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D-C CIRCUIT ANALYSIS

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D-C CIRCUIT ANALYSIS

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PREFACE

Direct-current circuit considerations are reflected in most aspects of electronic equipment design and maintenance procedure. For this reason, it is essential that those concerned with electronics possess a working knowledge of the essential relationships relating to d-c circuit analysis. Such knowledge and techniques are necessary and useful throughout the fields of communications and industrial electronics.

The intent of this book is to provide the fundamental concepts of direct-current analysis. The mathematical treatment employed has been kept simple, but the analyses are sufficiently extensive to permit the interested technician or student to develop a full comprehension of the pertinent theory. To insure this aim, an adequate amount of information is given relating to electrical laws in such form as to permit ready use; detailed descriptions of a relatively small number of selected major topics are presented, rather than treating a larger body of less important material; the topics, once given, are related to practical situations and equipment; drill is given in selected problems, so as to afford the reader additional profitable information and an opportunity to apply the principles he has learned; and, step by step diagrams provide clear cut concepts of the methodology involved in problem solution.

Specific attention is given to basic concepts (including a review of electrostatics); charge, electric current and potential difference; resistance and conductance; Ohm's law; resistance factors;

resistivity; conductivity; wire gages; simple d-c circuit analysis, including series, parallel and series-parallel arrangements; meters; power and power dissipation; combination d-c circuits; Kirchhoff's laws; superposition; Thévenin's theorem; and the Wheatstone bridge. Thus, a foundation is provided upon which more advanced concepts can be built.

Grateful acknowledgment is made to the staff of the New York Institute of Technology for its assistance in the preparation of the manuscript of this book.

April 1958
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A.S.

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Chapter 1

BASIC CONCEPTS

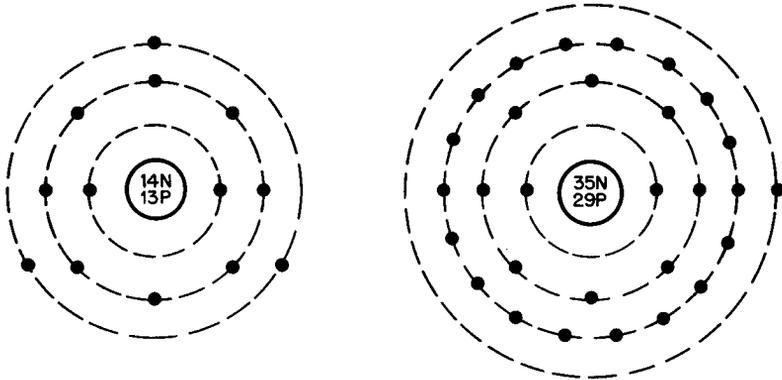
1. Characteristics of the Electron Theory of Matter

Early classical physics divided all earthly phenomena into two groups: *matter* and *energy*. Today, it is understood that this classification is highly artificial and that matter and energy are different manifestations of the same thing.

The fundamental building block of all matter is the *atom*. Various combinations of *atoms* make up *molecules*, a second building block in some kinds of matter. Some kinds of molecules consist of just a single atom — as for example in the rare gases like helium, argon, neon, and krypton. Other gases, like oxygen and hydrogen, are diatomic molecules, that is, the molecule consists of two atoms of the gas. The water molecule is triatomic, and consists of two atoms of hydrogen and one atom of oxygen. On the other hand, the structure of many solids, especially metals, does not involve the molecule as a building block. Instead the atoms of the metal are organized in groups called *crystals*.

Attempts to fabricate a physical picture of even a simple atom date back to the times of the ancient philosophers. Through the centuries, observed data gathered by physicists has finally brought into being an atomic concept that is probably very close to the truth. In the simplest sense, atoms are structures thought to be composed of three kinds of primary particles of electricity, *electrons*, *protons*, and *neutrons*, which are arranged within the atom in a definite manner.

Protons and neutrons make up the core or nucleus of the atom. Revolving around the nucleus at very high speeds, and divided among one or more orbits are as many electrons as there are protons in the nucleus. The numbers and arrangement of the particles differ in the different kinds of atoms and it is this that accounts



ALUMINUM ATOM

13 PROTONS, 14 NEUTRONS
3 RINGS 2-8-3
(FOR 13 ORBITAL ELECTRONS)

COPPER ATOM

29 PROTONS, 35 NEUTRONS
4 RINGS 2-8-18-1
(FOR 29 ORBITAL ELECTRONS)

Fig. 1. The atomic Structures of aluminum and copper.

for the differences in chemical and physical behavior that characterize the various elements. Each of the 102 chemical elements has its own kind of atom.

The atomic particles differ from each other in several ways. The *electron* (which is the fundamental particle of negative electricity or the fundamental *negative charge*) is the least massive of the three, having a relative mass of only about 1/1840 of either the proton or the neutron, specifically 9.106×10^{-28} gram. This small mass enables electrical forces to move electrons much more easily than the other particles. The minuteness of the amount of electricity corresponding to the fundamental negative charge (the electron) requires that a great number of electrons be moved (or act together) in order to obtain measurable electrical effects.

The proton is 1840 times more massive than the electron. Whereas the electrons are found in orbits that lie outside the nucleus of the atom, protons are always found within the nucleus

itself. The proton is considered to be the fundamental particle of positive electricity — i.e., the fundamental *positive charge*. The neutron has substantially the same mass as the proton but it has no electrical charge — i.e., it is electrically neutral. It is assumed to be made up of equal amounts of positive and negative electricity and forms a part of the nucleus of the atom.

There are additional particles on the modern physicist's list, which do not, at the present time, hold any interest for those primarily concerned with communications and hence will not be discussed here.

Every electron is like every other electron, every proton is like every other proton, and every neutron is like every other neutron. The distinguishing characteristics of the different elements are the arrangement and number of their protons, neutrons, and planetary electrons. Figure 1 shows the structure of an aluminum atom contrasted to that of a copper atom. Because they are not acted upon by electrical forces and are not involved in electrical phenomena, we will not have to discuss the neutron after this point.

2. Electrostatic Principles

The atoms of most substances are stable structures. The electrons are held in position in their orbits by two physical forces attributable to electrical effects. The force of attraction between electrons and protons (unlike charges) offsets the centrifugal force on the whirling electrons, and the force of repulsion between the electrons (like charges) serves to hold the electrons in their positions in the orbits. This behavior of particles within the atom is duplicated by large-scale electrical behavior and may be generalized by the statement: "*Unlike charges attract each other, whereas like charges repel each other.*"

The concept of attraction and repulsion between charges is an important one that serves as a basis of reference and explanation throughout the study of electricity and electronics. Coulomb defined the relationship of forces between two electric charges as:

$$F = \frac{q_1 q_2}{kd^2} \quad (1)$$

The formula indicates that the force F between two charges is directly proportional to the product of their magnitudes (q_1 and q_2 , respectively) and inversely proportional to the square of the

distance between them. (The word "charge" is used both to express a single fundamental quantity of electricity like an electron or an accumulation of such fundamental units on a "charged body".) In Equation 1, F represents the force in dynes,¹ q_1 and q_2 the strength of the respective charges in electrostatic units (esu), d the distance between charges in centimeters, and k , called the dielectric constant, is a proportionality constant that depends on the medium in which the charges are located ($k = 1$ in a vacuum).

Example 1. Two negative charges of 15 and 6 electrostatic units, respectively, are placed 3 centimeters apart in air ($k = 1$). What force of repulsion exists?

Solution.

$$F = \frac{q_1 q_2}{k d^2} = \frac{(15)(6)}{1 \times (3)^2} = \frac{90}{9} = 10 \text{ dynes}$$

An electrically neutral object possesses an equal number of electrons and protons. In general, all atoms are neutral electrically, because the number of protons is the same as the number of electrons. In metals, however, the outermost electrons may leave their orbits, causing a momentary imbalance. Such electrons are called *free electrons* and drift from atom to atom.

Should an atom lose or gain one or more electrons, it becomes an *ion*. If the atom gains electrons, it has a preponderance of negative charge and becomes a negative ion; if it loses electrons, it has a preponderance of positive charge and is a positive ion.

3. Electrostatic Charging

An electrically neutral rod of hard rubber may be given a negative charge by rubbing it briskly with fur; an electrically neutral glass rod may be given a positive charge by wiping it with dry silk. In the first case, electrons pass from the fur to the rod making the rod negatively charged (since it now has an excess of electrons) and leaving the fur positively charged (since it has lost electrons and now has excess protons). In the second case, electrons are rubbed off the glass rod onto the surface of the silk leaving the rod with an excess of protons and, therefore, positively charged.

Let us consider two metal spheres, mounted on short pieces of Bakelite attached to bases as in Fig. 2. They are assumed to be electrically neutral at the start. If a negatively charged rod is

¹A force of one dyne is approximately equal to a weight of one milligram.

touched to one of the spheres, some of the excess electrons on the rod will flow onto the surface of the ball, giving it a negative charge. Assume that a second rod, positively charged, is now used to transfer a positive charge to the second sphere. (What has actually occurred in this action is the transfer of electrons *from*

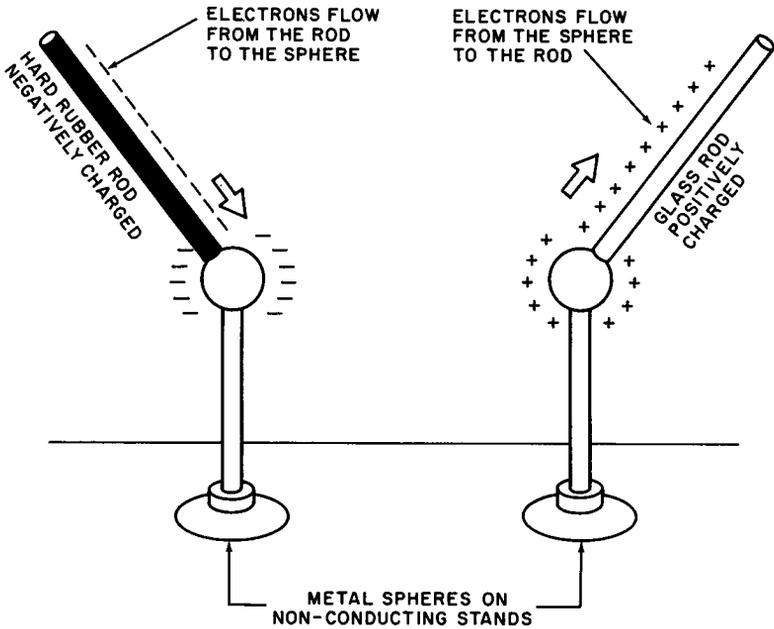


Fig. 2. Electrostatic charging.

the neutral sphere *to* the positive rod, partially neutralizing the charge on the rod, leaving the ball with a shortage of electrons and a net positive charge.) We now have two spheres, one of which is negatively charged, while the other is positively charged. The spheres appear to be identical in all respects, except for the invisible and different charges upon them.

4. Conductors and Insulators

If the two spheres in the charged state are connected to each other by a metallic wire, the excess electrons on the negatively charged ball will flow through the wire to the other sphere, as a

result of the attraction between unlike charges. It is known, however, that the individual electrons on the negatively charged body do not physically pass to the positively charged body. The phenomenon of electrical neutralization is explained by the fact that the wire contains a large number of free electrons, which are drawn to the positively charged ball by the force of attraction between unlike charges. At the same time, electrons pass into the wire from the negatively charged ball, the number of free electrons in the wire remaining constant. The actual net drift of the electrons is relatively slow, but charge is transferred instantaneously. A flow of *charge* between bodies having different static charges is one form of *electric current*. A current of this kind passes so briefly (only until neutralization occurs) that its mention at this point primarily serves as a transition from the preceding discussion of electrostatics to that of electrodynamics — the study of charge in motion — that is the basics of circuit analysis.

Some substances (*conductors*), particularly those classified as *metals*, permit electron flow to occur easily. Others, notably substances like Bakelite, glass, and wax, resist the flow of electrons to such an extent as virtually to prevent it from occurring at all. It is believed that conductors are naturally endowed with a large quantity of free electrons, which produce a flow of current when acted upon by electrical forces. Nonconductors (insulators) have very few free electrons. One of the best electrical conductors, silver, is estimated to have 1.68×10^{24} free electrons per cubic inch as contrasted to hard rubber, with some three free electrons for the same volume under normal conditions. Even between different metallic conductors, there are variations in the ease with which an electric current is passed.

5. Charge, Electric Current, and Potential Difference

The Coulomb. In our discussion of the electron theory of matter, we mentioned that the fundamental particle of negative charge was that carried by the electron. This charge is an extremely small quantity of electricity — its precise magnitude was only determined in the course of the present century — and larger units of charge have been established for convenience in computation. For example, the electrostatic unit of charge (esu) was chosen to make the force between two equal charges one centimeter apart exactly 1 dyne. Even this unit (which represents approximately

2×10^9 electronic charges) is small for practical purposes, and the practical unit of charge, the *coulomb* is 3×10^9 (three billion) times as large.

The Ampere. As developed earlier in our discussion of electrostatics, an electric current consists of a transfer of charge. It is measured in terms of the quantity of charge that passes a point in a conductor in a given time. A practical unit is based on the coulomb as the quantity of charge and the second as the unit of time. This unit is called the *ampere*, and represents a rate of flow of charge of 1 coulomb per second.

Potential Difference — the volt. Force is required to separate unlike charges or to bring like charges together. If the separation between unlike charges is to be increased, *work* must be done. The amount of this work is given by the product of the force exerted on the charge to move it and the distance that it has been moved. (This concept is analogous to the work done in lifting a weight from the ground.) This work can be recovered in the process of allowing the charge to return to its original position, in the same manner as work can be done by allowing a suspended weight to fall back to earth. In the case of the weight, the work required to lift it to a height equals the work recoverable in dropping it from the same height, or the *potential energy* of the suspended weight. Similarly, the work that is done in moving a charge is equal to the work recoverable in allowing the charge to return to its original (reference) position, or its *potential*.

Electrical potential is measured in terms of the work required to move a unit charge. The unit of charge is the coulomb and that of work, the *joule*. (1 joule is about $\frac{3}{4}$ foot-pound.) The practical unit of electrical potential, the *volt*, represents an expenditure (or production) of energy of one joule per coulomb of charge transferred. It is this energy that is dissipated in the process of passing charge through a resistance. Furthermore, it is not possible to cause an electric current to flow between two points unless they are at different electrical levels or potentials. The flow of current between the metal spheres mentioned earlier ceases as soon as the electrical levels of the spheres become the same, that is when the charges are neutralized. (The reader will find that Fig. 3 will clarify the preceding discussion.)

In order for a continuous current to flow, a *difference* of potential, or *electromotive force*, must be maintained. This is the function of batteries and generators — they raise quantities of

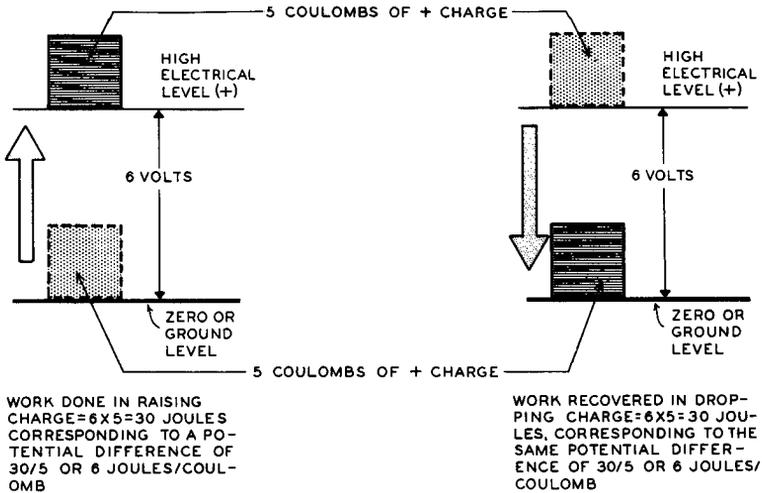


Fig. 3. Voltage as work done or recovered per unit charge transferred.

charge to higher electrical levels from which they can pass to lower ones, doing work in the process. This imparting of energy to charges is accomplished at the expense of mechanical energy in generators and of chemical energy in batteries.

6. Resistance and Conductance

As previously discussed, some materials are good conductors, others are poor conductors, and still others conduct hardly at all. When a substance conducts electricity well it is said to have *low resistance*; when it conducts poorly, its resistance is *high*. Again, we need a unit of measure for the amount of resistance.

We shall take as our basic unit of resistance the *ohm*. The ohm is a resistance across which there is a potential difference of 1 volt when the current in it is 1 ampere. The ohm may also be defined as the opposition offered to the flow of electricity by 1000 feet of No. 10 copper wire at 68° Fahrenheit (normal room temperature). Thus, 2500 feet of the same wire at room temperature have a resistance of 2.5 ohms. It is possible, on the other hand, to make a 10,000,000 ohm special *resistor* $\frac{1}{2}$ -inch in length. The resistor, of course, would not be made of copper wire, but probably of a mixture of carbon and clay, or other high resistance material.

The term "conductance" is sometimes used in discussions of circuitry. Conductance (G) is the reciprocal of resistance or

$$G = 1/R$$

A wire having a high resistance has a low conductance, and wire of low resistance is one of high conductance. The unit of conductance is the mho (ohm spelled backwards).

Example 2. What is the conductance of a wire having a resistance of 50 ohms?

Solution. $G = 1/R$ $G = 1/50 = 0.02 \text{ mho}$

7. Ohm's Law

George Simon Ohm discovered in the course of experiments that the current passing in a metallic circuit was always directly proportional to the applied potential (voltage) and inversely proportional to the circuit resistance. This law of electrical behavior is fundamental to circuit analysis and is the basis of most simple circuit problems. Stated in mathematical terms

$$I \text{ (current)} \sim \frac{E \text{ (potential)}}{R \text{ (resistance)}}$$

Ohm converted this proportionality into an equation by properly choosing the unit of resistance (which we did in the last section in terms of a physical measurement). He chose the magnitude of the resistance unit (the ohm) so that the following *equation* could be set up:

$$I \text{ (current in amperes)} = \frac{E \text{ (potential in volts)}}{R \text{ (resistance in ohms)}} \quad (2)$$

This equation is used in two other forms, both of which are transpositions of it:

$$E = I \times R \quad (3)$$

and

$$R = E/I \quad (4)$$

Example 3. The potential applied across the filament of a certain transmitting tube is 10 volts. If the resistance of the filament is 5 ohms, what must be the current through the filament?

Solution. The unknown is the current. Hence the use of Equation 2 is called for. Substituting:

$$I = 10/5 = 2 \text{ amperes}$$

Example 4. A pilot lamp uses 0.05 ampere at 6 volts. What is the "hot" resistance of the filament? (When the filament is cold its resistance is lower.)

Solution. The unknown is the resistance. This calls for Equation 4.

$$R = E/I = 6/0.05 = 120 \text{ ohms}$$

Example 5. What voltage is required to force a current of 0.15 ampere through a vacuum tube filament having a resistance of 165 ohms?

Solution. Because voltage is unknown, we can best use Equation 3.

$$E = 0.15 \times 165 = 24.75 \text{ volts}$$

Example 6. If the voltage applied to a circuit is doubled and the resistance of the circuit is increased to three times its former value, what will be the final current value?

Solution. From Equation 2, the original current, $I_1 = E/R$. To find the new current, I_2 , E is doubled and R is tripled:

$$I_2 = \frac{2E}{3R} = 2/3 \frac{E}{R} = 2/3 I_1$$

The final current is 2/3 the original value.

Ohm's Law Applied to Conductance. Earlier we defined conductance as the reciprocal of resistance using the equation $G = 1/R$. Substituting for R its equivalent given by equation 4:

$$G = \frac{1}{R} = \frac{1}{E/I} = \frac{I}{E} \quad (5)$$

This equation tells us that conductance is the ratio of the current in a conductor to the voltage that produces it. This ratio often is of use in solving problems that require the answer in terms of conductance, as in example 7.

Example 7. What is the conductance of a circuit if 6 amperes flow when a potential of 12 volts is applied to the circuit?

Solution. Use Equation 5.

$$G = I/E = 6/12 = 0.5 \text{ mho}$$

Note that the potential must be in volts and the current in amperes for the conductance to be in mhos.

8. Review Questions

- (1) Differentiate between a molecule and an atom.
- (2) Explain the modern theory of atomic structure.
- (3) Define the terms: free electrons, conductor, and insulator.

- (4) What is the Law of Electric Charges? Coulomb's Law?
- (5) Two negative charges of 10 and 30 esu are separated in air by 0.5 centimeter. What force, in dynes, exists between the two? Do they attract or repel each other?
- (6) What other terms are synonymous with difference of potential?
- (7) What is the practical unit of potential difference?
- (8) Define the conductance of a substance and give the mathematical expression of it.
- (9) State Ohm's Law in words. List the three mathematical forms of Ohm's Law.
- (10) How much current will pass through a resistance of 180 ohms when a potential of 220 volts is applied across it?

Chapter 2

RESISTANCE FACTORS

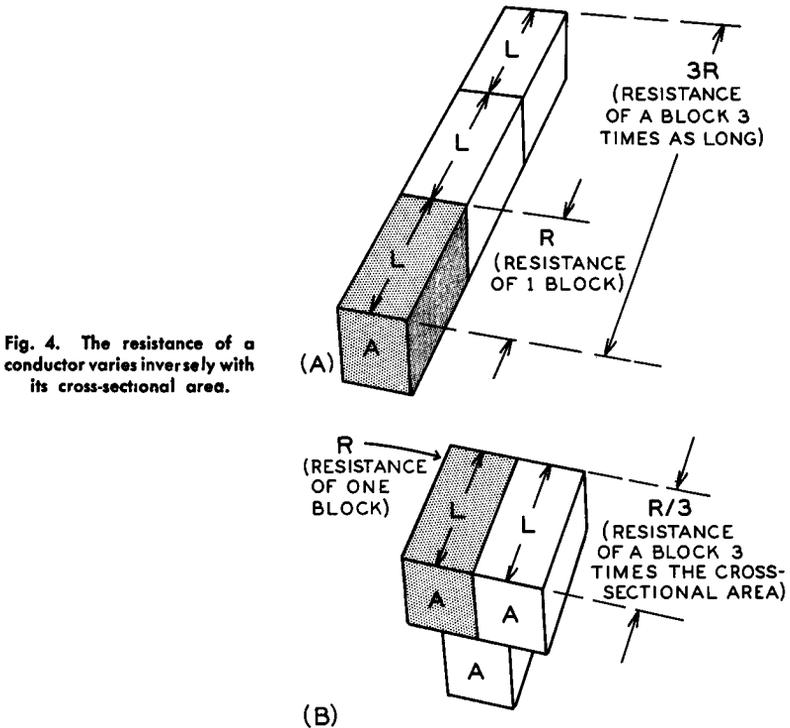
9. Factors Affecting Resistance

Because resistance and resistors play an essential role in communications equipment (a resistor may be inserted into an electrical circuit for many reasons including control of current flow, to produce differences of potential, etc.), it is necessary to know just what governs the amount of resistance to be expected in specific cases. Even the resistance of a nominally good conductor often becomes a factor in the correct operation of certain equipment. Resistance is a function of the material of which the component (or conductor) is made, its length, its cross-section area, and its temperature.

The Length of a Conductor. Resistance is directly proportional to conductor length. If one foot of a conductor of given cross-sectional area offers a certain degree of opposition to electric current, two feet of the same conductor will offer twice as much opposition. (See Fig. 4.) In terms of the electron theory, charge is transferred at a finite speed and any particle of charge passes a finite length of a conductor in a given time. During such a period the flow of charge is opposed by a certain resistance. Should the length of the conductor be doubled, the charge would be in motion twice as long and would be opposed twice as much.

Cross-section Area of a Conductor. If we take a conductor whose cross-section area is 1 square centimeter, and another piece

of the same material and the same length whose cross-section area is 2 square centimeters, we will find that the smaller conductor has twice the resistance of the larger one. This may be explained in terms of the free electrons available for conduction: the volume of the larger conductor is twice as great and there are twice as



many free electrons available in any unit length. In general, *resistance is inversely proportional to the cross-section area*, and since the cross-sectional area of a conductor is directly proportional to the square of its diameter, the *resistance of a conductor is inversely proportional to the square of its diameter*. (See Fig. 4.)

Thus, if the *diameter* of a conductor of given length is *doubled*, the resistance will be reduced to one-quarter of the previous value; if the *diameter* is *reduced* to one third, the resistance will be *increased* nine times.

The Circular Mil. The *circular mil* was selected because electrical conducting wires are usually round and possess comparatively small diameters that can easily be expressed in mils. A circular mil (c.m.) is merely the cross-sectional area of a round wire with a diameter of 0.001 inch. Because it is a circular *area* unit of measure, it is necessary only to square the number of mils given in the diameter to find the number of circular mils in a circle of that diameter. Thus, a conductor with a 1-mil diameter would have a 1-c.m. cross-section area; a 3-mil diameter, a 9-c.m. area;

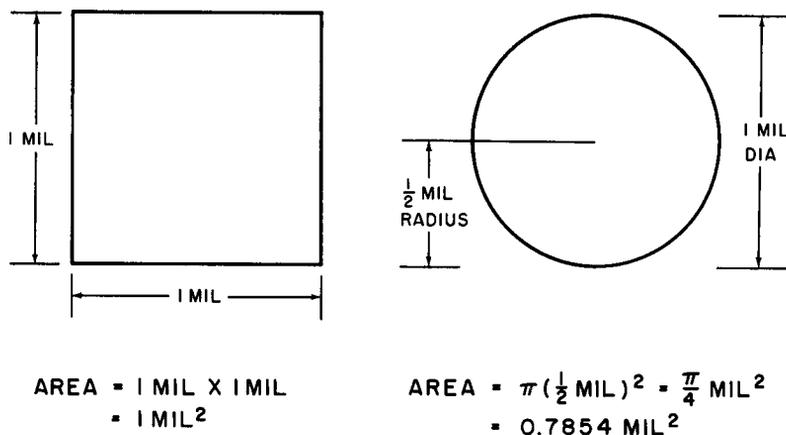


Fig. 5. Comparison of square and circular mils.

and a 40-mil diameter, a 1600-c.m. area. A round conductor of 1000 mils (1 inch) in diameter has a cross-section area of 1,000,000 c.m., which is equivalent to 0.7854 square inch. Conversely, one square inch is equivalent to 1,000,000/0.7854 or 1,273,200 c.m. This conversion factor permits calculation of the equivalent circular mils for a square or rectangular cross section of conductor. In square measure, then,

Area of a circular mil = 0.7854 area of a square mil (6)

The area of a square whose sides are 1 mil long would be a square mil. Figure 5 shows the relationship between the area represented by a circular mil and that by a square mil. As may be seen from the diagram, the square mil is a bit larger than the circular mil. The proportionality factor is given in Equation 6, and may be derived by comparing the areas of the two. The area of the square mil is 1 mil \times 1 mil or 1 mil². That of the circular

mil is πr^2 or $3.1416 \times (1/2 \text{ mil})^2 = (3.1416/4) \text{ mil}^2 = 0.7854 \text{ mil}^2$. The ratio of circular to square mil is thus 0.7854 to 1 as given above.

Material of the Conductor. The fact that different materials offer differing degrees of opposition to electric current was discussed in general terms in Chap. 1. The degree of opposition depends upon the atomic structure of a material, because the atomic structure determines the number of free electrons available for conduction. For example, an alloy called nichrome drawn into a piece of wire offers much more resistance to the flow of an electric current than a piece of copper wire of equivalent dimensions.

The Effect of Temperature. Electrical conductivity in most materials is strongly affected by changes in temperature. Metallic conductors, for the most part, have positive temperature coefficients of resistance, i.e., the resistance rises as the temperature rises; some nonmetals, like carbon, have negative coefficients. For copper, the metal most used in conductors, the temperature coefficient is positive. Certain alloys, such as manganin, show virtually no change in resistance with changing temperatures. They are excellent for use as instrument shunts or multipliers.

10. Resistivity

It is often desirable to know the resistances of electrical materials in terms of unit dimensions of length and cross-sectional area, because this information permits calculations and comparisons between the materials. The *resistivity* (ρ) of a given material is numerically equal to the resistance of a wire (or bus) of the material of unit length and cross-section area. The resistance per circular mil-foot, representing the resistance of a wire with a diameter of 1 mil (0.001 inch = 1 mil) and a length of 1 foot, is a commonly used unit for ρ . Resistivities of a number of conductors are given in Table 1.

A useful equation for calculation of resistance from resistivity is

$$R = \rho \frac{L}{A} \quad (7)$$

where R is the resistance of the wire in ohms, ρ is the resistivity in ohm-c.m./ft, L is the length of the wire in feet, and A its cross-sectional area in c.m.

TABLE 1
RESISTIVITIES OF COMMON ELEMENTS AND ALLOYS
AT 20°C (68°F)

<i>Elements</i>	<i>Resistivity</i> (ohm-c.m./ft)	<i>Alloys</i>	<i>Resistivity</i> (ohm-c.m./ft)
Silver	9.9	Brass	42.
Copper, annealed	10.37	German Silver	199.
Copper, hard drawn	10.7	Manganin	265.
Gold, pure	14.	Constantan	302.
Aluminum	17.	Excello	550.
Tungsten	33.	Nichrome	675.
Zinc	36.	Phosphor Bronze	57.
Nickel	47.		
Iron, cast	54.		
Lead	132.		

Example 8. What is the resistance of an annealed copper wire 200 feet long and 4 mils in diameter, at 20°C? What is the resistance of an annealed copper wire of the same length with a 40-mil diameter?

Solution. (a) $R = \rho \frac{L}{A} = \frac{10.4}{42} \times 200 = \frac{2080}{16} = 130 \text{ ohms}$

(b) $R = \rho \frac{L}{A} = \frac{10.4 \times 200}{40^2} = \frac{2080}{1600} = 1.30 \text{ ohms}$

11. Temperature Coefficients of Resistance

Communications equipment, especially field equipment, may be subjected to variations in temperature, hence a knowledge of how to compute the effect of temperature on the resistance elements is useful. The resistance of a conductor varies directly with its temperature coefficient, defined as the amount that the resistance increases (or decreases) with each degree rise in temperature for each ohm of resistance at the reference temperature. Normally, the changes due to temperature variation are slight and may be neglected in most practical applications. For accurate computation however, Equation 8 can be used:

$$R_f = R_i + R_i \times T_c (t_f - t_i) \quad (8)$$

where R_f is the resistance at the final temperature, t_f , R_i is the resistance at the initial temperature, t_i , and T_c is the temperature

TABLE 2

TABLE OF TEMPERATURE COEFFICIENTS OF RESISTANCE OF VARIOUS MATERIALS AT 20°C

<i>Material</i>	<i>Temperature Coefficient per ° C</i>	<i>Material</i>	<i>Temperature Coefficient per ° C</i>
Advance Metal		Lead	0.0038
(copper, nickel)	0.000018	Manganin (copper, manganese, nickel) ..	0.000006
Aluminum	0.00388	Mercury	0.00089
Bismuth	0.00435	Nichrome	
Brass (annealed)	0.002	(chromium, nickel)	0.0004
Carbon	-0.0003	Nickel	0.0062
Constantan		Phosphor Bronze	0.004
(copper, nickel)	0.000018	Silver	0.00377
Copper (drawn)	0.004	Steel, hard	0.0016
Excello	0.00016	Steel, soft	0.005
German Silver (copper, nickel, zinc)	0.00038	Tungsten	0.0046
Gold	0.0034	Zinc	0.0037
Iron, cast	0.006		

coefficient for the material. Typical values of the temperature coefficient of resistance are given in Table 2.

Example 9. A carbon resistor ($T_c = -0.0003$) of 300,000 ohms at room temperature (20°C) is heated to 50°C. What is its resistance?

Solution. $R_t = R_1 + R_1 \times T_c (t_t - t_c) = 300,000 + 300,000 \times (-0.003) (50 - 20)$
 $= 300,000 + (-2700) = 297,300 \text{ ohms}$

12. Conductivity

Conductivity is defined as the reciprocal of resistivity, and its use is sometimes convenient for comparison of various conducting materials. In practice, conductivities are measured with reference to a standard, the resistivity of 1 cubic centimeter of annealed copper at 20°C, which is arbitrarily defined as *100-percent conductivity*. (The standard has resistivity value of 1.724×10^{-6} ohm-c.m.) Utilizing this constant, the *percent conductivity* of other materials can be computed.

Example 10. Compute the percent conductivity of aluminum.

Solution. The resistivity of aluminum (from Table 1) is 17, and that of copper, 10.37. The relative *conductance* of aluminum is the *reciprocal* of the aluminum-copper resistivity ratio, or:

$$\begin{aligned} \text{relative conductance of aluminum} &= \frac{\text{resistivity of copper}}{\text{resistivity of aluminum}} \\ &= \frac{10.37}{17} = 0.61 \text{ or } 61\% \end{aligned}$$

13. American Wire Gage (AWG)

American wire sizes are designated by number in the American Wire Gage system (formerly Brown and Sharpe Gage). These numbers range from the largest sizes, 0000, 000, 00, 0, and 1, to 40, the smallest size, based on a constant ratio between diameters of successive gage numbers. The larger the number, the smaller the diameter of the wire and the smaller its cross-section area. The different sizes are selected so that progressively larger cross-section areas differ by the ratio of the square mil to the circular mil, a factor of roughly 1.27. The choice of factor is such that the cross-section area and, therefore, the resistance, either halves or doubles (approximately) for every three gage numbers, depending upon the direction of the change. The ratio of any cross-section area or resistance to that of a gage number differing by 10 is either ten to one or one to ten (approximately), again depending upon the direction of the change.

For purposes of rough calculation, a number 10 wire has these properties: (a) a resistance of about one ohm for each 1000 feet; (b) a diameter of about 1/10 of an inch or 100 mils; and (c) a cross-section area of about 10,000 circular mils. (Bearing in mind that the resistance is halved or doubled in three gage numbers, it can be seen from these figures that a number 7 wire would then have a resistance of 1/2 ohm for each 1000 feet and cross-section area of 20,000 circular mils; number 13 wire, a resistance of 2 ohms per 1000 feet and a cross-section area of 5000 circular mils; number 16, a resistance of 4 ohms per 1000 feet and a cross-section area of 2500 circular mils.)

Example 11. Compute the resistance of a 500-foot length of number 18 copper wire.

Solution. Number 19 wire would have a resistance of $2 \times 2 \times 2$ or 8 ohms per 1000 feet since it differs by 9 gage numbers from 10. The resistance of number 18 wire is then

$$\frac{8}{1.27} = 6.3 \text{ ohms per 1000 ft}$$

$$\text{or } \frac{6.3}{2} = 3.15 \text{ ohms for 500 ft}$$

Example 12. Determine the cross-section area in circular mils, the diameter, and the resistance per 1000 feet of number 8 copper wire.

Solution. The area in circular mils of number 10 wire is 10,000 c.m. (approx.)

(a) The circular mil area of number 7 wire is 20,000 c.m. (approx.) and the circular mil area of number 8 wire is $20,000/1.27 = 15,800$ c.m. (approx.)

(b) The diameter of No. 8 wire = $\sqrt{15,800} = 126$ mils (approx.) (c) The resistance of No. 7 is 0.5 ohm per 1000 feet. The resistance of 1000 feet No. 8 = $0.5 \times 1.27 = 0.635$ ohms (approx.).

14. Review Questions

- (1) Name three factors affecting the resistance of a conductor.
- (2) The resistance of a conductor is 50 ohms. What change in resistance will take place if its diameter is doubled?
- (3) Express the area of a circular mil in terms of a square mil.
- (4) Name three alloys whose resistance is hardly affected by changing temperatures.
- (5) A 100-foot piece of annealed copper wire has a 5-mil diameter. Find its resistance at 20°C.
- (6) The same copper wire is heated to 50°C. Determine the value of the new resistance.
- (7) What is the relationship between resistance and conductivity?
- (8) What is the conductivity of the wire in Question 5?
- (9) Name three properties of No. 10 copper wire.
- (10) What is the resistance of 2000 feet of No. 19 copper wire?

Chapter 3

SIMPLE D-C CIRCUIT ANALYSIS

15. Series Connection of Resistors

A complete circuit consists of a *source* of voltage, conductors, and one or more *loads* connected so that the current leaves the negative terminal of the source, flows through the *load*, and returns

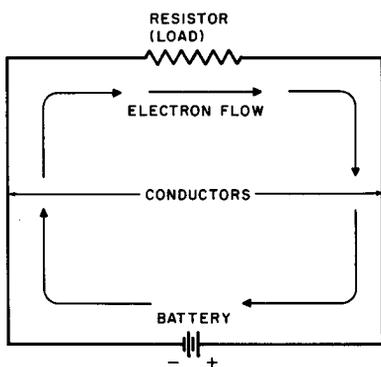


Fig. 6. A complete circuit.

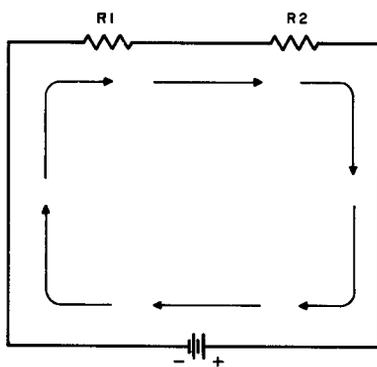


Fig. 7. A simple series circuit.

to the positive terminal of the source. For these simple exercises we shall use a *dry cell* or other "battery" as the source (strictly speaking, a battery is two or more cells). The *load* in an electrical circuit is understood to mean the device or devices that utilize

the electric current and are worked or operated by the current; Fig. 6 shows a complete circuit.

The Series Circuit. A simple series circuit consists of two or more loads connected end to end as shown in Fig. 7. Note that this is a complete circuit and the current flow starts from the negative terminal of the source, passes through the load, and returns to the positive terminal. Because no current is diverted at any point, the current in a series circuit must be *everywhere the same*, and R1 and R2 may be considered the equivalent of a single resistor having a resistance equal to the *sum* of R1 and R2. We can thus state in general, that *the total resistance of a series circuit equals the sum of the individual resistors or*

$$R_{\text{total}} = R1 + R2 + R3 + \text{etc.}$$

Example 13. Draw a simple diagram showing the method of connecting three resistors of equal value so that the total resistance will be three times the resistance of one unit.



$$R1 = R2 = R3$$

$$R_{\text{TOTAL}} = R1 + R2 + R3 = 3R1$$

Fig. 8. Circuit for Example 13.

Solution. If you connect three resistors of equal value in series, as in Fig. 8, the total resistance will be the sum of the three, or three times the resistance of one unit.

Fall of Potential. Suppose that we have three resistors connected in series, as shown in Fig. 9. In practical problems of this kind we assume, for simplicity, that the conductors have no resistance. Assume further that the resistors are of equal value and that the potential difference provided by the battery is exactly 9 volts. A voltmeter connected across points A and F, therefore, would read 9 volts, as would a voltmeter connected from point B to point E, because the conductors are assumed to have no resistance. Just as water moving through a pipe loses pressure the further it goes, electricity loses potential when passing through a resistor. The amount of potential lost (the *fall of potential* or *voltage drop*) depends upon both the resistance and the strength of the current.

Considering Fig. 9, the current is the same in each resistor, because this is a series circuit. All the resistors have the same value, hence voltage drop should be the same in each resistor. The total potential across the set of three is 9 volts, hence the voltage drop from B to C must be 3 volts, from C to D, 3 volts, and from D to E, 3 volts.

Note the signs at the ends of the resistors. Point B is just as negative as the negative end of the battery. Due to the voltage

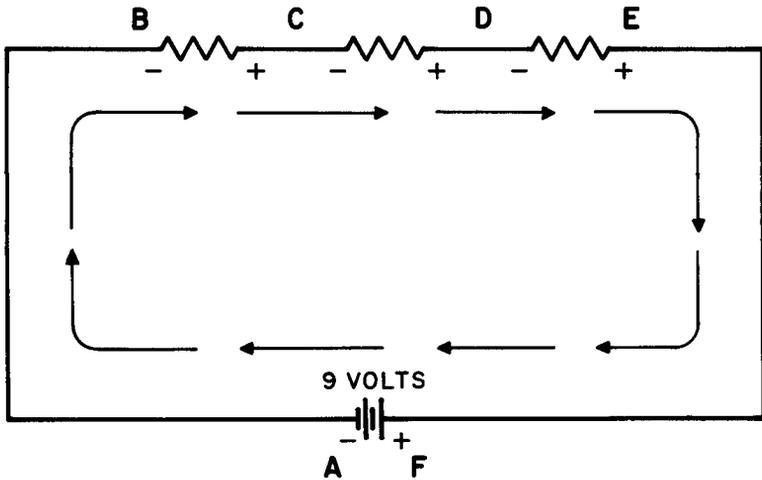


Fig. 9. Illustration of fall of potential.

drop from B to C, point C is a little more positive than point B. But, because there is a fall of potential from C to D, point C is a bit more negative than point D. This accounts for the fact that the same point, point C, can have both a minus and a plus sign. In other words, point C is more positive than point B, but more negative than point D.

We may state three rules that emerge from these considerations:

1. When electrons flow through a resistor, the end into which they flow becomes more negative than the end from which they emerge.
2. In a series circuit, the sum of the voltage drops across the individual resistors is equal to the total voltage applied by the source.

3. The voltage drop across any individual resistor may be found by applying Ohm's Law, Equation 2: $E = IR$, where E is the *voltage drop*, I is the *resistor current*, and R is the *resistance*.

Sample problems will help clarify these points.

Example 14. What is the sum of all the voltage drops around a simple direct-current series circuit, including the source?

Solution. Note the phrase "including the source." Viewed in this manner, the sum of all the voltage drops is zero. Here is the reason: Referring to Fig. 9, from B to C there is a drop of 3 volts, C to D a second drop of 3 volts, and from D to E another drop of 3 volts, making a total drop of 9 volts from B to E. But, going through the battery there is a rise of 9 volts that makes up for the 9-volt drop from B to E. Thus the total drop is zero.

Example 15: Three resistors, 5 ohms, 5 ohms, and 10 ohms, respectively, are connected in series with a 100-volt source. What is the voltage drop across the 10-ohm resistor?

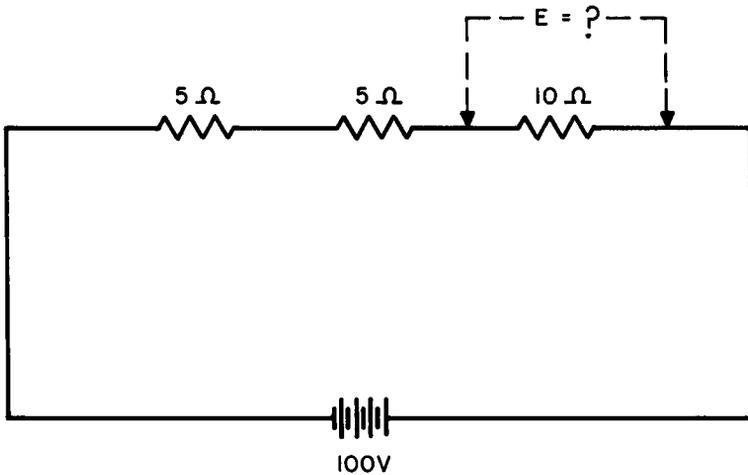


Fig. 10. Circuit for Example 15.

Solution. This problem may be solved in three steps. (Refer to Fig. 10.)

1. The three series resistors have a total resistance of 20 ohms.
2. The current flowing in the circuit is

$$I = E/R = 100/20 = 5 \text{ amperes}$$

3. The voltage drop across the 10-ohm resistor is

$$E = IR = 5 \times 10 = 50 \text{ volts}$$

D-C CIRCUIT ANALYSIS

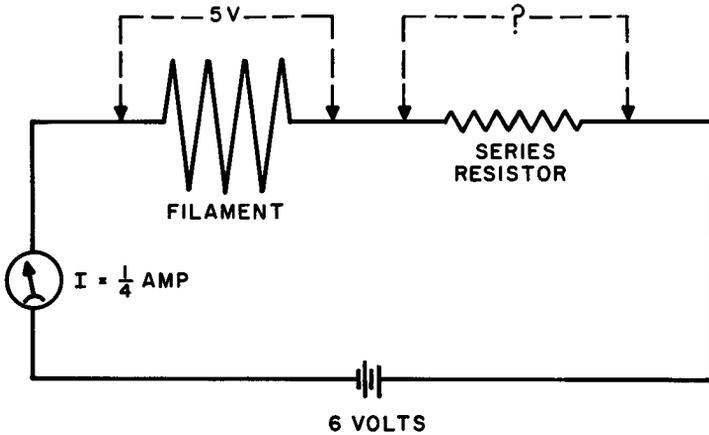


Fig. 11. Circuit for Example 16.

Example 16. If a vacuum tube having a filament rated at $\frac{1}{4}$ ampere and 5 volts is to be operated from a 6-volt battery, what is the value of the necessary series resistor?

Solution. Figure 11 shows the known factors clearly: 5 volts required across the filament, a 6-volt battery, and a current of $\frac{1}{4}$ ampere. What value of the series resistor will drop the voltage 1 volt (from 6 to 5) at a current of $\frac{1}{4}$ ampere? $R = E/I = 1/0.25 = 4$ ohms

Example 17. If two voltmeters are connected in series across a load, how would you be able to determine the total drop across the two instruments?

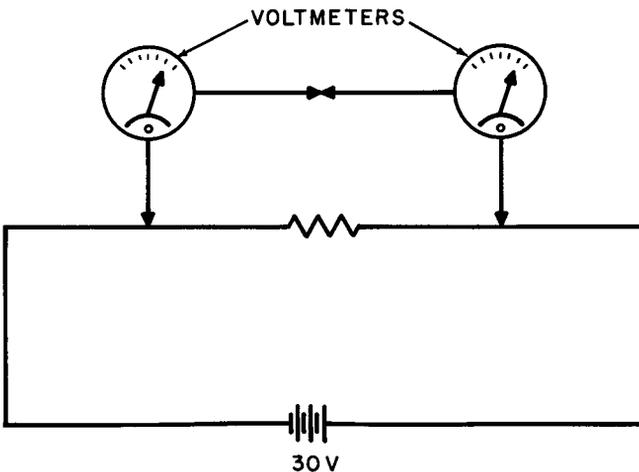


Fig. 12. Circuit for Example 17.

Solution. Figure 12 shows the circuit. For purposes of this problem, voltmeters may be considered to act just like ordinary resistors. The total voltage drop across the two meters in series is, therefore, equal to the sum of the drops across each of them. Because a voltmeter reading gives the drop across the meter, *the sum of the readings gives the total voltage drop.*

Example 18. A circuit is passing a current of 3 amperes. The internal resistance of the source is 2 ohms. The total external resistance is 50 ohms. What is the terminal voltage of the source?

Solution. All sources of current have a certain amount of internal resistance which must be considered as being in series with the external load. In this problem, the total circuit resistance is thus 52 ohms (2 plus 50). However, the terminal voltage of the source is responsible for the current of 3 amperes through the 50-ohm load, and is therefore equal to the voltage that appears across the load. The load is 50 ohms; applying Ohm's Law:

$$E = IR = 3 \times 50 = 150 \text{ volts}$$

Example 19. Two resistors are connected in series. The current through them is 3 amperes. R1 has a value of 50 ohms, R2 has a voltage drop of 50 volts across its terminals. What is the total impressed emf?

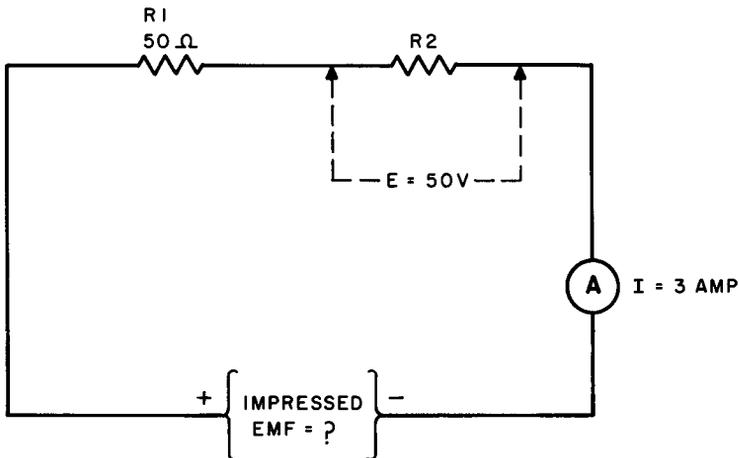


Fig. 13. Circuit for Example 18.

Solution. The known facts are shown in Fig. 13. This is again a problem that must be solved in three steps.

1. The total impressed emf must be equal to the sum of all the voltage drops around the series circuit. One of these drops is already given — 50 volts across R2.

2. Find the voltage drop across R1: $E_1 = IR = 3 \times 50 = 150$ volts

3. $E_{\text{total}} = E_1 + E_2 = 150 + 50 = 200$ volts

Example 20. A relay coil has a resistance of 500 ohms and is designed to operate on 0.125 ampere. If the relay is to operate from a 110-volt source, what value of series resistance is needed?

Solution. Find first the total resistance necessary to hold the current down to 0.125 ampere at 110 volts:

$$R = E/I = 110/0.125 = 880 \text{ ohms}$$

This is the total resistance needed. However, the relay coil has a resistance of 500 ohms, hence the additional resistance required is $880 - 500 = 380 \text{ ohms}$.

16. Parallel Connection of Resistors

A parallel circuit is one in which the current may take two or more different paths from the source through the load(s).

Figure 14 shows a simple parallel circuit in which the current divides at point A to flow in two parts on its way through R_1 and R_2 , rejoins at point B, and then returns to the positive terminal

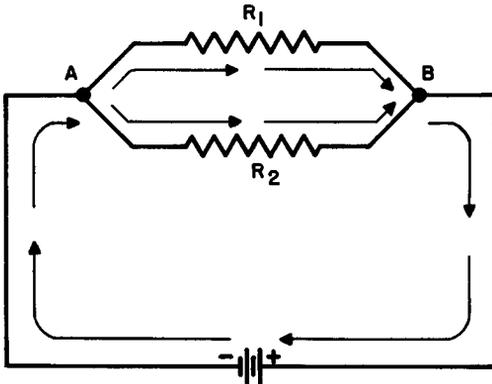


Fig. 14. A simple parallel circuit.

of the battery. The figure indicates another method of defining a parallel circuit as one in which one terminal of each element connects to a common point, forming one terminal of the network, and the other terminal of these elements connects to a second common point that forms the second terminal of the system.

Joint or Total Resistances. With R_1 and R_2 in Fig. 14 providing two paths through which the current may flow, the current finds *less* difficulty in returning to the source than if only one of the resistors were present. This means that a parallel circuit must

have *less resistance* than that presented by either one of its component resistors alone. There are three simple rules for finding the joint or total resistance of a parallel circuit:

1. For *two equal resistors* in parallel, the joint resistance is *half* the resistance of either component. For example, the joint resistance of two 100-ohm resistors in parallel is 50 ohms.

2. For *two unequal resistors* in parallel, the total resistance may be found from the simple equation:

$$R_{\text{total}} = \frac{R_1 \times R_2}{R_1 + R_2} \quad (10)$$

Thus, the joint resistance of a 2-ohm and an 8-ohm resistor in parallel is:

$$R_{\text{total}} = \frac{2 \times 8}{2 + 8} = \frac{16}{10} = 1.6 \text{ ohms}$$

Note again that the joint resistance is smaller than the *smallest* component.

3. If there are *more than two resistors* in parallel, the joint resistance must be found from the equation:

$$1/R_{\text{total}} = 1/R_1 + 1/R_2 + 1/R_3 + \text{etc.} \quad (11)$$

Examples of the use of Equation 11 will be found among the sample problems.

The Voltage Drop Across a Parallel Circuit. Again referring to Fig. 14, it is evident that points A and B connected directly across the battery must have a potential difference between them equal to that of the source. R_1 and R_2 are also connected to the same points, hence the voltage drop across each of them and across the combination must be equal to the emf of the source. This leads to the rule: *The voltage drop across each branch of a parallel circuit is the same.*

Current in the Branches of a Parallel Circuit. The current, which splits into two parts at point A, must add up to the original value at point B. This is true even with 5 or 10 resistors in parallel. Thus the rule: *The total current flowing in and out of the source must be equal to the sum of the currents in the branches.*

Of course, the individual current flowing in any one branch is determined by the voltage across that branch and the resistance of the branch. A simple example will help clarify these points.

Example 21. Two resistors, one of 2 ohms and the other of 8 ohms, are connected in parallel. Find their joint resistance, the voltage drop across each resistor, and the current in each resistor. Also find the total circuit current if the resistors are connected across a 16-volt source.

Solution. The joint resistance may be found by applying Equation 10.

$$R_{\text{total}} = \frac{2 \times 8}{2 + 8} = \frac{16}{10} = 1.6 \text{ ohms}$$

The voltage drop across each resistor is 16 volts by the rule given earlier. The total current is found by applying Ohm's Law to the joint resistance:

$$I_T = E/R = 16/1.6 = 10 \text{ amperes}$$

The current in the 2-ohm resistor:

$$I = E/R = 16/2 = 8 \text{ amperes}$$

Likewise, the current in the 8-ohm resistor:

$$I = E/R = 16/8 = 2 \text{ amperes}$$

Note that the individual currents add up to the total current, which is in accordance with the rule previously given.

Example 22: If resistors of 5, 3, and 15 ohms are connected in parallel, what is the total resistance?

Solution. Use Equation 11.

$$1/R = 1/5 + 1/3 + 1/15$$

Converting each fraction to one having 15 as the denominator:

$$1/R = 3/15 + 5/15 + 1/15$$

Adding the fractions:

$$1/R = 9/15$$

Inverting both sides:

$$R = 15/9 = 1.67 \text{ ohms}$$

Utilizing what has been said of the characteristics of a parallel circuit, another method may be used to calculate the total resistance of a group of resistors connected in parallel. This alternate method begins by assuming a voltage impressed across the terminals of the resistor group. Any value that is convenient for easy calculations will do. In order, the currents in each branch of the resistors are computed. The sums obtained are added to give the total current. Using Ohm's Law,

$$R_{\text{total}} = \frac{E \text{ assumed}}{I_{\text{total}}}$$

This method may be illustrated by using the values of Example 22 and solving by the alternate method.

Alternate Solution. Assume a voltage of 15 volts. (It is emphasized that this election is arbitrary.)

$$I_{R5 \text{ ohms}} = \frac{E \text{ assumed}}{5 \text{ ohms}} = \frac{15}{5} = 3 \text{ amperes}$$

$$I_{R3 \text{ ohms}} = \frac{E \text{ assumed}}{3 \text{ ohms}} = \frac{15}{3} = 5 \text{ amperes}$$

$$I_{R15 \text{ ohms}} = \frac{E \text{ assumed}}{15 \text{ ohms}} = \frac{15}{15} = 1 \text{ ampere}$$

$$I_T = 3 + 5 + 1 = 9 \text{ amperes}$$

$$R_T = \frac{E \text{ assumed}}{I_T} = \frac{15}{9} = 1.67 \text{ ohms}$$

(This method is particularly useful when the resistance values are awkward.)

Example 23. Draw a simple schematic diagram showing the method of connecting three resistors of equal value so that the total resistance will be one-third of one unit.

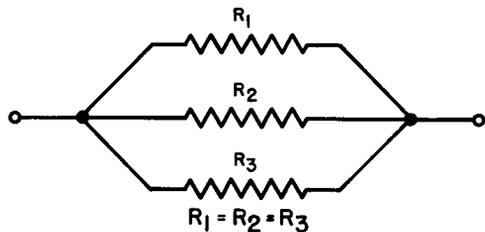


Fig. 15. Circuit for Example 23.

Solution. The joint resistance of *any number of equal* resistors in parallel may be determined by dividing the resistance of one of the components by the number of resistors in the circuit. Thus, for this problem, the correct hookup (Fig. 15) is a simple parallel one.

Example 24. If two voltmeters are connected in parallel, how may the total voltage drop across both instruments be determined?

Solution. Because the drop across parallel branches is equal for each branch, *each* of two parallel voltmeters will read the voltage across *both* instruments. *Both meters should therefore have exactly the same reading* (if they are accurate).

Example 25. If two ammeters are connected in parallel, how may the total current through the two meters be determined?

Solution. The ammeters make up the branches of a parallel circuit, so that the total current must be equal to the sum of the two individual meter readings. *Simply add the readings of the two meters.*

17. Series-Parallel Combinations

Definition. A circuit is said to be *series-parallel*, when it contains at least one series and one parallel circuit inter-connected as part of the same network. Figure 16 shows a typical series-

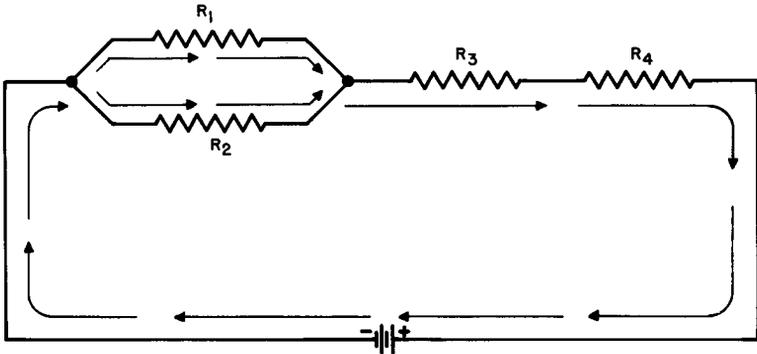


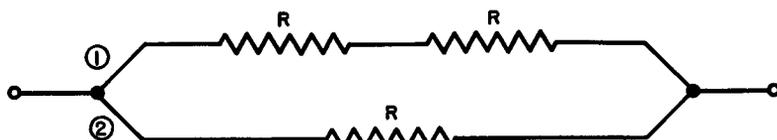
Fig. 16. A series-parallel circuit.

parallel circuit. R_1 and R_2 form the branches of a parallel circuit. R_3 and R_4 are in series with each other and with the parallel combination.

Handling Series-Parallel Problems. Solving problems of this nature requires separate handling of the series portions and the parallel portions, as far as possible. Each problem must be viewed as a whole and then broken down into component parts before attempting the solution. Some examples will help to get you started.

Example 26. Draw a simple schematic diagram showing the method of connecting three resistors of equal value so that the total resistance will be two-thirds of one unit.

Solution. The correct drawing appears in Fig. 17. Before referring to the solution, try to solve this problem by trying various connections and finding the total resistance of each of them. It cannot be a simple series circuit because here the resistance would be three times the value of one unit; it cannot be a simple parallel connection because this would have a total resistance of one-third the value of one unit. Hence, it must be a series-parallel combination because the required resistance falls between the largest value (series) and the smallest value (parallel) of total resistance.



BRANCH 1: $R + R = 2R$

BRANCH 1 & 2: $R_{TOTAL} = \frac{2R \times R}{2R + R} = \frac{2}{3} R$

Fig. 17. Circuit for Example 26.

Example 27. Draw a simple schematic diagram showing the method of connecting three resistors of equal value so that the total resistance will be $1\frac{1}{2}$ times the resistance of one unit.

Solution. See Fig. 18.

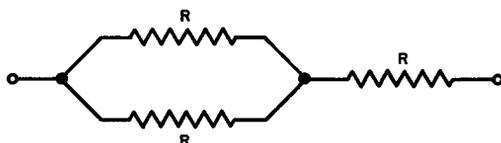


Fig. 18. Circuit for Example 27.

$$R_{TOTAL} = \frac{R}{2} + R = \frac{3R}{2} = 1\frac{1}{2}R$$

Example 28. Two resistors of 18 and 15 ohms are connected in parallel; a 36-ohm resistor is connected in series with this combination and a 22-ohm resistor is connected in parallel with this entire combination. If the total current flowing through the combination is 5 amperes, what is the current flow in the 15-ohm resistor?

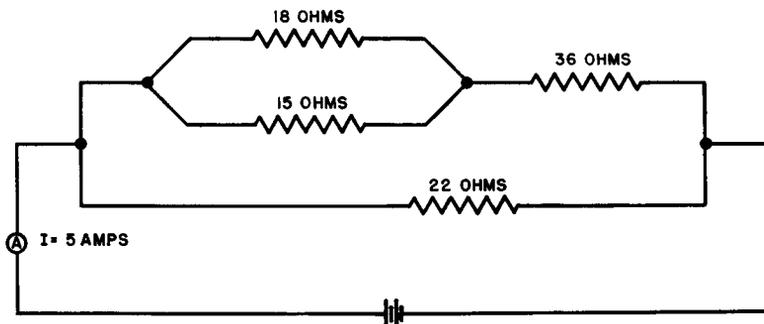


Fig. 19. Circuit for Example 28.

Solution. In slightly more complex problems of this sort, it is almost mandatory that the circuit diagram (Fig. 19) be drawn and studied before a solution is attempted.

1. Find the joint resistance of the entire upper branch:

$$R = \frac{R_1 \times R_2}{R_1 + R_2} = \frac{15 \times 18}{15 + 18} = 8.18 \text{ ohms}$$

To this add 36 ohms, obtaining 44.18 ohms.

2. Find the total resistance of the two big branches considered together.

$$R = \frac{44.18 \times 22}{44.18 + 22} = 14.69 \text{ ohms}$$

3. Knowing that the total current is 5 amperes, and the total resistance 14.69 ohms, we can find the impressed voltage across the entire combination.

$$E = IR = 5 \times 14.69 = 73.4 \text{ volts}$$

4. Now that the total voltage is known, we can find the current flowing in the upper branch only:

$$I = E/R = 73.4/44.18 = 1.66 \text{ amperes}$$

5. This current is divided between the 18-ohm and the 15-ohm resistance. To find the current in the 15-ohm resistance only, first find the voltage drop from A to B in Fig. 19:

$$E_{AB} = IR = 1.66 \times 8.18 = 13.58 \text{ volts}$$

6. Finally, the current in the 15-ohm resistor

$$I = E/R = 13.58/15 = 0.905 \text{ ampere}$$

18. Rheostats and Voltage Dividers

The Difference Between Rheostats and Voltage Dividers. A *rheostat* is a simple variable series resistor. It is symbolized by the characteristic zigzag line with an arrow through it. A *voltage divider* may be either a variable resistor or a group of fixed resistors connected *across* a source of potential and tapped to provide lower voltages than the source. Figure 20 clarifies this distinction.

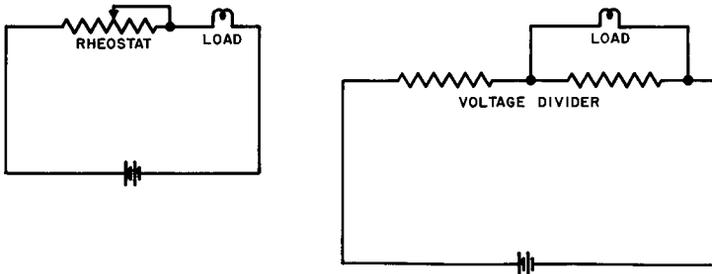


Fig. 20. Rheostat and voltage divider.

The Operation of the Voltage Divider. A rheostat, because it is a simple series resistor, is handled in the same manner as a series resistor in problems and computations. A voltage divider, however, must be treated in a somewhat different manner.

Example 29. Suppose that a voltage divider is set up as shown in Fig. 21. It consists of three fixed resistors connected across a 90-volt source. The load to be operated may be connected across A and D, A and B, B and C, and C and D. Assuming that the load draws a negligible current, what voltages are available?

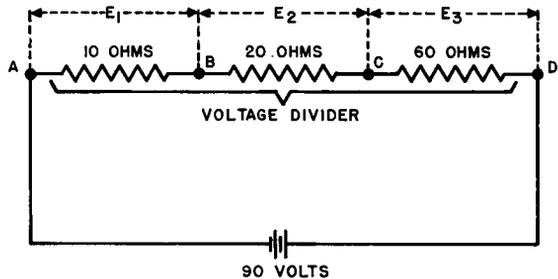


Fig. 21. Circuit for Example 29.

Solution. Consider first the voltage between A and B, E_1 . The resistance between these points is 10 ohms, which is $1/9$ of the total resistance. Hence, the voltage drop from A to B is $1/9$ of the impressed voltage, or 10 volts. Using the same reasoning, E_2 must be 20 volts and E_3 , 60 volts. Thus, a single voltage source may be used to provide a choice of four different voltages: 90 volts, 60 volts, 20 volts, and 10 volts.

Uses of Voltage Dividers. Voltage dividers are found in many circuits in radio and television receivers and transmitters. Some of the more important applications are volume and tone controls, power supply networks, and electrode supply networks for vacuum tubes.

19. Meters for Measuring Voltage, Current, and Resistance

General Construction. Meters used for measurement of voltage, current, and resistance almost always utilize magnetic principles in their operation. The D'Arsonval type of movement consists of a moving coil wound with fine wire suspended between a pair of permanent magnets. The indicating needle is secured to the coil, so that it swings across the dial face when current flowing through the coil causes it to rotate on its pivots. The movement is designed to make the number of degrees of rotation proportional

TABLE 3

<i>Current</i> Basic unit is the <i>ampere</i> (amp or A)		
Milliampere (ma)1/1000 ampere	(10 ⁻³ ampere)
Microampere (μa)1/1,000,000 ampere	(10 ⁻⁶ ampere)
<i>Voltage</i> Basic unit is the <i>volt</i> (v)		
Millivolt (mv)1/1000 volt	(10 ⁻³ v)
Microvolt (μv)1/1,000,000 volt	(10 ⁻⁶ v)
Kilovolt (kv)1000 volts	(10 ³ v)
Megavolt (Mv)1,000,000 volts	(10 ⁶ v)
<i>Resistance</i> Basic unit is the <i>ohm</i> (Ω)		
Kilohm (KΩ)1000 ohms	(10 ³ Ω)
Megohm (MΩ)1,000,000 ohms	(10 ⁶ Ω)
<i>General Prefixes</i>		
Micro-micro (μμ)One trillionth	(10 ⁻¹²)
Micro (μ)One millionth	(10 ⁻⁶)
Milli (m)One thousandth	(10 ⁻³)
Kilo (k)One thousand	(10 ³)
Mega (M)One million	(10 ⁶)

to the strength of the current in the coil. The sensitivity of this movement can be very high; a common type of meter found in radio repair shops will give a full-scale deflection for a coil current of only 0.00005 ampere. It might be added at this point that the normal D'Arsonval coil will burn out quickly if the current is higher than that for which it is designed. For this reason, a great deal of care must be exercised in using these instruments.

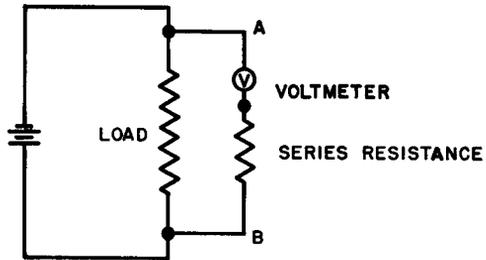
Units and Prefixes. A thorough understanding of the operation and connections of meters must be preceded by definitions of the common units of measure and their subdivisions. These are given in Table 3.

Voltmeters. A voltmeter is a *galvanometer* that has an internal series resistance, usually high in value. (A galvanometer is the general name for any instrument that detects the presence of electric current.) Generally, a D'Arsonval movement is used in series with a resistor (called a *multiplier*) that limits the flow of current through the coil to a safe value. A voltmeter is always connected *in parallel* with the points across which the potential is to be measured. Because of its series resistor, a voltmeter connected in

series with the load would cut the current in the load circuit down to a very small value, seriously affecting operation and giving a meaningless reading. Figure 22 shows the proper connection of a voltmeter for measuring the voltage drop across a load.

Remember that the D'Arsonval coil swings through an arc that is proportional to the *current* through the coil. In a voltmeter the current through the coil is proportional to the voltage applied be-

Fig. 22. Connection of a voltmeter.



tween points A and B in Fig. 22, and hence the deflection of the coil is proportional to the voltage.

A simple problem will show how to compute the value of a multiplier for use in converting a D'Arsonval galvanometer to a voltmeter.

Example 30. Suppose that a meter movement is available that gives a full-scale deflection when one milliampere flows through the coil. Suppose further that it is desired to convert this movement to a voltmeter that will have a full-scale deflection when 1000 volts is applied across it. What must be the value of the multiplier?

Solution. We know that the full-scale coil current is $I = 0.001$ ampere (converting from ma). Assume that the coil has zero resistance (this of course is not strictly true, however the resistance will be so small as to be insignificant). By Ohm's Law the resistance required in series with the movement to permit 0.001 ampere to flow with 1000 volts applied is

$$R = E/I = 1000/0.001 = 1,000,000 \text{ ohms}$$

Because the meter is linear, 500 volts will give half-scale deflection, 250 volts will give quarter-scale, 125 volts will give eighth-scale, and so on.

The dial may thus be calibrated in evenly spaced divisions. A 1-ma full-scale deflection meter when used as a voltmeter is said to have a sensitivity of 1000 ohms per volt, because 1000 ohms of series resistance is required for each volt that is to be measured.

Additional Voltmeter Problems. Let us look into some further examples of multiplier problems.

Example 31. If a 0-1 d-c milliammeter is to be converted into a voltmeter with a full-scale deflection of 100 volts, what value of multiplier resistor should be connected in series with the milliammeter? (Assume that the coil resistance is negligible.)

Solution. A 0-1 d-c milliammeter is a movement that gives full-scale deflection when 0.001 ampere flows through the coil. The series resistor or multiplier may be found from

$$R = E/I = 100/0.001 = 100,000 \text{ ohms}$$

Example 32. What is the sensitivity, in ohms per volt, of a meter movement giving full-scale deflection for a coil current of 50 microamperes?

Solution. The coil current on full-scale is 0.00005 ampere. Find the resistance necessary to limit the current to this value with 1 volt applied.

$$R = E/I = 1/0.00005 = 20,000 \text{ ohms}$$

Because a multiplier of 20,000 ohms must be used for each volt of applied potential, the meter is said to have a sensitivity of *20,000 ohms per volt*.

Ammeters and Milliammeters. These instruments measure current strength (I). The requirement is that they "count" the number of coulombs per second passing through the circuit. To

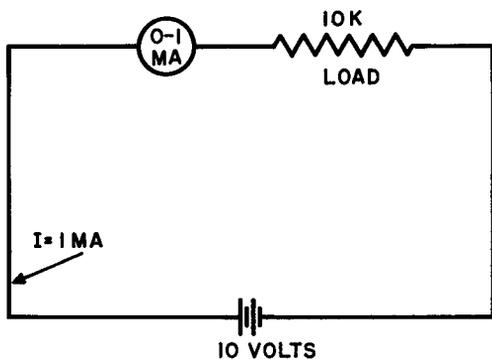


Fig. 23. Connection of an unmodified 0-1 milliammeter.

do this, they must be connected in *series* with the load, thus making it impossible to use a series resistor, which would impair the operation of the circuit. (Anything inserted in the line must have a very low resistance, if it is not to affect the circuit operation.)

In a circuit in which the current never gets larger than 1 ma, one could use a galvanometer that gives full-scale deflection for a

current of 1 ma (that is, a 0.1 milliammeter). Figure 23 shows a 0.1 milliammeter connected in a circuit in which the current is just 1 ma. However, if the same instrument were to be used in a circuit in which the current is normally about 100 ma, a modification would be required. As we have pointed out, a series multiplier cannot be used. To use a 0.1 milliammeter in a 100-ma circuit, a low *parallel* resistance called a shunt is connected to the meter.

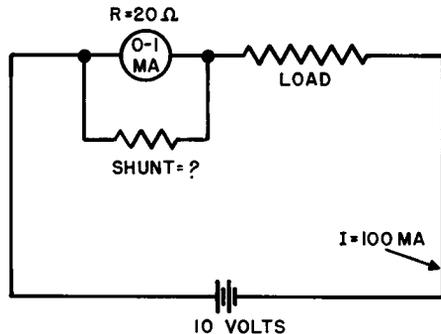


Fig. 24. The use of a shunt in extending the range of a milliammeter.

The shunt provides a parallel path of low resistance for the circuit current and does not alter the circuit constants at all. Figure 24 shows a 0.1 milliammeter with its shunt connected in a circuit carrying 100 ma. It is important to note here that the *coil resistance* now becomes a significant factor and must be used in the calculations that determine the size of the required shunt.

Example 33. Assume that the 0.1 milliammeter shown in Fig. 24 has a coil resistance of 20 ohms, and that it is to be used in a circuit where the maximum current is 100 ma. What should be the resistance of the shunt?

Solution. We can view the problem this way: The resistance of the shunt must permit it to carry 99 of the 100 milliamperes flowing the circuit, leaving only 1 milliamperes to flow through the coil, since this is its maximum rating. This means that the resistance of the shunt must be much *lower* than that of the coil; in fact, the shunt resistance can be only 1/99 of the coil resistance, or $R_s = 1/99 \times 20 = 0.202 \text{ ohm}$.

A convenient equation for solving all types of shunt problems is:

$$\frac{I_s}{I_c} = \frac{R_c}{R_s} \quad (12)$$

where I_s is the shunt current, I_c the coil current, R_c the coil resistance, and R_s the shunt resistance.

Alternate Solution. Solving the same problem by means of Equation 12 yields:

$$\frac{99}{1} = \frac{20}{R_s}$$

cross-multiplying

$$99R_s = 20$$

$$R_s = \frac{20}{99} = 0.202 \text{ ohm}$$

Ammeter problems are quite easily handled after correct analysis. The following example illustrates the procedure.

Example 34. A milliammeter with a full-scale deflection of 1 ma and a coil resistance of 25 ohms was used to measure an unknown current by shunting the meter with a 4-ohm resistor. It then read 0.4 ma. What was the unknown current value?

Solution. Find first what the current in the shunt would be if full-scale meter current was flowing (i.e., 1 ma in the meter coil). Use Equation 12.

$$\frac{I_s}{1} = \frac{25}{4} \quad I_s = \frac{25}{4} = 6.25 \text{ ma}$$

But the current in the coil is only 0.4 ma; hence, the current in the shunt really is:

$$I_s = 0.4 \times 6.25 = 2.5 \text{ ma}$$

Hence, the total current in the circuit is the sum of the coil and shunt currents or:

$$I = 2.5 + 0.4 = 2.9 \text{ ma}$$

Example 35. What should be the resistance of the shunt that will permit a 0-100 milliammeter to be used in a circuit where the maximum current is expected to reach 1 ampere? The coil resistance is 80 ohms.

Solution. Use Equation 12. 100 ma = 0.1 ampere. Thus, the meter coil may carry 0.1 ampere and the shunt will have to carry 0.9 ampere for a total of 1 ampere. Hence,

$$\frac{0.9}{0.1} = \frac{80}{R_s} \quad R_s = 80 \times \frac{0.1}{0.9} = 8.9 \text{ ohms}$$

Example 36. If two ammeters, connected in series in the same circuit read 5 amperes each, what is the total current flowing in the circuit?

Solution. Since the current in a series circuit is everywhere the same, the total current is the reading of *either* ammeter, or 5 amperes.

20. Power and Power Dissipation

The Heating Effect of Electricity. In passing through the resistance of a conductor, the electric current generates heat. The

actual amount of heat generated each second is proportional to the resistance in ohms and to the *square* of the current in amperes. In its simplest form, the equation for the rate of heating produced is

$$H = I^2R \quad (13)$$

In this form, the heat units are expressed in *joules per second* (watts). If it is desired to express the heat in *calories per second*, the equation becomes

$$H = 0.239 I^2R \text{ calories/second} \quad (14)$$

The rate at which heat is produced, *measured in terms of watts*, is of great interest in electrical work. The watt is the unit of *power* used in electricity and is the power dissipated in a resistive circuit carrying 1 ampere under a potential difference of 1 volt.

Electrical Power. According to the statement given above, the power (in watts) dissipated in any circuit is the product of the voltage drop (in volts) and the current (in amperes) or:

$$P = EI \quad (15)$$

If, for example, the voltage drop across a resistor is 10 volts and the resistor is carrying 0.5 ampere, the power dissipation in the resistor is *5 watts*.

Ohm's Law may be applied to Equation 15, and two other forms derived from it. Both these forms are valuable and should become part of the radioman's stock-in-trade.

$$P = EI$$

but since $I = E/R$, $P = E \times E/R$,

$$P = E^2/R \quad (16)$$

and, since $E = IR$, $P = IR \times I$,

$$P = I^2R \quad (17)$$

For example, the power dissipation (in watts) of a resistor of 20 ohms having a current of 0.25 ampere passing through it may be found from Equation 17.

$$P = I^2R = 1/16 \times 20 = 1.25 \text{ watts}$$

Power and Energy. The word "energy" as used scientifically is similar in meaning to the word "work." Energy is sometimes defined as the capacity to do work. Actually, the work done by

any ideal machine or device is the same as the energy used in the process and is measured in the same physical units. *Power*, on the other hand, is the *rate of doing work* or the rate at which energy is consumed. Thus, the power concept involves *time*, whereas energy does not.

Electrical power has the same basic meaning as mechanical power, and may also be measured in *horsepower*. One may be converted to the other by the relationship:

$$1 \text{ horsepower} = 746 \text{ watts}$$

Thus, a "quarter-horse" ($\frac{1}{4}$ -horsepower) electric motor uses energy at the rate of about 186 watts.

Measurement of D-C Power. The power dissipated in a resistor carrying dc is easily measured by means of a correctly connected ammeter and voltmeter as shown in Fig. 25.

The power in watts is found by multiplying the voltmeter reading by the ammeter reading. A single instrument that measures power directly in watts is called a *wattmeter*.

Power Dissipation in Resistors. An important application of power calculations is the determination of resistor sizes for specific circuits. These determinations are easily made by applying Equation 15, 16, or 17, as is evident in the solutions of the following problems.

Example 37. What should be the minimum power dissipation rating of a resistor of 20,000 ohms to be connected across a potential of 500 volts?

Solution. Apply Equation 16.

$$P = E^2R = 250,000/20,000 = 12.5 \text{ watts}$$

This is the actual power dissipation of the resistor. In practice it is necessary to have a safety factor of possibly 100% to avoid overheating. Doubling 12.5 watts gives this safety factor of 100%, therefore, a recommended power rating would be 25 watts.

Example 38. What is the maximum rated current-carrying capacity of a resistor marked "5,000 ohms, 200 watts"?

Solution. Applying Equation 17,

$$P = I^2R,$$

$$I = \sqrt{\frac{P}{R}}$$

$$I = \sqrt{\frac{200}{5000}} = 0.2 \text{ ampere}$$

Example 39. If the value of a resistance to which a constant emf is applied is halved, what will be the resultant proportional power dissipation?

Solution. Applying Equation 16,

$$P_1 = E^2/R$$

$$P_2 = E^2/(R/2)$$

$$= 2E^2/R$$

$$P_2 = 2P_1$$

Thus, if the resistance is halved with constant emf, the power dissipation will be *doubled*.

Power Dissipation in Parallel and Series Resistances. As a general rule, it is safe to assume that the power dissipation capability of several resistors, used either in series or parallel, is the

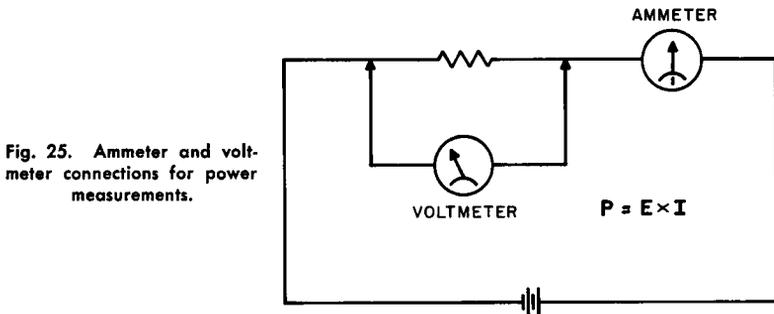


Fig. 25. Ammeter and voltmeter connections for power measurements.

sum of the ratings of the component resistors, provided they all have the same rating. For instance, two resistors rated 10 watts each, are capable of dissipating 20 watts when used together in series or parallel. Series or parallel connections are a common way to obtain higher power ratings, when highly-rated resistors are not available.

When the resistor ratings are different, it is necessary to analyze the overall picture to arrive at a definite conclusion. The example given below illustrates the general method of handling this type of problem:

Example 40. A 10,000-ohm, 100-watt resistor, a 40,000-ohm, 50-watt resistor, and a 5,000-ohm, 10-watt resistor, are connected in parallel. What is the maximum value of the total current through this combination that will not exceed the power rating of any of the resistors?

Solution. From Equation 16 we obtain:

$$E = \sqrt{PR}$$

This relation gives the maximum voltage that can be applied across a given resistor of a given maximum power rating. Because the voltage drop across any of the parallel resistances is the same as that across the combination, this will give us a value for the maximum voltage that may be applied.

Substituting the values given in the problem, we have

$$1. E = \sqrt{100 \times 10,000}$$

$$2. E = \sqrt{50 \times 40,000}$$

$$3. E = \sqrt{10 \times 5,000}$$

It is evident by inspection that the third figure gives the maximum voltage that can be applied across the parallel combination, because the resistor of lowest power rating determines the maximum safe applied voltage. Then

$$E = \sqrt{50,000} = 223.6 \text{ volts}$$

The currents in the individual resistors are, then:

$$1. 223.6/10,000 \text{ amps,} = 0.02236 \text{ amp} = 22.36 \text{ ma}$$

$$2. 223.6/40,000 \text{ amps} = 0.00559 \text{ amp} = 5.59 \text{ ma}$$

$$3. 223.6/5,000 \text{ amps} = 0.04472 \text{ amp} = 44.72 \text{ ma}$$

Adding these together to get the total current, we obtain *72.67 ma*.

Additional Computations Involving Power Dissipation. The computations utilizing the power formulae presented in the preceding pages are useful in many practical situations in communications. Radio technicians and engineers are often concerned, for example, with the power radiated by an antenna system. The Federal Communications Commission has specified requirements with respect to operation of various classes of transmitters. In some situations, a commercial station that operates at one power level during the day must reduce the level during nighttime operation. The steps that we have covered are useful in these computations as shown in the following examples.

Example 41. The daytime transmission-line current of a 10-kilowatt transmitter is 10 amperes. The transmitter is required to reduce to 2.5-kilowatt operation during evening hours. What is the value of transmission-line current during the night?

Solution. $P_{\text{day}} = I_{\text{day}}^2 R$, $P_{\text{night}} = I_{\text{night}}^2 R$

$$P_{\text{day}} = 10 \text{ kw, } P_{\text{night}} = 2.5 \text{ kw, } I_{\text{day}} = 10 \text{ amperes}$$

$$\frac{P_{\text{day}}}{P_{\text{night}}} = \frac{10}{2.5} = \frac{4}{1} = \frac{I_{\text{day}}^2 R}{I_{\text{night}}^2 R} = \frac{100R}{I_{\text{night}}^2 R}$$

$$4I_{\text{night}}^2 = 100$$

$$I_{\text{night}}^2 = 25$$

$$I_{\text{night}} = 5 \text{ amperes}$$

Example 42. If the daytime input power to a certain broadcast station antenna having a resistance of 15 ohms is 1500 watts, what would be the nighttime input power if the antenna current were cut in half?

Solution. The power formula may be used to determine the current at the 1500-watt level. Then, one-half of this current value will be used to determine the new power. Thus,

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{1500}{15}} = \sqrt{100} = 10 \text{ amperes}$$

The new power is calculated at one-half this current value, or 5 amperes.

$$P = I^2R = 5^2 \times 15 = 25 \times 15 = 375 \text{ watts}$$

Example 43. What is the antenna current when a transmitter is delivering 1 kilowatt into an antenna having a resistance of 20 ohms?

Solution. $I = \sqrt{\frac{P}{R}} = \sqrt{\frac{1000}{20}} = \sqrt{50} = 7.07 \text{ amperes}$

Example 44. The d-c input to the final stage of a transmitter is exactly 800 ma at 2500 volts. The antenna resistance is 16.4 ohms, and the antenna current is 9 amperes. If the plate efficiency equals output power divided by input power, compute the plate efficiency of the final amplifier.

Solution. The input power is $P_{in} = EI = 2500 \times 0.8 = 2000$ watts
The output power is $P_{out} = I^2R = 9^2 \times 16.4 = 1328.4$ watts

$$\text{Plate efficiency} = \frac{P_{out}}{P_{in}} = \frac{1328.4}{2000} = 0.6642 \text{ or } 66.42\%$$

Example 45. The ammeter at the base of a Marconi antenna has a certain reading. If this reading is increased 2.5 times, what is the increase in output power?

Solution. Since $P = I^2R$, the power varies as the square of the current. Thus, an increase in current of 2.5 times results in a power increase to 2.5^2 or 6.25 times the original power.

Example 46. A long transmission line delivers 10 kilowatts into an antenna. At the transmitter end, the line current is 10 amperes. At the coupling house, the current is 9.8 amperes. Assume the losses in the coupling system to be negligible and the line properly to be terminated, so that the line resistance equals the antenna resistance. Compute the power lost in the line.

Solution. Applying the suitable power formula:

$$R = \frac{P}{I^2} = \frac{10,000}{(9.8)^2} = 104.12 \text{ ohms line resistance}$$

The power at the input to the line is:

$$P_{in} = I^2R = 10^2 \times 104.12 = 10,412 \text{ watts}$$

The power lost in the line is equal to the power in minus power out.

$$P_{in} - P_{out} = 10,412 - 10,000 = 412 \text{ watts}$$

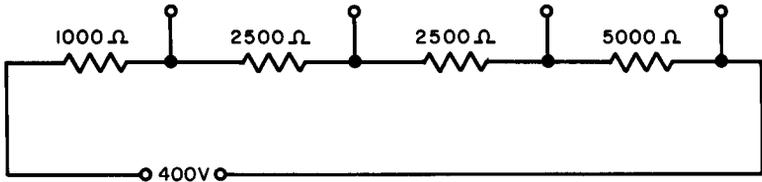
Example 47. The power input to a 72-ohm concentric line is 1 kilowatt. What is the current flowing in the line?

Solution.
$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{1000}{72}} = \sqrt{13.88} = 3.73 \text{ amperes}$$

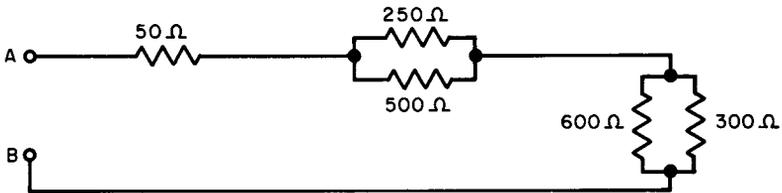
21. Review Questions

- (1) What is meant by the terms *series circuit*, *parallel circuit*, and *combination circuit*?
- (2) Three resistors of 10, 5, and 7.5 ohms, respectively, are connected in series. What is the total resistance of the combination?
- (3) The three resistors of Question 2 are connected in series across a 45-volt battery. What is the voltage drop across the 5-ohm resistor?
- (4) A vacuum tube has a filament rated at 0.15 ampere and 3 volts is to be operated from a 6-volt battery. What is the value of series resistor that must be added to ensure proper operation?
- (5) Give three variations of the equation expressing power dissipation in a d-c circuit in terms of E, I, and/or R.
- (6) What should be the minimum power dissipation rating of a resistor of 10,000 ohms to be connected across a potential of 250 volts?
- (7) What is the maximum rated current-carrying capacity of a resistor marked 2000 ohms, 10 watts?
- (8) A 50-ohm resistor is connected in series with a parallel combination made of a 25-ohm resistor and a 10-ohm resistor. The entire combination is connected across a 45-volt battery. What is the power dissipation in the 10-ohm resistor?
- (9) A milliammeter with a full-scale deflection of 1 ma and a resistance of 15 ohms was used to measure an unknown current by shunting the meter with a 3-ohm resistor. The meter then read 0.8 ma. What is the unknown current value?
- (10) Give the equation for solution of shunt problems involving meters.
- (11) A resistor of 250,000 ohms is connected in parallel with one of 500,000 ohms. Compute the resistance of the combination.
- (12) What is the power rating of the following resistors: (a) a 1/2-megohm resistor that can carry 5 ma; (b) a 500-ohm resistor that can carry 0.2 ampere; (c) a 25,000-ohm resistor that can carry 10 ma.
- (13) Calculate the current rating for each of the resistors given: (a) 1/4-watt, 1-megohm resistor; (b) a 10-watt, 1000-ohm resistor; (c) a 1/2-watt, 250,000-ohm resistor.
- (14) Find the equivalent resistance of three resistors of 10, 44, and 28 ohms, respectively, connected in parallel. Assume that 12 volts is applied to the combination, and determine the current through each resistor, the power dissipated in each resistor, and the total current in the circuit.
- (15) 250 ma flow in a given circuit with an applied potential of 50 volts. What is the resistance of the circuit?

- (16) Determine the voltages between each tap and the negative terminal in the following circuit.



- (17) Three resistors of 60,000 ohms, 15,000 ohms, and 8,000 ohms, respectively, are available. A value of 20,000 ohms is desired. Diagram the method of connection.
- (18) A device has a coil wound with wire that will be destroyed if more than 14 amperes are passed through it. The wire has a resistance of 0.025 ohm. What is the highest potential that can be placed across the coil without destroying it?
- (19) A microammeter reads 15 microamperes full scale and has an internal resistance of 140 ohms. Calculate (a) the voltage required for full-scale deflection and (b) the value of shunt resistance needed to make it possible to measure 150 ma?



- (20) Compute the effective resistance between A and B in the diagram. If a potential of 120 volts were applied between A and B, what current would flow?

Chapter 4

ANALYSIS OF COMBINATION D-C CIRCUITS

22. Circuit Characteristics

In our previous discussions, we have touched upon three basic methods of connecting a group of electrical components. These were, respectively, series circuits (consisting of two or more resistors connected end to end so that current must pass through one component to reach the beginning of the next), parallel circuits (consisting of two or more resistors connected side by side so that the current has several paths at one terminal and recombines at the opposite terminal), and combination circuits (consisting of either series-parallel circuits or parallel-series groupings, or both). Because complex communications circuits are often made up of simple combinations, it is important to specify and summarize the characteristics of each of these basic types. Ohm's Law applies to complicated circuits as well as to simple ones, and to each part of a circuit as well as to the whole. The simple circuit problems that we have presented thus far, permit us to make the generalizations given in Table 4.

Table 4 permits us to set up the procedures necessary to solve the more complicated combination circuits. The steps that follow should generally be performed in the order indicated.

(1) Analyze the basic circuit diagram and determine the parallel groups (including parallel groups that contain series elements). If necessary, redraw the circuit to indicate these groups

TABLE 4
SUMMARY OF CHARACTERISTICS OF D-C CIRCUITS

<i>Factor</i>	<i>Series Circuit</i>	<i>Parallel Circuit</i>	<i>Combination Circuits</i>
1. Applied Voltage (Unit is the <i>volt.</i>)	The voltage drop across a series combination equals the <i>sum</i> of the separate voltage drops.	The voltage drop across a parallel branch combination is the <i>same</i> across each branch.	Rules applying to series circuits apply to the series portions and rules applying to parallel circuits apply to the parallel portions.
2. Resistance (Unit is the <i>ohm.</i>)	The resistance of a series circuit is equal to the <i>sum</i> of the individual resistances comprising the circuit. $R = R_1 + R_2 + R_3 \dots$	The resistance of a parallel combination equals the reciprocal of the sum of the reciprocals of the resistances making up the parallel branches. The total resistance is less than the resistance of the smallest branch. $R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$	
3. Conductance (Unit is the <i>mho.</i>)	Not normally used for series circuit calculations.	$G = G_1 + G_2 + G_3 + \dots$ may be used in lieu of the resistance formula.	
4. Current (Unit is the <i>ampere.</i>)	Current in the series circuit is the <i>same</i> in all parts of the circuit.	The current in a parallel circuit equals the <i>sum</i> of the currents in each branch of the parallel circuit.	
5. Power (Unit is the <i>watt.</i>)	Total expended power is equal to the <i>sum</i> of the power dissipated by the individual resistors. $P = P_1 + P_2 + P_3$	Same as series circuit; expended power is equal to the <i>sum</i> of the power dissipated by the individual resistors. $P = P_1 + P_2 + P_3$	

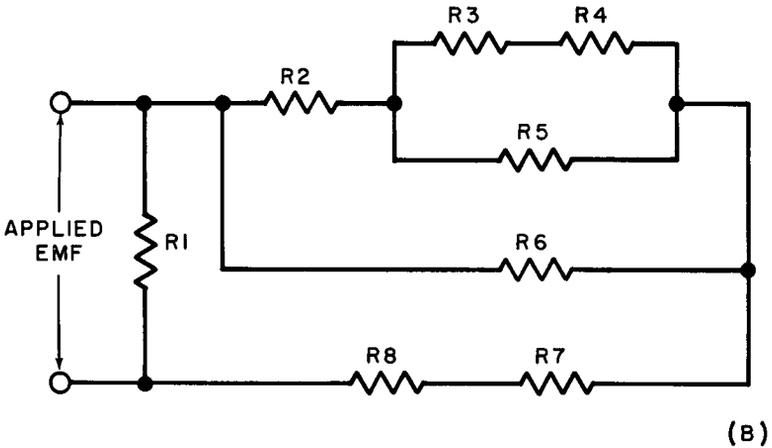
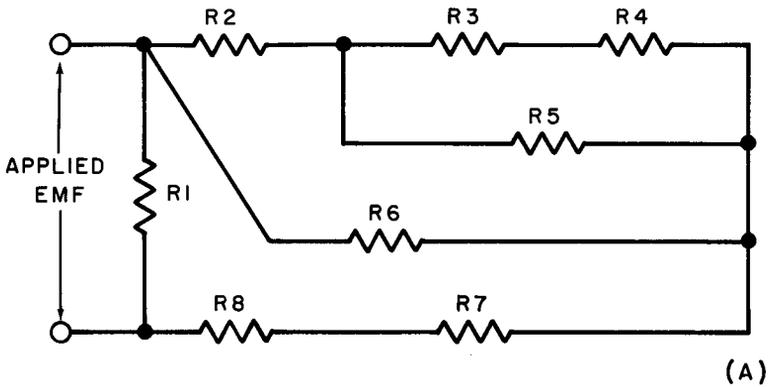


Fig. 26. Regrouping of complex combination circuit with single d-c source, for solution of the problem.

clearly. (Figure 26 shows how one such section might be redrawn to reduce an apparently complex combination to conventional form.)

(2) Calculate the resistance values for each group to obtain a single equivalent value of resistance. The reduction of a group of resistors might involve several steps. Figure 26A shows the group as a technician working on a communications device might sketch the circuit. Figure 26A shows how the circuit may be redrawn to reflect the arrangement of the resistors more clearly.

(3) Once the combination circuit is clearly delineated, the circuit should be reduced step by step in logical order. Let us

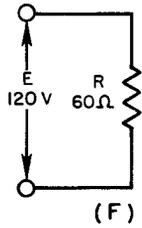
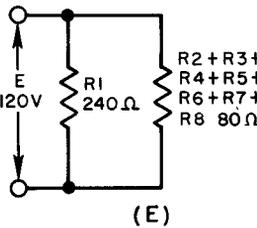
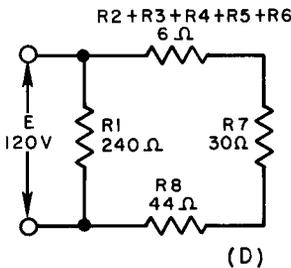
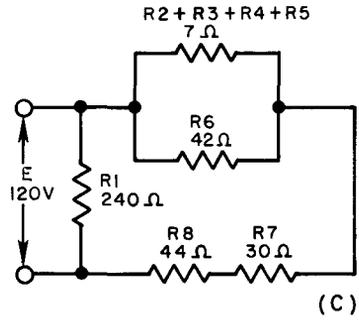
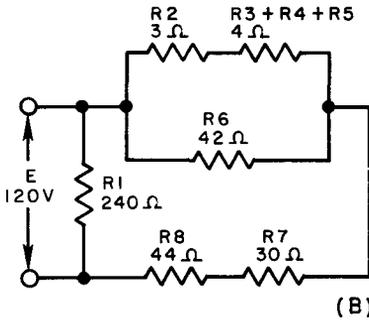
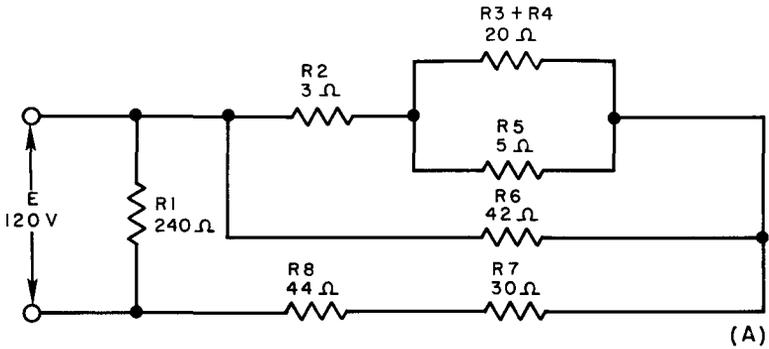


Fig. 27. Reduction of complex combination circuit Fig. 26 for solution of the problem.

assume that the applied voltage and the values of each resistor in Fig. 26 are known, and that the total line current, the voltages across each resistor, and the current through each resistor are to be determined. The required reduction steps are shown in Fig. 27.

(4) After the appropriate equivalent values are determined, Ohm's Law may be applied to complete the analysis. The separa-

tion of the combination circuits into small parts allows easy computation of the current, voltage, and resistance distribution. It is important to remember that *Ohm's Law may be applied to any part of a d-c circuit using the values of the voltage across that part of the circuit, its resistance, and the current through it.*

Example 48. The procedures are illustrated more fully by assigning values to the problems of Fig. 26 and checking the actual calculations. Assume $R_1 = 240$ ohms, $R_2 = 3$ ohms, $R_3 = 15$ ohms, $R_4 = 5$ ohms, $R_5 = 5$ ohms, $R_6 = 42$ ohms, $R_7 = 30$ ohms, $R_8 = 44$ ohms; applied voltage = 120 volts. (Simple values have been assigned to allow the steps to be followed easily.) Find the total current drawn by the network, the current through R_2 and the voltage drop across R_2 .

Solution. Following the steps of Fig. 27:

Step A: The equivalent resistance of R_3 and R_4 is $15 + 5 = 20$ ohms.

Step B: This resistance is in parallel with R_5 , so the equivalent resistance of R_3 , R_4 , and R_5 is

$$\frac{20 \times 5}{20 + 5} = \frac{100}{25} = 4 \text{ ohms}$$

Step C: Combining the above with R_2 : $4 + 3 = 7$ ohms

Step D: Since the equivalent resistance of R_2 , R_3 , R_4 , and R_5 is in parallel with R_6 , the equivalent resistance of the combination is:

$$\frac{7 \times 42}{7 + 42} = \frac{294}{49} = 6 \text{ ohms}$$

Step E: Combining the equivalent resistance R_2 , R_3 , R_4 , R_5 , and R_6 with R_7 and R_8 (in series with it): $6 + 30 + 44 = 80$ ohms

Step F: The 80-ohm equivalent resistance and R_1 are in parallel, and the total equivalence resistance may now be computed as:

$$\frac{80 \times 240}{80 + 240} = \frac{19,200}{320} = 60 \text{ ohms}$$

Step G: By Ohm's Law:

$$I = \frac{E}{R} = \frac{120}{60} = 2 \text{ amperes}$$

The following additional steps are required to find the current in, and the voltage drop across, R_2 . Note that we must retrace our steps.

Step H: E is applied to the parallel combination of R_1 and R_{2-6}

$$I_{R_{2-6}} = \frac{120}{80} = 1.5 \text{ ampere}$$

Step I: The equivalent R_{2-6} is 6 ohms

The voltage across this combination is:

$$E_{R_{2-6}} = IR = 1.5 \times 6 = 9.0 \text{ volts}$$

Step J:
$$I_{R6} = \frac{E}{R6} = \frac{9}{42} = 0.214 \text{ ampere}$$

Step K: The voltage across the R2-5 combination must be 9 volts also.

$$\text{Therefore, } I_{R2-5} = \frac{9}{7} = 1.286 \text{ ampere}$$

Step L: Referring to Fig. 27B

$$I_{R2} = 1.286 \text{ ampere}$$

$$E_{R2} = 1.286 \times 3 = 3.858 \text{ volts}$$

23. Algebraic Signs for Voltages and Current

As the study of electricity proceeds from fundamental series and parallel circuits to those of greater complexity, involving networks of resistance and more than one source of emf, Ohm's Law must be amplified and extended. This extension was accomplished by two electrical laws credited to Gustav Robert Kirchhoff (1824-1887). Before stating these laws, however, it is essential to understand how algebraic signs (+ and -) are applied to emf's and currents.

An algebraic sign in electricity generally indicates a *direction*. Since all directions are arbitrarily chosen relative to a conventional reference direction, the first step in the procedure is to establish the required reference.

Reference 1: The direction of an electric current is always taken from the more negative to the less negative point. This is not a new idea; it is merely a restatement of the fact that electricity flows from minus to plus.

Reference 2: When a current flows toward a given point, it is assigned a plus sign; when it flows away from the same point, it is assigned a minus sign. Thus, in Fig. 28, the current is shown as flowing from the negative side of the battery, through the resistors of the circuit, and back to the positive side of the battery (Reference 1). In addition, the current approaching point A is assigned a plus sign while the same value of current leaving point A is given a minus sign (Reference 2).

Reference 3: The voltage drop across a resistor is identified as negative (-) voltage because it subtracts from the total circuit voltage. In the example of Fig. 28, the voltage drops across the resistors are 20 volts and 80 volts, respectively. For the purpose

of tracing current around the circuit, they are identified by minus signs. For all other purposes, these signs may be omitted; in ap-

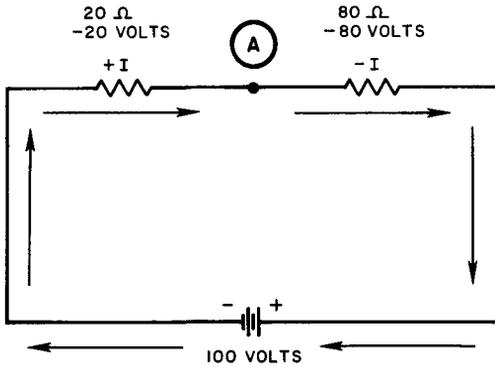


Fig. 28. Algebraic signs for voltages and currents.

plying Ohm's Law, for instance, there is no need for the assignment of algebraic signs.

Reference 4: In tracing the path of a current around a circuit, each time a source of voltage that aids the current flow is encountered, this source is assigned a plus sign; if the source opposes the current flow, it is assigned a minus sign. Consider the example shown in Fig. 29.

Any point in the circuit may be chosen as start of the path; using point A and proceeding around the circuit in the direction of the current flow, the emf's and voltage drops are as follows (the circuit current is designated as I):

$$\text{Starting from A: } -IR_1 + E_2 - IR_2 - E_3 + E_1$$

Note that E_1 and E_2 are designated as positive emf's, because they *aid* the flow of current, whereas E_3 is negative because it *opposes* the flow of current.

24. Kirchhoff's First Law

Kirchhoff's first law is not really unfamiliar at this point. Without having made an explicit statement of it, the law has been implied throughout the foregoing work on parallel circuits.

The algebraic sum of the currents at any junction of conductors is zero.

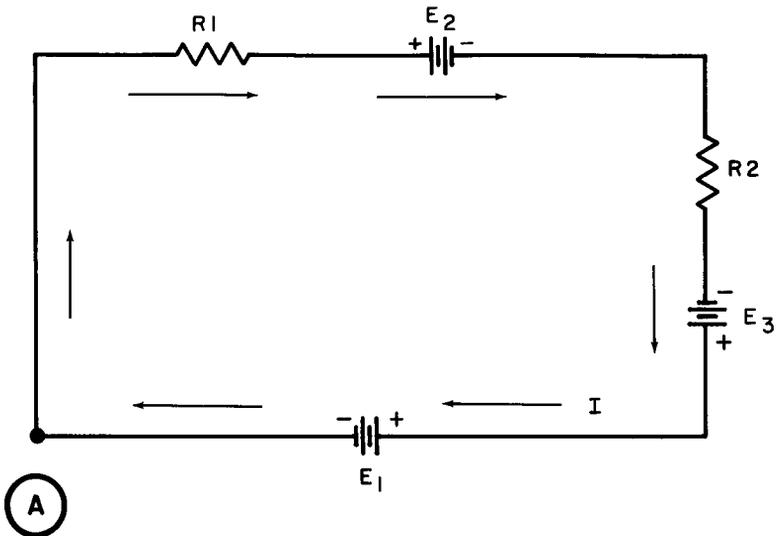


Fig. 29. Algebraic signs assigned to emf's and voltage drops.

Stated in other words, there is as much current flowing toward any given point in a circuit, as there is flowing away from the same point. This concept is almost self-evident; no point on a conductor can have a charge concentration built up upon it without limit, nor can more electrons flow away from a point than flow toward it. Either of these situations would lead to impossible conditions.

Using algebraic signs, an application of Kirchoff's first law to a branched circuit is shown in Fig. 30. Referring to point A and using the convention of algebraic current signs previously chosen (Reference 2), we have:

$$+ I_1 - I_2 - I_3 = 0 \quad (18)$$

This is another way of stating the long familiar idea that the sum of the currents in the branches of a parallel circuit is equal to the circuit current, because Equation 18 may be re-written as

$$I_1 = I_2 + I_3 \quad (19)$$

Exactly the same procedure may be applied to point B in Fig. 30 with identical results. I_2 and I_3 are now *approaching*

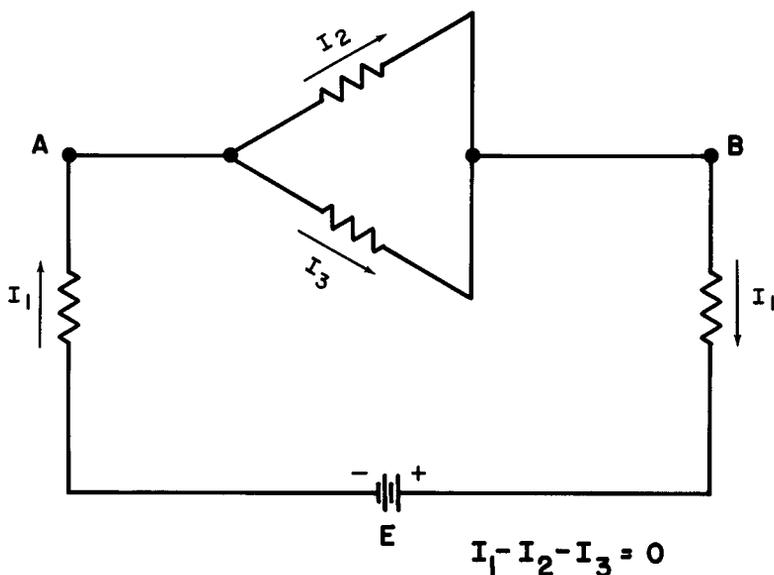


Fig. 30. Use of Kirchhoff's first law.

point B and are hence given plus signs; I_1 is now leaving point B and so is considered to be minus. Thus:

$$+I_2 + I_3 - I_1 = 0 \quad (20)$$

reducing to the more familiar form

$$I_2 + I_3 = I_1$$

which is exactly the same as Equation 19. Numerical examples of the application of the first law will be presented after discussing Kirchhoff's second law.

25. Kirchhoff's Second Law

The student has regularly used Kirchhoff's Second Law in solving simple Ohm's Law problems in series circuits. Again, this was not apparent as a new law, because it was applied as a self-evident axiom: *the sum of the voltage drops in a series circuit is equal to the source voltage*. Kirchhoff's second law is:

*The algebraic sum of the emfs and voltage drops
around any closed circuit is zero.*

In the example of Fig. 29, using the sequence starting at point A, Kirchhoff's second law would give us:

$$-IR_1 + E_2 - IR_2 - E_3 + E_1 = 0 \quad (21)$$

The advantage of the second law over the simple concept that the sum of the voltage drops in a series circuit is equal to the source voltage is that it permits *more than one source of emf to enter into the problem*. Consider the following problem as an application of Kirchhoff's second law:

Example 49. Calculate the current flowing in the circuit of Fig. 31. The internal resistance of the battery E_1 is 0.6 ohm, that of E_2 is 0.4 ohm, and that of E_3 is 1.0 ohm.

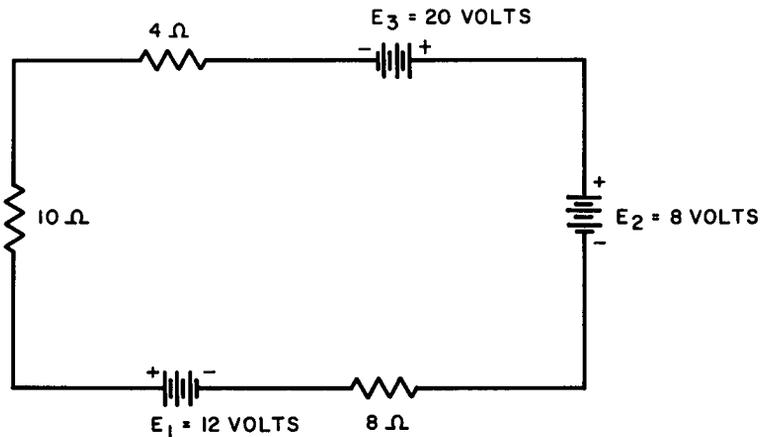


Fig. 31. Circuit for Example 49.

Solution. The internal resistances behave as series components so that the circuit should be redrawn to show these resistances as part of the circuit, as shown in Fig. 32.

Batteries E_1 and E_3 are acting in the same direction and together produce an emf of 32 volts *counterclockwise*; source E_2 has a potential of 8 volts *clockwise*. Thus, the differential emf is 24 volts *counterclockwise* and the current may therefore be assumed to flow in that direction. In more complex networks, it may not be possible to predict the direction of the current in this easy manner. This makes no difference, however, because an incorrect assumption merely results in a minus answer for the current. The negative sign for the current indicates that the assumed *direction* was not the right one; however, the *numerical answer* is not affected by the improper choice of direction.

D-C CIRCUIT ANALYSIS

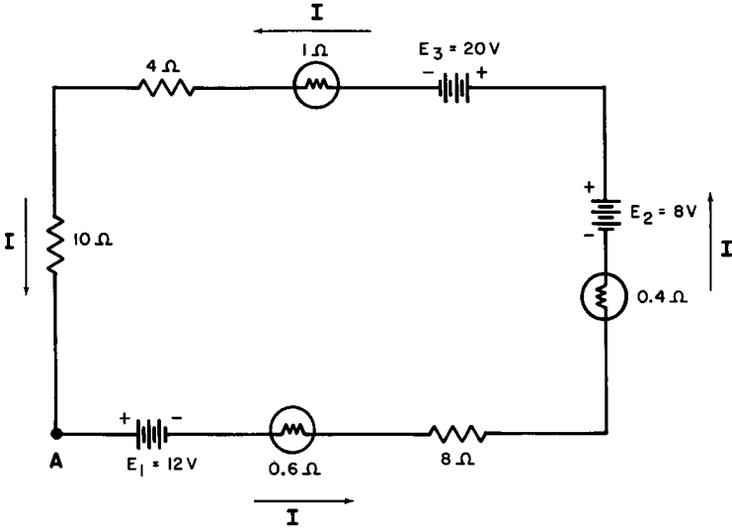


Fig. 32. Circuit for Example 49 redrawn to include the internal resistances of the batteries. (They are indicated by the circled resistor symbols.)

Starting at point A and following the current around counterclockwise, the application of Kirchoff's second law provides this equation:

$$+12 - 0.6I - 8I - 0.4I - 8 + 20 - 1I - 4I - 10I = 0 \tag{22}$$

thus

$$-0.6I - 8I - 0.4I - I - 4I - 10I = -12 + 8 - 20$$

changing signs

$$0.6I + 8I + 0.4I + I + 4I + 10I = 12 + 20 - 8$$

summing up both sides:

$$(24.0) I = 24$$

$$I = 1.0 \text{ ampere}$$

Most practical problems call for the use of both the first and second laws in their solution, as shown in the following example.

Example 50. In the circuit of Fig. 33, the internal resistances of the two batteries are 5 and 10 ohms, respectively, and the load resistance is 15 ohms. Find the current in the load resistance, and in each branch.

Solution. Applying the first law at point A:

$$I_3 - I_1 - I_2 = 0 \tag{23}$$

(I_1 and I_2 leave point A, whereas I_3 approaches point A.)

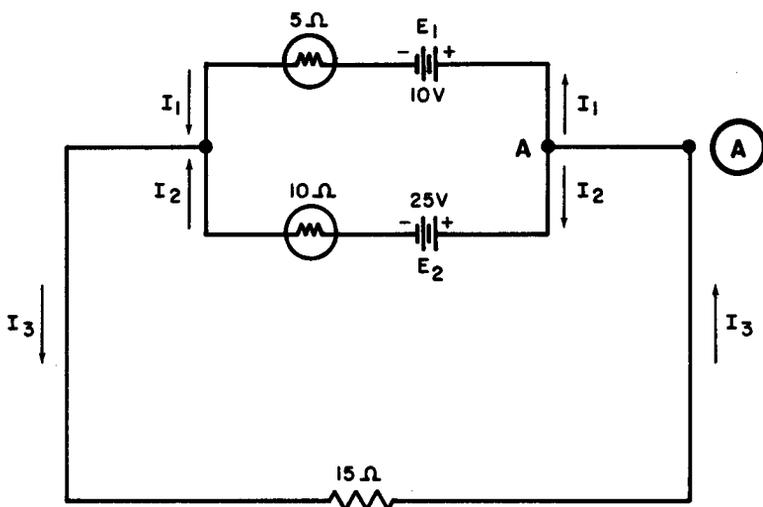


Fig. 33. Circuit for Example 50.

Applying the second law for the path through the upper battery and the load resistor starting from point A:

$$\begin{aligned}
 10 - 5I_1 - 15I_3 &= 0 \\
 5I_1 &= 10 - 15I_3 \\
 I_1 &= 2 - 3I_3 \qquad (24)
 \end{aligned}$$

Applying the second law for the path through the lower battery and the load resistor starting from point A:

$$\begin{aligned}
 25 - 10I_2 - 15I_3 &= 0 \\
 10I_2 &= 25 - 15I_3 \\
 I_2 &= 2.5 - 1.5I_3 \qquad (25)
 \end{aligned}$$

Substituting Equations 24 and 25 into Equation 23:

$$\begin{aligned}
 I_3 - (2 - 3I_3) - (2.5 - 1.5I_3) &= 0 \\
 I_3 - 2 + 3I_3 - 2.5 + 1.5I_3 &= 0 \\
 I_3 + 3I_3 + 1.5I_3 &= 2 + 2.5 \\
 5.5I_3 &= 4.5
 \end{aligned}$$

$$I_3 = 4.5/5.5 = 0.818 \text{ ampere}$$

To find I_1 , substitute the value of I_3 found in Equation 26 into Equation 24:

$$I_1 = 2 - 3(0.818) = 2 - 2.454 = -0.454 \text{ ampere}$$

Note that this value of I_1 comes out as a minus quantity, which means that the direction assumed for I_1 at the start was incorrect. Actually, I_1 approaches point A rather than flowing away from it. This is so, because the battery E_2 produces an emf that overrides that of E_1 , forcing the current to flow backward through the upper battery. This does not alter the *magnitude* of the result, however, since I_1 is equal to 0.46 ampere.

To find I_2 , substitute the value found for I_1 in Equation 26 into Equation 25:

$$I_2 = 2.5 - 1.5 \times 0.818$$

$$I_2 = 2.5 - 1.227$$

$$I_2 = 1.273 \text{ ampere}$$

As a check on these two values, add the results for I_1 and I_2 to ascertain whether or not the sum is equal to I_3 :

$$-0.454 + 1.273 = 0.819$$

The value of I_3 found in Equation 26 was 0.818 ampere, so our result checks to all but the last significant figure.

The student is strongly urged to solve the practice problems in Kirchhoff's Laws given at the end of this chapter. He will find them to be invaluable in practical work dealing with d-c resistive networks of any variety.

26. Superposition

Our last examples with Kirchhoff's Laws developed an interesting phenomenon. Frequently the current through a certain resistance is actuated by the combined effect of two sources of emf. For example, in Fig. 34, no direct application of Ohm's Law could be used to determine the voltage across R_3 , because the two sources involved *each* contribute to the resultant current through (and hence voltage drop across) it.

The application of Kirchhoff's Laws to problems of this kind results in cumbersome equations. An alternate method of solution is the application of Helmholtz' *principle of superposition*. A statement of this principle is *when there are two or more sources in a network acting simultaneously, the response at any point in the network is the algebraic sum of the responses produced at this point, if each source was considered to act separately*. The word *response* here may mean the current flowing through, or the voltage drop across, a resistance.

When applying this principle, all of the sources of the current or voltage are considered "dead" (replaced by a short circuit),

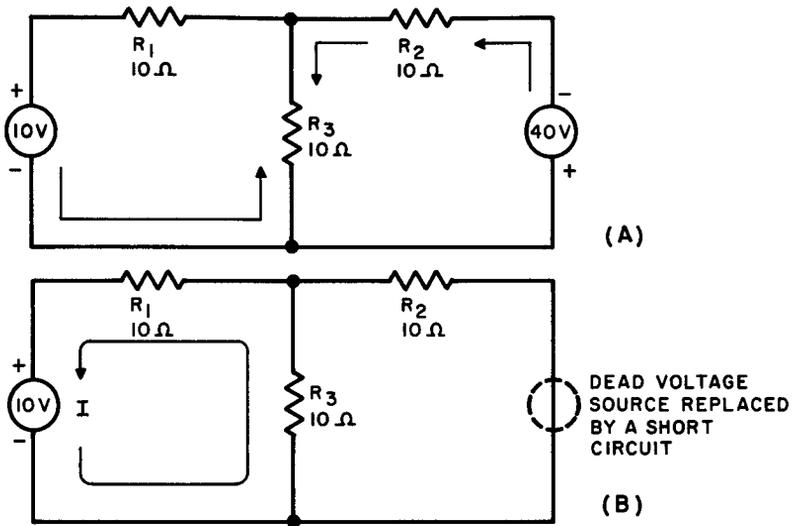


Fig. 34A and B. Circuit and solution of Example 51 by use of the principle of superposition (Part 1).

*except one.*¹ The circuit is solved in terms of this one remaining active source. Then this source is considered dead and the circuit is solved in terms of the effect of another source. When the effects of all of the sources have been computed separately, the results are added algebraically.

Example 51. In Fig. 34, find the voltage drop across R_3 and the current through it. Also determine the power dissipated in R_3 .

Solution. Let us first consider the 40-volt source as being dead and having negligible internal resistance. We will, therefore, replace it with a short circuit and solve in the usual way for the voltage across R_3 . (See Fig. 34B.) R_2 and R_3 are in parallel with each other and R_1 is in series with the combination. The resistance of R_2 and R_3 in parallel, $R_{2,3} = 5$ ohms. Hence, the resistance of the circuit, $R = R_1 + R_{2,3} = 10 + 5 = 15$ ohms, and the current $I = E/R = 10/15 = 2/3$ ampere $\uparrow +$

The current in R_3 is $1/2 I$, therefore, the voltage drop across it due to the 10-volt source is $E_{R_3} = 2/3 \times 1/2 \times 10 = 3\ 1/3$ volts $\uparrow +$

(The arrows indicate the direction of electron current or the polarity of the voltage across R_3 .)

Now consider the 10-volt source to be dead, and solve for the drop across R_3 using the 40-volt source, as in Fig. 34C. Here R_1 and R_3 are in parallel

¹ The internal resistances of the batteries or other sources remain in the circuit as resistors.

D-C CIRCUIT ANALYSIS

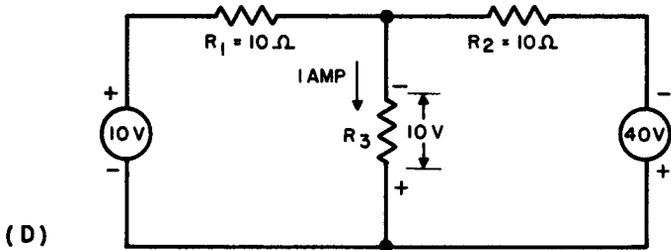
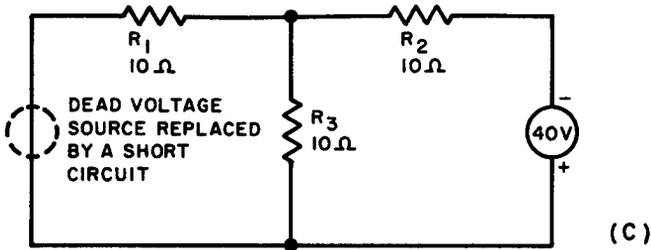


Fig. 34C and D. Circuit and solution of Example 51 by use of the principle of superposition (Part 2).

and R_2 is in series with the combination. Hence, the resistance $R_{1,2} = 5$ ohms, and the total resistance $R_{1,2,3} = 10 + 5 = 15$ ohms.

$$I = \frac{E}{R} = \frac{40}{15} = 2 \frac{2}{3} \text{ amperes } \downarrow +$$

The current in R_3 is again $1/2$, hence the voltage drop across it due to the 40-volt source is:

$$E'_{23} = 2 \frac{2}{3} \times \frac{1}{2} \times 10 = 13 \frac{1}{3} \text{ volts } \downarrow +$$

The actual voltage drop across R_3 due to both sources is given by the algebraic sum of the drops due to the separate sources. Arbitrarily assigning a minus sign to the downward-directed values.

$$3 \frac{1}{3} \text{ volts} - 12 \frac{1}{3} = -10 \text{ volts}$$

The current in R_3 is found similarly

$$\frac{1}{3} - \frac{4}{3} = -\frac{3}{3} = -1 \text{ ampere}$$

The power dissipated is

$$E \times I = 1 \times 10 = 10 \text{ watts}$$

To find the power dissipated in R_3 , one cannot use the principle of superposition directly, but must determine the current and the voltage drop first. This is the case because voltage and current are linear functions, whereas power is a product (square) function.

27. Thévenin's Theorem

One of the most powerful tools available to the technician for the solution of complex network problems is a theorem due to Thévenin. This theorem can be stated in many different ways. Its meaning is clearest when it is broken down into several parts. Thévenin's theorem is generally employed to find the current flow-

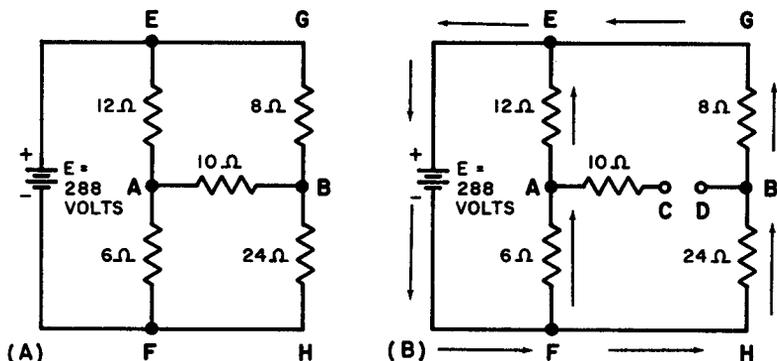


Fig. 35A and B. Application of Thévenin's theorem (Part 1).

ing in one branch of a complex network. Figure 35A shows such a network, in which it is desired to find the current flowing through the 10-ohm resistor (branch A-B).

1. The first part of Thévenin's theorem instructs us to open the branch in which the current flows and calculate the potential difference across the break. (See Fig. 35B.)

$$I_{EF} = 288/18 = 16 \text{ amperes}$$

$$I_{GH} = 288/32 = 9 \text{ amperes}$$

Next find the potential of points A and B referred to the negative end of the source.

$$E_A = I_{EF} \times 6 \text{ ohms} = 16 \times 6 = 96 \text{ volts}$$

$$E_B = I_{GH} \times 24 \text{ ohms} = 9 \times 24 = 216 \text{ volts}$$

$$\text{The voltage between A and B} = 216 - 96 = 120 \text{ volts}$$

2. The second portion of the theorem requires that the source of voltage be removed and replaced with a jumper having a resist-

ance equal to the internal resistance of the source. (If the source is to be considered as having zero resistance, the jumper is a short circuit. We will take this point of view here.) Having done this, we are now to calculate the resistance of the network looking back from the break. This is shown in Fig. 35C. At first glance, this

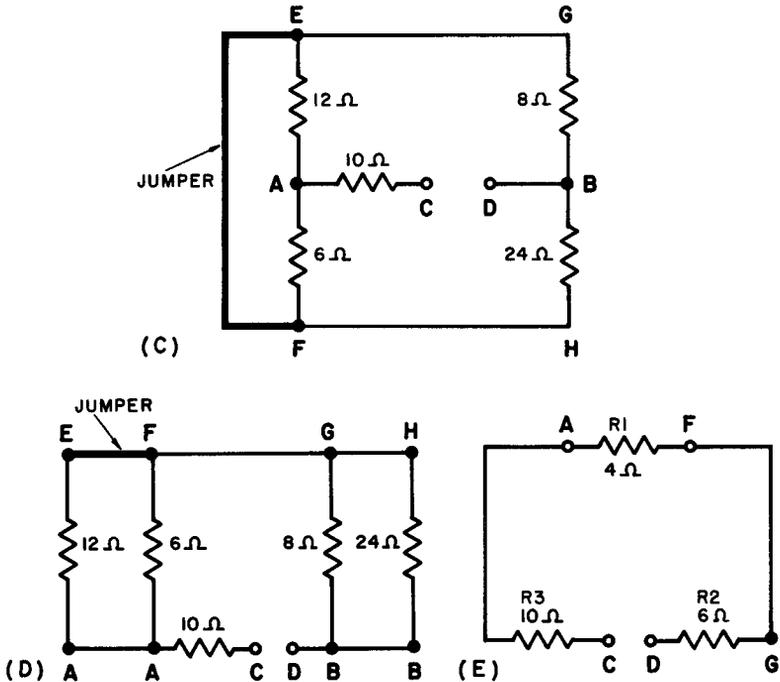


Fig. 35C, D, and E. Application of Thévenin's theorem (Part 2).

complicated resistive network appears difficult to resolve, but with the aid of the identifying letters, it may be redrawn in the familiar form shown in Fig. 35D. It is only necessary to solve this simple series-parallel circuit to find the joint resistance presented to the break terminals C and D. Thus the joint resistance (R1) of the branches EA and FA is:

$$\frac{1}{R_1} = \frac{1}{12} + \frac{1}{6} = \frac{3}{12}$$

$$R_1 = 4 \text{ ohms}$$

The joint resistance (R_2) of branches GB and HB is:

$$\frac{1}{R_2} = \frac{1}{8} + \frac{1}{24} = \frac{4}{24}$$

$$R_2 = 6 \text{ ohms}$$

Thus, the circuit is reduced to the equivalent arrangement of Fig. 35E. Its resistance is:

$$R = R_1 + R_2 + R_3 = 10 + 4 + 6 = 20 \text{ ohms}$$

3. The last part of Thevenin's theorem states that the current in the branch may now be found by dividing the voltage across the break by the resistance looking into the network from the break. The current in the branch AB of the original circuit thus is:

$$I_{AB} = \frac{120 \text{ volts}}{20 \text{ ohms}} = 6 \text{ amperes}$$

The student may convince himself of the value of Thevenin's theorem by attempting to use straightforward Ohm's Law procedures on the network just described. He will find that the equations involved are quite difficult and require tedious manipulation.

28. The Wheatstone Bridge

The arrangement of resistors and a voltage source shown in Fig. 35A may be reduced to a relatively simple network that has great usefulness in making measurement by adjusting the resistance values so that the voltage drop between points A and B becomes zero. When this modification is made, the circuit is called a *balanced Wheatstone Bridge*. As an example, assume in Fig. 35A that resistor AF is changed to 36 ohms and that a sensitive galvanometer replaces resistor AB. These alterations are shown in Fig. 36.

Under the conditions shown in the diagram, the galvanometer reads zero because the voltage between points A and B is zero; this is shown in the following proof. Assume that the meter switch is open thereby causing an open circuit between A and B. The network is then a simple series-parallel arrangement. This permits us to find the current in the branches EF and GH:

$$I_{EF} = 288/48 = 6 \text{ amperes} \quad (28)$$

$$I_{GH} = 288/32 = 9 \text{ amperes} \quad (29)$$

Find the voltage drop across AF and BH, thus giving potentials at A and B, referred to the negative side of the battery:

$$E_{AF} = I_{EF} \times R_{AF} = 6 \times 36 = 216 \text{ volts}$$

$$E_{BH} = I_{GH} \times R_{BH} = 9 \times 24 = 216 \text{ volts}$$

Because the potentials of points A and B are the same, referred to the same side of the battery, the difference in potential between

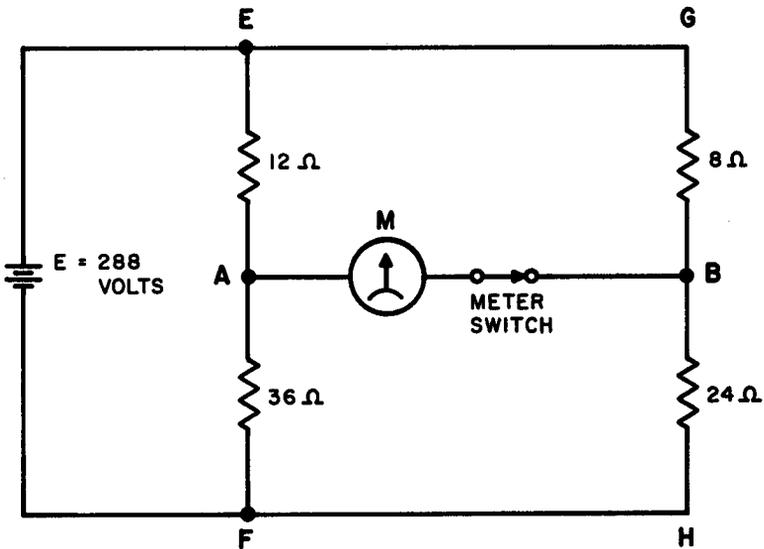


Fig. 36. Balanced Wheatstone Bridge.

them is zero, and no current flows through the meter when the switch is closed.

The value of the Wheatstone bridge becomes more evident when it is handled in more general terms. (Refer to Fig. 37.)

The general equation for a balanced Wheatstone bridge (i.e., galvanometer M reads zero when its switch is closed, indicating zero difference in potential between terminals A and B) is easily derived, as follows:

Designate the current in the R_1 - R_3 branch, I_1 , and that in the R_2 - R_4 branch, I_2 ; the voltage drop across R_1 , $E_1 = I_1 R_1$; the voltage drop across R_2 , $E_2 = I_2 R_2$.

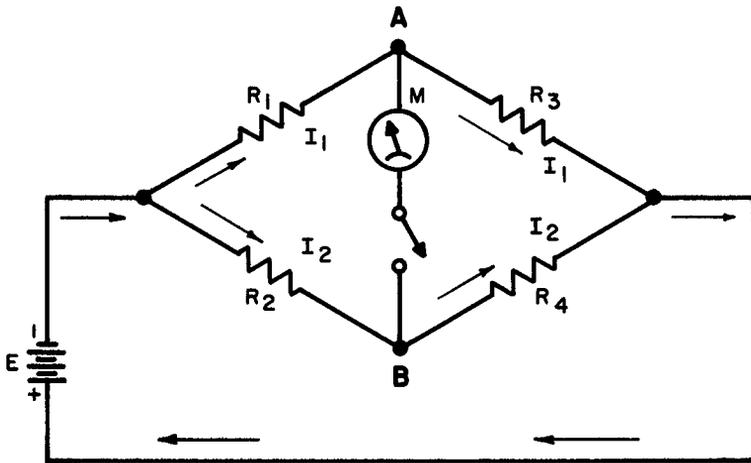


Fig. 37. General form of the Wheatstone Bridge.

If the potentials at A and B are equal, as required by the original conditions, $E_1 = E_2$ and

$$I_1 R_1 = I_2 R_2 \quad \text{or} \quad \frac{I_2}{I_1} = \frac{R_1}{R_2}$$

In a similar manner, we obtain the voltage drops E_3 and E_4 across R_3 and R_4 , respectively as:

$$E_3 = I_1 R_3$$

$$E_4 = I_2 R_4$$

But, since the potentials at A and B are equal, $E_3 = E_4$, and hence $I_1 R_3 = I_2 R_4$ or $I_2/I_1 = R_3/R_4$. These ratios of I_2/I_1 may be equated, thus:

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \quad (30)$$

This is the fundamental equation of the balanced Wheatstone bridge. It tells us that for a balanced bridge, the ratio of R_1 to R_2 is equal to the ratio of R_3 to R_4 .

The principal application of the balanced Wheatstone bridge is the determination of the value of an unknown resistance. This can be accomplished with excellent precision, if the known resistors used in the bridge are accurately rated.

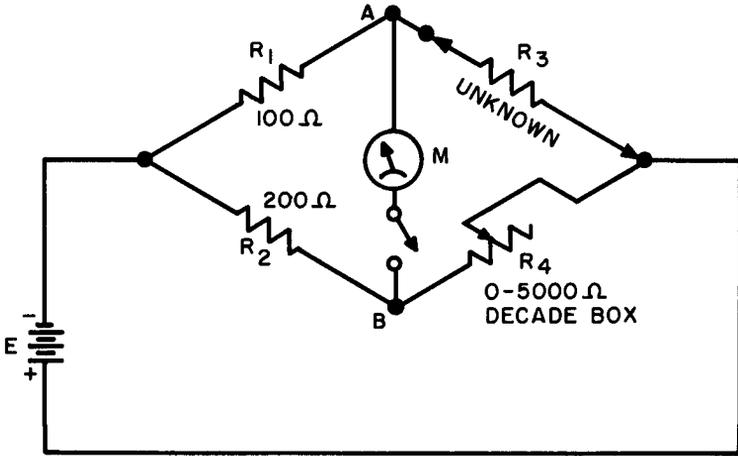


Fig. 38. A practical form of the Wheatstone Bridge.

Figure 38 shows one form that a practical Wheatstone bridge may take. R_1 and R_2 are precision resistors of 100.0 ohms and 200.0 ohms, respectively; R_3 is a resistance of unknown value to be measured by the bridge; R_4 is a calibrated variable resistor in the form of a precision decade box, or a high-precision helical potentiometer.

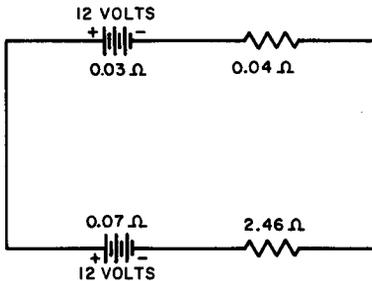
In use, R_4 is set at about the middle of its range, R_3 is connected across the terminals as shown, and the meter key is quickly tapped while the meter is observed. It is always advisable to start this process with a low-resistance shunt connected across the meter terminals. Otherwise, if the potential across AB should be relatively high, the delicate galvanometer coil might be damaged. R_4 is then adjusted in the direction that causes the meter deflection to approach zero; as the operator comes closer to this point, the shunt across the meter is removed, thus improving its sensitivity and the precision of the measurement. Finally, when the value of R_4 has been set so that there is no indication on the galvanometer, the bridge is considered balanced, and the resistance of R_4 is read off. Suppose that R_4 reads 2466.3 ohms at balance. The resistance of R_3 may then be calculated to the same precision by substituting all known values in Equation 30 and solving for R_3 .

$$\frac{100}{200} = \frac{R_3}{2466.3}$$

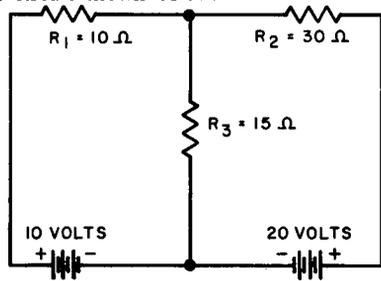
$$R_3 = \frac{1}{2} \times 2466.3 = 1233.2 \text{ ohms}$$

29. Review Questions

- (1) Four resistors, $R_1 = 22$ ohms, $R_2 = 18$ ohms, $R_3 = 30$ ohms, and $R_4 = 12$ ohms, are connected in series across a 60-volt battery. The internal resistance of the battery is 3.2 ohms. What is the current flow in the circuit? What does the value of the current drawn from the battery become, if a single 20-ohm resistor is connected across the battery so that it is in parallel with the other four resistors above?
- (2) Three resistors of 1.5 ohms each are connected in series across a battery whose internal resistance is 0.5 ohm. If the current through the circuit is one-half ampere, compute the terminal voltage of the battery.
- (3) Four resistors, $R_1 = 7.2$ ohms, $R_2 = 1$ ohm, $R_3 = 3.4$ ohms, and a resistor of unknown value, R_4 , are connected in series across a 6-volt battery whose internal resistance is 0.4 ohm. If a current of 1.42 amperes flows in the circuit, what is the value of R_4 ?
- (4) What is the value of current in the circuit shown below?

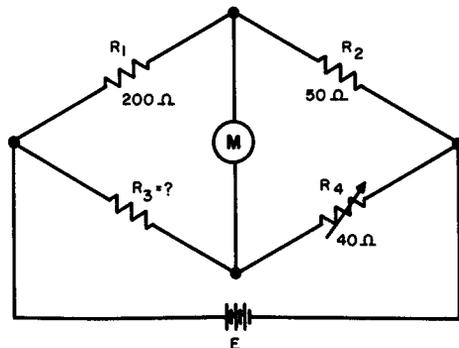


Circuit for Question 4.

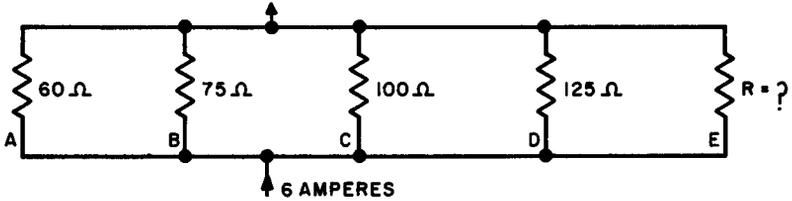


Circuit for Question 6.

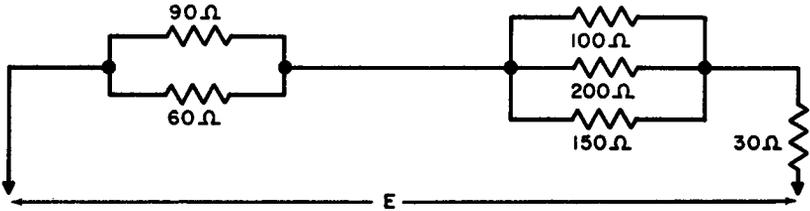
- (5) Two 12-volt batteries are connected in parallel to an 18-ohm load. If each battery has an internal resistance of 0.5 ohm, calculate the current through the load.
- (6) What is the value of current through R_3 in the diagram above? (Assume that the batteries have no internal resistance.)
- (7) In problem 6, compute the value of the current through R_3 if the battery connections of the 20-volt battery are reversed.
- (8) The Wheatstone bridge shown here is balanced when an unknown resistor is placed at R_3 . $R_1 = 200$ ohms, $R_2 = 50$ ohms, and $R_4 = 40$ ohms. What is the value of R_3 ?
- (9) Assume the values of the Wheatstone bridge shown in problem 8 are 10%



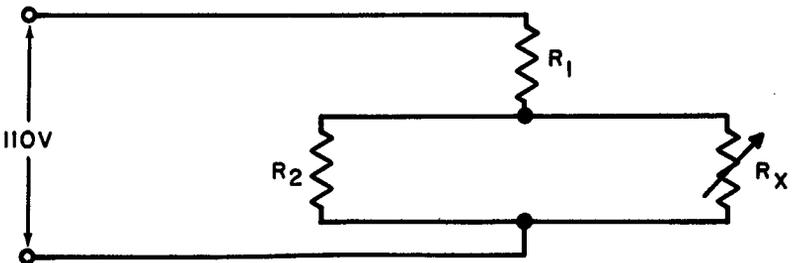
- higher than indicated (thus $R_1 = 220$ ohms, etc.) . What is the value of R_3 ?
- (10) A microammeter, with an internal resistance of 180 ohms, reads 50 microamperes at full-scale deflection. What voltage yields full-scale deflection?
- (11) Five resistors with the values shown are connected in parallel. The current through resistor A is 1.25 amperes. Compute (a) the current in resistors B, C, D, and E; (b) the equivalent resistance value of the combination; (c) the resistance of E.



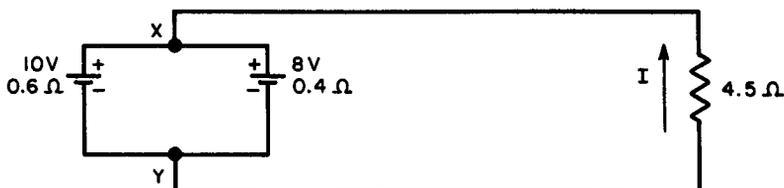
- (12) In the series-parallel circuit shown, the applied voltage is 125 volts. Calculate (a) the current in each resistor; (b) the total current; (c) the power dissipated in each resistor; (d) the total power dissipated.



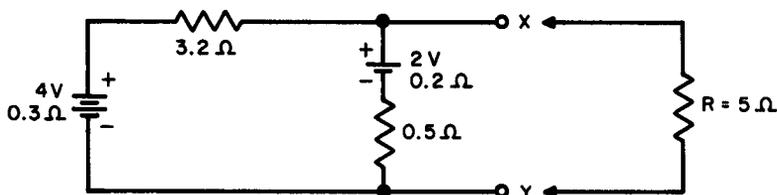
- (13) If the values of each resistor of Question 13 were doubled and the applied voltage remained constant, how would each value of the answer for (a), (b), (c), and (d) above vary? Compute the numerical values to check your answers.
- (14) In the circuit shown, R_1 has a value of 8 ohms, and R_2 a value of 12 ohms. What value must R_x have so that the power dissipated by the entire circuit is 880 watts?



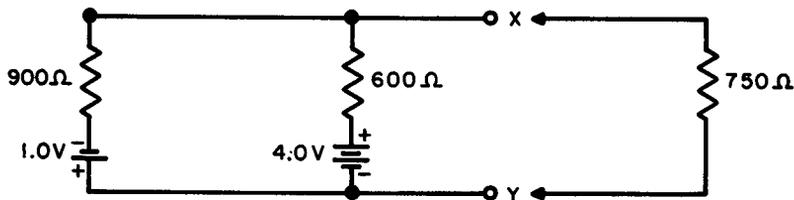
- (15) Using Kirchhoff's laws, determine current I across the 4.5-ohm resistor in the circuit shown. Compute the voltage drop from point X to point Y.



- (16) If the polarity of the 8-volt battery in Question 15 is reversed, calculate the current through the 4.5-ohm resistor.
- (17) Utilizing Thévenin's Theorem, compute (a) the voltage across terminals X and Y before R is connected into the circuit; (b) the effective resistance as measured between X and Y; and (c) the current through R when it is connected across X and Y.



- (18) Find the voltage, current, and power delivered by the circuit shown to the 750-ohm load attached to terminals X and Y.



- (19) Given the same circuit conditions shown in Question 18, compute the voltage, current, and power delivered by the circuit if the load resistance is 1500 ohms.
- (20) A Wheatstone bridge is arranged as shown in Fig. 38. Assume the bridge to be balanced when $R_1 = 5,000$ ohms, $R_2 = 3,100$ ohms, $R_3 = 50,000$ ohms. Compute the value of R_4 .

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