in an electric field of intensity E , as shown in Fig. 23. Electric field intensity is given by the equation $E = F/q'$, hence the force acting on the test charge q' is equal to Eq' . This force acts upon the test charge to move it from S_1 to S_2 , that is, over the distance Δs , which we shall take as being infinitesimally small. In order to stop the motion of the test charge, or to move it slowly in the opposite direction *without acceleration*, the force required would be $-F$. For this case, then, $-F = Eq'$ or

$$F = -Eq' \quad (46)$$

in which F is the force required to move q' in the direction S_2 to S_1 . If such a force is applied to the test charge and succeeds in moving it from S_2 to S_1 (a distance Δs), the work done is

$$W = F\Delta s = -Eq'\Delta s \quad (47)$$

The work done in moving the test charge against the electric field represents an increase in potential energy. Thus, the increment of electric potential, ΔV , is the increase of potential energy per unit charge or

$$\Delta V = \frac{W}{q'} = \frac{-Eq'\Delta s}{q'} \quad (48)$$

$$E = -\frac{\Delta V}{\Delta s} \quad (49)$$

The ratio $\Delta V/\Delta s$ is the rate of change of potential with distance in the direction of the electric field. The minus sign before the ratio indicates that *the electric intensity is the negative of the potential gradient, in the direction of the electric field*. Potential gradients are expressed in volts per meter or microvolts per meter in the mks system.

Let us summarize the characteristics and properties of equipotential surfaces as follows:

1. Equipotential surfaces never intersect in a finite field.
2. It is always possible to draw a series of lines indicating the direction of the field strength perpendicular to these surfaces.
3. The field is stronger where the surfaces are closer to each other.
4. The field strength is the negative of the potential gradient.
5. When two bodies bearing charges of opposite sign—such as a pair of charged plates—produce a field between them, this field may be quite uniform throughout the space separating them. In such arrangements, the equipotential surfaces are equally far apart.

28. Capacitance

When two conductors that are separated by an insulator are connected across a source of potential, as shown in Fig. 24, the plates become oppositely charged. When the activating potential is removed, the charge remains on the plates for as long a period as leakage through the insulating material or dielectric permit. This residual charge constitutes *stored electricity*, while the property of storage in an electric circuit is known as *capacitance*. In the elementary explanation of the storage ability of a *capacitor* (a device that exhibits capacitance), it is often erroneously stated that the energy of the stored charge resides in the *plates* of the capacitor, since

it is easy to discuss the charges thereon as *bound charges*. The mutual attraction of opposite charges superficially appears to offer a satisfactory answer to the question as to the mechanism of energy storage. That this is *not* a fact is demonstrated when the dielectric material is changed. For example, a given capacitor is capable of storing

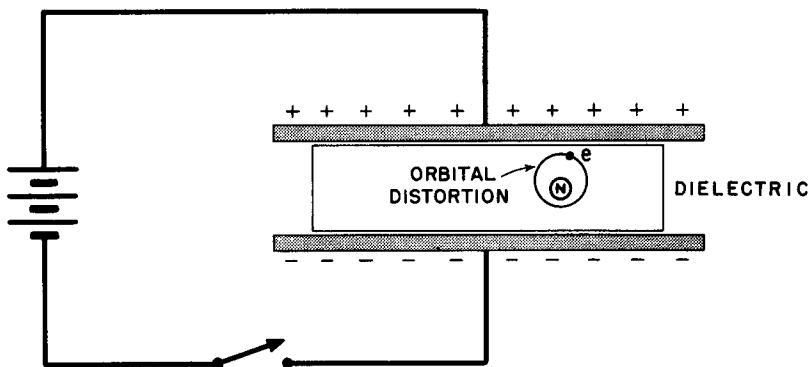


Fig. 24. How energy is stored in the dielectric of a capacitor.

a certain amount of energy when air is used as the dielectric; if glass is now substituted for the air *without changing the voltage of the charging source*, the storage ability of the same capacitor rises by a factor of at least five. Since neither the area nor separation of the plates has changed, this increase in energy content signifies that storage of energy occurs elsewhere. It is generally agreed that the energy of a capacitor resides in the dielectric rather than on the plates. The actual mechanism by which this is accomplished is not perfectly understood, but it seems quite certain the orbits of electrons circling the nuclei of the atoms of the dielectric are distorted by the electrostatic field between the plates. Any change in the normal shape or dimension of such orbits must represent potential energy similar to that of a stretched rubber band or a compressed spring.

As the voltage applied to a capacitor is increased, the charge increases in direct proportion. That is:

$$Q \sim V \quad (50)$$

where Q is charge magnitude and V is applied voltage. This proportionality is converted to an equation by multiplying the right side

by a constant of proportionality that we designate C, the capacitance of the capacitor (by definition).

$$Q = CV \quad (51)$$

where Q may be measured in coulombs and V may be measured in volts. Solving for C we have:

$$C = \frac{Q}{V} \quad (52)$$

The fundamental unit of capacitance is the *farad* (named for Michael Faraday, 1791-1867). One farad is the equivalent of one coulomb/volt and is defined as that capacitance which will accept a charge of one coulomb when the applied voltage is one volt.

The farad is a prohibitively large unit for practical use. In equipment where capacitors are used, it is common to encounter ratings in terms of microfarads ($1 \mu\text{f} = 10^{-6}$ farad) and micro-microfarads ($1 \mu\mu\text{f} = 10^{-12}$ farad).

An additional relationship that finds much application may be obtained by utilizing the fact that the one ampere of current flow is defined as one coulomb per second. Hence:

$$It = CV \quad (53)$$

where I is current in amperes, t is time in seconds, C is capacitance in farads, and V is voltage in volts.

Example: A $100\mu\text{f}$ capacitor is connected across a 300-volt d-c line. (a) Find its charge in coulombs when fully charged and (b) calculate how long it must be allowed to charge to reach full charge if the average current flowing into it is 0.1 ampere.

Solution: (a) $Q = CV$

$$Q = 100 \times 10^{-6} \times 300 = 3 \times 10^{-2} = .03 \text{ coulomb}$$

$$(b) t = \frac{CV}{I}$$

$$= 100 \times 10^{-6} \times 300/0.1 = .03/0.1 = 0.3 \text{ second}$$

29. The Energy Stored in Capacitor

The reality of energy stored in a capacitor is shown by the spark discharge from a charged capacitor or its ability to light an incandescent lamp briefly. The magnitude of this stored energy is a function of the capacitance of the capacitor and the voltage originally

used to charge it, provided that enough time has been allowed for a full charge to accumulate. It is obtainable from the equation

$$W = \frac{1}{2}CV^2 \quad (54)$$

If C is expressed in farads and V in volts, W must be expressed in joules or watt-seconds. This may be checked as follows: Since $C = Q/V$ (Equation 52),

$$W = \frac{1}{2} Q/V \times V^2 = \frac{1}{2} QV$$

If Q is in coulombs and V in volts, W is in joules.

Example: A manufacturer of photographic flashbulbs rates his Type A as requiring an energy 0.002 watt-second for reliable ignition. Will such a flashbulb fire when connected across a $50\text{-}\mu\text{f}$ capacitor that has been fully charged from a 12-volt dry battery?

Solution:

$$W = \frac{1}{2} CV^2$$

$$W = \frac{1}{2} \times 50 \times 10^{-6} \times 144 = 3.6 \times 10^{-3} = 0.0036 \text{ watt-second.}$$

Hence, the flashbulb will fire when energized by this capacitor.

30. The Capacitance of a Parallel Plate Capacitor

In Section 18, it was shown that the field intensity between two parallel, oppositely charged conducting plates is given by

$$E = \frac{\gamma}{\epsilon_0}$$

where γ is the charge per unit area in coulombs per square meter and ϵ_0 for a vacuum is the constant 8.85×10^{-12} coulomb 2 per newton-meter 2 . If q signifies the charge on either of the two plates and A the area of either plate (assuming that the plates have equal areas), then:

$$E = \frac{q}{\epsilon_0 A} \quad (55)$$

The electric field, and hence the potential gradient, between a pair of charged plates is essentially uniform. This means that the change of voltage (ΔV) over any number of small increments of distance (Δs) will be uniform. Hence, in the direction of the electric field we may write $E = V/s$, hence

$$V = Es \quad (56)$$

In this equation, V is the total potential difference between the two plates and s is the total distance between them. Hence:

$$V = Es = \frac{qs}{\epsilon_0 A} \quad (57)$$

From Equation 52, $C = Q/V$ and since in this case $Q = q$, then

$$C = \frac{q}{qs} = \frac{\epsilon_0 A}{s} \quad (58)$$

if A is in square meters and s is in meters, then C is in farads. When the dielectric between the plates is some substance other than air or a vacuum, Equation 58 must be changed to

$$C = K \frac{\epsilon_0 A}{s} \quad (59)$$

where K is the dielectric constant of the substance between the plates. Note the simplicity of these equations. You will recall that

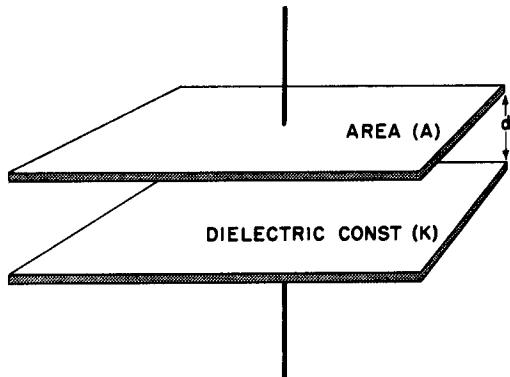


Fig. 25. A two-plate capacitor.

in Section 13 we eliminated the 4π from expressions derived from Coulomb's Law by defining the constant ϵ_0 . The reason for doing so is now apparent: if the quantity 4π had not been thus treated, it would appear in this practical equation for capacitance. As one studies further in electrostatics, the advisability of this step becomes increasingly evident.

Example: A capacitor of $0.02 \mu F$ such as that shown in Fig. 25 is required for a demonstration Tesla coil transformer. The dielectric is to be glass ($K = 6$)

and the plates are to be metal foil measuring 20 x 20 cm each. Calculate the thickness of glass required to obtain this capacitance.

Solution: Solve Equation 59 for s .

$$\begin{aligned}s &= K \frac{\epsilon_0 A}{C} \\&= 6 \frac{(8.85 \times 10^{-12})(0.04)}{0.02 \times 10^{-6}} = \frac{(5.31 \times 10^{-11})(0.4 \times 10^{-1})}{2 \times 10^{-8}} \\&= 1.06 \times 10^{-4} \text{ meter} \\&= 0.106 \text{ mm}\end{aligned}$$

31. The Measurement of Capacitance

There are three common methods of measuring capacitance: the ballistic method, the reactance method, and the bridge method. Only the ballistic method relies on electrostatic principles, hence it is the only one that we shall deal with here. A ballistic galvanometer is essentially a moving-coil instrument in which the coil has a substantial amount of inertia and is properly damped either by mechanical or electrical methods. The throw of such an instrument due to the impulse of a sudden flow of electricity through the coil is proportional to the *quantity* of electricity that has passed through it. If a capacitor is discharged through the coil of such an instrument, the discharge interval will be very short, since the resistance of the coil is usually small. With enough inertia, the assumption that the entire discharge occurs before the coil begins to move is an excellent approximation and the relationship is:

$$Q = KS \quad (60)$$

where Q is the charge in culombs and S is the throw of the ballistic galvanometer in cm. K is the "galvanometer constant."

Measurement of capacitance is made by connecting a circuit such as that given in Figure 26. The measurement may be made either on the charge or discharge of the capacitor. When SW-1 is moved to position A, and SW-2 is also on position A, the unknown capacitor C_x charges through the galvanometer whose throw may be noted. This is the *charge* reading. SW-1 may now be set to position B and the discharge throw reading observed. The discharge and charge throws are now averaged.

If S' is the average throw, C_x the unknown capacitance, and the average quantity of electricity is Q' , then from Equation 60

$$Q' = KS' \quad (61)$$

If the voltage applied by the charging source is V , we know from Equation 51 that Q' is also equal to $C_x V$. Equating these values of Q' ,

$$C_x V = K S' \quad (62)$$

A standard capacitor, C , of precisely known value is now substituted for C_x by moving SW-2 to position B and repeating the process just described. In this case, the deflection of the galvanometer,

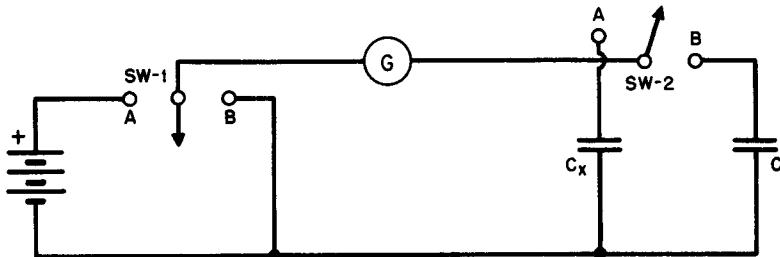


Fig. 26. Circuit for the measurement of capacitance by the ballistic method.

meter is S , the capacitance C , the galvanometer constant K , and the voltage V . This gives us a similar equation,

$$C V = K S \quad (63)$$

Dividing Equation 62 by Equation 63,

$$\frac{C_x V}{C V} = \frac{K S'}{K S}$$

The V 's and K 's drop out, leaving

$$C_x = C \frac{S'}{S} \quad (64)$$

32. Review Questions

1. Define the volt in terms of the work done in moving a unit charge. Explain why volts and joules/coulomb are equivalent.
2. Determine the maximum potential to which a sphere of 10-cm radius can be charged.
3. What is meant by potential gradient? What is the relationship between the potential gradient and the electric intensity of a field?
4. Explain why the surface of any charged conductor must be an equipotential surface.

5. Why is it impossible for equipotential surfaces in a finite field to intersect?
6. Why is it believed that the energy of a capacitor resides in the dielectric rather than on the capacitor plates?
7. Find the capacitance of an air-dielectric capacitor consisting of two plates each of 8-cm^2 area and separated by 3 mm.
8. What voltage applied to the terminals of a $0.5\text{-}\mu\text{f}$ capacitor will give it an energy content of 20 joules?
9. Describe the ballistic method of obtaining the capacitance of a capacitor.
10. Explain why the galvanometer constant drops out of the final equation for the ballistic method of determining capacitance. How could you determine the value of this constant for a given galvanometer?

Chapter 5

ELECTROSTATIC DEVICES AND APPLICATIONS

33. The Electrophorus

The electrophorus, devised by Alexander Volta, gave one of the earliest demonstrations of the magnitude of charge that can be built by *electrostatic induction* (Chapter 1). It consists of a disk of paraffin, hard rubber, or some similar insulating material and a metal disk movable by means of an insulated handle. (See Fig. 27.) There are four steps in its operation: First, the non-conducting material electrified negatively by friction, usually by rubbing it with cat's fur or flannel (A). The metal disk is now placed on top of the insulating base (B). Since the insulating material is far from perfectly smooth, the metal disk is actually spaced from it in most places, and contact between them is established only at a few points. A charge therefore appears on the metal plate by electrostatic induction, with the bottom of the disk positive and the top negative. A finger is then touched to the top of the disk (C), effectively grounding it. The excess negative charges are repelled to ground by the negative charge on the non-conductor, leaving the metal disk with a net positive charge. The finger is first removed, *then* the disk is lifted from the plate (D). When the disk is far enough away from the field of the plate, the charges upon it are free and a spark may be made to leap from the plate to a finger or through a glow-tube (neon, argon, etc) to produce a sharp, bright glow. The process may now be repeated indefinitely without any loss to the initial frictional charge. At first glance, this seems to be a violation of the Law of Conservation of Energy, since energy is repeatedly obtained in the form of a spark or

glow-tube flash without renewing the charge on the non-conducting plate. In this case, the energy comes from the mechanical work done in separating the disk against the attracting electric forces.

34. The Van de Graaff Generator

The principle of the electrophorus is applied in the operation of various types of static machines. The most spectacular and useful of these is the Van de Graaff generator diagrammed in Fig. 28. This

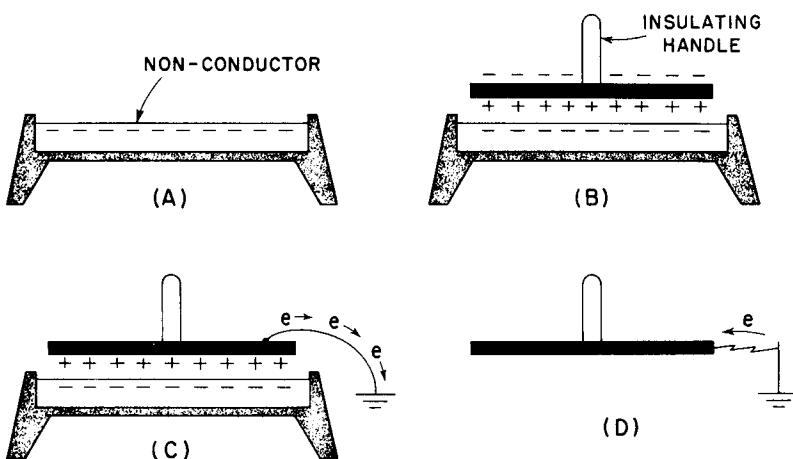


Fig. 27. Steps in the generation of charge with an electrophorus.

generator can supply upwards of two million volts and is extensively used in atomic research.

A large spherical, highly-polished metal cap is supported on an insulating column (not shown) high above the ground. A motor drives a paper or plastic belt in the direction shown at high speed (50 to 60 miles per hour). The charging process is initiated by a source of high potential B, which sprays a positive charge onto the belt via a metal comb C₁, which terminates in sharp points (Section 19). The positive charges are carried upward into the body of the sphere where they are picked off by comb C₂ and led to the metal cap. Here they distribute themselves on the outside surface. At the same time, electrode P₂ is charged negatively by induction from the ribbon, causing comb C₃ to acquire the same charge. Thus, a negative charge is sprayed on the downward moving ribbon at this point.

These charges are then carried out of the cap to the region of comb C4. The negative charges are taken off by C4 and transferred to electrode P1, which again induces a positive charge in the ribbon. At this time, the initiating source of potential may be removed, if desired, since the process is self-sustaining.

The field strength at the surface of the spherical cap is made as great as possible without a discharge taking place. The potential

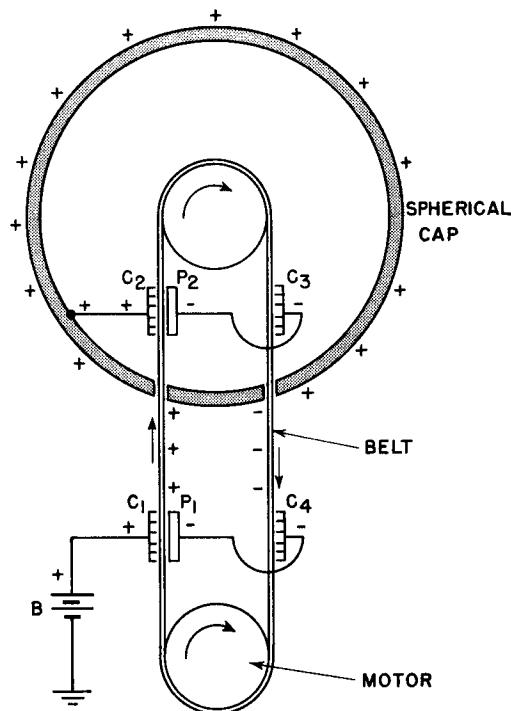


Fig. 28. Cross section of a Van de Graaff generator.

to which the sphere may be raised is proportional to the radius of curvature of its surface. For multimillion-volt machines (since corona discharge starts at fields of a few megavolts per meter), it is essential that the cap have a large diameter. The Van de Graaff generator at the Carnegie Institute in Washington is 19 feet in diameter and 55 feet high; in the case of the Massachusetts Institute of Technology, the diameter is 15 feet.

As in the case of the electrophorus, the energy developed in the spherical cap is the outcome of the work done in moving charges against repelling and attracting electrostatic forces. As the left side

of the belt moves toward the sphere, work must be done to overcome the repulsion of like charges; similarly, energy is expended in moving negative charges away from the sphere as the right side of the belt descends.

The intense electric field which surrounds the cap can change either positive or negative ions into high-speed, high-energy nuclear bullets for atomic research. Although other particle accelerators such as the cyclotron, the synchrocyclotron, the synchrotron, the betatron, and the bevatron have gained in popularity in recent years, the Van de Graaff generator is still a useful tool and is not yet obsolete.

35. The Earth's Electric Field — a Geophysical Problem

Although radar waves have reached far into space to reflect from the moon and return, and earthquake waves penetrate deep into the core of the earth to tell us something of its nature and structure, most of what we know of the earth is still gleaned from the thin blanket of tenuous gas which surrounds it. With such meager tools, it is no wonder that the problem of the origin and maintenance of the earth's electric field has never been solved completely. The information we possess is at best tentative, but as theoretical progress is made so will our ability to understand and manipulate the strings that control the earth's behavior improve. The research associated with the International Geophysical Year (particularly the satellite programs) has provided much new data.

The sea is an equipotential surface because it is an electrical conductor. As we move away from the surface of the earth, the electric potential rises to an average of about 100 volts per meter. This potential varies from place to place, from season to season, and to a great extent depends upon the topographical features of the terrain. The potential gradient is severely affected by rain and clouds and in times of thunderstorms may rise to over 10,000 volts per meter. With the realization that the picture is a statistical one, we may conceive of the earth as a huge conducting sphere carrying a negative charge of about half a million coulombs of electricity, which is so distributed over its surface as to yield a potential gradient of approximately 100 volts per meter.

Within the first 6 miles of ascent, the field strength of the earth's electric field falls to about 5% of its value near the surface. This can be accounted for only by considering that the conductivity of the atmosphere increases with increasing altitude. In an idealized pic-

ture of the upper reaches of the earth's atmosphere, at altitudes of not much more than 60 miles, the atmosphere is considered as a perfect conductor. The resistance of the entire blanket of air boxed in between the surface of the earth and the upper conducting sphere has been measured as approximately 200 ohms. The half-million "average" coulombs of the earth's charge would leak off in four or five minutes in the form of atmospheric currents if some replacement mechanism were not in constant action.

The energy content of the thunderstorms that are always in progress somewhere on the surface of the earth is more than enough to account for the half-million coulomb charge of the earth. On a

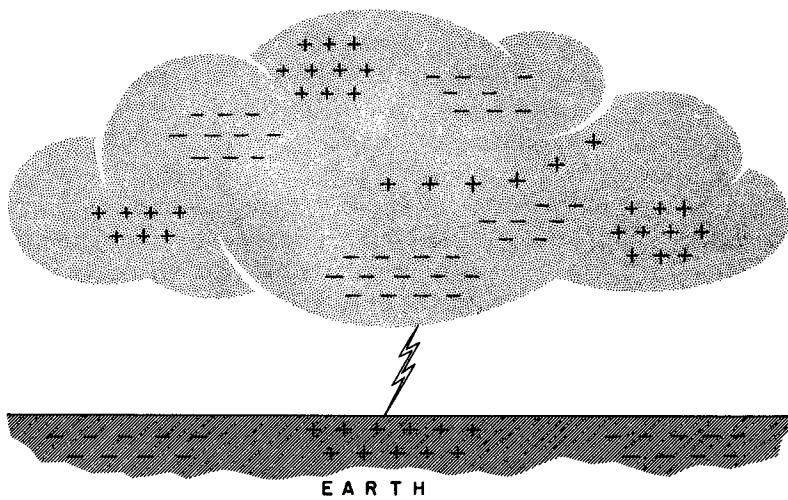


Fig. 29. Charge distribution of the earth and a cloud during a thunderstorm.

statistical basis again, best estimates give roughly 16 million thunderstorms per year and perhaps 100 flashes of lightning per second! Although a flash may have a life of but a fraction of a second, with a million volts or more and a hundred thousand amperes involved in a single stroke, it is easy to explain the existence of a year-round average charge.

The typical thunderhead cloud has a charge distribution like that shown in Fig. 29. As its bottom surface comes closer to the surface of the earth, an opposite charge is induced along the ground. When the difference of potential becomes great enough, a lightning discharge ensues.

Lightning is not made up of a single stroke. When a cloud-to-earth discharge is about to begin, a comparatively thin leader stroke starts from the cloud and reaches the earth in a succession of short jumps. After contact between the leader stroke and the ground, a return stroke begins, following the path previously formed by the leader. It is this stroke that carries the enormous currents characteristic of lightning; its velocity often approaches one-half the velocity of light. A single lightning flash may comprise several of these pairs of leader and return strokes.

The separation of electric charges in thunderclouds is attributed to the disruption of water droplets by violently ascending air currents. Laboratory experiments show that a drop of water meeting a vertically rising column of air takes on a positive charge, while the air carries the negative charges upward. Rain from the lower part of a thunderhead is therefore generally negatively charged, while that from the upper portions bears a positive charge. Another phenomenon that may also explain the polarities exhibited by the earth is that condensation of moisture in the atmosphere takes place more easily around negative nuclei or electrons than it does around positive nuclei, and the fall of such drops to the earth will give it a negative charge. In fair weather, as mentioned before, the earth is negatively charged; sometimes during violent thunderstorms, there is a reversal of the gradient in localized regions.

36. Review Questions.

1. Explain why it is possible to remove charge after charge from an electrophorus without recharging it.
2. Using the ratio of sphere diameters, how does the maximum voltage to which the Van de Graaff generator at the Carnegie Institute can be charged compare with that of the machine at the Massachusetts Institute of Technology?
4. With the help of a rough sketch, explain how a charge is built up on the sphere of a Van de Graaff generator.
5. What is the range of variation of the potential gradient of the earth from normal conditions to conditions of severe electrical disturbances?
6. What causes variations in the earth's potential gradient?
7. If the earth's charge were not being continuously maintained, how long would it take for complete discharge to occur? What do we believe is the charge-maintaining mechanism?
8. What is the normal polarity of the earth's charge?
9. Describe the method by which we believe electric-charge separation occurs in thunderclouds.
10. What is the sequence of events involved in the production of a lightning stroke?

INDEX

- Ampere, 19, 20
Arc discharge, 22
Atom, 2, 3, 9
Atomic particles, 2
Atomic structure, 2
Capacitance, 49, 50, 51, 52
Capacitor, 49, 50, 54, 55
energy stored, 51, 52
parallel-plate, 52
Charge, 2
density, 35
negative, 2, 4, 11
positive, 2, 4, 11, 12
static, 31
Charging, 10
Coefficient of viscosity, 36
Conductors, 4, 40
Corona, 36
Coulomb (units), 18, 19, 24
Coulomb's law, 12, 13, 17-19, 24,
 25, 153
 units for, 18, 19
Current intensity, 19, 20
Dielectric, 40, 50
 constant, 25, 41
 strength, 40
Dyne, 16, 17
Electric charge, 1, 4, 5, 6, 7
Electric field, 21, 22, 23, 24, 52
 earth's, 60-62
Electrification, 11, 12
Electrons, 2-5, 9
 charge of, 37, 38
 mass, 3
Electrophorus, 57-59
Electroscope, 7, 9-11
Electrostatic:
 detector, 6, 7
 induction, 10, 11, 57
 line of force, 47
 unit of charge, 18, 19
Energy, 19, 43, 49
 equipotential surface, 46-48, 60
 law of conservation of, 57, 58
Erg, 19
Esu, 18
Farad, 51
Field intensity, 23, 45
 theories, 21
Force, 16
Forces of attraction and repulsion, 2
Free electrons, 4, 9, 11
Galvanometer, ballistic, 54
Galvanometer constant, 54
Gauss' law, 27-34
 examples of the use, 29-34

INDEX

- Inertia, 3, 54
Insulators, 4, 40
Ion, 4, 9, 35, 60
Ionization, 35
Joule (unit), 19, 45
K (constant), 12, 13, 41
Leak, 35
Lightning, 61, 62
Lines of force, 22
Mass, 15
Matter, 1
Meter, 14
Millikan, Robert, 36
Neutral atom, 2
Neutrons, 2, 3
Newton (unit), 17
Newton's law of gravitation, 12
Ohm, 20
Oil-drop experiment, 36-39
Particle accelerators, 60
Potential, 19, 43
Potential gradient, 40, 48-51, 60
Proof plane, 6, 9
Protons, 2-5
Rubbing contact, 12
Static generators, 7
Stoke's law, 36
Thunderstorms, 62, 63
Triboelectric series, 11, 12
Unit systems, 14, 15, 16
 cgs, 16, 17
 English, 14
 mks, 17, 18
Van de Graaff, 7, 45
 generator, 5-8, 59
Volt, 19, 44
Weight, 15-17
Work, 19, 43