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Loop Plant Modeling:

Overview

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The loop plant is a fruitful area for operations modeling. Because it can have many configurations and detailed structures, and because over 30 percent of the total telephone work force interacts with it, it is physically and operationally complex, and modeling is often the only way to arrive at an understanding of the basic principles underlying its operation. This article provides an introduction to the loop plant modeling papers which follow and includes both a description of the loop plant and brief descriptions of the individual papers.

I. INTRODUCTION

The following ten papers describe various models of the telephone loop plant. The papers cover work done chiefly during the past decade, although the foundations of the work are often much older.

The loop plant, which is described in detail in Section II of this overview, is a fruitful area for operations modeling for two main reasons. First, since the loop network must extend everywhere customers use telephones, it is geographically dispersed and can have many configurations and detailed structures. Second, over 30 percent of the total telephone work force interacts with it, to select network paths and

connect customers, to rearrange and repair the network, to monitor and analyze service and costs, and to design and construct network additions. In such a physically and operationally complex network, modeling is often the best—and sometimes the only—way to arrive at an understanding of the basic principles underlying its operation.

The emphasis in these articles is on concepts rather than implementation, on understanding basic mechanisms rather than on ways these and other models can be used in day-to-day operations. For that reason, as well as limitations on space, no attempt has been made here to describe the many computer programs and operations systems utilizing the models.

The models described here are largely concerned with investment options, work activities, and the tradeoffs between them. This work has been collected at this time both because much of it is recent and has not been reported before and because it has now reached a stage when much of the loop plant has been successfully modeled, and interrelationships have begun to emerge. This is not to imply that the work is complete or that all the critical linkages have been made. Much remains to be done, but an underlying structure can be discerned, and a firm base for future work has been established.

In the next section of this overview a description of the loop plant is provided, both to aid in understanding the models and to avoid repeated definitions of the same terms. In the final section, brief descriptions of the individual papers are provided.

II. THE LOOP PLANT

The loop is the part of the telephone network connecting the customer to the nearest switching office. As used here, the term "loop plant" covers both the physical network, and the activities required to operate it.

2.1 *Loop network definitions*

AERIAL: Type of construction in which multipair cables are suspended from utility poles. Aerial construction may be used for feeder or distribution networks, but more frequently for the latter. It is less costly to relieve than buried or underground plant, which influences cable sizing¹ but it is more easily damaged. Maintenance and appearance considerations have led to a decline in new aerial construction techniques; in 1977 only 23 percent of the plant added in the Bell System was aerial. Overall, 28 percent of the pair miles are aerial.

ALLOCATION: The process of apportioning spare feeder facilities in groups of 25 or 50 pairs along a feeder route so that the placement of the next relief cable is deferred as long as economically practical. Allocation is a planning step; it does not result in an engineering work order.

ALLOCATION AREA: A geographical region subdividing the area served

by a feeder route. The allocation area is the recommended unit of the distribution network to be analyzed as a unit for relief or revision. Each allocation area is fed by a unique bundle of pairs called a pair group. Pair groups are defined for the analysis of congestion for relief and rearrangement timing² and for allocation of spare feeder facilities.³ Two sets of allocation areas are defined for a route: ultimate and existing allocation areas (q.v.).

ULTIMATE ALLOCATION AREA: A proposal for the ideal network configuration. An ultimate allocation area is a grouping from 1 to 5 distribution areas that have the same feeder transmission requirements (gauge and loading).

EXISTING ALLOCATION AREA: A reflection of the existing configuration of the network for monitoring activities and for establishing the base from which the ultimate design will take place. There may be up to 10 percent multiplying between existing allocation areas, with a goal of no multiplying. In size, existing allocation areas can vary from 500 to 2000 working telephone lines. The size has been defined so that the statistical fluctuation in operating cost information is minimized^{4,5} as well as to accommodate the wide range of actual network configurations encountered in practice.

BROAD GAUGE CABLE COSTS: Term used for the $A + BX$ approximation used in estimating the cost of placing and splicing a cable. In the expression, A is the total labor and material cost per sheath foot, independent of number of pairs; B is the material, placing, and splicing cost per pair foot; and X is the number of pairs. There will generally be a different set of A and B costs associated with the gauge of wire in the cable as well as with the type of construction—airial, buried, or underground. In addition broad gauge costs will vary from one location to another.

BURIED: Type of construction in which multipair cables are buried directly in the ground. Buried construction may be used for feeder or distribution networks. About 34 percent of the new construction in the Bell System was buried in 1977; at the present time, 21 percent of the pair miles are buried.

COMMITMENT: The process of physically connecting allocated feeder pairs to the distribution network.

CONNECT-THROUGH: An assignment option in which the loop connection between the central office and a premises—feeder, distribution, and drop—is left intact when telephone service is discontinued. In this way no action need be taken in the loop network when a new customer occupies the premises and requests service. The disadvantage is that, depending upon assignment rules, the pair may be designated as unavailable for other customers, increasing the number of idle pairs at relief.⁶

DISTRIBUTION AREA: A subdivision of an allocation area to which

feeder pairs are committed. A distribution area may be an area served by a point of interconnection (e.g., a serving area interface and a control point) or it may be a group of laterals administered together in multiple. A distribution area should contain between 200 and 600 ultimate living units; it is usually a "logical" or "natural" area with common user or geographical characteristics. There are no restrictions on multiplying between individual terminals or between distribution areas within an allocation area, unless the distribution area is administered as a Serving Area (q.v.).

DISTRIBUTION NETWORK: The system of cables and terminals to which individual telephone lines are connected, and which then is connected to the feeder route. Distribution cables are usually small, on the order of 50 to 300 pairs, and follow all the streets where subscribers are located. Most distribution networks are short, with a maximum length under $\frac{1}{2}$ mile. Distribution cables are not sized for periodic reinforcement as the feeder network is, but are constructed initially to satisfy estimated ultimate pair requirements. This minimizes construction activity close to subscribers and eliminates the need to closely monitor a large number of small cables. Individual distribution pairs appear in one or more distribution pedestals or terminals, discussed below.

DISTRIBUTION PEDESTAL: Point of interconnection between the drop leading to the customer's residence and the distribution network in buried plant. Pedestals may contain from 5 to 50 pairs.

DISTRIBUTION TERMINAL: Point of interconnection with the distribution network in aerial plant containing between 5 and 50 pairs.

FEEDER ROUTE: Major network of cables from central office to within $\frac{1}{2}$ mile or so of customers. Area within feeder boundaries is the first and largest subdivision of area served by a central office. For convenience 4 routes are often nominally designated "north," "south," "east," and "west," although in practice office configurations and local design procedures may result in wide variations from nominal. Physically, feeder routes consist of many multipair cables in parallel, which are interconnected by splices. Pairs from the route end cumulate toward the central office, so that at the central office all pairs in the route are concentrated. The reduction in route cross section as it gets farther from the central office is termed "tapering." At various points pair bundles leave the feeder route in laterals which connect the feeder cables to the distribution network. As discussed under "Resistance Design" (q.v.) up to four different gauges of wire may be used in the cables making up a feeder route. Under standard design procedures, an attempt is made to restrict any one loop to only two consecutive gauges (e.g., 26 gauge next to the CO, followed by 24 gauge farther out). This may, however, result in all 4 gauges appearing in parallel close to the central office. As discussed under "Feeder Relief" (q.v.), in the practical case of reinforcing part of

a feeder route, separate relief calculations must be made for all gauges appearing in a given cross section.

FEEDER RELIEF: Broadly, the process of making more pairs available at a given demand point. This may be done either by network rearrangement, so that unused pairs become available where needed,³ or by adding new cables.

When relief is provided, the number of pairs added should be that number that minimizes the total cost of the current plus all future relief projects. Opposing economic forces are at work which tend to both maximize and minimize the number of pairs added. The cost per pair is less with large cables, and the time before relief is required is increased. However, large cables represent large idle investments which will not be compensated by increased service earnings for an extensive period. The process of balancing these compensating forces to reach an economic optimum is termed "economic cable sizing."¹

In general, separate sizing calculations are required for each section of the feeder route, and for each gauge within the cross section. The size and complexity of such calculations often makes mechanized assistance desirable.⁷

The time at which relief should be provided to the network depends on the use of existing facilities. Use is usually characterized by "fill" of the individual feeder sections, where fill is defined as the ratio of pairs in use to pairs available. Two fills may be used: "assigned pair" fill, which includes all pairs connected between subscriber and central office, even if telephone service is not in use; and "working pair" fill, which includes only pairs connected to active customers. The difference between the two is the "idle assigned" level. As the inventory of spare facilities decreases, the probability that a network rearrangement will be required to satisfy an inward service order increases. The optimum time to provide relief occurs when the incremental costs of relief exactly equal the incremental rearrangement costs of congestion.^{2,8}

FEEDER SECTION: Feeder sections are linear segments of feeder routes which have been defined so that the number of pairs in the route may be matched to present and future demand as effectively and economically as possible. Sections may be defined to study duct sizing, to account for gauge requirements, and to accommodate major demand points. As discussed under "allocation area" (q.v.), ideally one feeder route section should be associated uniquely with each allocation area. While the number of sections can be made arbitrarily large, forecast uncertainties and difficulties in constructing and rearranging a feeder network which changes size and gauge frequently make it impractical and uneconomic to define and attempt to administer a network with a large number of sections.

FEEDER LATERAL: A branch cable connecting the distribution net-

work to the feeder route. A lateral may be ultimately sized or it may be relieved as growth exhausts pairs, but then less frequently than the backbone feeder.

LOOP: Connection between the subscriber's telephone and the central office. It takes its name from the pair of wires usually used, which traditionally constitutes a "loop" of direct current from the central office to the subscriber for signaling. A loop extends from the main distribution frame within the central office to the first connection point within the customer's premises, usually a voltage overload device called a "protector." Typically, the components are: "tip" cables from MDF to cable vault; feeder cable from vault to interface or distribution/feeder splice; distribution cable from that point to the distribution terminal; and "drop" wire from the terminal to the residence. The wires for any given loop may appear in only one, two, or all three types of plant (aerial, buried, underground) during the course of their route from central office to customer, and appear in multipair cables from over 3000 to 50 pairs in size.

According to a survey taken in 1973,⁹ the average length of a loop was 11,400 feet; of this, it is estimated that typically 2000 feet is in distribution cable, and the remainder in feeder.

MULTIPLING: The practice of having a cable pair appear (i.e., be accessible for service) at more than one place. Multiplying may be applied in distribution, feeder, or both networks. It was originally devised to provide flexibility with low service penetration and high party line content; its drawback is encouragement of rearrangements, which leads to increased faults and repair costs. While multiplying is useful in many environments, there are others where its penalties outweigh its advantages.⁵ For that reason, multiplying has been eliminated or sharply reduced in the Serving Area (q.v.) Concept.

PAIR GAIN: The general name applied when electronic systems provide several loops over a smaller number of wire pairs. Such systems employ frequency and time division multiplexing, switching, and other techniques.¹⁰ Various ways are used to specify the system characteristics; for example, in the *SLC-40* system 40 loops are obtained from 4 physical pairs; this may be called a "pair gain" of 36 pairs, or a "pair gain ratio" of $36/4 = 9:1$.

PERMANENT PLANT: Term loosely applied to either Serving Area (q.v.) design, or its predecessor, Dedicated Outside Plant.¹¹ In its strictest sense, it has been used to describe loops which are nowhere in multiple and, once connected to a residence, are never severed. As the result of experience, a somewhat more relaxed definition is emerging in which distribution pairs are not in multiple, and are never broken between terminal and interface, but may be disconnected from the feeder pair at an interface.

REARRANGEMENT: The process of disconnecting some pairs in cables at splices and reconnecting them to others. This activity is often required in conjunction with relief and is also required because of shifts in demand patterns.

RESISTANCE DESIGN: The name given to the method of insuring that the resistance and insertion loss of all loops is limited, so that any loop will be able to signal and send dialing information to the central office. In addition, resistance design rules have been chosen so that the resulting loop, trunk, and switching network as a whole provides adequate transmission performance. The standard resistance limit is 1300 ohms total (combined or looped resistance of the two pair wires). To achieve this limit for loops up to 6 or 7 miles in length, 3 wire gauges are commonly used: 26 gauge, 24 gauge, and 22 gauge. If used alone, they would correspond to maximum loops of (approximately) 15,000 feet, 24,000 feet, and 38,000 feet respectively. To conserve copper, the use of two gauges in tandem is recommended; that is, loops between 15,000 and 24,000 feet long, for example, would contain both 26 and 24 gauge conductors. Obviously, every loop cannot be individually tailored; theoretical savings in copper on any individual loop must be balanced against loss of flexibility and fragmentation of the route as a whole. Usually, as a result of such tradeoffs, 6 or 7 gauge change points are defined per feeder route.

Beyond 6 or 7 miles, up to as much as 20 miles, a fourth gauge of cable, 19 gauge, is available. However, both transmission and signaling limits are frequently met using finer gauge cable and electronic supplements to copper pairs.¹⁰ The low customer density and forecast uncertainty in this region of the network imposes restrictions on sectionalization and administration not covered by the following articles. (See Ref. 10 for further details.)

In addition to the resistance limit, resistance design rules include specifications designed to insure adequate transmission performance. The major ones are: loading (the application of an 88 mH inductance every 6000 feet to compensate for capacitance) should be applied on all loops longer than 18,000 feet; and bridged tap (cable in electrical parallel with the loop) should be limited to 5000 feet.

SERVING AREA: A distribution area administered under the Serving Area Concept (SAC).¹² Under this concept multiple appearances in the distribution network are sharply limited, and two or more pairs per ultimate living unit are usually provided from interface to terminal. All connections between the feeder network and the distribution network are made in a cross connection unit called a Serving Area "Interface." The purpose of serving area design is to reduce rearrangement and repair activities in the distribution plant by permanently establishing connections between a residence and the interface. All subsequent activities

then take place in the interface. Once a connection has been made to the feeder, two major operating modes are used: I, in which the connection is permanent, never broken; and II, in which an idle pair may be broken when no unassigned spares are available. At the present time, type II is more commonly encountered in practice.⁶

UNDERGROUND: Type of construction in which cables are pulled through tubes called ducts that are buried in the ground. When ducts are constructed, several are placed in parallel; since digging and street opening costs are large, particularly in the cities, enough are placed at one time so that reinforcement is infrequent. As available ducts near exhaustion, duct capacity can significantly influence the economic cable size.¹ Access points to underground cables and ducts are provided by manholes placed every 500 to 1000 feet. Due to its cost, underground construction is generally reserved for large concentrations of cables such as in a main feeder route. About 43 percent of the pair miles placed in 1977 were in ducts; about 51 percent of the Bell System loop plant is underground.

2.2* Functional description

The loop network must be capable of providing a transmission path between customers and central offices in response to widely varying and occasionally unexpected service requests, without requiring either excessive pairs or excessive work activities. To achieve this goal, the network must be continually monitored and, when activities or pairs exceed proper levels, steps taken to augment or rearrange the network.

Viewed over time, loop network activities are cyclical though aperiodic. Long intervals may elapse between basic reappraisals of allocation area, route, and office boundaries, including reestablishment of long range goals for design changes and area rehabilitations. More frequently, but still at several year intervals, major additions to the network may be designed and carried out. With comparable frequency, though not necessarily linked with cable additions, areas may be converted from multiple to permanent plant, new multiple or permanent distribution networks may be added, and in older ones major amounts of deteriorated cable and terminals may be replaced. Several times a year, small additions to and rearrangements of the network may be made. And, finally, daily reassociations of pairs and residences may occur throughout the distribution plant in response to service requests.

The models to be described were developed in response to the need to understand and quantify these cycles. They address such questions as: How long should the interval be between relief cables? How are op-

erating activities related to fill levels? What should be monitored? How is the level of the items monitored related to decisions to convert to permanent plant?

Although many of the individual models which are described in the papers which follow address individual parts of the overall process just described, none is completely independent. An important consideration underlying this body of work is that all of the functions and cycles of the loop plant be included.

III. OVERVIEW OF THE MODELS

3.1 Inventory levels

In the first paper that follows in this issue, a description of one of the first analytical models to be developed for the loop plant is given. In "A Simple Model for Studying Feeder Capacity Expansion," Freidenfelds describes how an inventory viewpoint may be used to determine the optimum size of a feeder relief cable, balancing the advantage of reduced cost per pair for larger cables against the extra "carrying charges" for temporarily unused capacity.

In the second paper, "Economic Evaluation of Subscriber Pair Gain System Applications," Koontz develops equations which expand the options of feeder relief to include electronically derived loops. As the pair gain systems described in the other section of this special issue become more widespread, such analyses will become increasingly important.

The relief of individual feeder routes affects the aggregate number of pairs over many such routes. In "A Model of Pairs Added at the Main Frames for a Large Entity," Blum and Bell develop a model of the combined effect of many independent relief decisions on aggregate pair levels.

How feeder pairs are apportioned among competing demand points can have a major effect on both day-to-day work activities, and on the frequency of relief. In "The Feeder Allocation Process," Marsh discusses models used in establishing broad guidelines for flexible and efficient feeder pair allocation and commitment strategies.

3.2 Models of operating activities

In "An Approach to Modeling Operating Costs in the Loop Network," Koontz describes work to date on modeling the level of day-to-day work activities in the loop plant as a function of administrative strategies, inventory levels, and network characteristics. In a related paper, "Optimal Operating Policies for Serving Areas Using Connect-Through Administration," Freedman examines how similar models in permanent plant areas can be used to analyze relief and rearrangement options to serving areas.

In "Loop Plant Work Operation Cost Models Using Semi-Markov Processes," Gibson uses Markov modeling to describe how the work content of one particular activity, the reassociation of pairs among several cable sheaths (a cable "throw") can be modeled as a function of pair status and other information.

In "Economic Design of Distribution Cable Networks," Stiles describes the tradeoffs in the distribution network between initial cost of construction and future costs, and shows how these tradeoffs can affect design decisions.

3.3 Network organization

In "Statistical Analyses of Costs in Loop Plant Operations," Dunn and Landwehr develop some of the statistical considerations governing the basic size of allocation areas, and the interval over which data from them should be monitored, in order for random statistical errors to be reduced to a satisfactory level. Aughenbaugh and Stump in "The Facility Analysis Plan: New Methodology for Improving Loop Plant Operations" then show how such data may be used in making basic decisions to convert geographical areas from multiple to permanent plant. They also describe a monitoring and control system called the "Facility Analysis Plan" which relates many of the work operations just discussed.

IV. ACKNOWLEDGMENTS

Many contributions to this work have been made by individuals not explicitly identified. To all those whose work laid the foundations for the present efforts, grateful acknowledgment is made.

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Loop Plant Modeling:

A Simple Model for Studying Feeder Capacity Expansion

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Using a very simple model of feeder cable sizing facilitates the discussion of many interesting questions: How sensitive are sizing decisions to various items of data? How does the need for expensive conduit affect cable size? What kind of economy-of-scale can be expected from consolidating routes into larger backbone configurations? What effect might randomness or uncertainty in the demand forecast have on sizing? How might sizing be affected by limits on available capital? The simple sizing model discussed assumes linear growth of demand over an infinite horizon in an isolated feeder section. The cost of cable or conduit is assumed to be composed of a fixed charge plus a cost per unit of capacity added.

I. INTRODUCTION

As described by N. G. Long¹ (this issue), the *feeder* portion of the overall loop plant consists of cables, conduit, and various other hardware. It provides communication paths, usually consisting of a pair of copper wires, between the central office and the distribution plant. Additional cables, and perhaps conduit to house those cables, are added to the feeder over time as existing spare is depleted by growth in demand. Optimally sizing such additional cables and conduit is an investment decision problem known as a *capacity expansion* problem.²

A sophisticated computer program, called EFRAP,³ has been developed for solving a more general version of the feeder capacity expansion problem than we shall consider here. Our aim in this paper is to develop a manageable "analytic" model of feeder sizing. While we thus ignore some aspects of the problem, such as demand in more than one gauge, which are included in the more sophisticated approach, we can more easily include others, such as the use of temporary pair gain systems (see

W. L. G. Koontz,⁴ this issue). Furthermore, a simpler model is easier to understand.

For our basic model we assume that demand for additional feeder pairs through some section of plant is increasing linearly at the rate g over an infinite horizon. The cost to install and maintain forever a cable of x pairs is assumed to be expressible as $a + bx$ dollars per year per foot. In Section II we make some observations on sensitivity and on economies of scale based on this model.

In the following section we study the problem from a *cost of the future*, or backward dynamic programming viewpoint. This makes it possible to analyze some complications such as conduit, partial conduit, removal of existing sheaths, and the use of temporary pair-gain systems.

In Section IV we show that when we allow nonlinear demand in the near term, the dynamic programming formulation becomes more "computational" in nature. Also, we briefly consider a generalization from the linear deterministic demand to a stochastic demand process with stationary independent increments. We show that, except when the expected growth is very low compared to its standard deviation, we essentially get the same results as with the deterministic model.

In Section V we show that when the current cable relief budget is limited, we might still be able to calculate cable sizes on a case-by-case basis provided we can estimate an appropriate Lagrange multiplier value.

Finally, in the last section we mention some other applications of the simple feeder sizing model.

II. THE BASIC MODEL

We focus our attention on a single link of the feeder network, called a *feeder section* (see Long,¹ this issue). We assume that the demand for additional feeder pairs at time t in the future is given by $D(t) = gt$. In general, the demand may not always be homogeneous—customers far from the central office being routed through this section may need a coarser gauge of wire. This more general case is treated in Ref. 3, but not here. We also assume that $D(t)$ includes a *fill-at-relief* margin to account for the fact that additional cable is placed—i.e., relief is provided—well before all pairs are actually in use (e.g., see Koontz,⁵ this issue).

We model the cost of a cable of size x , that is, one having x pairs, as $a + bx$ dollars per year per foot. This cost is an annual equivalent to the total present worth cost of supplying x pairs, taking into account the costs of material, maintenance, return on capital, and taxes over the life of the cable. In most studies, the details of these costs can be relegated to a side calculation in which an *annual charge factor* is developed relating equivalent annual costs to installed first cost for various classes of plant (see the Appendix). Once a cable is added, we assume, for pur-

poses of calculating costs, that it will be "maintained forever"; i.e., that it will be replaced at the end of its life by equipment of the same cost. This is done mostly as a matter of mathematical convenience. Cables tend to have very long lives (e.g., 45 years) and it makes little difference on a present worth basis precisely what is assumed.

Under these circumstances, we will clearly use equal-sized cables, x^* , which minimize the present worth cost

$$PW = \sum_{j=0}^{\infty} \left(\frac{a + bx}{r} \right) e^{-rjx/g} = \frac{(a + bx)/r}{1 - e^{-rx/g}} \quad (1)$$

where r is the discounting rate, and we have assumed that $a + bx$ is a continuous annuity, compounded continuously. Figure 1 plots a sample PW versus x . If we wish to consider only those discrete sizes which are actually available, the minimum can be found by trying several of them. We will show shortly, however, that only small errors result from small deviations in the size. For the rest of this paper it will be convenient to assume a continuum of sizes. We can easily show that PW is a convex function of x , and so, setting its derivative to zero yields an expression for the minimizing value, x^* ,

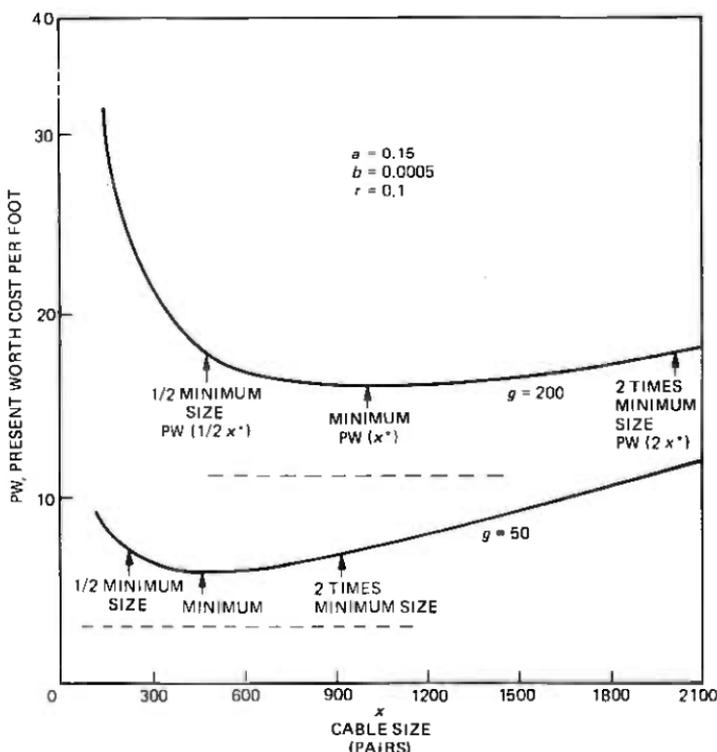


Fig. 1—Present worth cost versus size.

$$e^{rx^*/g} - \frac{rx^*}{g} - 1 = \frac{ar}{bg}$$

A quick approximation which is good for situations with short relief intervals $t^* = x^*/g$ is found by using a Taylor approximation for the exponential:

$$x^* \approx \sqrt{2 \left(\frac{a}{b}\right) \left(\frac{g}{r}\right)} \quad (2)$$

Figure 2 shows x^* versus g/r for several values of the a/b ratio, with the approximation displayed for $a/b = 300$.

2.1 Sensitivity to parameters

The sizing curves tend to be shallow. Figure 1 shows that even with size varying from $1/2$ to 2 times the optimum, the present worth varies by about 10 percent for the case of 200 pairs per year growth and 15 percent for the 50 pairs per year case. This point is even stronger if we consider that according to approximation (2), our estimate of growth rate would have to be in error by about a factor of four in order to make that much error in size!

Having made such a sweeping statement, we caution the reader that percent of present worth may not always be an appropriate measure of

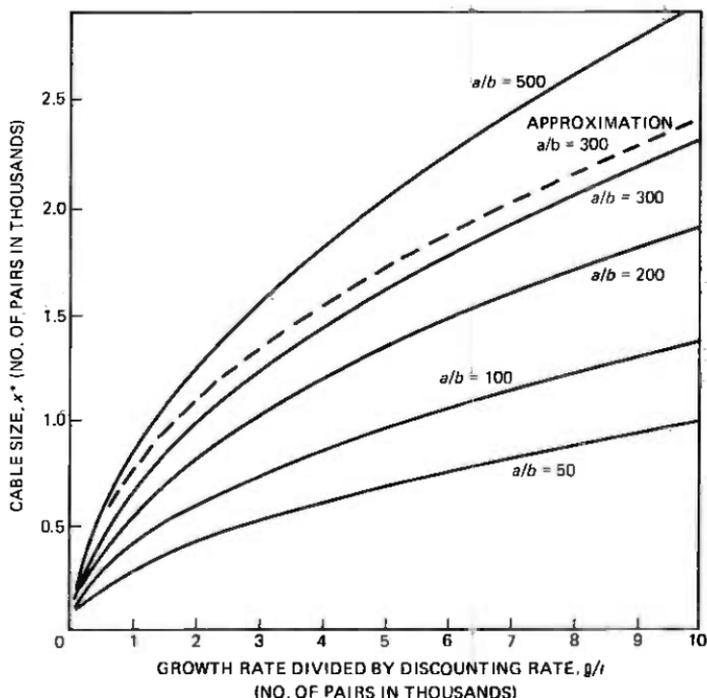


Fig. 2—Economic cable size.

the penalty for incorrect decisions. It may be more appropriate, for example, to first subtract obviously "uncontrollable" components from the total. One such component is the "b-cost" of facilities in service. That is, even if there were no "a" component in the $a + bx$ cost, we would install capacity continuously and *still* incur a present worth cost of

$$PW_b = \int_0^{\infty} b g t e^{-rt} = \frac{bg}{r^2}$$

Also, we are assuming that there is an initial shortage which implies that we *must* incur at least one "a-cost," $PW_a = a/r$, at time zero. For the examples of Fig. 1, we have

$$g = 200: \quad PW_a + PW_b = 11.5$$

$$g = 50: \quad PW_a + PW_b = 3.5$$

Dashed lines are shown in Fig. 1 at these levels. If these amounts are first deducted from present worth, the percentage present worth penalty for doubling or halving the optimal size jumps to about 33 percent.

2.2 Economies of scale

The reason we *have* a cable sizing problem is because of economies of scale in the cost of each cable. Here we have expressed that cost as $a + bx$. In general, any cost function which exhibits decreasing average cost per unit as the number of units increases is said to exhibit scale economies. We would like to buy more at once to take advantage of the lower unit cost but must balance that advantage against the penalty for having to tie up more capital sooner.

In a broader sense, we also speak of economies of scale as referring to the advantages of bigness. In the feeder relief problem, we might consider the potential advantages of using one large route in place of two parallel small ones. Our basic model can provide some insight. Figure 3 plots present worth cost versus growth rate (for the same cost parameters as in Fig. 1). The upper curve assumes that a 1000-pair cable will be used regardless of growth rate, while the lower curve assumes that an optimally sized cable will be used at each growth rate. It is straightforward to verify that for either curve the present worth cost per unit of growth decreases as the amount of growth served increases. For example, if we combine two parallel routes with a growth rate of 200 pairs per year into one with 400 pairs per year, we would save

$$PW_1 = 2(16.52) - 28.78 = \$4.26 \text{ per foot}$$

using optimally sized cables, or

$$PW_2 = 2(16.52) - 29.39 = \$3.65 \text{ per foot}$$

even if we had to use the 1000-pair cables in the combined route. That

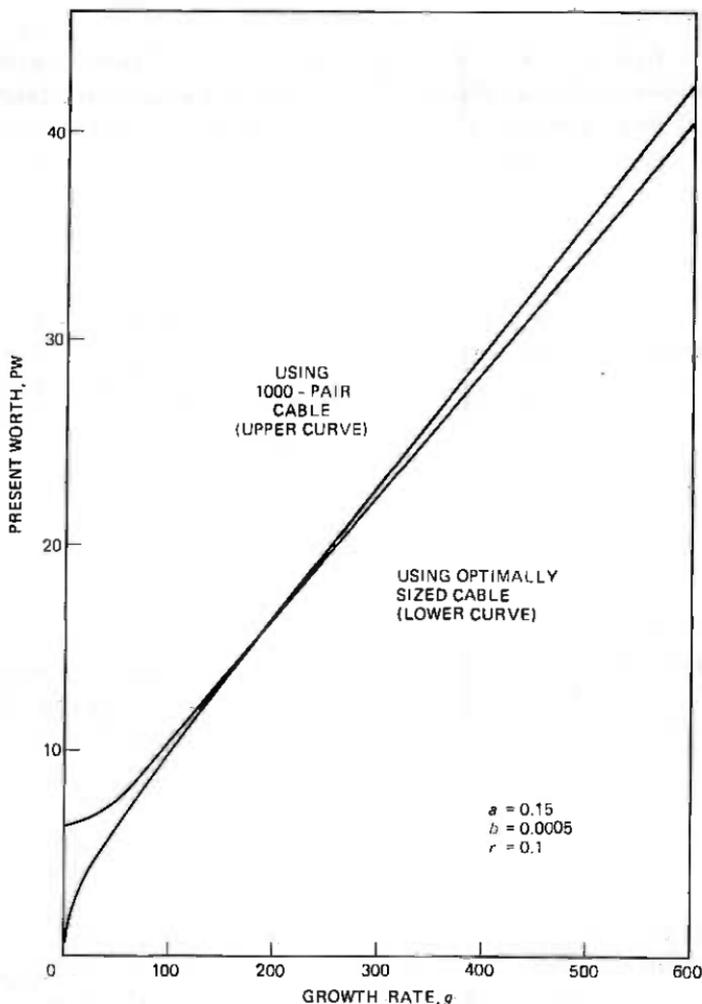


Fig. 3.—Economy of scale.

is, even though the 1000-pair cable is optimal for $g = 200$, and is non-optimal for $g = 400$, we still save by combining the routes. Intuitively, we can think of the savings in combining the routes as attributable in part to eliminating one a -cost at time zero (present worth of $0.15/0.1 = \$1.5$ per foot), in part to utilizing excess capacity faster, and in part to being able to take advantage of a larger, lower unit cost cable.

III. CONDUIT AND OTHER COSTS

Here we expand the basic model to consider the effects of various complications, such as impending conduit shortage, and extra buried cable costs. We still retain the assumption of linear growth in a single gauge. These extensions are based on the *cost-of-the-future* formulation.

3.1 Cost of the future formulation⁶

Instead of starting with equal cable sizes for all future relief, suppose we assume that relief cables will be sized x_0, x_1, x_2, \dots for a total present worth cost of

$$PW = \sum_{i=0}^{\infty} \frac{a + bx_i}{r} e^{-rt_i} \quad (3)$$

where $t_0 = 0$ and $t_i = \sum_{j=0}^{i-1} x_j/g$ for $i > 0$. We can also write (3) as

$$PW = \frac{a + bx_0}{r} + PW_F e^{-rx_0/g} \quad (4)$$

with

$$PW_F = \sum_{i=1}^{\infty} \frac{a + bx_i}{r} e^{-rt_i} \quad (5)$$

where $t_1 = 0$ and $t_i = \sum_{j=1}^{i-1} x_j/g$ for $i > 1$. In this form, we note that PW_F , the *cost of the future*, is independent of x_0 , the size of the initial cable. Given a sequence of cables for all but the initial shortage, or just its cost, PW_F , we can use (4) to find the optimal size of the initial cable. With a continuum of sizes, x_0 , available, the minimizing size is the one for which the derivative of (4) is zero, treating PW_F as a constant:

$$x_0^* = \frac{g}{r} \ln \frac{r^2 PW_F}{bg} \quad (6)$$

To actually minimize (4) over the entire sequence of relief cables, we must clearly use the minimal PW_F ; but that implies minimizing (5) which is mathematically identical to (3). Thus we have a recursive, or backward dynamic programming formulation. It can be shown that, starting with any positive value for PW_F , if we successively use (6) (truncating any negative sizes to zero) to get improved estimates of size and (4) to get improved estimates of PW_F , we converge to the optimal solution. A sample computation in the next section (Table Ia, first three columns) illustrates.

3.2 Including conduit

Suppose that placement of each cable, regardless of its size, uses up a conduit duct and that when all ducts are used up a new conduit system must be built at a cost of $\alpha + \beta N$ dollars, annual charge, per foot for N ducts.

A slight generalization of the formulation of Section 3.1 gives us a handy algorithm. Let PW_i be the total present worth cost of placing all cable and conduit starting from a time when there are i spare conduit ducts and no spare cable available; and let x_i be the corresponding optimal cable size. Note that these cables are numbered backward in time

unlike those of Section 3.1. Assuming that N ducts of conduit will be installed at a time, we can write

$$PW_i = \min_{x_i} \left\{ \frac{a + bx_i}{r} + PW_{i-1} e^{-rx_i/g} \right\} \quad (7)$$

for $i = 1, \dots, N$, and

$$PW_0 = \frac{\alpha + \beta N}{r} + PW_N \quad (8)$$

Of course, each minimizing x_i can be very quickly found by using the appropriate PW_F in (6). It can be easily shown that the PW_i of (7), and hence the optimal x_i , form a monotone sequence with x_i approaching the size minimizing (1) and PW_i the corresponding PW . In view of (8), the sizes must decrease as more spare cable spaces are available. That is, as a conduit system is filled, it becomes optimal to install larger cables to defer the impending cost of building another conduit system.

We can also find the optimal *conduit size* if we replace (8) with

$$PW_0 = \min_N \left\{ \frac{\alpha + \beta N}{r} + PW_N \right\}$$

It turns out that the term in brackets is unimodal in N , so that we can stop at the first local minimum. Table I shows a sample calculation using discounting rate, $r = 0.1$; cable cost, $a + bx = 0.15 + 0.0005x$; conduit cost, $\alpha + \beta N = 1.0 + 0.1N$ dollars per year per foot; and $g = 200$ pairs/year. Note the convergence of x_i^* and PW_i^* to the solutions of Section II in the first major iteration. Of course, we could have stopped at $i = 7$ since we had found the minimum with that calculation. Note also the rather rapid convergence even with an initial guess of $PW_0 = 1000$ compared to the optimal $PW_0 = 31.3$.

3.3 Buried cable, aerial cable, partial conduit

We can extend the above analysis to various situations such as the availability of spaces for direct burial of cable or pole-line spaces for additional aerial cable. We start with an estimate of PW_F , the present worth cost of all future relief after the initial spaces are used up. Since size depends on the logarithm of PW_F in (6), our decisions are not usually very sensitive to this value; and so, we might use PW_0 as calculated in Section 3.2, for example. We then size cables for the initially available spaces, starting with the last space, using (7).

The flexibility of this procedure is illustrated by considering the following *partial conduit* problem. Suppose we can install a buried cable plus a single conduit duct (costing an extra \$0.05 per foot, annual charge) at the current shortage. Then at the time of the next shortage, we must build manholes costing \$0.02 per foot annual charge and place a cable in the duct. From the following shortage onward we will build conduit and place cable as in Section 3.2, $PW_F = 31.3$ less the cost of the man-

Table I — Iterations for cable and conduit size

Ia: Using initial guess of $PW_0 = 1000$				
i (Number of ducts)	x_i^* (6)	PW_i^* (7)	$\frac{\alpha + \beta i}{r} + PW_i^*$ (8)	
1	3000	239.6	250.6	(Maximum size cable available is assumed to be 3000 pairs)
2	3000	70.0	82.0	
3	3000	32.1	45.1	
4	2333	23.2	37.2	minimum
5	1680	19.9	34.9	
6	1376	18.4	34.4	
7	1218	17.6	34.6	
8	1129	17.1	35.1	
∞	1004	16.5	∞	
Ib: Using $PW_0 = 34.4$ from Table Ia				
i	x_i^*	PW_i^*	$\frac{\alpha + \beta i}{r} + PW_i^*$	
1	2295	23.0	34.0	minimum
2	1664	19.8	31.8	
3	1368	18.3	31.3	
4	1213	17.6	31.6	
Ic: Using $PW_0 = 31.3$ from Table Ib				
i	x_i^*	PW_i^*	$\frac{\alpha + \beta i}{r} + PW_i^*$	
1	2282	22.9	33.9	minimum
2	1658	19.8	31.8	
3	1365	18.3	31.3	
4	1212	17.6	31.6	

The optimal solution is to build 3-duct conduits and place cables of 1365, 1658, and 2282 pairs as shortages occur.

holes already built, or $PW_F = 31.3 - 0.2/r = \$29.3$ per foot. These charges are shown on the schematic of Fig. 4. Our solution proceeds backward, starting with x_1 :

$$x_1^* = \frac{g}{r} \ln \frac{r^2 PW_F}{bg} = \frac{200}{0.1} \ln \frac{(0.1)^2 (29.3)}{(0.0005)(200)} = 2150$$

$$PW_1^* = \frac{a + bx_1}{r} + PW_F e^{-rx_1/g} + \frac{0.2}{r} = 24.3$$

$$x_2^* = \frac{g}{r} \ln \frac{r^2 (24.3)}{bg} = 1776$$

$$PW_2^* = \frac{a + bx_2}{r} + PW_1^* e^{-rx_2/g} + \frac{0.05}{r} = 20.9$$

Thus we should place a cable of 1776 pairs along with the conduit duct, and later fill the conduit duct with a cable of 2150 pairs for a total present worth cost of \$20.9 per foot. We note that this is considerably less than the \$31.3 for going directly to a conduit system.

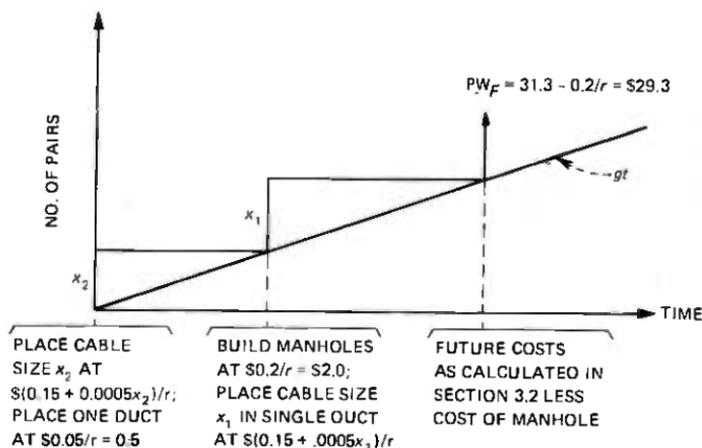


Fig. 4—A partial conduit problem.

It is interesting to ask how much of this savings is attributable to the availability of the partial conduit duct. If such a duct were not available, we might have buried a cable initially and gone directly to the conduit system when more capacity was required. The calculations for this case would be identical to some of those done earlier. In fact, the solution can be read from the $i = 1$ row in Table Ic. It is to use a 2282-pair cable for a total present worth cost of \$22.9 per foot. Thus the availability for the partial conduit duct saves about \$2.0 per foot in this example.

IV. MORE SOPHISTICATED DEMAND MODELS

Here we briefly consider two formulations with more realistic assumptions about demand. In the first, we allow demand to be some nonlinear, but still deterministic, function over the early part of a study. In the second we suppose that demand is a random process and that we wish to make decisions to minimize expected present worth cost.

4.1 Nonlinear demand in the near term

Let $D(t)$ be a nondecreasing function which represents the cumulative number of pairs required over $t = 0$ to T . Beyond T , $D(t) = D(T) + g(t - T)$. Let $PW(t)$ be the present worth cost of meeting all future demand starting from a shortage at time t . If $t > T$, we assume $PW(t) = PW_F$, independent of time. The optimal relief schedule can be found from the following dynamic program

$$PW(t) = \min_{\tau > t} \left\{ \frac{a + b[D(\tau) - D(t)]}{r} + PW(\tau)e^{-r(\tau-t)} \right\}$$

Note that we can build up the PW function by working backward from T , considering only discrete relief times (or, equivalently, discrete relief sizes).

Although it is computationally easy to solve if we keep T reasonably small, this formulation actually goes beyond our analytical model framework for this paper.

4.2 Random demand

Suppose the demand is not assumed to be known deterministically but is instead assumed to be generated by a known stochastic process. Here we only consider processes with *stationary independent increments*. Intuitively, that means the additional future demand (positive or negative) is statistically the same no matter what time or current demand level we start with. An example of such a process is random inward and outward movement of customers according to independent Poisson processes;⁷ another is Brownian motion.²

Our development will be heuristic rather than mathematically rigorous. Let τ_x be the (random) time until we first get x more customers than we currently have (i.e., the first-passage time). With no spare pairs at $t = 0$, τ_x is the time of the next shortage if we place a cable with x pairs. We would thus like to minimize the expected value of

$$PW = \frac{a + bx}{r} + PW_F e^{-r\tau_x}$$

where both PW_F and τ_x are random variables. Because of the statistical independence assumption, PW_F and $e^{-r\tau_x}$ are independent and the expected value of their product is the product of their expected values

$$E[PW] = \frac{a + bx}{r} + E[PW_F]E[e^{-r\tau_x}] \quad (9)$$

where $E[\cdot]$ denotes expected value. The reader might recognize the factor involving τ_x as the Laplace transform of the first-passage time, evaluated at r . We can think of τ_x as the sum of x independent, identically distributed first-passage times to one more unit of demand, τ_1 . The Laplace transform of the sum, τ_x , is the product of the individual Laplace transforms, thus

$$E[e^{-r\tau_x}] = (E[e^{-r\tau_1}])^x$$

Since the Laplace transform is a number between zero and one, we can define an *equivalent* (positive) *growth rate*, g_{eq} , such that

$$e^{-r/g_{eq}} \equiv E[e^{-r\tau_1}]$$

Then we can rewrite (9) as

$$\overline{PW} = \frac{a + bx}{r} + \overline{PW}_F e^{-rx/g_{eq}}$$

where the bar denotes expected value; and we have precisely the form of (4) with g_{eq} replacing g . That is, we can solve this stochastic problem exactly as we would a deterministic one if we only use the equivalent growth rate in place of the deterministic one.

To get an idea of how the equivalent growth rate relates to more familiar quantities, we have plotted g_{eq} versus g_{av} for various σ^2 in Fig. 5, where g_{av} is the expected number and σ^2 is the variance of the additional number of customers per unit time. These curves are derived in Ref. 3 for the Poisson inward/outward movement model. Their most notable feature is that unless the variance is *very* large compared to the average, the equivalent growth rate is only slightly larger than the average growth rate. Thus we conclude that randomness of this type may be ignored for most cable-sizing problems.

V. SIZING UNDER A BUDGET CONSTRAINT

What if, for some reason, we had to get by with less than the ideal overall feeder relief budget for some year? How should we modify our sizing? We model the situation as a constrained optimization. Letting i index all of the relief projects subject to the constraint, and assuming there is only one such constraint,

$$PW_{total} = \text{minimum}_{\text{all } x_i\text{'s}} \left\{ \sum_{\text{all } i} \frac{a + bx_i}{r} + PW_i e^{-rx_i/g_i} \right\} \quad (10)$$

subject to the budget constraint

$$\sum_{\text{all } i} \frac{a + bx_i}{r} \leq \beta \quad (11)$$

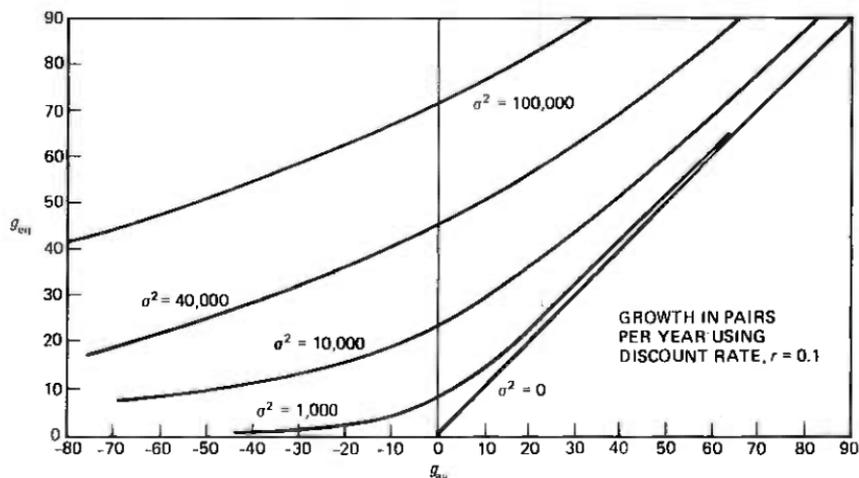


Fig. 5—Equivalent growth rate versus average growth rate.

Note that this is equivalent to constraining the total installed first cost if all annual charge factors are the same (see Section II); a slight modification would allow individual annual charge factors for each project.

Applying a Lagrange multiplier, λ , to the constraint (11) and adding it to the objective function (10),

$$PW_{\text{total}} = \min_{x_i\text{'s}} \sum_i \left\{ \frac{a + bx_i}{r} + PW_i e^{-rx_i/g_i} \right\} - \lambda \left(\beta - \sum_i \frac{a + bx_i}{r} \right)$$

or

$$PW_{\text{total}} = \sum_i \min_{x_i} \left\{ (1 + \lambda) \frac{a + bx_i}{r} + PW_i e^{-rx_i/g_i} \right\} - \lambda\beta$$

Carrying out each minimization,

$$x_i = \frac{g_i}{r} \ln \frac{r^2 PW_i}{bg_i} - \frac{g_i}{r} \ln(1 + \lambda) \quad (12)$$

Thus the optimal solution involves subtracting from the unconstrained optimum (first term), a number of pairs which is directly proportional to growth rate, and is increasing with λ . That is, if we can estimate an appropriate value of the Lagrange multiplier, λ , we can continue to do our sizing on a case-by-case basis even in this constrained situation, by simply replacing (6) with (12).

Of course a general formulation for dealing with budgetary constraints would be considerably more complex. It would include the possibility of different budget constraints in different periods so that we may want to install some cables early, for example, to avoid a pinch in later years. We may also wish to consider deferring construction out of a tightly constrained period at the expense of temporarily increasing the operating costs. (Recall that additional cable is installed before spare is completely exhausted. Some of that spare margin could be used up at a cost.)⁵

VI. FURTHER OBSERVATIONS

There are many problems which can conveniently be studied with a feeder sizing model of this type. We have touched on some; others include the following.

(i) How is the optimal size affected by relieving earlier or later than the nominal relief time? The reader may wish to check that any excess spare or pent-up demand at relief should be subtracted from or added to, respectively, the optimal size.

(ii) Would it pay to remove a small existing cable and replace it with a larger one instead of building conduit right away? A straightforward calculation will show that it is often economical provided the existing cable is small enough and the cost of removal is not too large.

(iii) Would it pay to relieve with pair-gain systems instead of wire pairs? That question is explored by Koontz⁴ in this issue.

(iv) What is the cost of losing a few feeder pairs, for example, because they are defective? The cost in feeder relief is essentially just the advancement of some appropriate PW_F .

Another use for this model has been to obtain approximate solutions within the more sophisticated EFRAP³ sizing algorithm.

We have given many specific feeder sizing problems in which the cost-of-the-future approach works. Of course, it will not *always* be helpful. Generally, it will only be helpful when we can define an appropriate cost of the future which is *independent of time* and at least *relatively independent of prior decisions*.

VII. ACKNOWLEDGMENTS

Many of the ideas presented here had their genesis in unpublished work at Bell Laboratories. Most notably, the simplest model [eqs. (1) through (2)] was studied by D. R. Hortberg, and the cost-of-the-future approach was used by C. E. Warren.

APPENDIX

Levelized Equivalent Annual Cost Associated With a Capital Expenditure— Annual Charge Factors

In calculating the cost of some equipment or service it is necessary to distinguish between costs that are classified as *expense* and *capital*. Due primarily to income tax laws, the impact of a capital expenditure includes not only the immediate cash flow, but additional future financial consequences as well. The Internal Revenue Service (IRS) classifies certain expenditures as *expense*; e.g., most routine service, maintenance, and items which are used up in less than a year. These expenses are immediately deductible from income in calculating income tax. Other expenditures, primarily associated with durable equipment, are classified as *capital*. Tax deductions for capital items are spread out over their useful life (i.e., the items are *depreciated*). The allowed depreciation schedule (i.e., how much can be deducted from income in each year) is liable to be quite complex, with current regulations allowing more deduction in earlier than later years (called *accelerated* depreciation). Furthermore, an *investment tax credit* (a reduction of tax obligation) is generally allowed in the year following a capital outlay. In addition to these tax consequences, the Bell System also includes the effects of its *accounting* system (*book depreciation* is generally different from tax depreciation).

Fortunately, for most outside plant studies, it is not necessary to keep track of these complex financial consequences in detail. All that is required is their present worth or, as we shall describe, their *levelized equivalent annual cost* (LEAC). For study purposes, it is generally adé-

quate to assume some standard financial consequences of the type described above for various classes of capital expenditure. For example, any underground cable with a given installed cost may be assumed to generate an identical stream of tax depreciation allowances, investment tax credits, book depreciation, etc. Thus it is only necessary to examine the detailed financial consequences for representatives of the various classes (e.g., cable of various types, conduit, and repeaters). The same present worth will apply to each member of the class. It is often convenient to scale the results of calculations for each category per dollar of installed first cost (IFC). Furthermore, it is useful to calculate a levelized equivalent annual charge (LEAC) for each category. The LEAC is defined so that the present worth of a constant annuity of LEAC dollars per year equals the present worth of the capital expenditure and all of its associated financial consequences. The LEAC per dollar of IFC, commonly called the *annual charge factor* (ACF), is calculated, perhaps with the aid of a computer program, and tabulated for all of the common outside plant capital expenditures.

In a particular outside plant study, the total financial impact of a capital expenditure is reflected by merely assuming that a constant annual charge of $LEAC = ACF \times IFC$ is incurred starting from the time an item is placed into service. It is commonly the case in outside plant studies that a capital expenditure represents a commitment to continue providing service into the indefinite future, replacing the given equipment by similar equipment at the end of its life (*repeated plant assumption*). In that case, it is appropriate to apply the LEAC from the time an item is placed until the end of the study period (which might be infinite, for example). This allows for valid economic comparisons of plant items with different service lives. The present worth of the LEACs of all capital expenditures plus the present worth of expense* items is called the *Present Worth of Annual Charges* (PWAC). This is taken to be the fundamental economic criterion—among plans providing equal service, the smaller the PWAC, the better.

The actual calculation of the ACFs varies according to the type of plant (e.g., different tax laws apply to short-life versus long-life plant, and to low-salvage versus high-salvage items), as well as to current tax laws (e.g., the investment tax credit seems to change regularly), and to Bell System or regulatory body policy (e.g., normalization or flow-through accounting for differences between book and tax depreciation). The following equations, taken from the "new greenbook,"⁸ are representative of the calculations involved.

The LEAC is the constant annuity whose present value over the service life, L , is

* In general, if there are differences in the revenues generated for the different alternatives under study, these differences should be treated in the same manner as differences in expense flows.

$$\begin{aligned}
 & \text{PW} \left(\begin{array}{c} \text{Capital} \\ \text{recovery} \end{array} + \begin{array}{c} \text{Income} \\ \text{tax} \end{array} \right) \\
 &= (1 + \phi)[\text{IFC} - S] \\
 &\quad - \phi \text{PW}(D_b) \\
 &\quad - \tau(1 + \phi)\text{PW}(D_\tau - D_b) \\
 &\quad - (1 + \phi)[\text{PW}(\text{TC}) - \text{PW}(\text{ATC})];
 \end{aligned}$$

where

$\text{PW}(\cdot)$ designates present worth;

ϕ is the *income tax factor*:

$$\phi = \left(\frac{\tau}{1 - \tau} \right) \left(1 - \delta \frac{i_d}{i} \right)$$

with

τ = income tax rate

δ = debt ratio

i_d = interest cost of debt

i = composite cost of debt and equity

IFC is the installed first cost at time zero

S is the net salvage obtained at the end of the service life

D_b is the book depreciation:

$$\left(\begin{array}{c} \text{Book} \\ \text{depreciation} \\ \text{in year } t \end{array} \right) = \frac{\text{IFC} - S}{L}$$

D_τ is the tax depreciation which varies from year to year according to

$$\left(\begin{array}{c} \text{Tax} \\ \text{depreciation} \\ \text{in year } t \end{array} \right) = \begin{cases} 2/L_\tau & \text{in year 1} \\ \left(1 - \frac{2}{L_\tau} \right) \left(\frac{2}{L_\tau} \right) \left(\frac{L_\tau + 1 - t}{L_\tau - 1} \right) & \text{in other years} \\ -S & \text{in year } L \end{cases}$$

but with the proviso that no further depreciation is allowed once the year-by-year total amount depreciated reaches the IFC

L_τ is *tax life*, generally 80% of L

TC is the investment tax credit (e.g., 10 percent of IFC in year 1)

ATC is the amortized tax credit:

$$\text{ATC} = \text{TC}/L$$

This formula applies under several assumptions:

(i) Tax credits are flowed through rather than normalized (last two terms would be different).

(ii) Tax depreciation is calculated according to double-declining balance in year 1 and sum-of-years digits thereafter (or there would be a different formula for year-by-year tax depreciation).

(iii) The asset depreciation range (ADR) system is allowed and $L \geq 3$ so that $L_r = 0.8L$.

(iv) Salvage is less than 10 percent of IFC so that allowable depreciation for tax purposes is the total IFC (otherwise, less year-by-year depreciation would be allowed with the difference made up in year L).

(v) The entire IFC is to be capitalized both for book and tax purposes (sometimes the IRS allows part of installation costs, capitalized on the books, to be treated as expense in tax calculations).

Further discussion of this formula or the assumptions behind it is beyond the scope of this paper. The interested reader is referred to the new "greenbook."⁸

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Loop Plant Modeling:

Economic Evaluation of Subscriber Pair Gain System Applications

By W. L. G. KOONTZ

(Manuscript received August 20, 1977)

In recent years, the cost of subscriber pair gain systems, i.e., systems which enable more than one subscriber to be served by a single cable pair, has decreased significantly in comparison with cable. Moreover, the operating expenses associated with an all-cable loop network have increased along with the cost of labor, particularly in areas of high customer mobility and uncertain growth. For these reasons, the application of pair gain systems has become an important consideration in loop plant design. This paper presents and discusses a series of mathematical models which can be used in the economic analysis of subscriber pair gain system applications. Given the forecast requirement for loop facilities, one may use these models to evaluate and compare alternatives for meeting this requirement on a present worth basis. The alternatives may include cable only, pair gain systems only, or a combined cable/pair gain alternative (deferred cable). These models have been applied in Bell Laboratories studies of the market for pair gain systems. They are now being incorporated into guidelines which will enable the operating companies to apply pair gain systems in an economic manner.

I. INTRODUCTION

Subscriber pair gain systems, which use carrier and concentrator techniques to reduce requirements for loop cable, have been available since the 1950s. Until recently, however, their high cost relative to cable has limited their application to very long rural routes requiring expensive coarse-gauge cable. Now, as a result of improved technology, the cost of pair gain is competitive with cable in the suburban, as well as the rural, environment. In addition, the reliability of pair gain systems has been

greatly improved and this has also contributed to their increased attractiveness. Thus, subscriber pair gain systems have become an important consideration in economically expanding the capacity of the loop network.

In this paper, a series of capacity expansion models which consider both pair gain and cable will be developed. These models are extensions of the models developed in Ref. 1. The capacity expansion models are used to develop the optimal strategy for adding capacity to the loop network with a combination of pair gain and cable. The optimal strategy for the basic model is developed in Section II and some specific cases are studied in Sections III and IV. The problem of network complexities and a simple method for dealing with this problem is discussed in Section V. Mathematical programming approaches, which have been implemented as computer programs, will be discussed in Section VI. Finally, some advanced models, which reflect the stochastic nature of subscriber demand and loop network activities, will be introduced in Section VII.

The operating companies have felt an increased need for guidance in the proper application of pair gain systems. The pair-gain/cable capacity expansion theory developed in this paper forms a basis for application guidelines and computer programs now used by the operating companies in planning pair gain system application. The theory has also been applied within Bell Labs to suggest new applications for pair gain systems and to develop improved designs for the loop network.

In order to follow the theoretical development, it will be useful to have additional background information regarding pair gain systems. This section will include, therefore, an overview of subscriber pair gain systems and their applications.

1.1 Subscriber pair gain systems

The basic structure of a subscriber pair gain system is illustrated schematically in Fig. 1. The system consists of a central office (CO) unit, located in the central office building, and a remote unit, located in the field. A given number, say L , of 1-party subscribers* are connected to the remote unit by individual wire pairs which will be called subscriber lines. The remote unit is connected to the CO unit by K wire pairs ($K < L$) which will be called CO links. The CO unit effectively converts the K links into L line appearances at the CO. The CO unit may be integrated into the switching equipment such that physical expansion of the K links into L lines is not required. The *pair gain*, which is defined as the dif-

* For the purposes of this paper, multiparty subscribers may be grouped into equivalent 1-party subscribers.

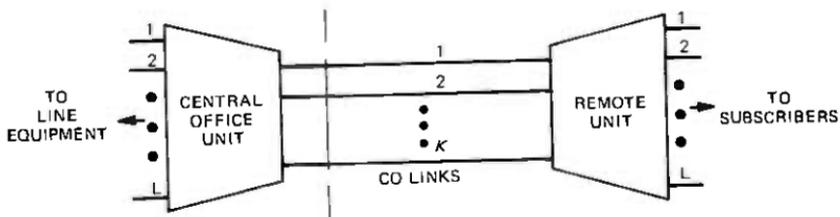


Fig. 1—Structure of a pair gain system.

ference $L - K$, is the net reduction in cable pair requirements achieved by the pair gain system.

There are two basic approaches to achieving pair gain. In a *carrier* system, time or frequency division multiplexing is used to derive additional voice and signaling channels over each CO link. For example, the *SLC*TM-1* (Subscriber Loop Carrier: $L = 2$, $K = 1$) derives a second line from a single wire pair by means of amplitude modulation. Another example is the *SLC*-40 ($L = 40$, $K = 2$), which uses a delta modulation scheme to derive 40 channels over 2 wire pairs (using digital repeaters).

The other basic approach is concentration. In a concentrator, each of the L subscribers has access through a switching network to either all, or a subset of, the K links. When a subscriber goes off-hook, an idle link is connected to his line. The *LSS* (Loop Switching System: $L = 192$, $K = 66$) is a concentrator in which each subscriber has access to 7 links. The *LSS* switching network employs miniature relays under microprocessor control. If no idle link can be connected to the off-hook subscriber, the call is blocked. A concentrator must be designed and operated to maintain a low probability of blocking, consistent with grade of service objectives.

A system does not have to be pure carrier or concentrator. The *SLM*TM ($L = 80$, $K = 2$) concentrates 80 lines down to 24 channels which are derived from 2 links via delta modulation.

Additional discussion of pair gain techniques is beyond the scope of this paper. The interested reader may find more information in Ref. 2. In the balance of this paper, pair gain systems will be entirely characterized by L , K , and cost parameters.

* The actual pair gain systems referred to in this paper are Bell System products. Similar systems are available through the general trade.

1.2 Pair gain system applications

Clearly, pair gain systems reduce the need for subscriber cable pairs and therefore the obvious application of pair gain is as an alternative to additional cable. However, the determination of an economic policy for pair gain application is not simply a matter of deciding whether to use pair gain or cable. Loop network capacity expansion is a dynamic process involving the questions of when to add capacity and how much new capacity to add. The pair gain alternative adds the question of by what means to add new capacity.

Consider a route which is experiencing growth and whose existing capacity is exhausted. Any of the following alternatives may be appropriate:

- (i) Place a new cable.
- (ii) Place one or more pair gain systems, using existing cable pairs as links.
- (iii) Place one or more pair gain systems initially, using existing cable pairs as links. When these systems exhaust, remove them and place a new cable.

Alternative (i) is the classic "all cable" solution which is emphasized in Ref. 1. Alternative (ii) is often called a permanent pair gain solution, since the pair gain systems are not removed. Alternative (iii) is called a temporary pair gain solution in which the relief cable is deferred, but once it is placed, the pair gain systems are removed. Generally speaking, the cost of the pair gain system relative to cable must be lower to justify (ii) rather than (iii). Thus (ii) is prevalent primarily on long rural routes while (iii) is more characteristic of suburban applications.

In the theoretical development which follows, neither of the above alternatives will be assumed *a priori*. Rather, a general formulation will be developed and it will be shown that each of these alternatives may be optimal under different circumstances.

Some "special" applications will be touched upon in Section VII. First of all, the application of single channel pair gain to the provision of second line service will be analyzed. Secondly, the application of pair gain systems as an alternative to network rearrangements will be studied. These latter results are quite preliminary and are included to stimulate further work.

II. BASIC CAPACITY EXPANSION MODEL

A basic model for loop network capacity expansion using pair gain and cable is derived in this section. Specifically, the model expresses the total PWAC (present worth of annual charges) associated with a generalized pair gain application policy. The minimum PWAC policy will be computed and its properties will be examined.

2.1 PWAC model

The PWAC model will be derived for the simplified route illustrated schematically in Fig. 2. The route consists of a single feeder section between the CO and the remote terminal site for the pair gain systems. It is assumed that at time $t = 0$, no pair gain systems are in place and the existing cable has just exhausted. Subscriber demand is assumed to be growing linearly with growth rate g .

The generalized application policy is as follows: From time $t = 0$ to $t = T$ ($T \geq 0$), additional capacity will be provided by means of pair gain. At time $t = T$, all pair gain systems are removed and a relief cable of size S is placed. Both T and S are design parameters to be optimized. The optimal values of T can be related to the three alternatives discussed in Section I as follows:

- (i) $T = 0$ (all cable)
- (ii) $T = \infty$ (permanent pair gain)
- (iii) $0 < T < \infty$ (temporary pair gain)

It is implicitly assumed that the existing cable can supply the pairs necessary for links.

During the time interval $[0, T]$, a pair gain system cost is incurred. This cost generally includes the cost of the pair gain equipment. It is assumed here that pair gain systems are "rented" from a central "supplier" for a given annual charge. This annual charge is incurred for each pair gain system from the time it is installed until the time it is removed. The amount of the annual charge depends upon the cost of the pair gain equipment, its service life, characteristics of the supplier, and other factors. The details of computing the annual charge will not be discussed here.

The installation and removal costs are incurred whenever a pair gain system is installed or removed. Depending on the tax status of these costs, they may be treated as one time charges or levelized over the period during which the pair gain system remains at a particular location. The details of computing installation and removal charges will also be omitted from this discussion.

In the basic derivation which follows in this section, the total pair gain system cost will be expressed as an annual charge rate $\gamma(t, T)$. The annual charge rate is time varying since additional pair gain systems may be installed during $[0, T]$. The annual charge rate may also depend on T if installation or removal charges are levelized over the period during which the pair gain systems are applied. Note that one time charges will result in impulses in $\gamma(t, T)$.

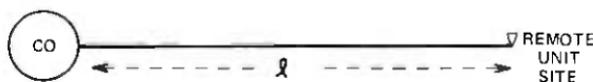


Fig. 2—Basic pair gain application.

The cable cost is expressed as an annual charge which begins at time T and continues forever. The annual charge is assumed to be of the form $(A + BS)\ell$ where S is the number of cable pairs provided and ℓ is the length of the cable. This cable will exhaust at time $t = S/g$ (assume $S/g > T$). All cash flows beyond this time are represented here by an equivalent present worth cost of the future C_F which is incurred at time $t = S/g$.

Figure 3 illustrates the cash flow assumed for the pair gain/cable capacity expansion model. The total PWAC for pair gain, cable, and all future relief is given by

$$\text{PWAC} = \int_0^T \gamma(t, T) e^{-rt} dt + \frac{1}{r} e^{-rT} (A + BS)\ell + C_F e^{-rS/g} \quad (1)$$

where r is the convenience discounting rate. In the linear growth case, with no conduit or other complications, the future capacity expansion starting at time $t = S/g$ is identical to the one starting at time $t = 0$. If the same T and S are used *ad infinitum*, $C_F = \text{PWAC}$ so that

$$\text{PWAC} = (1 - e^{-rS/g})^{-1} \times \left[\int_0^T \gamma(t, T) e^{-rt} dt + \frac{1}{r} e^{-rT} (A + BS)\ell \right] \quad (2)$$

For $T = 0$, eq. (2) reduces to the PWAC equation for the corresponding cable sizing problem.

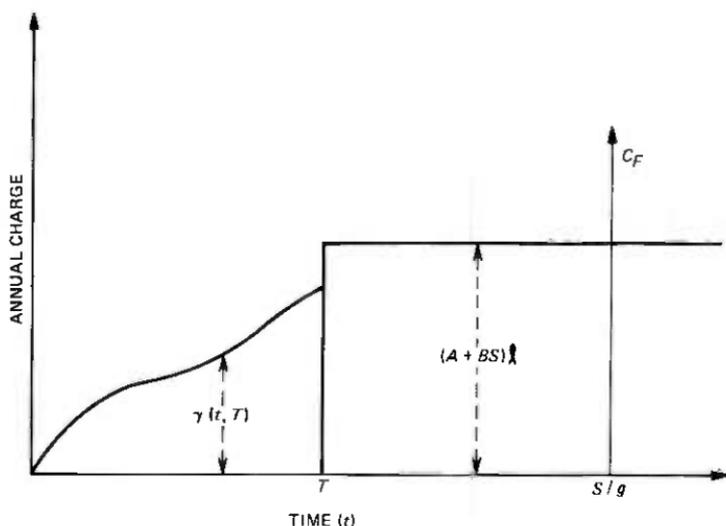


Fig. 3—Cash flow for general pair gain/cable policy.

2.2 Minimum PWAC policy

A set of equations for the optimal solution, (T^*, S^*) follows from the necessary conditions

$$\left. \frac{\partial \text{PWAC}}{\partial T} \right|_{\substack{T=T^* \\ S=S^*}} = 0 \quad (3)$$

and

$$\left. \frac{\partial \text{PWAC}}{\partial S} \right|_{\substack{T=T^* \\ S=S^*}} = 0 \quad (4)$$

The basic equations which follow from carrying out the differentiation and rearranging are

$$\gamma(T^*, T^*) + \int_0^{T^*} \frac{\partial \gamma(t, T)}{\gamma T} \Big|_{\substack{T=T^* \\ S=S^*}} e^{-rt} dt = (A + BS^*)\ell \quad (5)$$

and

$$e^{rS^*/g} - rS^*/g - 1 = \frac{r}{gB} \left[A + re^{rT^*} \int_0^{T^*} \ell^{-1} \gamma(t, T^*) e^{-rt} dt \right] \quad (6)$$

These equations, although somewhat complex, can be readily interpreted. Equation (5) requires that the pair gain systems be removed when the effective annual charge for pair gain equals the annual charge for the relief cable. Equation (6) is the standard cable sizing equation except that a positive term has been added to the cable A cost. This means that when temporary pair gain systems are used, the relief cable is oversized in comparison to the all cable solution.

Equations (5) and (6) define the general solution to the pair gain/cable capacity expansion model. In Sections III and IV, some specific cases will be explored.

III. SINGLE CHANNEL APPROXIMATION

The first case to be studied is an approximation to a single channel pair gain system ($L = 2, K = 1$). Let $\gamma(t, T) = \gamma gt$ where the constant γ is roughly interpreted as the annual charge per pair gained. This approximation ignores installation and removal charges and the effect of discretization. These effects will be considered in Section IV. For this special case, eqs. (5) and (6), after some manipulation, become (dropping the * notation)

$$\gamma g T = (A + BS)\ell \quad (7)$$

and

$$e^{rS/g} - 1 = \frac{\gamma}{B\ell} (e^{rT} - 1) \quad (8)$$

from which it can be determined that the optimal S satisfies

$$e^{\beta(\alpha+rS/g)} - \beta e^{rS/g} = 1 - \beta \quad (9)$$

where

$$\alpha = \frac{rA}{gB} \quad (10)$$

and

$$\beta = \frac{B\ell}{\gamma} \quad (11)$$

The α parameter of eq. (10) appears in the cable sizing equations derived in Ref. 1. The β parameter of eq. (11) is the ratio of incremental cable cost to cost per pair gained.

Equation (9) can be easily solved by standard numerical techniques. Figures 4 and 5 illustrate the solution as a function of β for various values of the other parameters. The optimal deferral period T is plotted in Fig. 4 and the optimal cable size S is plotted in Fig. 5. These curves illustrate some important points about the application of pair gain. First of all, the optimal deferral period increases with β . This result reflects the fact that when the cable cost is high relative to pair gain (e.g., when the loop length ℓ is large), longer deferrals are economical. As β increases to 1, T increases without bound and $\beta \geq 1$ corresponds to a permanent application of pair gain. Secondly, the curves illustrate the impact of other

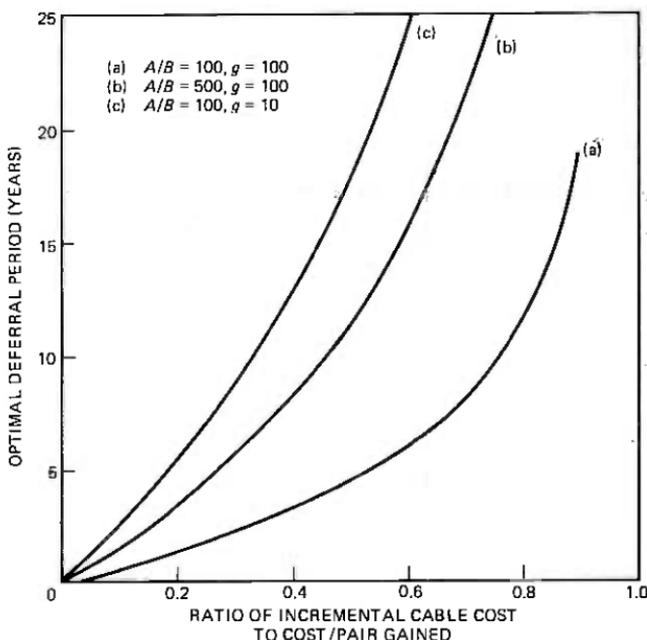


Fig. 4—Optimal deferral period curves.

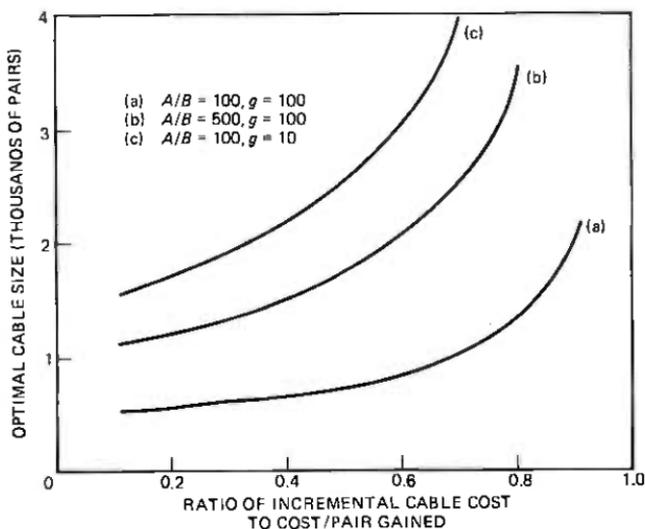


Fig. 5—Optimal cable size curves.

important parameters: A/B and the growth rate g . For larger A/B or less growth, longer deferrals are economical. Thus, the prime areas for pair gain application are those with slow growth and a high "fixed cost" for cable placement. Finally, the curves in Fig. 5 show quantitatively the increase in relief cable size that results from cable deferral with pair gain.

Note that for any $\beta > 0$, pair gain can be economically applied. Of course, it is not practical to apply pair gain for a very short time because of the cost of installation and removal. However, this result suggests that if installation/removal costs are low enough, short term deferrals will pay.

The results derived above show that, under certain assumptions, the all-cable solution is optimal only for very small β , the temporary pair gain solution is optimal for $\beta < 1$, and the permanent pair gain solution is optimal for $\beta \geq 1$. Even if $\beta < 1$, however, permanent pair gain may "prove in," i.e., compare favorably on a PWAC basis with the all-cable solution. Consider the following example:

$$\begin{array}{ll}
 A = \$0.167/\text{ft} & \gamma = \$50./\text{pair gained} \\
 B = \$0.00167/\text{ft} & r = 0.07 \\
 \ell = 20 \text{ Kft} & g = 50 \text{ lines/yr.}
 \end{array}$$

The all-cable solution is obtained by solving the standard cable sizing equation,

$$e^{rS/\ell} - rS/g - 1 = \frac{rA}{gB} \quad (12)$$

which yields $S = 347$ pairs. The PWAC for the all-cable solution, which is obtained from eq. (2) with $T = 0$, is \$554K. The PWAC for the permanent carrier solution is given by

$$\begin{aligned} \text{PWAC} &= \int_0^{\infty} \gamma g t e^{-rt} dt \\ &= \frac{\gamma g}{r^2} \end{aligned} \quad (13)$$

which, for this example, is \$510K. Therefore, permanent carrier proves in by \$44K over the all-cable solution.

On the other hand, the optimal policy for this example is to use pair gain for 10.72 years and then place a 702 pair cable. The optimal PWAC is \$430K which is an additional \$80K savings. Thus, temporary pair gain must always be considered, even for long routes where permanent pair gain proves in. Of course, there may be additional benefits which favor permanent rather than temporary application. In the next section, however, it will be shown that some of these benefits can and should be accounted for in the economic analysis.

IV. LUMPED PAIR GAIN MODELS

The single channel approximation in Section III does not adequately represent larger "lumped" pair gain systems. A lumped system provides pair gain in discrete steps. For example, one unit of *SLC-40* provides a pair gain of 38 (40 lines - 2 links). Also a lumped pair gain system incurs substantial installation and removal costs. In this section, more complex forms of $\gamma(t, T)$ will be developed to represent lumped systems.

4.1 Annual charge model for lumped systems

The cost of a lumped pair gain system can be characterized by three components, an annual charge a , an installation charge I , and a removal charge R . The annual charge represents the cost of the pair gain equipment (both CO unit, remote unit, and repeaters) annualized over its effective service life. The installation charge is incurred whenever a system is installed and is assumed to be levelized over the period during which the system remains in place. The removal charge is assumed to be a one time charge which occurs when a system is removed.

For example, suppose a system is installed at time $t = 0$ and removed at time $t = T$. Then the total PWAC for the system application is

$$\begin{aligned} \text{PWAC} &= \int_0^T [a + m(T)I] e^{-rt} dt + R e^{-rT} \\ &= M(T)I + \int_0^T a e^{-rt} dt + R e^{-rT} \end{aligned} \quad (14)$$

where $M(T)$ is a factor giving the present worth of annual costs associated with each dollar of capital which is to be recovered over T years. For example, if simple straight-line depreciation is used for both book and tax purposes,

$$M(T) = 1 + \phi(1 - (1 - e^{-rT})/rT) \quad (15)$$

where ϕ is the income tax factor (Ref. 3). With modern tax laws, eq. (15) is liable to be considerably more complex, but the above form can be used here for illustrative purposes.

It may be that taxes will be calculated on the basis of an average value of T rather than the actual value. In this case, $M(T)$ is a constant and the PWAC calculations are much simpler. In the derivations which follow, however, the more general case, where the annual charge factor for I depends on T , will be assumed.

Now consider the route of Fig. 2 and assume that the demand is met by placing a sequence of N pair gain systems, each having a pair gain of η . The n th system ($1 \leq n \leq N$) is installed at time $t = \tau_n$ where $\tau_n = (n - 1)\eta/g$. At time $t = \tau_{N+1} = N\eta/g$, all N systems are removed and a cable of size S is placed. When the cable exhausts, the relief cycle is repeated. The annual charge for the n th system is given by

$$\text{a.c.} = \begin{cases} a + m(\tau_{N+1} - \tau_n)I + \delta(t - \tau_{N+1})R & \tau_n \leq t \leq \tau_{N+1} \\ 0, & \text{otherwise} \end{cases} \quad (16)$$

where $\delta(t)$ is a Dirac delta function. The total annual charge function $\gamma(t, N)$ (for convenience, N is used as a control variable rather than T) is therefore given by

$$\gamma(t, N) = \begin{cases} na + \sum_{k=1}^n m(\tau_{N+1} - \tau_k)I, & \tau_n \leq t < \tau_{N+1} \\ N\delta(t - \tau_{N+1})R & t \geq \tau_{N+1} \end{cases} \quad (17)$$

4.2 Optimal relief policy using lumped pair gain systems

The total PWAC of the relief policy outlined above follows from Eq. (2):

$$\text{PWAC} = (1 - e^{-rS/g})^{-1} \times \left[\int_0^{N\eta/g} \gamma(t, N)e^{-rt} dt + \frac{1}{r} e^{-rN\eta/g} (A + BS)\ell \right] \quad (18)$$

For a given value of N , the optimum cable size is obtained as the solution of

$$e^{rS/g} - rS/g - 1 = \frac{r}{gB} \left[A + re^{rN\eta/g} \int_0^{N\eta/g} \ell^{-1} \gamma(t, N)e^{-rt} dt \right] \quad (19)$$

The optimal policy is determined by solving eq. (19) for $N = 0, 1, \dots$, and choosing the solution which minimizes the PWAC given by eq. (18). This process can be programmed quite easily, since the integral in eqs. (18) and (19) reduces to a summation, i.e.,

$$\int_0^{N\eta/g} \gamma(t, N) e^{-rt} dt = \frac{1}{r} \sum_{n=1}^N a e^{-r\tau_n} + m(\tau_{N+1} - \tau_N) I (e^{-r\tau_N} - e^{-rN\eta/g}) + N \left(R - \frac{a}{r} \right) e^{-rN\eta/g} \quad (20)$$

4.2.1 Example

The following data will be used to illustrate the above process

$A = \$167/\text{ft}$	$a = \$2500$	$\eta = 50$
$B = \$.00167/\text{ft}$	$I = \$1000$	$r = 0.07$
$\ell = 15 \text{ Kft}$	$R = \$500$	$g = 50 \text{ lines/yr}$
$\phi = 0.7$		

It is assumed that $M(T)$ is given by eq. (15). Table I lists the solutions to eq. (19) and the PWAC from eq. (18) as a function of N . The minimum PWAC solution is obtained for $N = 5$. Thus, the optimum pair gain policy is to install one pair gain system per year until 5 systems have been placed, and then, at the end of year 5, remove the 5 systems and place a 520 pair cable. The total PWAC, including all pair gain installation, carrying, and removal charges and cable cost, is \$373K.

The PWAC for the all-cable solution ($N = 0$) is \$416K and the PWAC for the permanent carrier solution ($N = \infty$) is \$553K. Therefore, in this example, permanent pair gain does not prove in over cable, but temporary pair gain provides significant savings. If the relative costs of cable

Table I — Lumped pair gain example

Number of systems	Cable size (pairs)	PWAC (\$1000s)	Optimum solution
0	346	416	
1	365	398	
2	392	386	
3	429	378	
4	472	374	
5	520	373	*
6	571	374	
7	624	375	
8	679	378	
9	735	381	
10	792	385	
∞	—	553	

and pair gain are varied, however, the result may change. For example, if $\ell = 25$ Kft, the solutions are:

- (i) All cable ($N = 0$)—\$693K
- (ii) Optimum ($N = 19$)—\$527K
- (iii) Permanent pair gain ($N = \infty$)—\$553K

Now permanent pair gain proves in over cable, but temporary pair gain is still optimal. If ℓ is further increased to 40 Kft, permanent pair gain is optimal. On the other hand, if ℓ is reduced to 5 Kft, the all-cable solution becomes optimal.

These results parallel the results obtained from the single channel approximation in Section III. Thus, the general nature of the optimal pair gain application policy is not affected by the considerations of installation and removal cost and the discrete sizes of pair gain systems.

V. RELIEF PROJECT DEFERRAL—A PRACTICAL APPROACH

Up to now, consideration has been limited to the simple network illustrated in Fig. 2. In practice, however, loop networks are much more complex.⁴ This section discusses some of the complexities of the loop network which must be considered and provides a simplified approach to dealing with them. In Section VI, more sophisticated mathematical programming approaches are outlined.

5.1 Loop network complexities

The simple network of Fig. 2 consists of a single cable section and a single point at which pair gain systems may be placed. A real loop network is composed of many interconnected cable sections and many potential pair gain system sites. In general, the capacity of the network may be expanded by placing additional cable or deploying pair gain systems throughout the network. Even for a moderately complex network, the number of alternatives for providing additional capacity is enormous.

Specifically, whenever a facility shortage occurs anywhere in the network, one or more of the following steps may be taken.

- (i) Place additional cable (where? how much?)
- (ii) Place or remove pair gain systems (where? how many? what kind?)

Clearly, the basic model of Section II cannot handle this complex problem. On the other hand, a truly general formulation is not practically solvable even by sophisticated mathematical programming techniques (see Section VI). Thus, it is necessary to simplify the general problem to one which is amenable to available techniques. In Section 5.2, it is shown that the problem can be simplified to the extent that a variation of the approach developed in Sections II–IV can be applied.

5.2 Deferral of a feeder relief project

Although feeder relief cables are sized independently for each feeder section,¹ the actual provision of relief is through a sequence of relief projects. A project generally provides relief for a feeder route for a period of at least two years by relieving one or more feeder sections. The consolidation of section relief cables into route relief projects is a practical measure which strikes a balance between the PWAC penalty for advancing the relief of some sections and the costs of complex splicing between sections as well as project overhead.

It will be assumed that a relief project is indivisible and its make-up (cable sizes, etc.) and cost are fixed. The only variable is the time at which the project is placed. It is also assumed that the project clears all shortages in the network. Finally, it is assumed that there is a single site at which pair gain systems may be located so as to reduce the cable pair demand in the sections requiring relief.

Figure 6 illustrates this formulation of the problem. The remote unit site is connected to the CO through a series of feeder sections and the relief project spans one or more of these sections. Subscribers beyond the remote unit site may be served by pair gain systems resulting in a net reduction in demand in the feeder sections shown.

Under these assumptions, the optimization problem is greatly simplified. Whenever a shortage occurs, only two options are available:

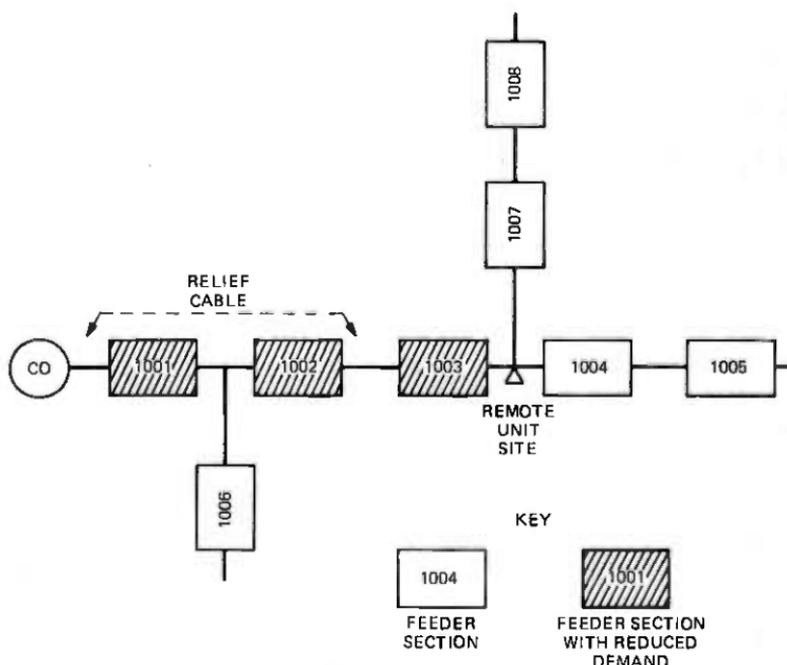


Fig. 6—Relief project deferral.

(i) Place an additional pair gain system

(ii) Place relief cable and remove all pair gain systems.

Moreover, once the project is done, there are no further decisions until the next project comes along. Thus, the basic question is how many systems should be placed, or, equivalently, how long should the project be deferred.

5.3 PWAC model

As in Section IV, it is assumed that a sequence of N pair gain systems are placed at times $\tau_1, \tau_2, \dots, \tau_N$ and, at time τ_{N+1} , the N systems are removed and the project (cable) is placed. It is necessary, however, to generalize the definition of the τ_n to be the time at which the pair demand just exceeds the capacity of the cable network augmented by $n - 1$ pair gain systems. For τ_n defined this way, the pair gain annual charge function $\gamma(t, N)$ is still given by eq. (17). If the annual charge for the relief project is A , then the total PWAC is given by

$$\text{PWAC} = \int_0^{\tau_{N+1}} \gamma(t, N) e^{-rt} dt + \frac{1}{r} A e^{-r\tau_{N+1}} + C_F e^{-rT_F} \quad (21)$$

where T_F is the time at which the demand exceeds the capacity of the relief project and C_F is the PWAC for all future relief.

It is assumed that, for the optimal (minimum PWAC) value of N , $\tau_{N+1} < T_F$ and that both C_F and T_F do not depend on N . This assumption is reasonable for small N . The consequences of relaxing this assumption are discussed in Section 5.5.

The τ_n are determined by the demand/facility relationships in the feeder sections spanned by the relief project. Let the demand in the k th feeder section be given by:*

$$d_k(t) = d_k(0) + g_k t \quad (22)$$

If $n - 1$ pair gain systems are in place, and each realizes a pair gain of η , then the demand in each feeder section is reduced by $(n - 1)\eta$. Therefore, if the k th feeder section contains S_k pairs, it will exhaust at time τ_n^k where

$$d_k(0) + g_k \tau_n^k - (n - 1)\eta = S_k$$

or

$$\tau_n^k = \frac{S_k + (n - 1)\eta - d_k(0)}{g_k} \quad (23)$$

Since something must be done as soon as any feeder section exhausts, it follows that

* The linear demand assumption is not necessary, but it simplifies the discussion.

$$\tau_n = \min_k \tau_n^k \quad (24)$$

The form of eq. (24) may be simplified somewhat if it is assumed that one section, say section cr , always exhausts first. Then, if the time scale is chosen such that $S_{cr} = d_{cr}(0)$, τ_n is given by

$$\tau_n = (n - 1)\eta/g_{cr} \quad (25)$$

where g_{cr} is the growth in the section which is exhausting.

5.4 Optimal policy

The optimal number of pair gain systems, N , is chosen so as to minimize the partial PWAC, P_N , given by the first two terms of eq. (21) as

$$P_N = \int_0^{\tau_{N+1}} \gamma(t, N) e^{-rt} dt + \frac{1}{r} A e^{-r\tau_{N+1}} \quad (26)$$

The minimization can be carried out by enumeration as in Section IV. If eq. (25) holds, however, a set of curves can be generated from which the optimal N can be determined given A and g_{cr} .

Since a more expensive project can be economically deferred for a longer period of time, it follows that the optimal N increases with A . The N th breakpoint, A_N , which is that value of A at which the optimal number of systems changes from $N - 1$ to N , is given by

$$\begin{aligned} & \int_0^{\tau_{N+1}} \gamma(t, N) e^{-rt} dt + \frac{1}{r} A_N e^{-r\tau_{N+1}} \\ &= \int_0^{\tau_N} \gamma(t, N - 1) e^{-rt} dt + \frac{1}{r} A_N e^{-r\tau_N} \end{aligned}$$

or

$$A_N = r(e^{-r\tau_N} - e^{-r\tau_{N+1}})^{-1} \times \left[\int_0^{\tau_{N+1}} \gamma(t, N) e^{-rt} dt - \int_0^{\tau_N} \gamma(t, N - 1) e^{-rt} dt \right] \quad (27)$$

A family of curves for A_N as a function of N and g_{cr} can be generated from eq. (27).

Equation (27) can be greatly simplified if it is further assumed that $m(T) = m$. In this case, A_N is given by

$$\begin{aligned} A_N = r & [e^{-r(N-1)\eta/g_{cr}} - e^{-rN\eta/g_{cr}}] \\ & \times \left\{ \sum_{k=1}^N \frac{a'}{r} [e^{-r(k-1)\eta/g_{cr}} - e^{-rN\eta/g_{cr}}] + N R e^{-rN\eta/g_{cr}} \right. \\ & \quad \left. - \sum_{k=1}^{N-1} \frac{a'}{r} [e^{-r(k-1)\eta/g_{cr}} - e^{-r(N-1)\eta/g_{cr}}] \right. \\ & \quad \left. - (N - 1) R e^{-r(N-1)\eta/g_{cr}} \right\} \quad (28) \end{aligned}$$

where $a' = a + mI$. After some manipulation, eq. (28) becomes

$$A_N = (a' - rR)N + \frac{r}{1 - e^{-r\eta/g_{cr}}} R \quad (29)$$

Figure 7 is a sample plot of the A_N as a function of g_{cr} for a pair gain system characterized by

$$a' = \$2500$$

$$R = \$500$$

$$\eta = 50$$

with $r = 0.07$. These curves can be used to determine the optimal N for a given project. For example, if the critical section growth is 150 pairs per year and the annual charge for the relief project is \$10K, then 3 systems should be placed.

Note that the curves are linear. This is because, except for small g_{cr} , the exponential in eq. (29) can be replaced by its linear approximation, i.e.,

$$A_N \approx (a' - rR)N + \frac{R}{\eta} g_{cr} \quad (30)$$

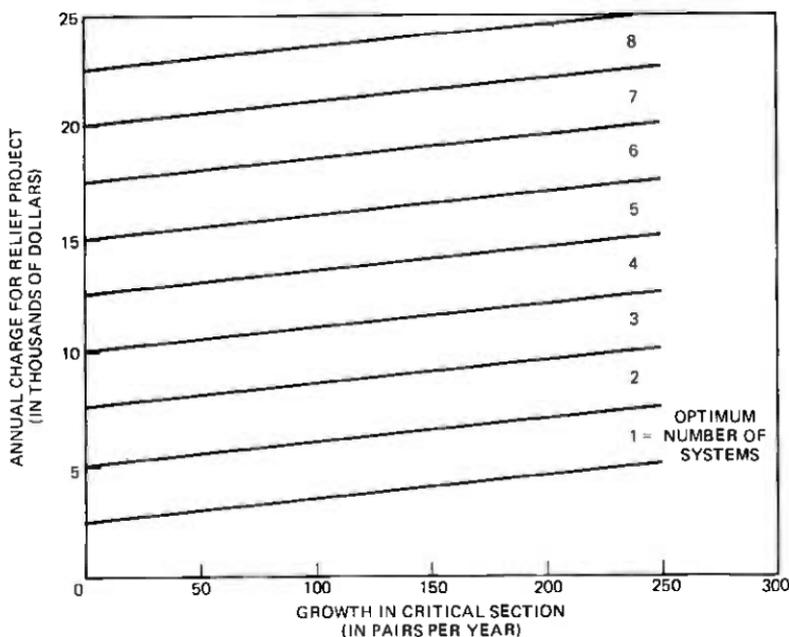


Fig. 7—Design chart for relief project deferral.

5.5 Extended deferrals

It has been assumed that the relief project is designed with no consideration toward the use of pair gain. The results of Sections II-IV, however, indicate that cable sizes are increased when pair gain is employed. Thus, when a project is deferred, that project, and all future projects, should be resized. For short (about one year or less) deferrals, the resizing is not very significant. If a project is deferred for more than one or two years, however, it should be redesigned.

When a deferred project is revised, larger cables will generally be called for [see discussion following eq. (6)] and the project annual charge will, therefore, increase. For this reason deferral of the revised project with additional pair gain systems should be considered. This repeated deferral/revision procedure is basically an iterative solution to a combined relief sizing and timing problem. It is a practical approach, however, and could be implemented by the operating telephone companies.

VI. MATHEMATICAL PROGRAMMING APPROACHES

In this section, the application of mathematical programming techniques to the pair gain/cable network capacity expansion program is discussed. These more powerful techniques can be applied to solve the important problem of *where* to place pair gain systems.

Although the methods developed in Sections II-V do not address the pair gain location problem, their importance in developing a more sophisticated approach should not be overlooked. In particular, they indicate the kinds of solutions (e.g., permanent pair gain, temporary pair gain) which may be obtained and the general conditions which favor a particular solution. These insights suggest simplifications which lead to tractable computer algorithms such as those described in this section.

The mathematical programming algorithms are described only briefly here. The reader is referred to Refs. 5 and 6 for more detail.

6.1 Permanent lumped pair gain

If temporary pair gain is precluded, the optimization problem becomes somewhat simpler. Whenever a shortage occurs in the network, either a cable or a pair gain system must be placed. The problem is further simplified if it is assumed that the cable size and the location of the pair gain system are determined on the basis of the pair gain system configuration at the time of the shortage and the projected demand. Under this assumption, each decision point (shortage) has only two alternatives: place a cable of a given size in the short section or install a pair gain system at a given location. Thus, the problem becomes a search of a binary decision tree.

The Long Feeder Route Analysis Program (LFRAP)⁵ solves the above

problem by a mathematical programming algorithm known as branch and bound. The LFRAP user provides data describing the network topology, existing cable facilities, and projected demand. The LFRAP output lists the sequence of placements of cable and pair gain systems which has the minimum PWAC over all sequences spanned by the binary decision tree.

Because of the restrictive assumptions on sizing cables and locating pair gain systems, the LFRAP solution is not truly optimal. The solutions have been shown to be quite good, however, and the restrictions are necessary in order to limit the computational requirements of the program.

6.2 Deferral of presized cables

Another way to simplify the problem is to assume that cable sizes have been predetermined and treat them as constant quantities. If, in addition, a discrete time scale is adopted, it becomes feasible to consider both installation and removal of pair gain systems.

Consider a one-year interval. If the facilities (cable and pair gain) in place at the beginning of the year do not meet the demand at the end of the year, shortages will occur. These shortages must be satisfied by some combination of cable placement and reconfiguration of the pair gain systems (installation, removal, relocation). The optimal pair gain configuration is the one which minimizes the total charge for the year including

- (i) Annual charges for additional cable,
- (ii) Annual charges for pair gain systems, and
- (iii) Installation and removal charges for pair gain systems.

The optimal configuration can be determined by branch and bound. Each node in the decision tree corresponds to a candidate location for one or more pair gain systems. The decision to be made is how many systems should be in place at that node during the one year interval. The sequence of decisions determines the pair gain configuration, which, in turn, determines the total charge for pair gain and undeferred cable.

This formulation of the problem has been implemented as an experimental computer program.⁶ The program obtains cable size data from the Exchange Feeder Route Analysis Program (EFRAP)⁷ and computes the optimal configuration for each year of a prescribed study period. The sequence of configurations corresponds to a sequence of pair gain system installations and removals, at various locations, interleaved with cable placements. If any pair gain systems are installed, then one or more cables are deferred, resulting in a PWAC savings.

6.3 Comments

Compared to the approaches of Sections II through V, mathematical programming is very powerful. Indeed, it is the only approach which addresses the network aspect of the problem. In some cases, such as when right-of-way limitations drastically reduce the number of potential pair gain system locations, the network question is academic. Thus, a sophisticated program is not always justified. However, in complicated networks, where the planning engineer is free to choose from many alternatives, the impact of pair gain system location is very difficult to judge. It appears, therefore, that both programs such as LFRAP and charts like Fig. 7 have a place in pair gain application planning.

VII. MODELS FOR FUTURE STUDY

This paper will be concluded with a glimpse of some recently proposed models for pair gain applications which do not fit the pattern established earlier. Specifically, the application of pair gain to provide temporary second line demand and the use of pair gain to avoid facility modifications will be discussed.

7.1 Provision of second lines

In residential areas, second lines are commonly requested to provide service for teen-aged children, for burglar alarms, or simply for convenience. Because of potential second line demand, the distribution cable network⁴ is sized to provide a minimum of two pairs per ultimate living unit. If second line penetration is low, many of these pairs will be unused, but they must still be provided since, at any given time, any subscriber may request a second line.

On the other hand, second line service can be provided by a single channel pair gain system. That is, the distribution network can be sized to provide one pair per ultimate living unit, and the second line can be provided by installing a single channel system at the subscriber's premises.

A rough calculation of the economics of providing second lines in this manner is fairly simple. It will be assumed that, in a given wire center serving area, the second line demand is constant over time and all second lines are provided by single channel pair gain. Under this assumption, the central office units can be treated as permanent facilities. The remote units will move from house to house in response to the second line demand.

Let p be the penetration of second line demand, i.e., in an area containing H living units, there will be pH second lines. Also, let T be the average duration of second line service at a given location. The total annual charge for pair gain, which is the total annual charge for providing second line service, consists of a pair gain equipment charge and an an-

nual turnover charge. For each single channel system, the annual charge, a , includes the annual charge for both the central office and remote equipment and the levelized cost of installing the central office unit. The turnover charge for each system follows from the assumption that the remote unit is removed from one location and reinstalled elsewhere every T years. Thus, the turnover charge per system is approximately $(I + R)/T$ where I and R are the installation and removal costs for the remote unit. The total annual charge per system is, therefore,

$$a + (I + R)/T$$

and, assuming p systems per living unit, the pair gain annual charge per living unit γ is

$$\gamma = p[a + (I + R)/T] \quad (31)$$

In order to judge the economics of the pair gain second line policy, one would compare eq. (31) with the annual charge per living unit for providing the same service with cable. The annual charge for the all-cable policy includes the marginal cost of the second distribution pair (which must be provided for every living unit) and the cost of additional feeder cable pairs.

Equation (31) indicates that the cost of the pair gain policy is directly proportional to the second line penetration. Thus, the policy is most likely to prove in areas of low second line penetration. Also, it should be noted that the turnover cost may be neglected in some cases. If, under the all-cable policy, it is necessary to install or even just terminate a second service wire, the net installation charge for the remote unit (i.e., remote unit installation charge minus drop installation charge) may be zero or negative. If I and R can be neglected, it is not necessary to estimate T in order to evaluate the economics of the policy.

7.2 Avoidance of facility modifications

In a congested network, inward service orders are often blocked, i.e., a facility modification must be made in order to provide service. As discussed in Ref. 8, a facility modification is a minor rearrangement of the network which entails costs in addition to the normal cost of providing service. Rather than disturb the network, however, one may elect to provide the service with a single channel pair gain system. In this section, a cost model for this kind of pair gain application will be outlined. The analysis will draw upon results derived in Ref. 8.

Consider a geographic area administered as multiple outside plant⁴ and served by X feeder pairs. Assume that demand in the area is growing linearly at rate g and let $S(t)$ be the number of spare feeder pairs [$S(0) = S$]. The probability that an inward order at time t is blocked is given approximately by⁸

$$\text{Pr}\{\text{BLK}\} = e^{-kS(t)/X} \quad (32)$$

where k is the apparent access group size (i.e., serving terminal size). Assume that, whenever an inward order is blocked, it will be served by a single channel pair gain system. Then, if $n(t)$ is the number of units of pair gain in the area [$n(0) = 0$], the rate of increase of $n(t)$ is

$$\dot{n} = g e^{-kS/X} \quad (33)$$

Since only unblocked inward orders use up spare feeder pairs, the rate of change in $S(t)$ is

$$\dot{S} = -g(1 - e^{-kS/X}) \quad (34)$$

Note that both $S(t)$ and $n(t)$ are modeled as continuous deterministic variables governed by a pair of differential equations. A more rigorous, but much more difficult approach would be to model them as discrete valued random processes driven by random arrivals and departures.

Equations (33) and (34) can be solved in closed form and the solutions are

$$S(t) = \frac{X}{k} \ln [1 + (e^{kS_0/X} - 1)e^{-kg t/X}] \quad (35)$$

and

$$n(t) = \frac{X}{k} \ln [1 + e^{-kS_0/X}(e^{kg t/X} - 1)] \quad (36)$$

If the only charge for the single channel pair gain system is the annual charge a , the total annual charge for pair gain will be $an(t)$.

If pair gain is not used, spare feeder pairs are used up at a rate of g so that $S(t) = S_0 - gt$ and blockages occur at a rate of $\lambda e^{-kS/X}$ where λ is the inward order rate. Thus, the annual charge for the all-cable alternative is

$$\lambda C_{\text{BLK}} e^{-k(S_0 - gt)/X}$$

where C_{BLK} is the average cost of a facility modification, until the spare is used up or relief is provided.

Under the pair gain policy, the spare feeder pairs are never exhausted. Rather, as time goes on, a larger and larger proportion of subscribers are served with single channel carrier. However, under either the cable or pair gain policy, the annual charge eventually increases to a point where relief cable is justified.

7.2.1 Example

Consider a pair group characterized by the following parameters:

$$X = 2000 \text{ pairs}$$

$$k = 20 \text{ terminations}$$

$$S_0 = 500 \text{ pairs}$$

$$g = 100 \text{ pairs/yr}$$

$$\lambda = 500 \text{ orders/yr}$$

Assume that $C_B = \$100$ and $a = \$50/\text{yr}$. Figure 8 is a plot of the annual charge function for both the cable and pair gain approach. In this case, the pair gain cost is uniformly lower.

7.3 Comment

The applications discussed in Section 7.1 and 7.2 have been implemented in a few areas of the Bell System. However, because of the random nature of pair gain installation and removals in these applications, conventional analysis methods like those of Sections II through VI cannot be applied. The material in this section represents an initial attempt to establish a mathematical foundation for these kinds of applications.

VIII. SUMMARY

This paper has presented a mathematical theory of pair gain appli-

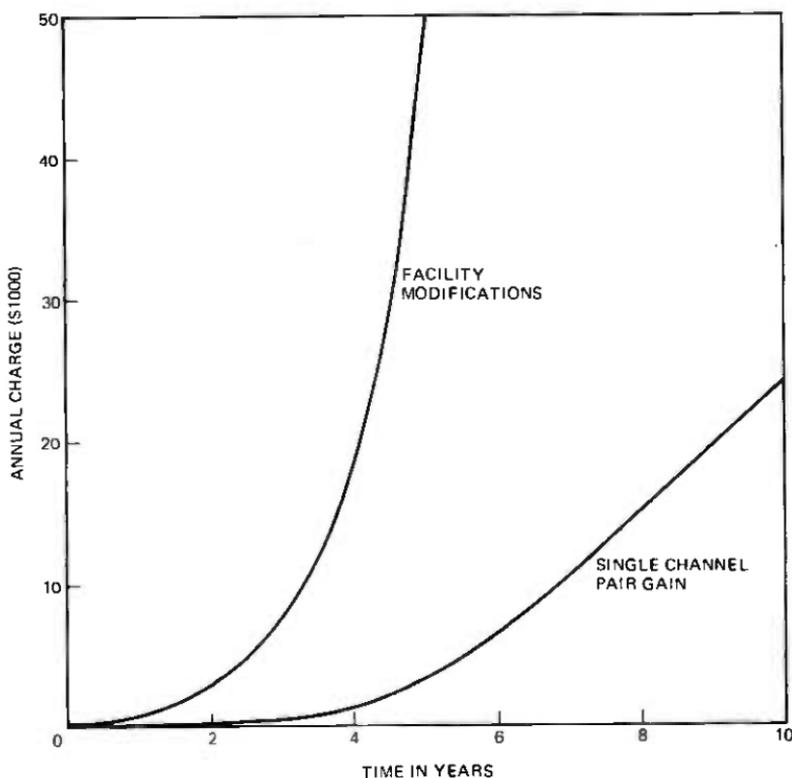


Fig. 8—Annual charge comparison of pair gain and facility modifications.

cation. The approach has been to adopt a framework based on the PWAC criterion and use it to develop both useful concepts and methods. These concepts are fairly consistent with the way pair gain has been traditionally applied. However, there are some important differences, particularly with regard to temporary versus permanent applications.

It has been shown that, under the conditions assumed by the model, the optimal strategy will be either all cable, permanent pair gain, or temporary pair gain (deferred cable). Although each of these strategies has been applied to real networks, it has not always been clear which one is best for a given situation. The operating companies now have guidelines and computer programs which will help them choose the lowest cost alternative. These pair gain planning tools have been developed as the result of mathematical modeling efforts such as those presented in this paper.

New applications, such as provision of second lines and avoidance of facility modifications, have been identified and studied. Although these applications are not very prevalent today, their importance will undoubtedly increase as improving technology continues to reduce the cost of pair gain relative to cable. Mathematical models like those developed here can serve as a guide for introducing new applications and developing future pair gain systems.

IX. ACKNOWLEDGMENTS

Many people have contributed to the results presented in this paper and their contributions are gratefully acknowledged. The author would like especially to thank R. G. Hinderliter, who suggested this area of work, N. G. Long, who encouraged the preparation of this paper, and the members of the Loop Transmission Engineering Center and the Loop Transmission System Laboratory who have participated at one time or another in the analysis of pair gain applications.

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Loop Plant Modeling:

A Model of Cable Pairs Added at the Main Frames for a Large Entity

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(Manuscript received August 17, 1977)

In estimating annual construction budgets and work forces required for a large entity, such as an operating company, it is desirable to know the total annual number of cable pairs to be added at the main frames in all of the feeder routes in the entity. These pairs are the result of many independent relief decisions made in the individual feeder routes. A model is presented which relates the total annual number of cable pairs added at the main frames to aggregate relief timing and sizing design parameters, an aggregate demand forecast and the total assigned and available pairs at the main frames in all of the routes in the entity. Because the model does not require individual route data or specific relief projects, it can be used as a "top-down" check on the "bottom-up" requirements determined by aggregating a list of identified relief projects. It is also useful for estimating cable pair requirements when specific relief projects are not yet identified.

I. INTRODUCTION

Growth in the number of telephone subscribers requires the periodic addition of cable pairs in the feeder network.¹ The feeder network comprises the large backbone cables which funnel cable pairs from the distribution network back toward the local wire center. The feeder cables are terminated in the wire center on a main distributing frame which serves as the interface between the loop network and the switching equipment.

At the main frame, the feeder cables are grouped into separate feeder routes which serve disjoint geographical areas within the boundaries of the wire center. Each route at the main frame is composed of a number

of available pairs, some of which are already assigned to existing subscribers served by the feeder route. The ratio of assigned pairs to available pairs at the main frame is called the main frame fill of the feeder route. The fill at other points along the route is defined in a similar manner.

When the fills in a portion of the feeder route become too high, new cable must be added. Most relief cables extend cable pairs already terminated on the main frame further out into the route. If the main frame fill of the route is also high, the new cable will be terminated on the main frame. This relief process is a complex one, involving both economic² and physical³ considerations in the particular feeder route.

Individual relief timing⁴ and sizing² decisions are made throughout the year by engineers based in local district engineering offices. A typical district has responsibility for about 100 main feeder routes. Districts are combined into administrative entities called areas which contain about 4 districts and 400 feeder routes. A typical operating company contains about 4 areas and 1600 feeder routes.

The aim of this paper is to present a model which has been derived to estimate the total number of cable pairs added on the main frames during a year in a large entity such as an area or company. The number of these pairs has historically been related to the total amount of feeder cable used in the entire feeder network during the year, and hence are an important component of an entity's annual construction budget and work force estimates. The model uses aggregate timing and sizing design parameters, an aggregate demand forecast and known aggregate main frame data such as total available pairs and total assigned pairs in the entity.

The model can be used to estimate future cable pair requirements for a large entity without identifying individual feeder routes or individual relief jobs. Hence the model is useful for estimating future cable pair requirements when specific relief projects are not yet identified, and for providing a simple "top-down" check on the "bottom-up" requirements obtained by aggregating a "market list" of identified relief projects. This check is useful because of the large number of feeder relief projects required each year in a large entity such as an area or company.

II. SUMMARY

A model is derived to estimate the total increase in available pairs at the main frames next year in all of the routes in a large entity. The model can be applied recursively to estimate the total increase in available pairs for any future year.

The problem of concern is formulated in Section III. Known aggregate main frame data are defined for an entity in terms of analogous data from the individual feeder routes. The basic approach used to derive the model

is outlined in Section 3.3 and involves modeling route relief at the main frame as a binomially distributed random variable where success connotes terminating a new cable on the main frame next year. Most of the modeling effort involves calculating the probability of relief (success) next year. Assumptions used to derive the model are conveniently summarized by defining an "analog entity" in Section 3.4.

The model is derived for an analog entity in three stages in Section IV. The initial result assumes that the fill at relief—the main frame fill in the route when a new cable is terminated—and cable size terminated at next relief will be the same as the fill at relief and cable size terminated at last relief in all of the routes in the analog entity. Adjustments to this steady state result are then derived in Sections 4.2 and 4.3 which enable the model to account for both changes in fill at relief and changes in cable size terminated.

The general model for estimating the increase in available pairs (ΔP) next year in an entity is

$$\overline{\Delta P} = \frac{G}{\hat{A}} + (1 - \theta)P \left(1 - \frac{F}{\hat{A}} \right) + N\Delta S$$

where

$$\hat{A} = \frac{W}{P - \frac{1}{2}NS}$$

is an estimate of the average fill at last relief for the entity. The estimates are expressed in terms of known aggregate main frame parameters for the entity which are either observed at the beginning of next year or forecast for next year:

- N = number of routes
- W = total assigned pairs
- P = total available pairs
- S = average cable size terminated
- G = growth in assigned pairs next year
- F = average fill at next relief
- ΔS = change in average cable size terminated next year

The parameter θ depends on the impedance of the entity to a change in fill at relief.

Confidence intervals are calculated for the model in Section 4.5 as a function of the number of routes in the entity for the idealized case of identical routes. The resulting error bounds represent the best possible model performance in actual entities.

Finally, a validation study is described in Section V where the performance of the model is evaluated in actual entities. This study also

provided the means for empirically selecting a universal value of 0.6 for the impedance parameter θ . An error bound of 12 percent at the 90 percent confidence level was determined for the $\theta = 0.6$ model when applied to a typical company-sized entity of 1600 routes. The corresponding error bound at the 50 percent confidence level, or probable error, was 5 percent.

III. MODEL FORMULATION

A model for estimating the total increase in available pairs at the main frames in a given entity next year will be derived. This parameter is an indicator of the total feeder cable requirements in the entity next year. The goal of the modeling effort is to estimate this parameter using known aggregate main frame data and to determine the statistical accuracy of the estimate when used in actual entities.

Known aggregate main frame data for an entity include:

- number of feeder routes
- total assigned pairs
- total available pairs
- average cable size terminated
- growth forecast
- average fill at next relief
- change in average cable size terminated

These data are either aggregated annually from the feeder routes which compose the entity, or are forecast directly for the entity. The above parameters will be formally defined in terms of analogous route parameters in Section 3.2.

3.1 Basic route parameters

A feeder route is described at the main frame by the following set of basic route parameters (denoted by lower-case letters) which are either observed or forecast at the beginning of next year (see Fig. 1):

- | | | |
|--|---|---|
| w = assigned pairs | } | observed at the
beginning of next year |
| p = available pairs | | |
| s = cable size terminated | | |
| a = fill at last relief | | |
| g = growth in assigned pairs next year | | |
| f = fill at next relief | | |

The ratio w/p is the main frame fill at the beginning of next year. The last cable terminated at the main frame contained s pairs and was placed in service when the main frame fill, or fill at last relief, was a . A new cable will be terminated in the route when the main frame fill reaches the fill at next relief, f . Initially, we will assume that the new cable contains the same number of pairs as the last cable terminated (s). This assumption will be relaxed in Section 4.3.

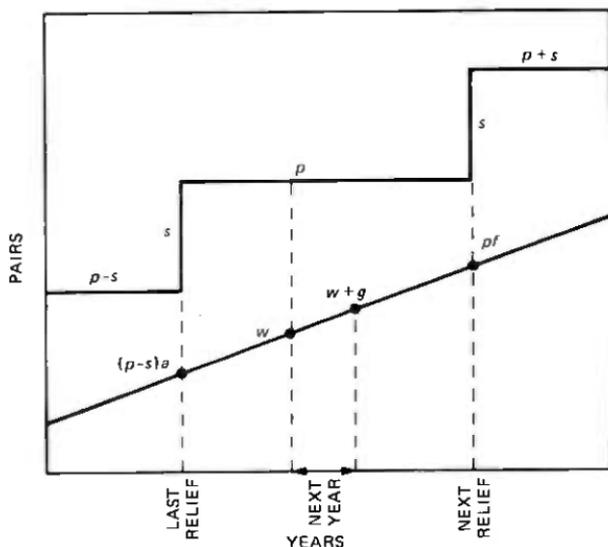


Fig. 1—Feeder route parameters.

3.2 Known aggregate main frame data

Known aggregate main frame data (denoted by upper-case letters) compiled from the individual feeder routes at the beginning of next year are

N = number of feeder routes

$W = \sum w_j$ = total assigned pairs

$P = \sum p_j$ = total available pairs

$S = \frac{1}{N} \sum s_j$ = average cable size terminated

The parameters W and P are simply aggregate facility data which have historically been collected to monitor the usage of the feeder network at the main frames. The ratio W/P is the main frame fill of the entity at the beginning of next year. The average cable size terminated (S) is an aggregate sizing design parameter.

The following parameters are forecast for the entity for next year:

$G = \sum g_j$ = growth in assigned pairs next year

$F = \frac{1}{P} \sum p_j f_j$ = average fill at next relief.

G is a standard demand forecast and F is an aggregate timing design parameter. These parameters are typically forecast directly for the entity rather than aggregated from the forecasts of the analogous parameters in each route.

The definition of N , W , P and G are intuitively aggregates of the analogous feeder route parameters. The definition of the aggregate de-

sign parameters S and F will become clear during the derivation of the model (Section IV). Note that there is no known aggregate parameter which is analogous to the fill at last relief (a) in a single route (see Section 3.1). This unknown aggregate parameter will be introduced with the definition of the analog entity in Section 3.4.

The basic approach used to derive a model for estimating the total increase in available pairs next year in an entity is described in the next section.

3.3 Basic approach

The increase in available pairs (Δp) next year in a route is either s or zero, depending on whether a new cable is terminated next year or not. Hence Δp can be expressed as χs where χ is the relief indicator for the route next year, defined as

$$\chi = \begin{cases} 1 & \text{if a new cable is} \\ & \text{terminated next year} \\ 0 & \text{otherwise} \end{cases}$$

The increase in available pairs next year in an entity can be expressed as

$$\begin{aligned} \Delta P &= \sum \Delta p_j \\ &= \sum \chi_j s_j \end{aligned} \quad (1)$$

The minimum value of $\Delta P = 0$ corresponds to the unlikely case of no new cable being terminated next year ($\chi_j = 0 : j = 1, N$), while the maximum value of $\Delta P = NS$ corresponds to the equally unlikely case of a new cable being terminated in every route next year ($\chi_j = 1 : j = 1, N$).

The relief indicator (χ) for a route next year can be modeled as a binomially distributed random variable where success corresponds to a new cable being terminated next year ($\chi = 1$). Treating the cable size (s_j) as a known parameter, the expected increase in available pairs next year in the entity can be expressed as

$$\overline{\Delta P} = \sum \lambda_j s_j \quad (2)$$

where $\lambda_j = \bar{\chi}_j$ is the probability of relief next year for route j .

Intuitively, the probability of relief next year should depend on how far the route is through its relief cycle at the beginning of next year. The fraction through the relief cycle (r) decomposes the relief cycle (t) into two components; the time since last relief (rt) and the time to next relief $[(1-r)t]$. The route will be relieved next year if the time to next relief (observed at the beginning of next year) is one year or less. If r is modeled

as a random variable, then the probability of relief next year,

$$\begin{aligned}\lambda &= \mathcal{P}\left((1-r)t \leq 1\right) \\ &= \mathcal{P}\left(r \geq 1 - \frac{1}{t}\right)\end{aligned}\quad (3)$$

depends on the tail of the distribution of the fraction through the relief cycle at the beginning of next year.

The route is said to be in the steady state if the fill at next relief is the same as the fill at last relief ($f = a$). In this case (see Fig. 2), the relief cycle of the route—the number of years between last and next relief—is

$$t^{ss} = \frac{sa}{g}\quad (4)$$

and the fraction through the relief cycle at the beginning of next year is

$$r^{ss} = \frac{w - (p-s)a}{sa}\quad (5)$$

If the fill at next relief (f) forecast at the beginning of next year is not equal to the fill at last relief (a), then the relief cycle becomes

$$t = \frac{p(f-a) + sa}{g}\quad (6)$$

If $f > a$ (see Fig. 3), then the relief cycle is increased and the fraction

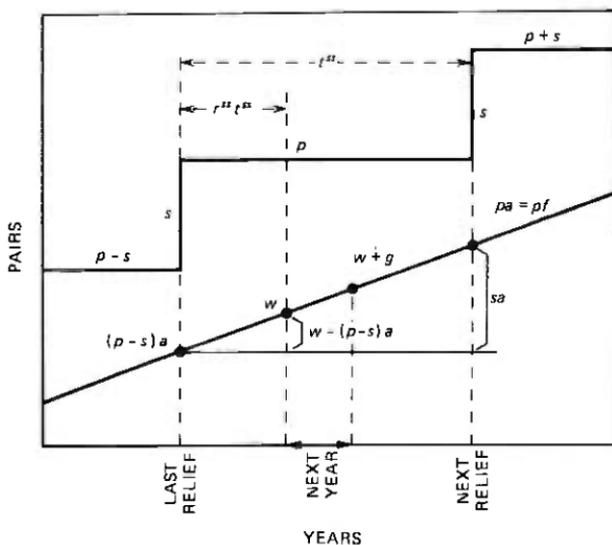


Fig. 2—Feeder route in steady state ($f = a$).

through the relief cycle is decreased such that their product, $rt =$ time since last relief, remains constant. Because the time since last relief is also equal to $r^{ss}t^{ss}$, we have $rt = r^{ss}t^{ss}$ or

$$r = r^{ss} \frac{t^{ss}}{t} \quad (7)$$

If r^{ss} is modeled as a random variable, then the probability of relief next year is

$$\begin{aligned} \lambda &= \mathcal{P} \left(r \geq 1 - \frac{1}{t} \right) \\ &= \mathcal{P} \left(r^{ss} \geq \frac{t}{t^{ss}} - \frac{1}{t^{ss}} \right) \end{aligned} \quad (8)$$

Hence r^{ss} —the fraction through the relief cycle if the route is in the steady state ($f = a$)—is the basic random variable which will be used to derive the model.

The distribution of r^{ss} determines the distribution of w —the assigned pairs in the route at the beginning of next year—which can be expressed in terms of r^{ss} by solving eq. (5) for w ,

$$w = ap + as(1 - r^{ss}) \quad (9)$$

This is the only basic route parameter defined in Section 3.1 which is modeled as a random variable. All other parameters (p, s, a, g, f) are as-

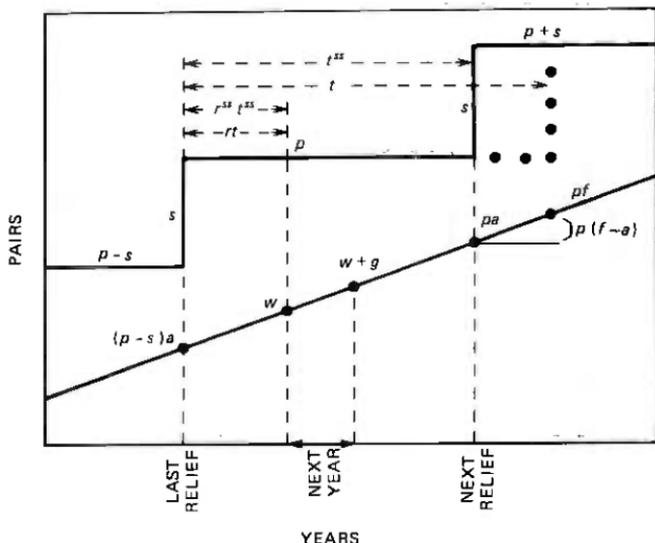


Fig. 3—Feeder route with a change in fill at relief ($f > a$).

sumed to be deterministic. Hence the length of the relief cycle [t^{ss} and t , see eqs. (4) and (6)] are deterministic, but the fraction through the relief cycle [r^{ss} and r , see eqs. (5) and (7)] are random variables.

3.4 Model assumptions—the analog entity

Several assumptions are required to derive a model for the total increase in available pairs (ΔP) next year in an entity in terms of the known aggregate main frame data (N, W, P, S, G, F) defined in Section 3.2. These assumptions are needed because the distribution of the aggregate data over the routes in the entity ($w_j, p_j, s_j, a_j, g_j, f_j : j = 1, N$) is in general not known. The assumptions are:

- A1: All of the routes in the entity are assumed to have the same fill at last relief ($a_j = A : j = 1, N$) where A is an unknown parameter that must be estimated in terms of known aggregate data.
- A2: If the routes in the entity are in the steady state condition ($f_j = A : j = 1, N$), then the fractions through the relief cycle of the routes ($r_j^{ss} : j = 1, N$) are independent and identically distributed uniform random variables on the unit interval. In other words, if a route is selected at random from routes in the steady state, then we are equally likely to observe any value of r^{ss} between 0 (just after relief) and 1 (just before relief).

An entity satisfying these two conditions will be referred to as an analog entity. The model will be derived for an analog entity in the next section. The performance of this model in actual entities will be studied in Section V.

IV. AVAILABLE PAIR INCREASE MODEL

A model for estimating the increase in available pairs next year in an analog entity is derived. This model can be applied recursively to obtain an estimate of the increase in available pairs for any future year.

4.1 Steady state model

The expected increase in available pairs in the analog entity next year [eq. (2)] is

$$\overline{\Delta P} = \sum \lambda_j s_j$$

where s_j is the cable size terminated and λ_j is the probability of relief next year in route j . If the routes in the analog entity are in the steady state condition (i.e., $f_j = A : j = 1, N$), then $t_j = t_j^{ss}$ [see eqs. (4) and (6)] and the probability of relief next year [eq. (8)] is

$$\begin{aligned} \lambda_j &= \mathcal{P} \left(r_j^{ss} \geq \frac{t_j}{t_j^{ss}} - \frac{1}{t_j^{ss}} \right) \\ &= \mathcal{P} \left(r_j^{ss} \geq 1 - \frac{1}{t_j^{ss}} \right) \end{aligned}$$

Since r_j^{ss} is assumed to be uniformly distributed on the unit interval (assumption A2), we have

$$\lambda_j = \begin{cases} 1 & \text{if } t_j^{ss} < 1 \\ \frac{1}{t_j^{ss}} & \text{if } t_j^{ss} \geq 1 \end{cases} \quad (10)$$

The physical significance of $t_j^{ss} < 1$ is that the last cable terminated (s_j) was sized so small that the route requires another relief cable in less than one year. This is not a normal engineering practice and it will not be considered in the model.

The expected available pair increase in route j for the case $t_j^{ss} \geq 1$ is

$$\begin{aligned} \overline{\Delta p_j} &= \lambda_j s_j \\ &= \frac{s_j}{t_j^{ss}} \\ &= \frac{g_j}{A} \end{aligned} \quad (11)$$

This result follows from eqs. (10) and (4) and assumption A1. Hence if s_j pairs are terminated every t_j^{ss} years, then s_j/t_j^{ss} pairs will be terminated on average next year. Equivalently, if cables were sized for only one year's growth, then g_j/A pairs would be terminated next year.

The expected increase in available pairs in the analog entity next year is obtained by summing eq. (11) over all routes,

$$\begin{aligned} \overline{\Delta P} &= \sum \frac{g_j}{A} \\ &= \frac{G}{A} \end{aligned} \quad (12)$$

where G is the entity growth forecast for next year. In the following sections, we derive adjustments to this simple steady state model by relaxing some of the restrictive assumptions leading to this result.

4.2 Change in fill at relief

The steady state assumption will now be relaxed by allowing the forecast fill at next relief in route j (f_j) to be different from the fill at last relief (A). Such a change could take place if locally selected fills at relief were used in place of a constant fill at relief policy represented by A .

4.2.1 Instantaneous change in fill at relief

If the fill at next relief is changed from A to f_j at the beginning of next year, then the probability of relief next year in route j [eq. (8)] is

$$\lambda_j = \mathcal{P} \left(r_j^{ss} \geq \frac{t_j}{t_j^{ss}} - \frac{1}{t_j^{ss}} \right) = \begin{cases} 0 & \text{if } t_j > t_j^{ss} + 1 \\ \frac{1}{t_j^{ss}} + 1 - \frac{t_j}{t_j^{ss}} & \text{if } 1 \leq t_j \leq t_j^{ss} + 1 \\ 1 & \text{if } t_j < 1 \end{cases} \quad (13)$$

This result follows because r_j^{ss} is uniformly distributed on the unit interval by assumption A2. If $f_j \gg A$ such that a year or more is added to the relief cycle (case $t_j > t_j^{ss} + 1$), then the route will not be relieved next year ($\lambda_j = 0$). On the other hand, if $f_j \ll A$ such that the relief cycle is reduced to less than one year (case $t_j < 1$), then the route will be relieved next year ($\lambda_j = 1$).

The expected increase in available pairs in route j for the case $1 \leq t_j \leq t_j^{ss} + 1$ is

$$\begin{aligned} \overline{\Delta p_j} &= \lambda_j s_j \\ &= \left(\frac{1}{t_j^{ss}} + 1 - \frac{t_j}{t_j^{ss}} \right) s_j \\ &= \frac{g_j}{A} + p_j \left(1 - \frac{f_j}{A} \right) \end{aligned} \quad (14)$$

This result follows after substituting eq. (13) for λ_j and eqs. (4) and (6) for t_j^{ss} and t_j . Note that Δp_j is greater than the steady state value (g_j/A) if $f_j < A$ and is less than the steady state value if $f_j > A$.

The expected increase in available pairs next year in the analog entity is obtained by summing eq. (14) over all routes,

$$\begin{aligned} \overline{\Delta P} &= \sum \left(\frac{g_j}{A} + p_j \left(1 - \frac{f_j}{A} \right) \right) \\ &= \frac{G}{A} + P \left(1 - \frac{1}{P} \frac{\sum p_j f_j}{A} \right) \\ &= \frac{G}{A} + P \left(1 - \frac{F}{A} \right) \end{aligned} \quad (15)$$

where F is the average fill at next relief for the original entity. The previous steady state result [G/A , eq. (12)] is adjusted by a factor that depends on the change in fill at relief. The adjustment is positive if $F < A$ and negative if $F > A$.

4.2.2 Impedance to a change in fill at relief

The last result [eq. (15)] was derived by assuming an instantaneous change in fill at relief from A to f_j in all routes in the analog entity at the beginning of next year. In practice, we should not expect an instantaneous change. It is not practical or economical to defer a job (by increasing the fill at next relief) which is already under construction or for which material has already been ordered. On the other hand, it is not possible to suddenly advance a job (by decreasing the fill at next relief) that had been planned for further into the future because major feeder relief jobs must be engineered a year or more in advance to allow time for job approval, ordering material and construction.

These considerations can be accounted for in the model by introducing the concept of the impedance of the analog entity to a change in fill at relief. The effect of this impedance is to cause the fill at relief to be changed gradually rather than instantaneously in all routes. In this case, the average fill at next relief (F') for the analog entity is somewhere between A and F —the average fill at next relief for the original entity. Hence F' can be expressed as

$$F' = \theta A + (1 - \theta)F \quad (16)$$

for some θ between 0 and 1. The parameter θ is related to the impedance of the analog entity to the change in fill at relief. If $\theta = 0$, the analog entity offers zero impedance and the change is instituted instantaneously in all routes ($F' = F$). If $\theta = 1$, then the analog entity offers infinite impedance and the change is never instituted ($F' = A$).

The expected available pair increase in the analog entity next year based on F' rather than F is

$$\begin{aligned} \overline{\Delta P}(\theta) &= \frac{G}{A} + P \left(1 - \frac{F'}{A} \right) \\ &= \frac{G}{A} + (1 - \theta)P \left(1 - \frac{F}{A} \right) \end{aligned} \quad (17)$$

Note that θ attenuates the effect of the change in fill at relief on the expected increase in available pairs. Equation (17) reduces to our previous result [eq. (15)] if the fill at relief is changed instantaneously in all routes ($\theta = 0$). An empirical value of θ will be determined in Section V based on the observed behavior of actual entities to a change in fill at relief.

4.3 Change in average cable size terminated

In all of the results presented so far, we have assumed that the same sized cable is terminated whenever a particular feeder route is relieved. In practice (see Ref. 2), changes in many factors can cause future cable sizes to be different from those placed in the past. In this case, the new cables terminated next year would cause a change in average cable size

terminated from its value at the beginning of the year (S). We will now derive another adjustment to the model by determining the impact of a change in average cable size (ΔS) on the expected increase in available pairs next year.

The change in average cable size terminated next year can be defined in the original entity as

$$\Delta S = \frac{1}{N} \sum \chi_j (c_j - s_j) \quad (18)$$

where N is the number of routes in the entity, χ_j is the relief indicator function for next year, s_j is the cable size terminated at last relief and c_j is the cable size to be terminated at next relief in route j . We will assume that a change in average cable size next year is forecast directly for the entity.

If the cable sizes ($c_j : j = 1, N$) are carried over to the routes of the analog entity, then the expected increase in available pairs next year can be expressed as

$$\begin{aligned} \Delta P &= \sum \chi_j c_j \\ &= \sum \chi_j (s_j + c_j - s_j) \\ &= \sum \chi_j s_j + \sum \chi_j (c_j - s_j) \\ &= \sum \chi_j s_j + N \Delta S \end{aligned}$$

The first term is the increase in available pairs in the analog entity if there is no change in average cable size next year. The model for its expected value is given in eq. (17). If average cable size is forecast to change by ΔS next year, then the increase in available pairs will change by $N \Delta S$.

The general model is obtained by adding $N \Delta S$ to eq. (17):

$$\overline{\Delta P}(\theta) = \frac{G}{A} + (1 - \theta)P \left(1 - \frac{F}{A}\right) + N \Delta S \quad (19)$$

This result represents the final adjustment to the original steady state model of G/A . The general model can be applied recursively to obtain the expected increase in available pairs for any future year.

4.4 Fill at last relief model

When defining the analog entity, we assumed that the fill at last relief was the same in all routes. A value for this parameter (A) is needed to calculate the expected increase in available pairs next year in the analog entity [see eq. (19)]. In this section, we derive an estimate of A in terms of known aggregate main frame data (see Section 3.2).

Because A is the fill at last relief in the analog entity, its estimator can only depend on parameters which can be observed at the beginning of

next year. These include the basic route parameters ($w_j, p_j, s_j : j = 1, N$) and the corresponding aggregate parameters (N, W, P, S).

The basic relationship in the analog entity between the route parameters (w_j, p_j, s_j) and A is expressed in eq. (9),

$$w_j = Ap_j - As_j(1 - r_j^{ss}) \quad (20)$$

Both r_j^{ss} and w_j are modeled as random variables and r_j^{ss} is assumed to be uniformly distributed on the unit interval.

Summing eq. (20) over all routes in the analog entity yields

$$N\bar{w} = AP - ANS(1 - R^{ss})$$

which can be solved for A to obtain

$$A = \frac{N\bar{w}}{P - NS(1 - R^{ss})} \quad (21)$$

In the last two equations, $\bar{w} = (\sum w_j)/N$ and

$$R^{ss} = \frac{1}{N} \sum (s_j/S)r_j^{ss} \quad (22)$$

are random variables and N, P and S are observed aggregate parameters at the beginning of next year. If \bar{w} is observed to be W/N at the beginning of next year, then a random variable (α) can be defined as an estimator of A where

$$\alpha = \frac{W}{P - NS(1 - R^{ss})} \quad (23)$$

An estimate of A is then the expected value of α .

The expected value of α does not have a simple form because the random variable R^{ss} —which is analogous to the fraction through the relief cycle in a single route—is approximately normally distributed. This follows from a generalized version of the Central Limit Theorem, called Lindebergs Theorem,⁵ for uniformly bounded independent random variables. In our case, R^{ss} is the sample mean of the independent random variables $[(s_j/S)r_j^{ss} : j = 1, N]$ which are uniformly bounded by $k = s_{\max}/S$ where s_{\max} is the largest manufactured cable size. The expected value and variance of R^{ss} can be calculated from eq. (22) and the expected value ($1/2$) and variance ($1/12$) of r_j^{ss} ,

$$\begin{aligned} \mathcal{E}(R^{ss}) &= \frac{1}{N} \sum (s_j/S)\mathcal{E}(r_j^{ss}) \\ &= \frac{1}{2} \\ \text{var}(R^{ss}) &= \frac{1}{N^2} \sum (s_j/S)^2 \text{var}(r_j^{ss}) \\ &= \frac{1}{12N^2} \sum (s_j/S)^2 \end{aligned} \quad (24)$$

An estimate of A , \hat{A} , can be obtained by replacing R^{ss} in eq. (23) by its expected value of $1/2$, yielding

$$\hat{A} = \frac{W}{P - \frac{1}{2}NS} \quad (25)$$

This is a reasonable estimator for A if the variance of R^{ss} is small. A bound on $\text{var}(R^{ss})$ can be derived from the bound on s_j/S . Recall that $s_i/S \leq k$, which together with eq. (24) yields

$$\text{var}(R^{ss}) \leq \frac{k^2}{12N}$$

Using a value of $k = s_{\max}/S = 4$ which corresponds to a typical average cable size terminated $S = 900$ pairs and the largest manufactured cable size $s_{\max} = 3600$ pairs yields $\text{var}(R^{ss}) \leq 0.003$ for a typical area sized entity of 400 routes. Hence for large entities, \hat{A} [eq. (25)] is a reasonable estimator of A .

This estimator will be used for A in the general model [eq. (19)] for estimating the available pair increase next year in the analog entity. A confidence interval for the model will be derived in the next section for the idealized case of an entity of identical routes. The performance of the model in actual entities will be analyzed in Section V.

4.5 Idealized model accuracy

A confidence interval for the expected increase in available pairs next year will be derived for an entity of N identical routes where the probability of relief next year and the cable size terminated are the same in all routes. The calculated error bounds for this idealized case are representative of the best possible model performance in actual entities and will be compared to observed model errors in Section V.

If the probability of relief next year and the cable size terminated are the same in all routes, then the increase in available pairs next year can be represented as a sum of N independent and identically distributed binomial random variables,

$$\Delta P = \sum \chi_{js}$$

The expected value and variance of ΔP are

$$E(\Delta P) = N\lambda s \quad (26)$$

$$\text{var}(\Delta P) = Ns^2\lambda(1 - \lambda) \quad (27)$$

where λ is the probability of relief next year and s is the cable size ter-

minated in all routes. If N is large, ΔP is approximately normally distributed by the De Moivre-Laplace Theorem.⁵

A δ -confidence interval for the relative deviation of ΔP from its expected value can be obtained from the probability statement,

$$\mathcal{P} \left(\left| \frac{\Delta P - \mathcal{E}(\Delta P)}{\mathcal{E}(\Delta P)} \right| < \epsilon \right) \geq \delta$$

which can be expressed in terms of a unit normal random variable,

$$\mathcal{P} \left(\left| \frac{\Delta P - \mathcal{E}(\Delta P)}{(\text{var}(\Delta P))^{1/2}} \right| < \epsilon \frac{\mathcal{E}(\Delta P)}{(\text{var}(\Delta P))^{1/2}} \right) \geq \delta$$

If β is selected such that

$$\mathcal{P} \left(\left| \frac{\Delta P - \mathcal{E}(\Delta P)}{(\text{var}(\Delta P))^{1/2}} \right| < \beta \right) \geq \delta$$

then the error bound ϵ can be expressed as

$$\epsilon = \beta \frac{(\text{var}(\Delta P))^{1/2}}{\mathcal{E}(\Delta P)}$$

Substituting eqs. (26) and (27) for $\mathcal{E}(\Delta P)$ and $\text{var}(\Delta P)$ yields

$$\epsilon(N, \lambda) = \beta \left(\frac{1 - \lambda}{\lambda N} \right)^{1/2} \quad (28)$$

This result will be used in the next section as a scale factor to compare observed errors in entities with different numbers of routes and average probabilities of relief.

Error bounds $\epsilon(N, \lambda)$ at the 50 percent ($\delta = 0.5$, $\beta = 0.675$) and 90 percent ($\delta = 0.9$, $\beta = 1.645$) confidence levels for an entity of identical routes with a probability of relief next year of $\lambda = 0.2$ are given in Table I as a function of the number of routes in the entity. These results are representative of the best possible model performance in actual entities.

V. VALIDATION STUDY

A validation study was performed to determine the statistical accuracy of the model when applied in actual entities. This analysis will also provide the means of calibrating the model by empirically selecting a

Table I — Idealized error bounds for available pair increase model ($\lambda = 0.2$)

Number of routes	Confidence level	
	50%	90%
100	14%	33%
200	10	23
400	7	16
1600	3	8

value for θ based on the observed behavior of actual entities to changes in fill at relief.

5.1 Validation plan

Route facility charts provide the data required to perform the validation study. These charts are maintained for each route in some areas to record the past history of assigned and available pairs and to project future growth and relief requirements at the main frame. A typical route facility chart is shown in Fig. 4. The assigned and available pairs in the route at the main frame are counted at year-end and posted on the facility chart. An increase in available pairs in the route caused by terminating a new cable on the main frame is shown in the month that the pairs were made available for service. In this case, an estimate of the fill at relief can be made by linearly interpolating between the year-end counts of assigned pairs made before and after the service date of the job.

The validation study consisted of applying the model in a number of sample entities and then analyzing the statistics of the model errors. The aggregate main frame data ($N, W, P, S, G, F, \Delta S$) required by the model can be compiled from facility chart data on each of the routes in a sample entity. The equations in Sections 3.2 and 4.3 were used with one exception: The average fill at next relief (F) was calculated using only routes that were relieved next year. Since historical data were used in the study,

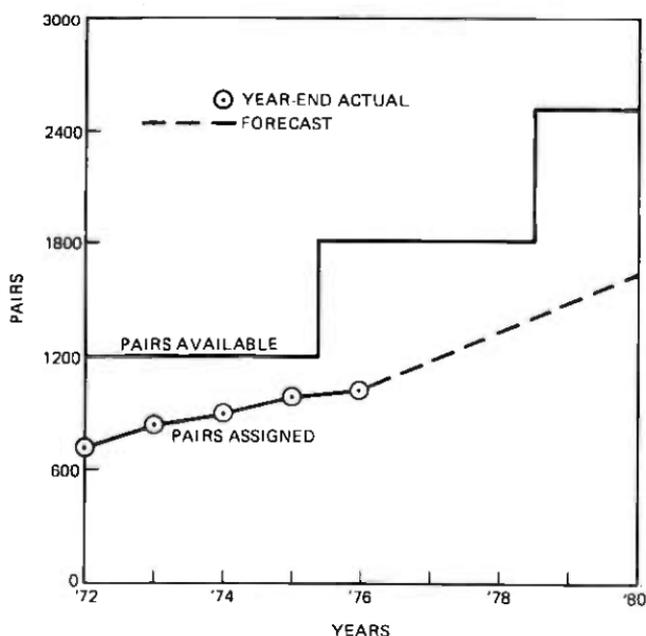


Fig. 4—Route facility chart.

actual values could be used for G , ΔS and F rather than forecast values which had the desirable effect of eliminating forecast errors from the validation study.

The general model [eq. (19)] was then used to calculate an estimate of the increase in available pairs next year in the sample entity for various values of θ between zero and one. An estimated value (\hat{A}) of the fill at last relief (A) required by the model was calculated using eq. (25). The actual model error for each value of θ ,

$$e(\theta) = \left| \frac{\overline{\Delta P}(\theta) - \Delta P}{\overline{\Delta P}(\theta)} \right|$$

was then scaled using the equation

$$\tilde{e}(\theta) = e(\theta) \frac{\epsilon(200,0.2)}{\epsilon(N,\lambda)}$$

where $\epsilon(N,\lambda)$ is given by eq. (28) and λ is the fraction of routes in the sample entity which were actually relieved next year. The scaling is intended to make errors comparable in sample entities with different numbers of routes (N) and probabilities of relief (λ). The scale factor is arbitrarily defined to be unity when $N = 200$ and $\lambda = 0.2$.

The distribution of scaled model errors over the sample entities was determined for each value of θ . The behavior of the 90 percent point and the 50 percent point, or median, of these distributions as a function of θ was used as the basis for selecting the best value for θ . The 50 percent point of the distribution of signed scaled errors was also studied as a function of θ to help detect model biases. Similar studies were performed on subsets of the sample entities to determine if the model could be calibrated using a universal value of θ .

5.2 Validation study results

Facility chart history on available and assigned pairs from over 650 routes were gathered from three different areas (see Table II). History was sought from year-end 1968 to year-end 1975 but this amount of history was not available in all routes.

Sample entities were defined within an area for each year by selecting routes whose growth in assigned pairs during the year was at least one percent of the total assigned pairs at the beginning of the year. The validation study was restricted to only growth routes because the con-

Table II — Route data

Area	Number of routes
A	283
B	211
C	168

cepts of timing and sizing relief are most meaningful in a growing route.

This process yielded 21 sample entities (i.e., 7 different years in 3 different areas). The actual number of routes in each sample entity are shown in Table III. Model errors were calculated as described in the validation plan (Section 5.1). The 90 and 50 percent points of the distribution of scaled errors are plotted versus θ in Fig. 5. The 50 percent point of the distribution of signed scaled errors—which ideally should be zero—is also shown to help detect model biases.

The best value of θ for estimating the increase in available pairs is 0.6 which minimizes the 90 percent point of the distribution of scaled errors. At this value of θ , the 50 percent point of the distribution is nearly minimized, and the 50 percent point of the distribution of signed scaled errors is almost zero. Similar analyzes performed for the 7 sample entities in each area yielded values of θ between 0.5 and 0.7. These results indicate that it is possible to calibrate the model with a universal value of $\theta = 0.6$.

The anticipated accuracy of the best model ($\theta = 0.6$) when applied in

Table III — Number of routes in sample entities

Area	'69	'70	'71	Year '72	'73	'74	'75
A	155	166	184	197	197	184	219
B	60	81	98	128	171	166	166
C	42	50	75	81	92	88	125

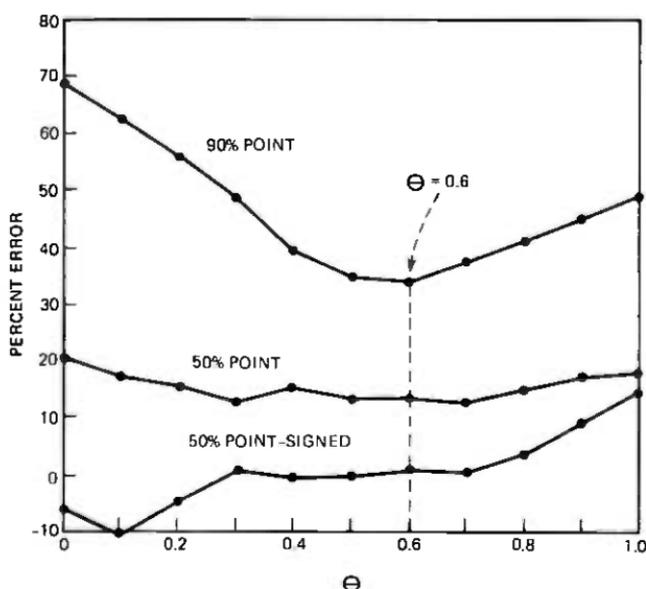


Fig. 5—Scaled model errors ($N = 200$, $\lambda = 0.2$).

Table IV — Anticipated error bounds for the available pair increase model ($\theta = 0.6$, $\lambda = 0.2$)

Number of routes	Confidence level	
	50%	90%
100	19%	48%
200	13	34
400	10	24
1600	5	12

an entity of N routes with an average probability of relief of λ can be expressed in terms of the scaled errors as

$$e(N, \lambda) = \bar{e}(\theta = 0.6) \frac{\epsilon(N, \lambda)}{\epsilon(200, 0.2)}$$

where $\epsilon(N, \lambda)$ is given in eq. (28) and $\bar{e}(\theta = 0.6)$ is read from Fig. 5 for either the 50 percent or 90 percent confidence level. Anticipated error bounds as a function of the number of routes in entities with a probability of relief of 0.2 are summarized in Table IV. These results are somewhat higher than the corresponding error bounds which were calculated for an entity of identical routes (see Table I) because of route-to-route variations in the sample entities.

VI. ACKNOWLEDGMENTS

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Loop Plant Modeling:

The Feeder Allocation Process

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Allocation is an engineering process by which economical use of spare feeder capacity in the loop network may be planned. The basic concept is to apportion the available spare feeder pairs along an entire feeder route such that placement of the relief cable is deferred for the section of the route with the shortest time to relief, thus reducing capital costs. Since rearrangements are planned on a route basis, rather than made expediently, operating costs are also reduced. This paper first illustrates the concept of allocation by applying a simple manual technique to a small example. Then a more sophisticated method, which has been computerized, is described. Finally, a generalized mathematical description is given.

I. INTRODUCTION

An important part of an operating company outside plant engineer's job is to decide how best to use the spare capacity in the feeder network.¹ This activity is called *feeder administration*. A major part of feeder administration is the *allocation* of spare feeder pairs.

1.1 The basic concept of allocation

Allocation is the process of planning the use of spare feeder capacity (i.e., spare feeder cable pairs) along an entire feeder route. The purpose of allocation is to make the best, or most economical, use of both existing and future feeder facilities, and to enable the engineer to plan splicing configurations which will avoid complexity and reduce as much as possible the need for costly future rearrangements.

The basic concept of allocation is to apportion the available spare pairs along the entire route, according to forecast growth rates. Doing so may

economically defer feeder relief which leads to higher feeder utilization (the fraction of the feeder actually in use).

1.2 Dual view of the feeder

Consider the schematic of a feeder route in Fig. 1. The feeder route is like an expressway for cable pairs which extends out from the central office. Lateral cables are spliced to the feeder cable at various points along the route and these lateral cables are joined to distribution cables which are connected to subscribers.

The distribution plant along the route has been organized geographically into regions called *allocation areas*. Allocation areas are used for feeder planning and administration; they provide the basis for engineering manageable portions of the route, rather than trying to look at the entire route all at once. Conversely, the use of allocation areas encourages the feeder engineer, when solving local feeder/distribution problems, to consider the requirements of the rest of the route. Allocation areas are also used for monitoring facility problems, described elsewhere in this issue.²

The feeder itself has traditionally been broken up into segments called *feeder sections*. The feeder sections, which include all of the cable and conduit in a cross section between two points (usually important man-holes) along the route, are used for relief planning and design.³ The reason for this is that new relief cables are economically sized and placed in one or more sections at a time and then are spliced together with other cables to provide pairs from the central office to the various areas served by the network.

The other way of looking at the feeder facilities is the *pair group*. A pair group is a "bundle" of feeder pairs (not necessarily in the same cable sheath) which extend outward from the central office to a specific portion of the distribution plant, a single allocation area. Thus there is a one-

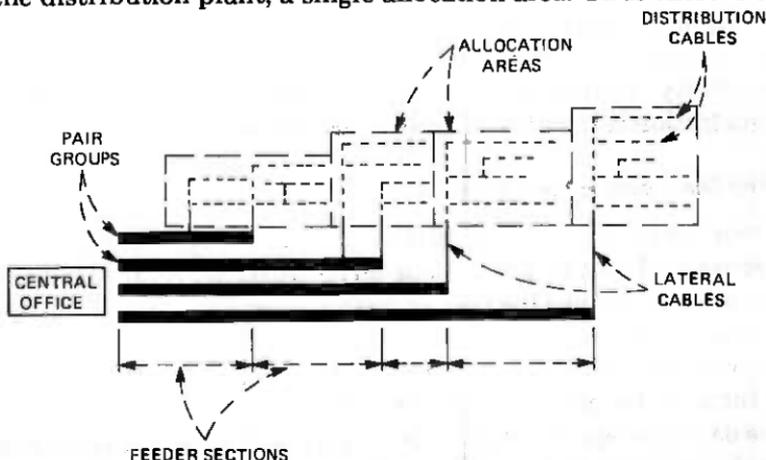


Fig. 1—Feeder route schematic.

to-one relationship between pair groups and allocation areas; the feeder complements (groups of pairs, usually in multiples of 25 pairs) in a particular pair group serve the allocation area associated with that pair group. Pair groups, then, are used primarily for feeder administration. Notice that a single section may have several pair groups passing through it and each pair group generally passes through several sections.

II. A SIMPLE APPROACH TO ALLOCATION

How should spare feeder complements be apportioned (or allocated) to the allocation areas along the route? This depends on the *lifetimes* of both the feeder sections and pair groups. "Lifetime" as used here does not mean the time until the feeder plant is replaced, but rather the time until additional plant must be provided. This is also known as fill time, time to exhaustion, or time to relief.

A section's lifetime is based on the number of spare pairs in the cross section and the growth into and through the section. A pair group's lifetime, however, is based on the number of spare pairs and the growth in that pair group only. Thus the feeder section lifetimes are usually different from the pair group lifetimes.

2.1 An illustrative example

Consider the example* shown schematically in Fig. 2. Here there are three pair groups feeding three allocation areas: AA1, AA2, and AA3. There are also three feeder sections: 2101, 2102, and 2103. Notice that

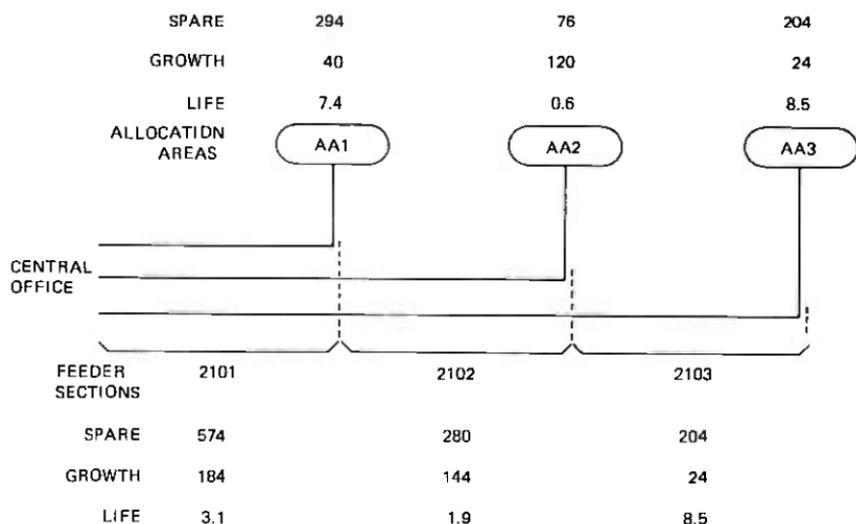


Fig. 2—An example route showing allocation area and feeder section lifetimes.

* All of the feeder routes in this paper are simplified in order to make clearer examples. Actual feeder routes frequently are composed of 20 or more sections.

the *ideal lifetime* (the number of spare pairs divided by the growth rate) of allocation area AA2 (and its associated pair group) is only 0.6 year. The section 2102 ideal lifetime, however, is 1.9 years. Farther out in the route, both allocation area AA3 and section 2103 have large ideal lifetimes of 8.5 years. It would appear that allocation area AA3 might be able to share some of its spare pairs with allocation area AA2. The only alternative to a rearrangement such as this is to place a relief cable in section 2102 within 0.6 year.

Notice that the ideal lifetime of section 2102 (1.9 years) is the shortest section lifetime in the route. This section is referred to as the *critical section*. It is this section that will exhaust first if growth occurs as forecast and if all possible rearrangements are made to defer relief. That is, if all spare pairs can be used, then the time to exhaustion for section 2102 could be up to 1.9 years. But this section will exhaust sooner given the way the spare pairs are presently distributed to the allocation areas. Relief can be deferred in the critical section as much as possible by apportioning the spare pairs in the critical section to the various pair groups running through it such that they all have identical ideal lifetimes.

Since the ideal lifetime of AA3 is greater than the critical section's lifetime, then a *surplus* of spare pairs is associated with or allocated to AA3. In the same way, a *deficit* of spare pairs is allocated to AA2. *Load balancing* is the process of reallocating spare pairs from areas with a surplus to those with a deficit. Load balancing often helps to identify necessary physical rearrangements in the feeder network.

2.2 A procedure for load balancing

A load balance worksheet has been developed for use by outside plant engineers. Figure 3 illustrates its application to the example route of Fig. 2. The entries on lines A through H correspond to those in Fig. 2, discussed in the previous section.

On line J is the *ideal allocation lifetime*. This is the lifetime each allocation area (pair group) should have in order to theoretically balance the route and defer the next relief for as long as possible. The ideal allocation lifetime is the same as the most critical section's lifetime for those sections beyond and including the critical section. Thus, both areas AA2 and AA3 have ideal allocation lifetimes of 1.9 years. For the allocation areas fed from sections between the most critical section and the next most critical section, the ideal allocation lifetime is the lifetime of the second critical section.

The *theoretical spare pair allocation* (line K) for each allocation area is the allocation area's growth rate multiplied by its ideal allocation lifetime. Note the entries in Fig. 3.

Finally, each allocation area's spare pair surplus/deficit is the difference between the number of spare pairs it currently has and its theo-

LOAD BALANCE WORKSHEET

DATE _____
WIRE CENTER _____
ROUTE _____

A. SECTION NUMBER	2101	2102	2103							
B. ALLOCATION AREA	AA1	AA2	AA3							
C. SPARE PAIRS IN ALLOCATION AREA	294	76	204							
D. ACCUMULATED SPARE PAIRS IN CROSS SEC. (ACCUM. C FROM FAR END)	574	280	204							
E. GROWTH IN ALLOCATION AREA	40	120	24							
F. ACCUMULATED GROWTH IN CROSS SEC (ACCUM. E FROM FAR END)	184	144	24							
G. IDEAL CROSS SECTIONAL LIFETIME (D÷F)	3.1	1.9	8.5							
H. CRITICAL SECTION(S)	#2	#1								
J. IDEAL ALLOCATION LIFETIME	3.1	1.9	1.9							
K. THEORETICAL SPARE PAIR ALLOCATION (E × J)	124	228	46							
L. SPARE PAIR SURPLUS/ DEFICIT (C-K)	170	-152	158							

Fig. 3—Load balance worksheet for example route in Fig. 2.

retical allocation. For this example the entries are 170 pairs for allocation area AA1, -152 for AA2, and 158 for AA3. Every deficit must be eliminated by using some surplus if the ideal allocation lifetimes of line J are to be achieved. Of course, it is not possible to use just *any* surplus; for example, here the 170 pairs in AA1 do not extend far enough out on the route to be used in AA2 (see Fig. 2).

Since the section and allocation area data are entered on the load balance worksheet from the central office outward, the table implicitly contains information on the configuration of the route. Therefore the load balance worksheet not only indicates spare pair surpluses and deficits, but it also shows how deficits can be resolved. Since AA3 is farther out in the route than AA2, then AA3's pair group passes through section 2102 and a transfer of surplus spare pairs could be made.

Why can't the 170 pair surplus in AA1 be used in AA2? Unless there are usable dead pairs in section 2102, there is no way to extend AA1's surplus spares without placing relief. How should the 170 pair surplus in AA1 be used? Ideally these pairs should be "held back" or reserved for use upon relief of the critical section. Unless section 2101 will be relieved along with section 2102, these pairs should not be allocated to allocation area AA1.

III. A MORE SOPHISTICATED APPROACH

An attractive feature of the load balance worksheet just described is its simplicity and consequential ease of preparation. Due to this simplicity, however, there are several important characteristics of the feeder which are not considered by the manual load balance worksheet: dead pairs, unallocated pairs, objective fills, and varying growth rates.

3.1 Improvements in the representation of the feeder route

First there are *dead pairs*, which are cable pairs that do not extend all the way back to the central office. Thus they cannot be used to provide service until they are spliced to cables which do reach the central office. Dead pairs are therefore a capital investment which is not making a return. Outside plant engineers need to know when and where it is practical and economical to use dead pairs, since this can defer feeder relief.

Unlike dead pairs, *unallocated pairs* are connected all the way back to the central office. They are not, however, associated with any allocation area and frequently are not even spliced to any lateral cable. The engineer should be aware of the number and locations of unallocated complements. In some cases, too many pairs may remain unallocated; relief can be deferred if the unallocated complements are allocated and then committed to allocation areas approaching exhaustion. On the other hand, there may be too few unallocated pairs in a section adjacent to a critical section on the central office side. Unless this section is to be relieved simultaneously with the critical section, then a certain number of its pairs should be reserved for future use. Upon relief of the critical section, these unallocated pairs are spliced to the new cable, thus providing central office pairs through and beyond the former critical section.

Next consider spare lifetimes. The ideal lifetime calculated using the load balance worksheet is simply the number of spare pairs divided by the growth rate. A more realistic lifetime, or *time to relief*, may be determined if an *objective* or maximum *fill* is used in the calculation. This fill is the percentage of cable pairs at a particular point which are practical and economical to use at the time just prior to relief. A typical value is 85 percent. (See Ref. 4, for a discussion of optimal objective fills, i.e., the economic fill at relief.) The time to relief, then, is

$$\text{Time to relief} = \frac{\left(\begin{array}{c} \text{objective} \\ \text{fill} \end{array} \right) \times \left(\begin{array}{c} \text{number of} \\ \text{pairs available} \end{array} \right) - \left(\begin{array}{c} \text{number of} \\ \text{pairs assigned} \end{array} \right)}{\text{growth rate}}$$

where the number of pairs available includes all central office pairs, i.e., spare, defective, and assigned (working) pairs.

Finally, the load balance worksheet uses a single growth rate for each allocation area. This is frequently a reasonable assumption, but some-

times growth rates change considerably over time. For example, as a developing area saturates with subscribers, the growth slows. On the other hand, if the nature of an area is changing from single-family to multifamily residential, growth will increase. Or there may be a single "impact" growth due to a new office building. Thus a more realistic approach is to make use of some sort of growth function.

An experimental computer program has been developed which takes into account these characteristics of a route. In order to show how this technique is more flexible than the manual load balance worksheet, an example is presented next. A generalized mathematical description is given in the appendix.

3.2 Computerized load balancing

As an illustration of the computerized table, consider the example route in Fig. 4. Here there are five feeder sections and six allocation areas. In addition to the pair groups feeding the allocation areas, there is an unallocated 50 pair complement which extends out to section 2104. There are also 100 dead pairs in section 2105.

Table I summarizes both the fill count and growth forecast for the allocation areas. Note that in most cases the growth rate is declining. For example, the forecast for allocation area AA4 is 100 pairs growth for the first year (12/77 to 12/78), 70 pairs the next year, 50 pairs per year for the next two years, and then 40 pairs per year after 12/81.

Figure 5 shows output from the computer program for this route. By examining this output the reader should gain a further understanding of the allocation process.

The leftmost column lists the allocation area (AA) and feeder section (FS) designations, along with abbreviations for unallocated (UNAL) and dead (DD) pairs. Feeder sections along with their associated allocation area(s), etc., are separated by horizontal lines, with the most distant

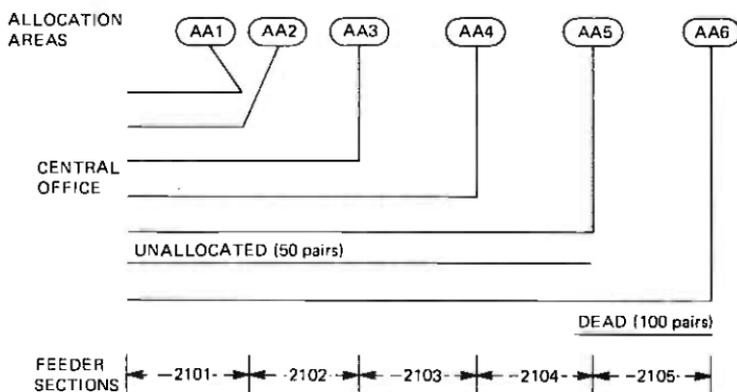


Fig. 4—Schematic of second example route.

Table I — Fill count and forecast for route in Fig. 4

Allocation area	Number of pairs available	Number of pairs assigned*	Objective fill	Growth forecast			Annual rate after 12/81
				12/77 to 12/78	12/78 to 12/79	12/79 to 12/81	
AA1	700	462	90	25	25	50	25
AA2	1500	1243	90	40	40	60	25
AA3	2450	1742	85	70	80	140	40
AA4	1850	1541	85	100	70	100	40
AA5	1000	771	85	55	40	80	30
AA6	400	228	90	30	20	40	10

* Fill count of 12/77.

section at the top of the table, and the section closest to the central office at the bottom.

Note, for example, section 2104, the second section from the top of the table. This section feeds allocation area AA5.

The second column is PAIRS AVAIL, the number of pairs available. For an allocation area this refers to the total number of central office pairs

LOOP FEEDER ANALYSIS TABLE

C.O. — WHIPPANY
ROUTE — 2A
DATE — 12/21/77

FACILITY COUNT (BASE) DATE: 12/77

AA FAS	PAIRS AVAIL	PAIRS ASGND	PRES FILL	OBJ FILL @RLF	GROW MARGN	---GROWTH FORECAST---			YRS TO RLF	EQUALIZED YRS	SUR PAIRS	SUR DEF	
						PER1 1278	PER2 1278	PER3 1281					
AA6	400	228	57.0	90	132	30	20	40	10	8.2	1.8	304	96
UNAL	0			90	0						0.0	0	0
DD/BT	100			90	90								
2105	500	228	45.6	90	222	30	20	40	10	17.2			
AA5	1000	771	77.1	85	79	55	40	80	30	1.6	1.8	1008	-8
UNAL	50			86	43						0.0	0	50
2104	1450	999	68.9	86	254	85	60	120	40	3.8			
AA4	1850	1541	83.3	85	31	100	70	100	40	0.3	1.8	1994	-144
UNAL	0			86	0						0.0	0	0
2103	3300	2540	77.0	86	285	185	130	220	80	1.8			
*** CRITICAL SECTION #1 — EXHAUST DATE 979 ***													
AA3	2450	1742	71.1	85	340	70	80	140	40	5.2	2.9	2299	151
UNAL	0			85	0						1.1	149	-149
2102	5750	4282	74.5	85	625	255	210	360	120	2.9			
*** CRITICAL SECTION #2 — EXHAUST DATE 1080 ***													
AA1	700	462	66.0	90	168	25	25	50	25	6.7	3.3	605	95
AA2	1500	1243	82.9	90	107	40	40	60	25	2.9	3.3	1513	-13
UNAL	0			87	0						0.4	84	-84
2101	7950	5987	75.3	87	900	320	275	470	170	3.3			
*** CRITICAL SECTION #3 — EXHAUST DATE 381 ***													

C.O.

Fig. 5—Computer output for example route in Fig. 4.

associated with the allocation area. This is, of course, the number of central office pairs in the allocation area's feeder pair group. (There are 1000 pairs available in AA5.) For unallocated pairs, the entry indicates the number of unallocated pairs ending in the section. Unallocated pairs passing through the section but ending farther out in the route are not included in this value. The number of dead pairs are included in each section in which they appear. There is an unallocated 50 pair complement ending in section 2104, but there are no dead pairs. There are 100 dead pairs in section 2105.

The number of pairs available in the feeder section is the total number of pairs in the cross section. (There are 1450 in section 2104.) This includes all of the pairs in pair groups feeding the section's allocation areas and the allocation areas beyond. All unallocated pairs in the section and beyond it, as well as dead pairs within the section, are accumulated also.

PAIRS ASGND refers to the number of pairs assigned. These are mostly working pairs but also include some pairs associated with specific subscriber locations but which are temporarily idle. These data are entered by allocation area and are accumulated by feeder section. (Neither dead nor unallocated pairs can be assigned.)

The next column (PRES FILL) provides the present fill for allocation areas and feeder sections. This is simply

$$\text{Present fill} = \frac{\left(\begin{array}{c} \text{current number of} \\ \text{assigned pairs} \end{array} \right)}{\left(\begin{array}{c} \text{number of pairs} \\ \text{available} \end{array} \right)}$$

When calculated for an allocation area, it refers only to the pairs in the allocation area's pair group. For feeder sections, however, the entire cross section is considered: all pair groups and unallocated pairs passing through, as well as dead pairs in the section.

The objective fill at relief (OBJ FILL@RLF) was described earlier. This fill level is determined by the engineer for the allocation area pair groups.⁴ Then, using these values, the program computes the objective fills for feeder sections and unallocated and dead complements.

The growth margin (GROW MARGN) is the number of spare pairs which can be used to provide service and bring the fill to the objective fill. It is

$$\text{Growth margin} = \left(\begin{array}{c} \text{objective} \\ \text{fill} \end{array} \right) \times \left(\begin{array}{c} \text{number of} \\ \text{pairs available} \end{array} \right) - \left(\begin{array}{c} \text{number of} \\ \text{pairs assigned} \end{array} \right)$$

The growth forecasts are next and for this program there may be from one to four columns. The simplest form is a single annual growth rate for each allocation area. Whenever the growth forecast reflects changes

in the growth rate over time, as in this case, the more flexible form of input may be used. The engineer then specifies up to three periods of time (PER1, etc.) and the growth that is expected to occur during each period within each allocation area. An annual growth rate (ANN RATE) is required for all time after the growth periods.

For example, consider again allocation area AA4. The first growth period extends from the facility count date of 12/77 to 12/78, one year in length. (Any time period is acceptable.) During this first period, the forecast growth is 100 pairs. During the next period, one year in length, the forecast growth decreases to 70 pairs. Then over the next two years the forecast growth is 80 pairs. (This is a smaller annual rate, of course, than the previous period.) Finally, after 12/81, the forecast annual growth rate for allocation area AA4 is 40 pairs per year.

The growth forecasts for a feeder section are an accumulation of the forecasts for all allocation areas fed by and beyond the section.

Using the growth margins and growth forecasts, the program computes the time (in years) to relief (YRS TO RLF), which is actually the time to reach the objective fill. This is determined for both allocation areas and feeder sections. As discussed earlier, the times to relief for a particular section and the allocation areas it feeds are frequently different. This may indicate that there is a better way to allocate existing spare pairs.

It is in the next three columns, however, that the ideal allocation becomes evident. EQUALIZED YRS and PAIRS represent the load balanced times to relief and number of pairs available for each allocation area and for the unallocated pairs in each section. The load balanced times to relief are based on the critical sections' times to exhaustion, as discussed earlier. Using these values, the equalized number of pairs available, i.e. the ideal allocation for theoretical load balance, may be determined. It is

$$\text{Equalized number of pairs available} = \frac{\left(\begin{array}{c} \text{number of} \\ \text{pairs assigned} \end{array} \right) + \left(\begin{array}{c} \text{growth over equalized} \\ \text{time to relief} \end{array} \right)}{\text{objective fill at relief}}$$

In the last column (SUR, DEF), the allocation surplus or deficit is printed. It is

$$\text{Allocation surplus/deficit} = \left(\begin{array}{c} \text{number of} \\ \text{pairs available} \end{array} \right) - \left(\begin{array}{c} \text{equalized number} \\ \text{of pairs available} \end{array} \right)$$

A negative value (deficit) for an allocation area indicates that there are too few pairs allocated to it. This allocation area will require relief sooner than if the ideal allocation were made to it. A positive value (surplus) for an allocation area indicates that too many spare pairs are available for the area. Its time to relief will be greater than its critical section's, and therefore, some other allocation area will have a deficit.

For example, allocation area AA6 in section 2105 has an allocation surplus of 96 pairs. The time to relief for this allocation area is 8.2 years. Since it is beyond the most critical section (2103), its equalized time to relief is 1.8 years. Therefore, by allocating more than 1.8 years of spare pairs to this allocation area, other allocation areas are shortchanged.

Consider AA4 fed from section 2103. This section is the most critical section and its time to relief is 1.8 years. Allocation area AA4, however, has only 0.3 of a year to go. The deficit of -144 pairs indicates that 144 additional pairs should be allocated to this allocation area so that it might last as long as its critical section.

In each feeder section, the allocation surplus/deficit for unallocated pairs is also determined. The entire unallocated 50 pair complement ending in section 2104 is surplus. This complement should be allocated to an allocation area with a deficit.

Deficits of unallocated pairs may occur in the sections which are adjacent (on the central office side) to a critical section. Such is the case for section 2102 which precedes the most critical section 2103. Ideally there should be 149 unallocated pairs in section 2102. Upon relief of the critical section these unallocated pairs would be spliced to the new cable and then allocated to allocation areas farther out in the route.

Now, by taking a look at the route as a whole, what potential changes in the present allocation can the engineer identify?

The most critical deficit is in AA4. Not only is the deficit large (-144) but the time to relief for this allocation area is extremely short (0.3 year). It should be possible (though not necessarily economical) to eliminate this deficit using allocation surpluses in sections beyond 2103, i.e., those sections above it in the table. Since there is a surplus of 96 spare pairs in AA6 in section 2105, there may be a spare 100 pair complement which could be transferred from AA6 to AA4. Furthermore, the 50 unallocated pairs ending in section 2104 could be allocated to AA4 since they pass through section 2103.

Rather than using the 150 pairs just discussed, could 150 pairs be obtained from AA3 (151 pair surplus) in section 2102? Since this section is closer to the central office, the pairs in AA3's pair group do not extend far enough out into the route to be allocated to AA4. If, however, there were 150 dead pairs in section 2103, then the central office pairs might be extended by splicing them to the dead complements.

The surplus in AA3 should, however, be used to satisfy the deficit of unallocated pairs (-149) in section 2102. A 150 pair spare complement should be reserved for use upon the relief of section 2103.

Similarly, perhaps 100 pairs of AA1's allocation should be held back to satisfy the unallocated deficit in section 2101. This is less important, though, because the relief of critical section 2 is considerably later than for critical section 1. Furthermore the exhaust dates of critical sections 2 and 3 indicate that they may be relieved simultaneously.

3.3 Implementation of the ideal allocation

The determination of an ideal load balance does not complete the allocation process. Next the outside plant engineer must study practical and economical rearrangements suggested by the idealized feeder configuration.

Since it is probably not economical to perfectly balance an entire route, the engineer generally begins with the most serious shortage. This first involves a detailed examination of various cable records, such as underground schematics, to determine precisely what type of cable splicing work is required to move spare complements from an allocation area with a surplus to another with a deficit. The engineer determines how many pairs are to be rearranged, which particular spare complements are to be used, which splice closures must be opened, and whether a stub (short length of cable) will have to be placed in the manhole. The objective is to minimize cost, of course, but there is also a consideration of factors such as present manhole congestion, which is difficult to quantify. Several alternatives may be examined.

After practical rearrangements are identified, an economic evaluation should be made. This is a comparison of the rearrangement costs (cable splicing labor and materials) with the savings of deferred relief (due to delayed capital expenditure). Only if the rearrangement proves economical will the engineer prepare work orders for its implementation.

IV. A FEEDER ADMINISTRATION PACKAGE

The allocation process is part of a feeder administration package now being used by outside plant engineers in the operating telephone companies. This package includes a user-oriented manual containing guidelines, procedures, engineering tools, and documentation to aid these engineers in their jobs. In addition to these materials, there is also a comprehensive training course.

V. CONCLUSIONS

The allocation process may be applied to apportion existing spare pairs in a feeder route. This can help to identify rearrangements of the network which will defer relief, leading to higher utilization and reduced capital costs. Similarly, these techniques are useful in planning the allocation of an upcoming relief cable.

Whether existing or relief pairs are involved, allocation establishes a plan for the use of spare facilities over the entire route. Then fewer rearrangements will be made and those which are made will be cost effective. Expedient rearrangements, often involving working pairs, will be required less frequently. Thus, not only will operating costs be reduced, but further complication and congestion of the feeder plant will be avoided.

VI. ACKNOWLEDGMENTS

The recent developments in feeder administration reflect the work of a number of outside plant engineers throughout the Bell System. Many of their ideas were brought together by the Feeder Task Force of the Bell System Design and Utilization Committee.

Specific contributions to the concepts described in this paper were made by W. N. Bell (Bell Laboratories), G. J. Dean and D. W. Post (New Jersey Bell), R. A. Wallfred (Northwestern Bell), and J. G. Funk (Western Electric).

APPENDIX

A Mathematical Description of the Load Balancing Process

This appendix mathematically describes the load balancing process for a generalized feeder route. The computer program described in Section 3.2 is based on this model. The route is a single linear path with allocation areas, dead pairs and unallocated pairs. Allocation area growth forecasts may vary deterministically over time. Prespecified objective fills are permitted. Multigauge and multipath routes are not considered here.

A.1 Define route configuration and growth forecast

Let N_s be the number of feeder sections in the route. Let i be the index for the i th feeder section, with $i = 1$ representing that section most distant from the central office, and $i = N_s$ the section closest to the central office.

For each section, let N_a^i = number of allocation areas fed from the i th section (as opposed to through it).

Let j be the index for the j th allocation area fed by the i th feeder section such that $j = 1$ to N_a^i , if $N_a^i \neq 0$. (If $N_a^i = 0$, then no allocation areas are fed by the i th section.)

For the j th allocation area fed from the i th feeder section (i.e., for each allocation area), the following are given:

P_{av}^{ij} = number of pairs available

P_{as}^{ij} = number of assigned pairs

A growth forecast for each allocation area is also given. As shown earlier, a simple annual growth rate may be used, with

G_f^{ij} = annual growth rate

This is frequently a reasonable assumption, but sometimes growth rates change considerably over time. A piecewise linear growth forecast is defined by

n = number of growth time periods (0,1,2, . . .)

If $n > 0$, then the following are given:

t_k = length of k th time period

G_k^{ij} = growth during k th time period, for $k = 1$ to n

At the end of the last time period, the annual growth rate G_r^{ij} is assumed to take effect. (If $n = 0$, then only the annual growth rate is given.) In order to determine a realistic lifetime, the objective fill at relief for each allocation area, F^{ij} , is also given.

The items above are given for each allocation area. The following items must also be given for each (the i th) feeder section,

P_d^i = number of dead pairs

P_u^i = number of unallocated pairs ending in this section

A.2 Accumulate section data from allocation area data

The growth forecasts for a feeder section are an accumulation of the forecasts for all allocation areas fed by and beyond the section.

$$G_r^i = \sum_{m=1}^i \left[\sum_{j=1}^{N_a^m} G_r^{mj} \right], \quad \text{for } i = 1, \dots, N_s$$

and, if $n > 0$,

$$G_k^i = \sum_{m=1}^i \left[\sum_{j=1}^{N_a^m} G_k^{mj} \right], \quad \text{for } i = 1, \dots, N_s \text{ and } k = 1, \dots, n$$

The number of pairs available in the feeder section is the total number of pairs in the cross section. This includes all of the pairs in pair groups feeding the section's allocation areas and the allocation areas beyond. All unallocated pairs in the section and beyond it, as well as dead pairs within the section, are also accumulated.

$$P_{av}^i = \sum_{m=1}^i \left[\sum_{j=1}^{N_a^m} P_{av}^{mj} + P_u^m \right] + P_d^i, \quad \text{for all } i \quad (i = 1, \dots, N_s)$$

Similarly, the number of assigned pairs in the section is

$$P_{as}^i = \sum_{m=1}^i \left[\sum_{j=1}^{N_a^m} P_{as}^{mj} \right], \quad \text{for all } i$$

A.3 Calculate time to relief

The *growth margin* is a generalization of spare pairs which includes the objective fill,

$$\text{Growth margin} = \left(\frac{\text{objective fill}}{\text{fill}} \right) \times \left(\frac{\text{number of pairs available}}{\text{pairs available}} \right) - \left(\frac{\text{number of pairs assigned}}{\text{pairs assigned}} \right)$$

For each allocation area the growth margin is

$$M^{ij} = F^{ij} P_{av}^{ij} - P_{as}^{ij} \quad \text{for all } i, j$$

Next, similar calculations are made on a section basis. The objective section fills are derived from the objective fills of all allocation areas fed by and beyond the section and the objective fills of all unallocated pairs beyond the section. For the section most distant from the central office ($i = 1$),

$$F^1 = \frac{\sum_{j=1}^{N_a^1} F^{1j} P_{av}^{1j}}{\sum_{j=1}^{N_a^1} P_{av}^{1j}}$$

and for $i > 1$,

$$F^i = \frac{\sum_{k=1}^i \sum_{j=1}^{N_a^k} F^{kj} P_{av}^{kj} + \sum_{k=1}^{i-1} F^k P_u^k}{\sum_{k=1}^i \sum_{j=1}^{N_a^k} P_{av}^{kj} + \sum_{k=1}^{i-1} P_u^k}$$

where objective fills for the unallocated pairs ending in the i th section and for the dead pairs in that section are set equal to that section's objective fill. The growth margins for unallocated and dead pairs, respectively, are

$$M_u^i = F^i P_u^i$$

$$M_d^i = F^i P_d^i, \quad \text{for all } i$$

Then, the grow margin for the i th section is

$$M^i = \sum_{k=1}^i \left[\sum_{j=1}^{N_a^k} M^{kj} + M_u^k \right] + M_d^i, \quad \text{for all } i$$

Now the *time to relief* for each feeder section and each allocation area may be calculated, using the growth margins and the growth forecasts, i.e.,

$$\text{Time to relief} = \frac{\text{growth margin}}{\text{growth rate}}$$

Since the growth forecast is a piecewise linear function, so too is the time to relief. For feeder sections when the growth margin is less than the forecast growth for the first time period, i.e.,

$$M^i < G_1^i$$

the time to section relief is

$$T^i = \frac{M^i}{G_1^i/t_1}$$

Similarly, for allocation areas when

$$M^{ij} < G_1^{ij}$$

then

$$T^{ij} = \frac{M^{ij}}{G_1^{ij}/t_1}$$

For feeder sections whenever

$$G_1^i \leq M^i \leq \sum_{k=1}^n G_k^i$$

then

$$T^i = \frac{\left(M^i - \sum_{k=1}^{m-1} G_k^i\right)}{G_m^i/t_m} + \sum_{k=1}^{m-1} t_k$$

where m satisfies

$$\sum_{k=1}^{m-1} G_k^i \leq M^i \leq \sum_{k=1}^m G_k^i, \quad (1 < m \leq n)$$

Similarly, for allocation areas, whenever

$$G_1^{ij} \leq M^{ij} \leq \sum_{k=1}^n G_k^{ij}$$

then

$$T^{ij} = \frac{\left(M^{ij} - \sum_{k=1}^{m-1} G_k^{ij}\right)}{G_m^{ij}/t_m} + \sum_{k=1}^{m-1} t_k$$

where m satisfies

$$\sum_{k=1}^{m-1} G_k^{ij} \leq M^{ij} \leq \sum_{k=1}^m G_k^{ij}, \quad (1 < m \leq n)$$

Finally, for feeder sections whenever

$$M^i > \sum_{k=1}^n G_k^i$$

then the time to relief is

$$T^i = \frac{\left(M^i - \sum_{k=1}^n G_k^i\right)}{G_r^i} + \sum_{k=1}^n t_k$$

Similarly for allocation areas whenever

$$M^{ij} > \sum_{k=1}^n G_k^{ij}$$

then

$$T^{ij} = \frac{\left(M^{ij} - \sum_{k=1}^n G_k^{ij}\right)}{G_r^{ij}} + \sum_{k=1}^n t_k$$

As an example, consider the case where

$$n = 3$$

and

$$G_1 + G_2 < M < G_1 + G_2 + G_3$$

(The feeder section and allocation area superscripts are deleted for simplicity in this example.) The time to relief, T , shown graphically in Fig. 6, is

$$T = \frac{M - (G_1 + G_2)}{G_3/t_3} + (t_1 + t_2)$$

A.4 Calculate the ideal allocation adjustment (surplus or deficit)

Next the feeder section times to relief, T^i , are used to identify the *critical sections*. The first (or most) critical section is that section with the shortest time to relief. Let I_1 be the feeder section index of the i th critical section. Then

$$I_1 = j \text{ such that } T^j = \min_{1 \leq k \leq N_s} (T^k)$$

The second critical section is the section between the first critical section and the central office with the least time to relief. Thus

$$I_2 = j \text{ such that } T^j = \min_{I_1 < k \leq N_s} (T^k)$$

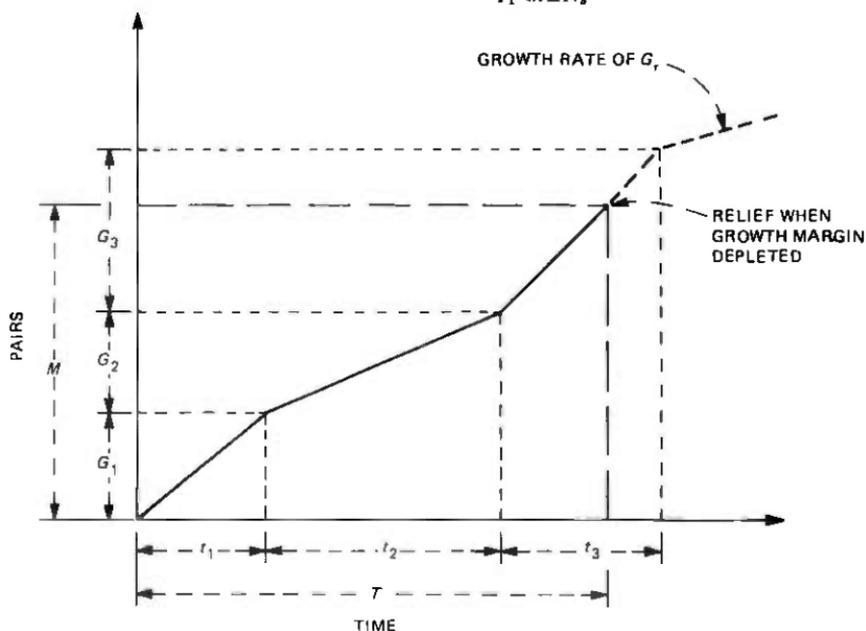


Fig. 6—Time to relief (T) for $n = 3$ and $G_1 + G_2 < M < G_1 + G_2 + G_3$.

The general form for this equation is

$$I_m = j \text{ such that } T^j = \min_{I_{m-1} < k \leq N_s} (T^k)$$

for

$$m > 1 \text{ until } I_m = N_s$$

The number of critical sections, N_c , is the value of m when

$$I_m = N_s$$

Ideally, the relief of the route's most critical section can be deferred for the greatest time if all allocation areas fed by and beyond this section have the same time to relief. The *equalized time to relief* for the allocation areas (and the sections) fed by and beyond the first critical section is

$$E_T^{ij} = E_T^i = T^{I_1}, \quad \text{for } i = 1 \text{ to } I_1 \\ \text{and } j = 1 \text{ to } N_a^i$$

For the k th critical section ($k > 1$)

$$E_T^{ij} = E_T^i = T^{I_k}, \quad \text{for } k = 2 \text{ to } N_c, i = I_{k-1} + 1 \text{ to } I_k, \\ \text{and } j = 1 \text{ to } N_a^i$$

The *equalized growth margin* is the number of pairs each allocation area will grow over its equalized time to relief. It is used to determine how many pairs should be available to an allocation area so that it will not require relief until its equalized time to relief.

The function for the growth over time is derived from the function for the time to relief for allocation areas. For

$$E_T^{ij} < t_1$$

the equalized growth margin over this time is

$$E_M^{ij} = \frac{G_X^{ij}}{t_1} E_T^{ij}$$

For the range

$$t_1 \leq E_T^{ij} \leq \sum_{k=1}^n t_k$$

then

$$E_M^{ij} = \frac{G_m^{ij}}{t_m} \left(E_T^{ij} - \sum_{k=1}^{m-1} t_k \right) + \sum_{k=1}^{m-1} G_k^{ij},$$

where m satisfies

$$\sum_{k=1}^{m-1} t_k \leq E_T^{ij} \leq \sum_{k=1}^m t_k, \quad (1 < m \leq n)$$

Finally for

$$E_T^{ij} > \sum_{k=1}^n t_k$$

then

$$E_M^{ij} = G_r^{ij} \left(E_T^{ij} - \sum_{k=1}^n t_k \right) + \sum_{k=1}^n G_k^{ij}$$

The *equalized pairs available* for each allocation area is simply the number of available pairs each allocation area would have if it were perfectly load balanced. It is

$$\text{Equalized pairs available} = \frac{\left(\begin{array}{c} \text{number of} \\ \text{pairs assigned} \end{array} \right) + \left(\begin{array}{c} \text{equalized} \\ \text{growth margin} \end{array} \right)}{\text{objective fill at relief}}$$

Thus,

$$E_{av}^{ij} = \frac{P_{as}^{ij} + E_M^{ij}}{F^{ij}}$$

Finally, the *ideal allocation adjustment* (surplus or deficit) for allocation areas is the difference between actual and equalized pairs available,

$$\text{Ideal allocation adjustment} = \left(\begin{array}{c} \text{number of} \\ \text{pairs available} \end{array} \right) - \left(\begin{array}{c} \text{equalized number} \\ \text{of pairs available} \end{array} \right)$$

or

$$A^{ij} = P_{av}^{ij} - E_{av}^{ij}$$

Deficits of unallocated pairs may occur in the sections which are adjacent (on the central office side) to a critical section. Upon relief of the critical section these unallocated pairs would be spliced to the new cable and then allocated to allocation areas farther out in the route.

It is also possible for there to be too many unallocated pairs in a particular section. Thus the unallocated pair allocation adjustment must be determined for every feeder section.

First the equalized growth margin for unallocated pairs must be determined in each section. This is the growth that will occur between the equalized times to relief between adjacent critical sections. This time difference between any adjacent sections i and $i - 1$ is

$$E_T^i - E_T^{i-1}, \quad \text{for } i = 2 \text{ to } N_s$$

For

$$E_T^i - E_T^{i-1} > 0, \quad (i > 1, \text{ implicitly})$$

the equalized growth margin for unallocated pairs is the growth occurring from time E_T^{i-1} to E_T^i .

Two evaluations must be made of the function for the equalized growth margin in a section, E_M^i , over time. (This function is similar to the function for allocation area equalized growth, above.) The function is given below with the time E_T^i (rather than E_T^{i-1}) as the independent variable. For $E_T^i < t_1$,

$$E_M^i = \frac{G_1^i}{t_1} E_T^i$$

For

$$t_1 \leq E_T^i \leq \sum_{k=1}^n t_k$$

then

$$E_M^i = \frac{G_m^i}{t_m} \left(E_T^i - \sum_{k=1}^{m-1} t_k \right) + \sum_{k=1}^{m-1} G_k^i$$

where m satisfies

$$\sum_{k=1}^{m-1} t_k \leq E_T^i \leq \sum_{k=1}^m t_k, \quad (1 < m \leq n)$$

Finally, for

$$E_T^i > \sum_{k=1}^n t_k$$

then

$$E_M^i = G_r^i \left(E_T^i - \sum_{k=1}^n t_k \right) + \sum_{k=1}^n G_k^i$$

Then the equalized growth margin for the unallocated pairs in a section is

$$E_{M_u}^i = E_M^i - E_T^{i-1}, \quad \text{for } i = 2, \dots, N_s$$

if the section farther from the central office has a shorter time to relief, i.e.,

$$E_T^i - E_T^{i-1} > 0$$

If, however, the section closer to the central office has the shorter life-time, i.e.,

$$E_T^i - E_T^{i-1} \leq 0$$

or if the section is the most distant one from the central office ($i = 1$), then

$$E_{M_u}^i = 0$$

After the unallocated equalized growth margins are determined, the equalized available pairs for unallocated pairs may be calculated,

$$E_{av_u}^i = \frac{E_{M_u}^i}{F^i}$$

Finally the ideal allocation adjustment for unallocated pairs is

$$A_u^i = P_u^i - E_{avu}^i$$

A.5 Glossary of symbols

A.5.1 Route data

Given:

- N_s = number of feeder sections
- n = number of growth time periods
- t_k = length of k th time period

Calculated:

- I_m = section index of m th critical section
- N_c = number of critical sections

A.5.2. Section data

Given:

- N_a^i = number of allocation areas fed by each section
- P_d^i = number of dead pairs in each section
- P_u^i = number of unallocated pairs ending in each section

Calculated:

- G_k^i = forecast growth during k th time period
- G_r^i = forecast growth rate after all specified time periods
- P_{av}^i = number of pairs available
- P_{as}^i = number of pairs assigned
- F^i = objective fill at relief
- M^i = growth margin
- M_u^i = growth margin for unallocated pairs
- M_d^i = growth margin for dead pairs
- T^i = time to relief
- E_T^i = equalized time to relief
- E_M^i = equalized growth margin
- E_{Mu}^i = equalized growth margin for unallocated pairs
- E_{avu}^i = equalized pairs available for unallocated pairs
- A_u^i = ideal allocation adjustment (surplus or deficit) for unallocated pairs

A.5.3 Allocation area data

Given:

- P_{av}^{ij} = number of pairs available
- P_{as}^{ij} = number of pairs assigned
- G_k^{ij} = forecast growth during k th time period
- G_r^{ij} = forecast growth rate after all specified time periods

F^{ij} = objective fill at relief

Calculated:

M^{ij} = growth margin

T^{ij} = time to relief

E_T^{ij} = equalized time to relief

E_M^{ij} = equalized growth margin

E_{av}^{ij} = equalized pairs available

A^{ij} = ideal allocation adjustment (surplus or deficit)

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Loop Plant Modeling:

An Approach to Modeling Operating Costs in the Loop Network

By W. L. G. KOONTZ

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A large share of loop network modeling effort is aimed toward characterizing operating costs. These costs arise because of the day-to-day activities associated with providing loop facilities. This paper considers those activities which occur as a result of inward service orders, i.e., requests for a cable pair. The models are designed primarily to reflect the impact of changes in the feeder portion of the network on operating cost. These models provide a basis for systems for administering the loop feeder network. Applications of the models are illustrated by examples.

I. INTRODUCTION

This paper, along with several other papers in this issue (Refs. 2-6), is concerned with the mathematical modeling of operating costs in the loop network. Specifically, models which predict the occurrence of loop network "activities" will be developed. These activities, together with their associated costs, constitute a cash flow which is a major component of the cost of the loop network. The goal of this modeling effort is to determine the effect of network design parameters on the operating cost so that the design of the network can be optimized on a total cost basis.

As is evident from the number of related papers in this issue, the concept of loop network operating cost is quite important. Moreover, there are several approaches to modeling operating cost as well as several areas of application of the models. In a very general sense, a loop network operating cost model predicts the sequence of activities (and the resulting cash flow) in the loop network. Many of these activities occur in direct response to inward service orders (see Ref. 1 for definition of terms).

Other activities, such as those relating to network troubles, may either be spontaneous or related to service order activity. Particular models may differ with regard to the types of activities modeled and the parameters of the loop network included in the model. This paper focuses on modeling activities directly related to inward service orders and the impact of parameters of the feeder network on these activities. These models can be applied in the feeder network design process to answer questions such as how to allocate feeder facilities and when to provide feeder relief.

Refs. 2 and 3 consider a broader class of loop network activities and place more emphasis on the impact of changes in the distribution network, notably conversion to interface design. Ref. 4 is also concerned with distribution design, but the emphasis there is on optimizing the parameters of a particular design: the Serving Area Concept. In Ref. 5, the approach used is similar to the one used here, but the object is to determine the optimum strategy for assigning facilities to inward service orders. Finally, Ref. 6 deals with modeling a particular activity in terms of its fundamental components.

Section II of this paper is an overview of the service order process which illustrates the kinds of loop network activities which may result from an inward service order. Section III presents the basic model for multiple outside plant (MOP). In Section IV, this model is extended to include use of the Connect-Through (CT) plan, which is discussed in Section II. Applications of the models are illustrated by means of examples.

II. INWARD SERVICE ORDERS AND LOOP OPERATING COST

Whenever a request for service, i.e., an inward service order, is received, a cable pair must be provided to connect the customer's premises to the local central office. The provision of this pair may involve one or more "activities" involving Operating Company personnel and equipment. These activities are the basic source of loop network operating cost. In this section, the process of providing a pair will be discussed in some detail in order to show how these activities arise.

2.1 Reassignable plant

Reassignable plant will be considered first. In reassignable plant, any pair which is not actually serving a customer (i.e., "working") is considered available for assignment (i.e., "spare").

Consider an inward order for residential service* at a given address. A particular *serving terminal*, in which several pairs (usually 10 to 50) are terminated, is associated with this address. If one or more pairs in

* Assume POTS unless otherwise indicated.

this serving terminal is spare, one will simply be chosen for assignment to the new customer. The connection is completed by having a *service wire* or "drop" connected from the customer's premises to the spare pair at the serving terminal. These operations (i.e., assign pair and connect drop) are the minimum effort required to provide service in reassignable plant.

If there is no spare pair in the designated serving terminal, the inward order is said to be "blocked." In this case, additional operations will be required. There are several alternatives. Figure 1 illustrates one possibility known as a line and station transfer (LST). Customer *B*, whose designated serving terminal is T_2 , needs service, but T_2 contains no spare pairs. There is a spare pair (P_2) in T_1 , however, and pair P_1 , which currently serves customer *A* out of T_1 , also appears in T_2 . Therefore, *A* can be transferred to P_2 , freeing P_1 to serve *B*. But what does this involve? Connecting the drop from P_1 to *B* is unavoidable. However, moving the drop at T_1 (a move which must be carefully coordinated with changes in the central office) is extra work which would not be required if a spare was present.

Another alternative is to connect a drop from *B* to P_2 at terminal T_1 . This is known as wiring out of limits (WOL) and involves the extra effort to secure the drop at the poles adjacent to T_1 and T_2 and any intermediate poles. WOLs are also trouble prone and unsightly. Other alternatives include multiple LSTs, clearing defective pairs, and application of single channel carrier (Ref. 7). All involve extra cost.

From this discussion, it is apparent that *avoidable* operating expenses in reassignable plant are triggered by blocked inward orders or *blockages*. Thus in Section III the emphasis will be placed on modeling blockages and the costs of LSTs, WOLs, etc. necessary to "clear the blockage."

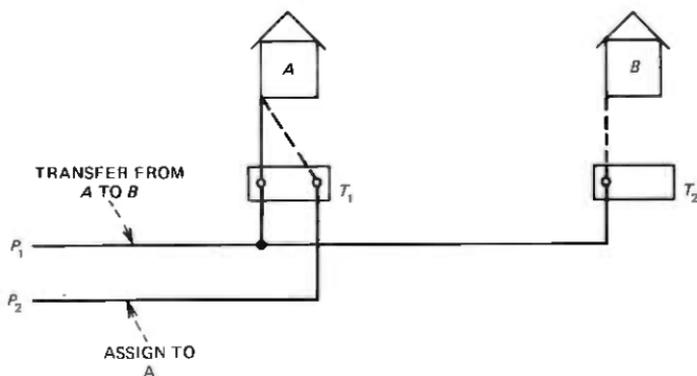


Fig. 1—Line and station transfer.

2.2 Connect-through administration

Connect-through or CT administration is the policy of leaving assigned pairs connected after service is discontinued. This policy is based on the assumption that a vacated premises will be reoccupied in a short time and that the new occupant will request telephone service. If a vacated premises is reoccupied and a pair is connected through to that premises, the installation activity is limited entirely to station work (installing stations, inside wiring, etc.). This is known as reusing a CT or, simply, a reuse. Under CT administration, a reuse is the simplest loop operation which may result from an inward order.

Clearly, it is simpler, and therefore less costly, to reuse a CT rather than assign a spare. Savings due to reuses make the CT plan economically attractive. Moreover, the advent of PhoneCenters increases the potential savings due to reuses. Under the PhoneCenter concept, the customer may elect to obtain station equipment at a local PhoneCenter and install it using previously installed jacks. Thus, in many cases, PhoneCenters will eliminate the need for station work at the customer's premises. If, in addition, it is not necessary for the installer to connect a pair for service, the installer visit is eliminated. Therefore, in a PhoneCenter environment, the savings due to a reuse, relative to the cost of assigning a spare, are greater by approximately the cost of the installer visit (i.e., the travel time).

Under CT administration, a pair may be in any of three states: working, spare, or CT. A CT pair is connected to a premises but not working. Both working and CT pairs are said to be assigned and CT pairs are sometimes called idle assigned pairs. Although CT pairs are available for assignment to the premises to which they are connected, they may or may not be considered available for assignment elsewhere.

Breaking a CT pair, or a BCT, is the process of assigning a CT pair to a new premises. A BCT involves both disconnecting and connecting a drop, either in the same terminal or in different terminals. A BCT is generally more complex than assigning a spare but less complex than an LST or a WOL (note that an LST or a WOL may involve a BCT). The rate of occurrence of BCTs depends not only upon customer movement and the configuration of the network, but upon the specific CT policy which is applied.

Variations on the basic CT plan are defined in terms of the treatment of CT pairs. Generally, CT pairs are divided into two categories: expendable CT pairs (i.e., those which can be reassigned to a new premises) and reserved CT pairs. These categories are recognized in the assignment preference list which reflects the policy for assigning pairs to inward service orders. An assignment preference list might look like the following:

1. Reuse CT pair.
2. Assign a spare pair.

3. Break an expendable CT pair.
4. Perform an LST, WOL, etc.
5. Break a reserved CT pair.

The operating cost under the CT plan depends on how the expendable and reserved CT categories are defined. Thus, the CT model must reflect this categorization.

One way to categorize CT pairs is to establish a *reserve time* such that only pairs which have been in the CT state for a period of time longer than the reserve time are expendable. This convention will be adopted in the derivation of the CT model in Section IV. If the reserve time is zero, then all CT pairs are expendable and a BCT will always be done in preference to an LST, WOL, etc. On the other hand, if the reserve time is infinite, then no CT pairs are expendable and a BCT will occur only as a last resort. The CT model can evaluate the effect of varying the reserve time between these two extremes.

2.3 The Serving Area Concept

The Serving Area Concept (SAC, Ref. 10) is a relatively new way to structure the loop network. Under SAC, a minimum of two distribution cable pairs are provided between each living unit and a serving area interface (SAI), which serves from 200 to 600 living units. Feeder cable pairs are also terminated at the SAI and a facility is provided for service by connecting the appropriate distribution pair to a feeder pair.

SAC operation is quite different than conventional design. For example, nearly all activity occurs at the SAI rather than at individual serving terminals. Although SAC is mentioned here for completeness, it will not be dealt with in detail in this paper. For a detailed discussion of operating costs under SAC, the reader is referred to Ref. 5.

2.4 Operating cost convention

This section will be concluded with a discussion regarding the way operating costs will be expressed in this paper.

There is a certain minimum cost required to provide a pair for service. In reassignable plant, the minimum cost is the cost of connecting to a spare pair in the designated serving terminal. Under the CT plan, the minimum cost activity is a reuse, provided the inward service order results from reoccupancy of a vacated premises. Even in CT plant, first occupancy of a new premises will necessitate at least connection to a spare pair.

It will be the convention of this paper to state the cost of an activity relative to the cost of the simplest (minimum cost) activity required to serve the inward order. This convention will be explained further as it is applied in Sections III and IV.

III. BASIC MODEL FOR REASSIGNABLE PLANT

Although most of the loop network today is operated under some kind of CT policy, reassignable plant is more straightforward and is a better starting point for the development of operating cost models. Moreover, many of the elements of the reassignable plant model carry over to more complex models.

3.1 Allocation areas and pair groups

The models derived here and in Section IV all assume the same geographic organization of the loop network. The geographic area served by the central office is divided into elemental units called *allocation areas*.^{1,8} An allocation area generally contains 500–2000 customers. Each allocation area is associated with a *pair group* consisting of those pairs which are either available for assignment to customers in the allocation area or can be made available through simple work operations (e.g., splicing). Ideally, no pair should be available for assignment in more than one allocation area. In practice, a pair which appears in more than one allocation area is associated with one of the allocation areas according to a "tie breaking rule" which will not be discussed here. The relationship between allocation areas and pair groups is illustrated in Fig. 2.

The operating costs in an allocation area are assumed to depend only on parameters of the allocation area and its pair group. Thus, the allocation area is the largest unit which has to be modeled. The operating costs for a larger area are determined by summation.

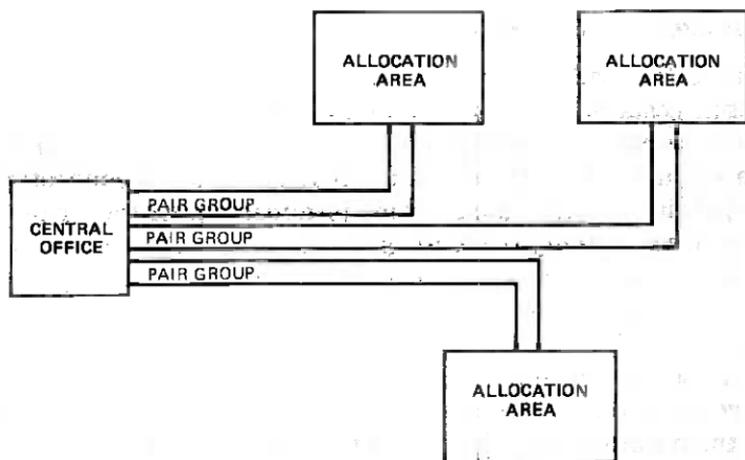


Fig. 2—Allocation areas and pair groups.

3.2 Allocation area model for reassignable plant

In reassignable plant, each inward service order results in either assignment of a spare pair in the designated serving terminal or a more complex operation (LST, etc.), which will be called "clearing a blockage" or, simply, a "blockage." In accordance with the cost convention stated in Section II, the cost of connecting to a spare in the designated serving terminal is assumed to be zero. For further simplification, it is assumed that the cost of clearing a blockage is the same for all blockages and is equal to the average cost of clearing a blockage. Thus, the operating cost incurred as the result of an inward service order is either zero or C'_{BLK} , the cost of clearing a blockage *relative to the cost of connecting to a spare in the designated serving terminal* (the prime is used to emphasize the relative nature of the cost factor).

A blockage is modeled as a probabilistic event and the probability that an inward service order is blocked is denoted $P(BLK)$. It is further assumed that inward service orders occur at a given constant rate, λ . Therefore, the expected operating cost per unit time, b , is given by

$$b = \lambda C'_{BLK} P(BLK) \quad (1)$$

The inward service order rate is a forecast quantity which usually must be derived from forecasts at the central office level. The cost of clearing a blockage may be estimated using techniques discussed in Refs. 2 and 6. Both of these quantities are assumed to be given here, leaving the probability of blockage as the key quantity to be derived.

3.2.1 Basic hypergeometric blocking probability model

Figure 3 illustrates a simple allocation area configuration. The allocation area is served by a pair group containing n feeder pairs. It is assumed that all n pairs are available for assignment within the allocation area. If the pair group contains defective pairs or pairs which have not been distributed to serving terminals, these pairs are not included in n .

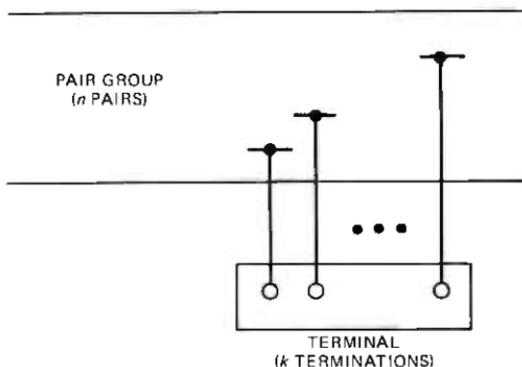


Fig. 3—Structure of hypergeometric blocking probability model.

The n pairs are distributed randomly among an unspecified number of k -pair serving terminals. It is assumed that w of the n pairs are working.

An inward move will be blocked if all k pairs in the designated serving terminal are working. Thus, the probability of blockage is the probability that k pairs, selected at random from n pairs, are all working or

$$P(\text{BLK}) = \frac{\binom{n-k}{w-k}}{\binom{n}{w}} \triangleq H(n, w, k) \quad (2)$$

Figure 4 is a plot of $P(\text{BLK})$ given by eq. (2) as a function of working pair fill ($f = w/n$) for various values of k with $n = 1000$. Note that the probability of blockage increases sharply with fill in the high fill region and is quite sensitive to terminal size. In fact, if k is much less than n and w (as it usually is), eq. (2) can be approximated by

$$P(\text{BLK}) \approx (w/n)^k \quad (3)$$

The basic hypergeometric model [eq. (2)] can be extended to more complex network configurations. At this point, however, a simple example illustrating an important application of the operating cost model will be presented.

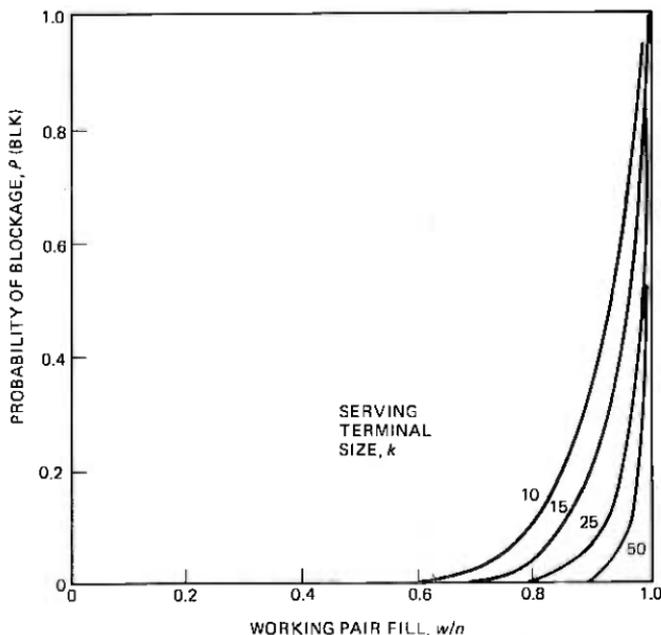


Fig. 4—Probability of blocking in reassignable plant.

Example: economic fill at relief

Suppose that the working pair fill of a given allocation area is increasing monotonically with time so that, at some point, additional pairs must be provided to the allocation area. The optimum fill at which new pairs are added, or the *economic fill at relief* (EFAR), is chosen to minimize the total cost of providing service. It is assumed that the cost of relief can be expressed as a levelized equivalent annual charge (LEAC, see Appendix to Ref. 9) which begins when the new pairs are added and that enough new pairs are added to reduce the operating cost to a negligible level for all future time. Under these assumptions, the total present worth cost of providing facilities is given by

$$PW = \int_0^T \lambda C'_{BLK} P(BLK) e^{-rt} dt + \frac{LEAC}{r} e^{-rT} \quad (4)$$

where T is the time at which relief is placed and e^{-rt} is the present worth factor. Note that, since fill is increasing with time, $P(BLK)$ is a function of t . A necessary condition for economic fill at relief is the following:

$$\lambda C'_{BLK} P(BLK) = LEAC \quad (5)$$

[Equation (5) is derived from the condition $dPW/dT = 0$.] Thus, relief should be placed at that point where the operating cost reaches the annual charge for the relief pairs.

For $\lambda = 100$ orders per year, $C'_{BLK} = \$100$ and $LEAC = \$5000$ per year, relief should occur when $P(BLK) = 0.5$. If the pair group size n is 1000 pairs and the terminal size k is 10 terminations, the economic fill at relief is approximately 0.90 as shown in Fig. 4.

This example is quite artificial because of the numerous simplifying assumptions made. In particular, a relief project almost always affects more than one allocation area and the sum of the allocation area operating costs must be compared with the relief cost. However, the example does illustrate the basic idea of economic fill at relief, one of the primary applications of the operating cost model.

3.2.2 Extension to multiple terminal sizes

Up to now, it has been assumed that only one size of serving terminal appears in the allocation area. Suppose, instead, that there are N_i terminals of size k_i for $i = 1, 2, \dots, m$. The probability of blockage given that the inward order occurs at a terminal of size k_i is $H(n, w, k_i)$ (eq. 2) so that the overall probability of blockage is given by

$$P(BLK) = \sum_{i=1}^m H(n, w, k_i) P(k_i) \quad (6)$$

where $P(k_i)$ is the probability that the inward order occurs at a terminal of size k_i .

If it is assumed that an inward order is equally likely to occur at any terminal, then

$$P(k_i) = N_i / \sum_{j=1}^m N_j \quad (7)$$

On the other hand, it may be more reasonable to assume that serving terminals are sized according to demand so that an inward order is more likely to occur at a larger terminal. Thus, eq. (7) may be replaced by

$$P(k_i) = k_i N_i / \sum_{j=1}^m k_j N_j \quad (8)$$

Another approach to modeling an allocation area containing a mix of terminals is to define an *equivalent terminal size* k_{eq} such that

$$P(\text{BLK}) = H(n, w, k_{eq}) \quad (9)$$

Although no analytic relation between k_{eq} and the k_i and N_i has been derived, k_{eq} can be chosen to fit $P(\text{BLK})$ to observed values or values obtained by computer simulation.

In summary, for reassignable plant designed under the multiple outside plant doctrine, operating costs are the result of blockages. The probability of blockage is the critical factor for determining operating cost [eq. (1)]. The probability of blockage, which has been derived from a simple model of the loop network in an allocation area, depends primarily on working pair fill and terminal size.

IV. EXTENDED MODEL FOR CONNECT-THROUGH PLANT

In this section, the basic model for reassignable plant will be extended to include areas operating under the CT plan. The extension is necessary mainly to include the impact of reusing and breaking CT pairs as discussed in Section II. It is also necessary to distinguish between working and assigned pairs and model their trajectories over time.

4.1 Allocation area model for CT plant

Figure 5 illustrates the "flow" of inward service orders under the CT plan. The inward service orders are sorted into two categories: those which correspond to reoccupancy of a vacated premises* and those which correspond to first occupancy of a new premises. If a vacated premises is reoccupied and a CT pair is assigned to that premises, then the CT pair is reused and no cost is incurred. If no CT pair is assigned to the premises, a new pair must be assigned. This operation is called a reterminate connection (RTC) and incurs a cost, C'_{RTC} . Even though an RTC may be accomplished by connecting to a spare pair in the designated serving

* In this discussion, the term premises is used in a general sense to denote a potential point of demand for service.

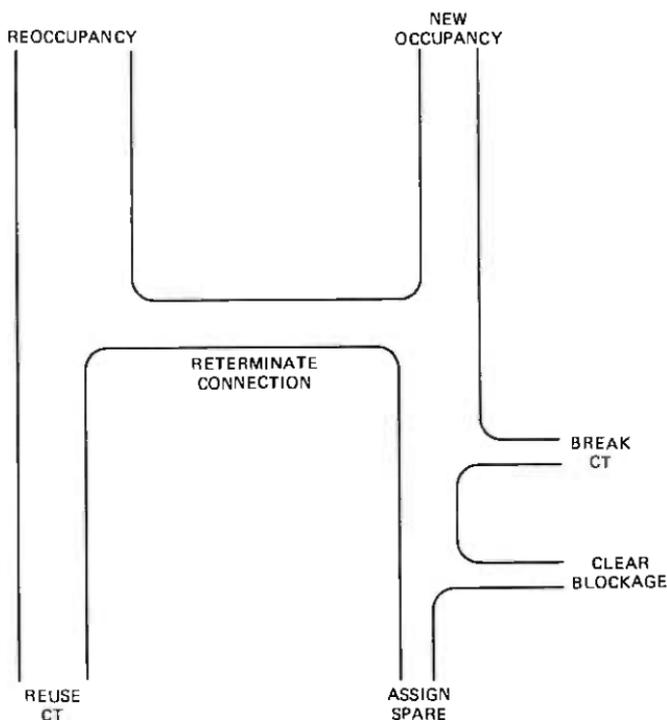


Fig. 5—Inward service order flow in CT plant.

terminal, the cost of this operation *relative to the cost of a reuse* must be counted, since a reuse is the simplest operation required to provide a pair for a reoccupancy.

As shown in Fig. 5, inward orders which result in RTC are lumped with inward orders corresponding to new occupancies. These orders are served by either assigning a spare pair in the designated serving terminal or breaking a CT pair (BCT), or clearing a blockage. No cost is associated with assigning a spare since this is the simplest operation required to provide a pair for a new occupancy and since the cost of assigning a spare to a reoccupancy has been accounted for by C'_{RTC} . The cost of breaking a CT pair, C'_{BCT} , and the cost of clearing a blockage, C'_{BLK} , are given relative to the cost of connecting to a spare in the designated serving terminal. Note that C'_{BLK} is defined the same way for the reassignable plant model.

Let RTC , BCT , and BLK be the rate of occurrence of reterminate connection, break CT, and clear blockage in a given allocation area. Then the operating cost per unit time, b , in this allocation area is given by

$$b = RTC \cdot C'_{RTC} + BCT \cdot C'_{BCT} + BLK \cdot C'_{BLK} \quad (10)$$

Note that eq. (10) is consistent with eq. (1) since, in reassignable plant, $RTC = BCT = 0$ and $BLK = \lambda \cdot P(BLK)$.

4.2 Probability of blockage and BCT

Let λ_{IN} be the rate of new occupancies and let $\lambda_{NET} = \lambda_{IN} + RTC$. BCT and BLK in eq. (10) can be expressed as

$$BCT = \lambda_{NET}P(BCT) \quad (11)$$

and

$$BLK = \lambda_{NET}P(BLK) \quad (12)$$

where $P(BCT)$ and $P(BLK)$ are the probabilities of the BCT and blockage events, respectively. These probabilities can be derived by extending the results of Section III.

First consider the probability that there is no spare pair in the designated serving terminal. Since only those pairs which are not assigned are considered spare, this probability is given by $H(n, a, k)$, where a is the number of assigned pairs [see eq. (2)].*

If the designated serving terminal contains no spare pair, but contains at least one expendable CT pair (see Section II), a BCT will occur. It is assumed that a CT is expendable if it has been idle for a designated reserve time, τ_R , or longer. It is further assumed that the ages of the CT pairs (i.e., the time they have been idle) are independent, exponentially distributed random variables with parameter τ_V . The parameter τ_V may be interpreted as the mean vacancy time of a premises in the allocation area. The probability, $P(EXP)$, that a CT pair is expendable is given, therefore, by

$$P(EXP) = e^{-\tau_R/\tau_V} \quad (11)$$

If there is no expendable CT or spare in the designated serving terminal, then it is necessary to either clear the blockage (LST, etc.) or break a reserved CT. In order to simplify the model it is assumed that breaking a reserved CT is equivalent to clearing a blockage. Since reserved CT pairs are broken only as a last resort, the error due to this assumption is only significant at high working pair fill (i.e., $w/n \approx 1$).

Now consider the conditional probability, $P(BLK/\overline{SPR})$, of a blockage given that there is no spare pair in the designated serving terminal. This is taken to be the probability that k pairs, selected at random from a population of a pairs, w of which are working and $a - w$ of which are CT with expendability probabilities given by eq. (11) are all either working or nonexpendable. This probability is given by

* It is assumed that a single terminal size, k , is in use. The results can be extended to multiple terminal sizes as in Section III.

$$P(\text{BLK}/\overline{\text{SPR}}) = \prod_{i=1}^k \left[\underbrace{\frac{w-i+1}{a-i+1}}_{\substack{\text{probability} \\ \text{that } i\text{th} \\ \text{pair is} \\ \text{working}}} + \underbrace{\left(1 - \frac{w-i+1}{a-i+1}\right) \left(1 - e^{-\tau_R/\tau_V}\right)}_{\substack{\text{probability that } i\text{th pair} \\ \text{is a reserved CT}}}\right]$$

$$= H(a, w', k) \quad (12)$$

where

$$w' = w + (a - w)(1 - e^{-\tau_R/\tau_V}) \quad (13)$$

Note that w' is the sum of the working pairs and the expected number of reserved CT pairs. The probability of blockage can now be computed as

$$\begin{aligned} P(\text{BLK}) &= P(\text{BLK}/\overline{\text{SPR}})P(\overline{\text{SPR}}) \\ &= H(a, w', k)H(n, a, k) \\ &= H(n, w', k) \end{aligned} \quad (14)$$

Equation (14) differs from eq. (2) only in the replacement of working pairs, w , with "equivalent working pairs," w' .

The probability that the designated serving terminal contains at least one expendable CT pair, given that it contains no spare pair, is $1 - H(a, w', k)$, so that

$$P(\text{BCT}) = (1 - H(a, w', k))H(n, a, k) \quad (15)$$

Figure 6 is a plot of $P(\text{blk})$ and $P(\text{BCT})$ as a function of the ratio τ_R/τ_V with the other parameters fixed at the values stated in the figure. This figure is a rough illustration of how the reserve time, which is a control variable, can influence the operating cost.

Actually, eq. (15) is the probability that an expendable CT pair in the designated serving terminal is broken. As discussed earlier, however, some blockages may include breaking expendable CT pairs in conjunction with clearing a blockage or breaking reserved CT pairs. The total BCT probability, denoted $P_{\text{TOT}}(\text{BCT})$, is taken to be

$$P_{\text{TOT}}(\text{BCT}) = P(\text{BCT}) + P(\text{BLK})(a - w)/(n - w) \quad (16)$$

In deriving eq. (16), it is assumed that, when a blockage occurs, the pair ultimately assigned is selected at random from the $n - w$ nonworking pairs, $a - w$ of which are CT pairs. These additional BCT are treated as blockages for the purpose of computing operating cost. However, they are included with the other BCT in the RTC model discussed in Section 4.3 and in modeling the trajectory of assigned pairs over time in Section 4.4.

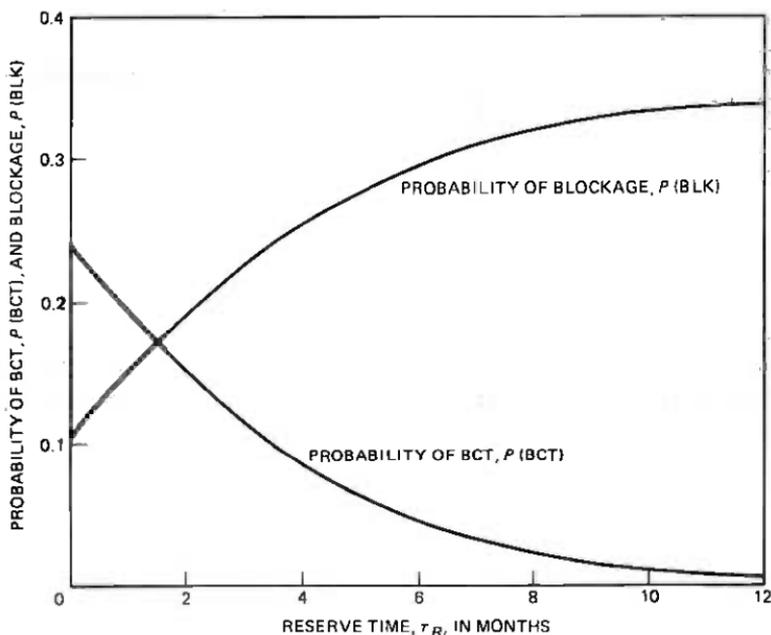


Fig. 6—Probability of BCT and blockage in CT plant.

4.3 Reterminate connection model

As discussed in Section 4.1, a reterminate connection occurs when a vacated premises is reoccupied and no pair is assigned to the premises. Thus, an RTC is the ultimate consequence of a BCT. This observation is the basis of the RTC model developed in this section.

Whenever a BCT occurs, an entity is created which corresponds to a vacated premises which has no pair assigned to it. An RTC occurs when one of these entities becomes reoccupied. An RTC does *not* occur when a vacated premises which has an assigned pair is reoccupied (this is a reuse) or when a new premises is occupied for the first time (see Fig. 5). Let $ENT(t)$ be the number of entities defined above which exist at time t . The RTC rate is taken to be

$$RTC(t) = ENT(t)/\tau_V \quad (17)$$

where τ_V is again interpreted as the mean vacancy time for an unoccupied premises.

Whenever an RTC occurs, an entity is destroyed, i.e., there is one less vacated premises with no pair assigned. Thus, the number of entities at time t satisfies

$$\frac{d}{dt} [ENT(t)] = BCT_{TOT}(t) - RTC(t) \quad (18)$$

where the time variation of the BCT and RTC rate has been explicitly

indicated. Note that the total BCT rate, which follows from eq. (16), is used. The RTC model is obtained by combining eqs. (17) and (18)

$$\tau_V \frac{d}{dt} [\text{RTC}(t)] + \text{RTC}(t) = \text{BCT}_{\text{TOT}}(t) \quad (19)$$

Figure 7 is a block diagram of the CT model as it stands at this point. The parameter λ_{IN} is the rate of new occupancy (see Fig. 5). New occupancies combine with RTC to form the net inward order rate, λ_{NET} (see Section 4.2). Net inward orders result in either assignment to a spare pair in the designated serving terminal (not shown in Fig. 7), breaking an expendable CT in the designated serving terminal [eq. (15)], or clearing a blockage [eq. (14)]. A fraction of the blockages results in additional BCT [eq. (16)]. The relationship between the RTC rate and the total BCT rate is illustrated in the frequency domain, for convenience.

4.4 Assigned and working pair trajectories

In the reassignable plant model, it is sufficient to model $w(t)$ as a specified function of time. The CT model is more complex, however, since $w(t)$ and $a(t)$ cannot be modeled independently.

Both $w(t)$ and $a(t)$ are modeled as responses to a given driving function, $p(t)$, which can be thought of as the number of premises in the allocation area. It is assumed that a vacant premises becomes occupied at rate $1/\tau_V$ and an occupied premises becomes vacant at rate $1/\tau_O$.* The following state equations are taken to characterize $w(t)$ and $a(t)$:

$$\frac{d}{dt} w(t) = [p(t) - w(t)]/\tau_V - w(t)/\tau_O \quad (20)$$

$$\frac{d}{dt} a(t) = [1 - P_{\text{TOT}}(\text{BCT})][p(t) - a(t)]/\tau_V \quad (21)$$

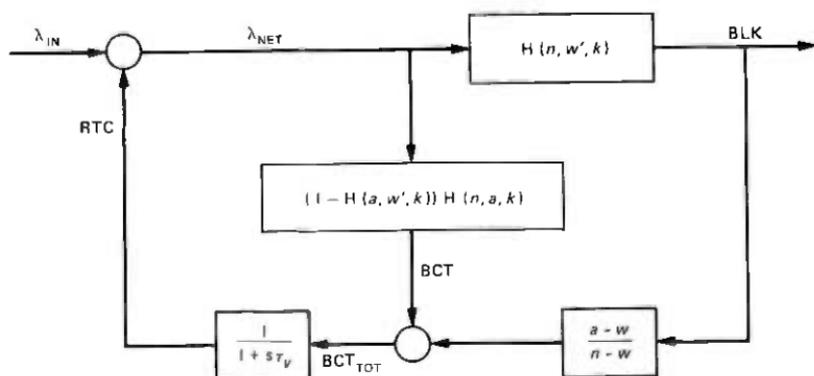


Fig. 7—Block diagram of CT model.

* The same symbol, τ_V , is used elsewhere in the CT model, although it has not been shown that all of these "vacancy times" are identical.

Whereas eq. (20) is a straightforward dynamic model, eq. (21) deserves further explanation. First of all, the quantity $[p(t) - a(t)]/\tau_V$ is the rate at which unoccupied premises which have no pair assigned become occupied. This rate includes new occupancies and RTC, i.e.,

$$[p(t) - a(t)]/\tau_V = \lambda_{IN}(t) + RTC(t) \quad (22)$$

or

$$\lambda_{NET}(t) = [p(t) - a(t)]/\tau_V \quad (23)$$

The rate of increase of $a(t)$ is $\lambda_{NET}(t)$ less the total BCT rate, hence eq. (21).

$P_{TOT}(BCT)$ is a rather complex, nonlinear function and eq. (21) must be solved numerically. This is done in the following example.

Example: economic fill at relief in CT plant

In this example, the optimal time to relieve an allocation area operating under the CT plan is computed. It is assumed that when relief occurs, the number of available pairs increase such that for time $t > T$, the time of relief, $P(BLK) = P(BCT) = 0$. Thus, there are no blockages or BCT for $t > T$. However, there will be RTC. For $t > T$, eq. (19) becomes

$$\tau_V \frac{d}{dt} [RTC(t)] + RTC(t) = 0$$

so that

$$RTC(t) = RTC(T)e^{-(t-T)/\tau_V} \quad (24)$$

for $t > T$. The present worth of the cost of all RTC which occur for $t > T$ is given by

$$\begin{aligned} PW_{RTC}(T) &= \int_T^{\infty} RTC(t)e^{-rt} dt \\ &= \frac{\tau_V C'_{RTC} RTC(T) e^{-rT}}{1 + r\tau_V} \end{aligned} \quad (25)$$

The total present worth cost for the allocation area is given by an extended version of eq. (4).

$$\begin{aligned} PW &= \int_0^T (RTC \cdot C'_{RTC} + BCT \cdot C'_{BCT} + BLK \cdot C'_{BLK}) e^{-rt} dt \\ &\quad + \frac{\tau_V C'_{RTC} RTC(T) e^{-rT}}{1 + r\tau_V} + \frac{LEAC}{r} e^{-rT} \end{aligned} \quad (26)$$

The optimal time to place relief follows from the condition $dPW/dT = 0$, as in section 3.2.1. Now

$$\frac{dPW}{dT} = [RTC(T) \cdot C'_{RTC} + BCT(T) \cdot C'_{BCT} + BLK(T) C'_{BLK}] e^{-rT}$$

$$+ \frac{\tau_v C'_{RTC}}{1 + r\tau_v} \left(\frac{d}{dt} [RTC(t)]|_{t=T} e^{-rT} - rRTC(T) e^{-rT} \right) - LEAC e^{-rT} \quad (27)$$

so that the optimal T must satisfy

$$\left(\tau_v \frac{d}{dt} [RTC(t)]|_{t=T} + RTC(T) \right) \frac{C'_{RTC}}{1 + r\tau_v} + BCT(T) \cdot C'_{BCT} + BLK(T) \cdot C'_{BLK} = LEAC \quad (28)$$

or, using eq. (19),

$$BCT_{TOT}(T) \frac{C'_{RTC}}{1 + r\tau_v} + BCT(T) \cdot C'_{BCT} + BLK(T) \cdot C'_{BLK} = LEAC \quad (29)$$

Equation (29) is the analog of eq. (5) for the extended model. Note that, through the first term of eq. (29), BCTs are given an additional cost penalty to account for future RTCs.

The optimum relief time, T , is determined by numerically minimizing PW given by eq. (26). Some sample results are listed in Table I. Sample plots of the trajectories of RTC, BCT, BCT_{TOT}, and BLK (Fig. 8) and $w(t)$ and $a(t)$ (Fig. 9) are also shown.

Table I — Example of optimal relief time for CT plant

Allocation area parameters			
Available pairs	n	1200 pairs	
Premises (initial)	$p(0)$	1030 prem.	
Assigned pairs (initial)	$a(0)$	1000 pairs	
Working pairs (initial)	$w(0)$	920 pairs	
Premises growth	g	10 prem./mo.	
Mean vacancy time	τ_v	3 mo.	
Mean occupancy time	τ_o	24 mo.	
Serving terminal size	k	10 term.	
Cost factors			
Retermine connection	C'_{RTC}	\$ 25	
Break CT	C'_{BCT}	\$ 10	
Clear blockage	C'_{BLK}	\$100	
Convenience rate	r	0.01/mo.	
Optimal time of relief			
LEAC* (\$ per month)	Reserve time (months)	Relief time (months)	Present worth cost (\$)
2500	0	28	206433
2500	2	26	209808
2500	12	24	214102
2500	120	24	214271
1250	0	23	108529
1250	2	21	110411
1250	12	19	112740
1250	120	19	112833

* Levelized equivalent annual charge.

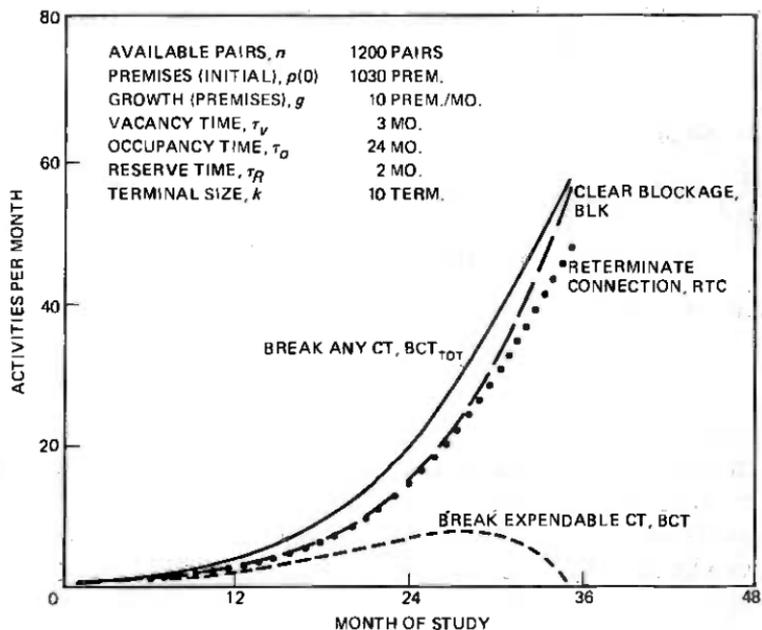


Fig. 8—Record of activity rates for CT model.

V. SUMMARY AND APPLICATION

This paper has presented a basic approach to modeling inward service order related operating costs in the loop network. These models provide

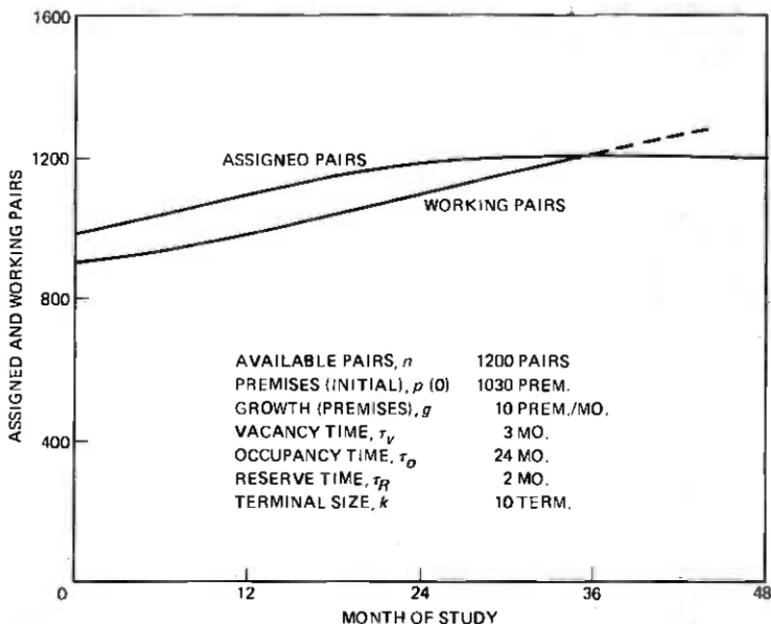


Fig. 9—Assigned and working pair trajectories.

both a theoretical basis and a practical method for the development of systems for administering the loop network. One such system is the Economic Feeder Administration and Relief (EFAR) computer program. EFAR computes the optimal time to place relief feeder cable. EFAR also evaluates the economic impact of transferring available pairs among pair groups. The EFAR algorithm is based on the reassignable plant model developed in Section III. Some variation of the more general CT model of Section IV will be incorporated into future releases of EFAR.

During the initial EFAR field trial, the reassignable plant model was tested by comparing its predictions to observed blockage rates. As a result of this test, heuristic modifications were added to the model. Further tests are proposed for the CT model. Both data collected from actual loop network operation and data derived from computer simulation will be used.

Compared with, say, the cost of placing new cable, loop network operating costs are very difficult to model. This is simply because the models must reflect a large number of small events rather than one large event. Thus, it is unreasonable to expect the kind of accuracy one could achieve in estimating the cost of a major construction project. Nevertheless, it is even more unreasonable to ignore operating costs—they are a significant part of the total cost of providing loop facilities.

VI. ACKNOWLEDGMENT

The original work at Bell Laboratories on loop network operating cost models was done by E. P. Klein, now of AT&T. In addition to the references cited, others who have contributed to the general knowledge in this area include J. O. Bergholm, M. J. Krone, and D. B. Lubner.

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Loop Plant Modeling:

Optimal Operating Policies for Serving Areas Using Connect-Through Administration

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Connect-Through administration is the policy of leaving the pair from a customer's premises to the central office intact when the customer disconnects from the network. A pair in this idle state is called a connect-through (CT) pair. In a serving area (the geographical entity in which all customers are served through an interface connecting the feeder and distribution parts of the network), growth may lead to the condition where no spares remain in the interface. It then becomes necessary to consider breaking CT pairs or providing additional spare pairs (relief). In this paper, two related operating decisions are examined. First, in order to determine under what conditions relief is more economical than breaking CTs, models are developed to compare the expected operating cost due to breaking CTs with relief costs. Second, when breaking a CT is the preferred procedure, it is shown that the optimal policy is to break the CT with the smallest instantaneous reuse probability, given by the hazard function of the premise vacancy time.

I. INTRODUCTION

Connect-Through administration is the policy of leaving the loop from a customer's premises to the central office intact when the customer disconnects from the network. This idle, but reserved, pair is called a connect-through (CT) pair. The savings from avoiding the disconnection operation and from having the pair available for reuse when (and if) a new customer moves into the same location may be counterbalanced by the fact that with fewer spare pairs available for new customers, costly loop network reconfigurations will be required more often.

Recent emphasis has been placed on understanding the effects of CT administration because of the decision to establish PhoneCenters in the Bell System. In a PhoneCenter environment, a customer may obtain the telephone in a PhoneCenter store. In many cases the customer can also connect the telephone to the network through jacks previously installed in his residence. This eliminates the need for any work inside the residence by the installer. If, in addition, a CT pair to this residence is available, no installer work of any kind is required. The savings from having a CT pair available for reuse are then much greater since the installer trip can be eliminated. Still, the trade-off between reuse savings and loop network reconfiguration costs must be evaluated to determine an optimal policy.

A serving area is a geographical entity (200 to 600 living units) served by feeder pairs terminated in a single interface (see Long,¹ this issue). When all of the feeder pairs in the interface are either working (in service) or CT, a new customer who cannot reuse a CT can then only be served by breaking a CT reserved for another location or by making more feeder pairs available at the interface (relief). In this paper, models are developed to determine under what conditions CT pairs should be broken in preference to providing relief, and to provide an optimal policy for deciding which CT to break when one is to be broken.

The question of whether to break a CT or to relieve is attacked by determining an optimal time for relief; this time is found by trading off operating costs and relief costs. Models for the operating costs of loop plant being administered under a CT plan are developed based on a linear growth, birth-and-death Markov model for customer demand. Expressions are developed for the expected number of CT and working pairs over time, and the average operating cost over time. Assuming that the system follows these expected trajectories exactly, the times of spare exhaust (the first time there are no spares remaining in the interface) and working exhaust (when all of the feeder pairs into the interface are working) can be calculated. The operating cost function is found to be a piecewise linear function of time which is discontinuous at the time of spare exhaust, when it becomes necessary to start breaking CT connections.

Relief timing is determined by the time at which the operating costs first exceed the levelized equivalent annual charges of relief plus post-relief operating costs (see Koontz,² this issue). Two types of relief are considered. The first affects only a single serving area (e.g., making additional pairs available at the interface). Solutions for this optimal relief time as a function of the system parameters are developed. The second type of relief provides additional feeder pairs to an entire allocation area (a group of two to five serving areas). This relief timing is optimized by considering the sum of the operating costs in each serving area.

The problem of which CT to break, when breaking is indicated, is solved by taking the one with the smallest instantaneous reuse rate. This rate is given by the hazard function of the premise vacancy time distribution. In addition to the exponential vacancy time distribution (equivalent to the Poisson demand model used in the relief timing models), modifications are incorporated to model three empirical observations about vacancy time.³ First, different categories of premises are allowed since, for example, first lines and additional lines would have different vacancy time distributions. Second, the fact that some CT pairs are un reusable is modelled by permitting abandonment of premises. Third, the observation that the vacancy times have a decreasing hazard rate³ is modelled by allowing the parameter of the exponential distribution to be a random variable. The optimal strategy in this model is shown to be breaking a CT which is the oldest in its category, with the choice of category depending on the ages of the oldest CT in each category.

II. RELIEF TIMING MODELS

2.1 Customer demand model

Demand for pairs is assumed to be the net result of individual customers moving into and out of "premises" according to independent Poisson processes. Section 2.1.1 examines the case where the number of premises is time-invariant, an appropriate model for non-growth areas. In Section 2.1.2 the number of premises is allowed to grow linearly over time.

2.1.1 Saturating exponential growth model

The system under consideration consists of a single serving area, served (by definition) by a single interface. It is assumed that there are a fixed number, p , of potential points of demand ("premises") in the serving area. The actual number of premises is assumed known, although in most situations it will be estimated from other data. Each premise without service ("vacant") generates inward moves according to a Poisson distribution of parameter λ , and each in-service ("working") premise generates outward moves by a Poisson distribution of parameter μ . The values of these parameters are not directly obtainable, but can be estimated from other data as will be described in Section 2.2.1. The reciprocals of these parameters are, respectively, the mean vacancy time, τ_v , and the mean occupancy time, τ_o . At a time when there are w working premises in the system, the expected inward and outward movement rates for the serving area as a whole are

$$\begin{aligned}\lambda_{TOT}(w) &= \lambda(p - w) \\ \mu_{TOT}(w) &= \mu w\end{aligned}\tag{1}$$

The state of the demand model at any time is represented by the number of working pairs in the interface. Since each working premise requires a single feeder pair, the number of working pairs is equal to the number of working premises. The number of working pairs increases by one with every inward move, and decreases by one with every outward move. Thus the rate of change in the expected number of working pairs equals the difference between the inward and outward movement rates:

$$\frac{dw}{dt} = \lambda(p - w) - \mu w \quad (2)$$

The solution to this differential equation is

$$w(t) = w(\infty) + (w(0) - w(\infty))e^{-(\lambda + \mu)t} \quad (3)$$

where

$$w(\infty) = p\lambda / (\lambda + \mu)$$

is the steady-state number of working pairs. The exact probability of there being w working pairs at time t is derived in Feller⁴ but is not necessary here since the present approach will deal only with expected values.

2.1.2 Linear growth model

Consider the saturating exponential growth model, with the number of premises allowed to vary with time instead of being fixed. In particular, assume that the number of premises grows linearly with time, so that

$$p(t) = G_p t + p(0) \quad (4)$$

The values of the constants G_p and $p(0)$ are not directly measurable; in Section 2.2.1 their estimation from other available data is described. With the number of premises time-varying, the differential equation for w [eq. (2)] still holds, but its solution is now

$$w(t) = C_1 e^{-(\lambda + \mu)t} + \frac{\lambda}{\lambda + \mu} \left(G_p t + p(0) - \frac{G_p}{\lambda + \mu} \right) \quad (5)$$

where

$$C_1 = w(0) - \frac{\lambda}{\lambda + \mu} \left(p(0) - \frac{G_p}{\lambda + \mu} \right)$$

For large t , the first term goes to zero so that the effect of the initial number of working pairs becomes negligible. Then the number of working pairs also increases linearly with time at a rate smaller than the premise growth rate. This can be represented as

$$w(t) = G_w t + w_0 \quad (6)$$

where

$$G_w = \frac{\lambda G_p}{\lambda + \mu} \quad (7)$$

and

$$w_o = \frac{\lambda}{\lambda + \mu} \left(p(0) - \frac{G_p}{\lambda + \mu} \right) \quad (8)$$

The parameters G_w and w_o for a given area will generally be obtainable from telephone company data. The parameter G_w is an estimate of the working pair growth and w_o represents the number of working pairs at the beginning of the study period. It will be assumed in the rest of this paper that the system has been operating for a sufficient time so that the exponential term of eq. (5) is negligible and the growth is linear.

2.2 CT levels over time

To determine the expected number of CT pairs, z , as a function of time, two phases have to be considered. The spare assignment phase (while the number of spare pairs in the interface is positive) lasts until the time of spare exhaust, T_s . The CT breaking phase lasts until the time of relief, T_R . As will be seen later, T_R must be between T_s and the time of working exhaust (when all of the feeder pairs are in service), T_w .

2.2.1 Spare assignment phase

When there are spares remaining in the interface, no CT pairs will have to be broken in order to provide service. An inward move will reuse a CT if there is one associated with its premises; otherwise, it will be assigned to a spare pair. Since an outward move always leaves a CT pair, the expected rate of increase in the number of CT pairs will equal the difference between the outward order rate, μ_{TOT} [eq. (1)], and the reuse rate. The reuse rate equals the inward order rate, λ_{TOT} , [eq. (1)] times the probability that an inward order will result in a reuse. Since the Poisson model implies that each vacant premise is equally likely to generate the next inward order, this probability is equal to the fraction of vacant premises which have CT pairs. Thus,

$$\frac{dz}{dt} = \mu w - \lambda(p - w) \left(\frac{z}{p - w} \right) \quad \text{for } t \leq T_s \quad (9)$$

Since this equation is only valid while there are spares remaining, the time of spare exhaust must be determined. Assuming that the system follows (2) and (9) exactly, T_s is found from

$$w(T_s) + z(T_s) = n \quad (10)$$

where n is the number of feeder pairs in the interface.

It should be noted that the derivations of eqs. (9) and (10) contain implicit approximations. First, T_s is not the expected time of spare ex-

haust (this requires first passage time calculations) but is the time when the expected number of spares becomes zero. This is a good approximation to the expected time of spare exhaust when growth is considered. Second, $z(t)$ is not exactly equal to the expected number of CT pairs at time t since a rigorous derivation from state probabilities would have to include the distribution of spare exhaust times. Again the approximation is sufficiently close for the models in this paper.

For the linear growth demand model, eqs. (4) and (6) are substituted into (9) and (10) and the large t approximation applied to get

$$\begin{aligned} z(t) &= \frac{\mu}{\lambda} \left[G_w t + w_o - \frac{G_w}{\lambda} \right] \\ &= G_z t + z_o \quad \text{for } t \leq \frac{n - w_o - z_o}{G_w + G_z} \end{aligned} \quad (11)$$

If the past history of the system has progressed according to the model, then

$$z_o = \frac{\mu}{\lambda} \left(w_o - \frac{G_w}{\lambda} \right) \quad (12)$$

and

$$G_z = \frac{\mu}{\lambda} G_w \quad (13)$$

Since z_o and G_z can generally be obtained from telephone company data, they can be used along with w_o and G_w to estimate the parameters G_p , $p(0)$, λ and μ . Equations (7), (8), (12), and (13) are solved simultaneously, yielding

$$\begin{aligned} G_p &= G_w + G_z \\ p(0) &= w_o + z_o + \frac{G_w + G_z}{G_w G_z} (G_z w_o - G_w z_o) \\ \lambda &= \frac{G_w G_z}{G_z w_o - G_w z_o} \\ \mu &= \frac{G_z^2}{G_z w_o - G_w z_o} \end{aligned} \quad (14)$$

These estimates will be used in the remainder of this paper.

2.2.2 CT breaking phase

After the spares are exhausted, every inward order results in either a reuse or the breaking of a CT. At this point, every nonworking pair in the interface will be a CT, so that

$$z(t) = n - w(t) \quad T_s \leq t \leq T_R \quad (15)$$

For the linear model, this becomes

$$z(t) = n - G_w t - w_0 \quad T_s \leq t \leq T_R \quad (16)$$

Fig. 1 illustrates the equations describing the linear model.

2.3 Operating cost models

This section uses the results of the previous section to estimate the expected operating costs over time. The *expected operating cost per inward move*, C_{IM} , is defined as the sum, over all possible operations to provide service, of the product of the cost per operation and the probability that an inward order requires that operation. Let C'_R , C'_S , and C'_B be the absolute costs per reuse, spare assignment and breaking a CT, respectively. Then

$C_{IM}(t)$

$$= \begin{cases} C'_R \left[\frac{z(t)}{p(t) - w(t)} \right] + C'_S \left[1 - \frac{z(t)}{p(t) - w(t)} \right] & t < T_s \\ C'_R \left[\frac{z(t)}{p(t) - w(t)} \right] + C'_B \left[1 - \frac{z(t)}{p(t) - w(t)} \right] & T_s \leq t \leq T_R \end{cases} \quad (17)$$

The *expected operating costs over time*, $b(t)$, are defined as the product of C_{IM} and the inward order rate [from eq. (1)]. To simplify the resulting equations, costs measured relative to the cost of a reuse (denoted C_R , C_S , and C_B) can be used in eq. (17) in place of absolute costs. It can be

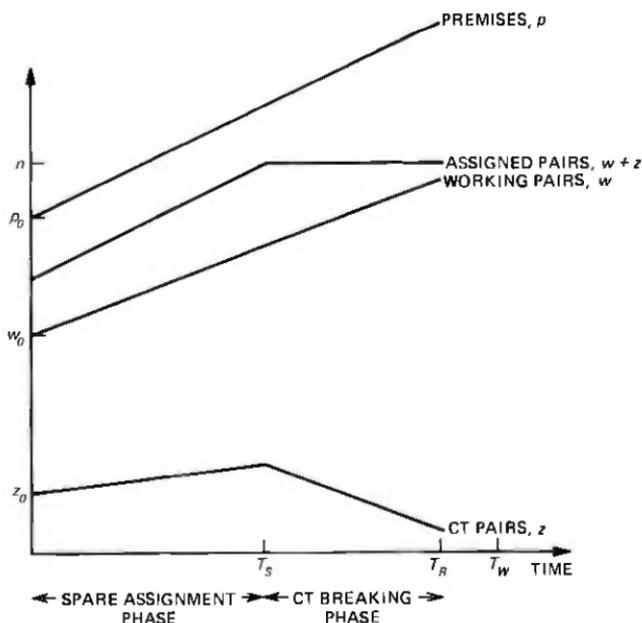


Fig. 1—Linear growth model.

shown⁵ that this substitution does not affect the relief timing or strategy decisions based on the models. Since $C_R = 0$, the operating costs over time become:

$$b(t) = \begin{cases} C_S \lambda (p(t) - w(t) - z(t)) & t < T_s \\ C_B \lambda (p(t) - n) & T_s \leq t \leq T_R \end{cases} \quad (18)$$

For the linear model, these operating costs are

$$b(t) = \begin{cases} C_S G_p & t < T_s \\ C_B [\lambda (p(0) - n) + \lambda G_p t] & T_s \leq t \leq T_R \end{cases} \quad (19)$$

where $p(0)$, G_p , and λ can be estimated through eq. (14). That is, until the time of spare exhaust, the operating cost is due to the constant rate at which spares are assigned, which is equal to the premise growth rate. Once the spares exhaust, the operating costs increase linearly over time as more and more CTs must be broken to provide service. Although the operating costs given by eq. (19) were derived as expected values of the costs, they will be subsequently used as if they were deterministic, an acceptable approximation for the models in this paper.

2.4 Relief timing calculations

The optimal timing for relief projects is determined by trading off relief and operating costs. It can be shown² that the economically optimal time for relief occurs when the difference in operating costs of the system immediately before and after relief becomes as large as the levelized equivalent charge for relief (LEAC).⁶

Consider first, relief of a single serving area (typically this is accomplished by transferring unneeded spare pairs from a nearby interface). From eq. (19), it can be seen that the operating costs during the spare assignment phase are independent of the number of spares, so that relief should not be performed before spare exhaust. Also, at the time of working exhaust, some sort of relief must be done if service is to be provided at all. After relief, the system will again be in the spare assignment phase. Let $\bar{b}(t)$ denote the operating costs during the CT breaking phase, and \underline{b} denote the initial post-relief (spare assignment) costs. Then the optimal relief time is the smallest t such that

$$\begin{aligned} \bar{b}(t) - \underline{b} &\geq \text{LEAC} \\ T_s &\leq t \leq T_w \end{aligned} \quad (20)$$

where LEAC is the levelized equivalent charge of the relief project.

There are three possible solutions to the minimization of t subject to (20). They are

- 1: $t^* = T_s$ if $\bar{b}(T_s) - \underline{b} \geq \text{LEAC}$
- 2: $t^* = T_w$ if $\bar{b}(T_w) - \underline{b} \leq \text{LEAC}$
- 3: t^* is found from $\bar{b}(t) - \underline{b} = \text{LEAC}$ otherwise

For the linear model, this becomes

- 1: $t^* = T_s$ if $(C_B - C_S)G_p \geq \text{LEAC}$
 - 2: $t^* = T_w$ if $(C_B - C_S)G_p + C_B\lambda \left(z_o + \frac{G_z}{G_w}(n - w_o) \right) \leq \text{LEAC}$.
 - 3: $t^* = [\text{LEAC} + C_S G_p - C_B(G_p + \lambda(w_o + z_o - n))][C_B\lambda G_p]^{-1}$
otherwise
- (22)

These cases are illustrated in Fig. 2.

In general, serving areas are administratively grouped into allocation areas, consisting of from two to five serving areas, and often the entire allocation area will be relieved at once (see Marsh,⁷ this issue). Let the parameters for serving area i be denoted by the subscript i and let N_s be the number of serving areas in the allocation area. Assuming that relief of individual serving areas is not feasible, the optimal relief time for the allocation area is the smallest t such that

$$\sum_{i=1}^{N_s} b_i(t) \geq \text{LEAC} + \sum_{i=1}^{N_s} \underline{b}_i \quad (23)$$

$$\hat{T}_s \leq t \leq \hat{T}_w$$

where $\hat{T} = \min_i T_i$

Since some of the serving areas may not have reached spare exhaust at

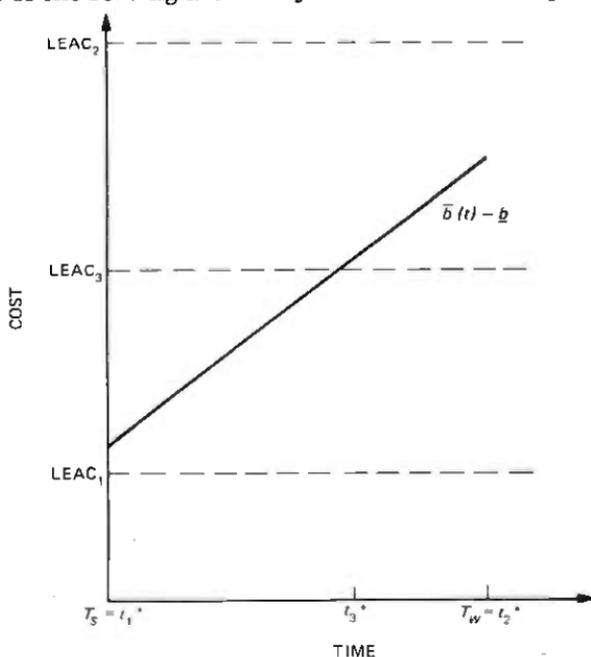


Fig. 2—Relief timing for one serving area.

the time of relief, the left side of (23) cannot specify whether pre- or post-spare exhaust costs should be used. This would be determined for each serving area from the limit in eq. (11).

III. CT BREAKING STRATEGY

Given that no spares are available at an interface, so that some CT must be broken in order to provide service to a new customer, the question of interest is which CT to break. The policy chosen should be the one which minimizes the present worth of the operating costs. It is shown in the appendix that a policy of breaking the one with the smallest instantaneous reuse probability is an excellent approximation to a minimum present worth strategy. Estimates of the reuse probabilities for each CT pair will depend on the model used for the demand for service. In particular, the exponential premise vacancy time distributions are allowed to be more general than before.

Let $f(t)$ be the probability distribution function of vacancy time at a premises, and $F(t)$ be the cumulative distribution function. Then the instantaneous reuse rate for a CT pair which has been idle for time t is given by the hazard function

$$h(t) = \frac{f(t)}{1 - F(t)} \quad (24)$$

and the probability of reuse in a small amount of time, dt , is given by $h(t)dt$. The hazard function is used in reliability theory as the measure of instantaneous failure rate, where $f(t)$ is the lifetime distribution of a system component.⁸ In the above model, premises vacancy time is analogous to the component lifetime and a reconnection at a vacant premises corresponds to the component failure. Following this analogy, the time that a pair has been idle as a CT will be referred to as its age. If a CT is to be broken, the one with the smallest reuse probability, and thus the smallest $h(t)$ should be chosen.

Four different vacancy time distributions are considered here. In addition to the commonly used exponential function, modifications to allow categorization, abandonment, and variability in the rate parameter are considered.

3.1 Exponential model

The exponential distribution is commonly used for modeling phenomena such as vacancy times due to its analytic simplicity. The Poisson demand model of Section II is equivalent to exponential vacancy and occupancy times. For a premises with an exponential vacancy time distribution of parameter λ ,

$$\begin{aligned} f(t) &= \lambda e^{-\lambda t} \\ h(t) &= \lambda \end{aligned} \quad (25)$$

Thus, the instantaneous reuse rate for any premises is constant over time, independent of when the premises became vacant. In addition, since all premises in a serving area are assumed to have the same demand parameter, λ , the reuse probabilities are the same for every CT in the interface. Thus, randomly selecting which CT to break is as good a strategy as any. This unappealing result leads to several modifications of the basic model.

3.2 Categorized exponential model

One modification of the exponential model is to reject the assumption that all premises in a serving area have the same demand parameter. Since a premises is defined as any potential point of demand for service, the characteristics of a premises should affect at least its mean vacancy time. For example, a second line would certainly have a longer expected vacancy time than a first line. Four other categorizations appropriate for premises within a serving area which have significant differences in the vacancy time parameters are³:

- (i) Type of dwelling (apartment/single family residence)
- (ii) Reason for disconnect (moving within a city/leaving city)
- (iii) Occupation (business/professional/military)
- (iv) Customer estimated date for reestablishment of service (less than two weeks/more than two weeks)

By using various combinations of categorizations, up to 48 different categories could be defined. If all premises in category i have exponential vacancy time distributions with parameter λ_i , the instantaneous reuse rates become

$$h_i(t) = \lambda_i \quad (26)$$

The optimal CT strategy is therefore to break any CT in the category which has the smallest λ_i (i.e., largest mean vacancy time).

3.3 Categorized exponential with abandonment model

A phenomenon which the above models do not take into account is the unreusability of some CT pairs (this is known as abandonment). This may be due either to physical abandonment of a premises or to changes in address designations which cause plant assignment procedures to ignore reuse possibilities. Assume that the vacancy time distribution of nonabandoned premises in category i is exponential with parameter λ_i and that the probability of abandonment is q_i . Then,

$$f_i(t) = (1 - q_i)\lambda_i e^{-\lambda_i t} \quad \text{for } t < \infty$$

$$h_i(t) = \frac{\lambda_i(1 - q_i)e^{-\lambda_i t}}{1 - (1 - q_i)(1 - e^{-\lambda_i t})} \quad \text{for } t < \infty \quad (27)$$

The reuse probabilities thus depend on t , the age of the CT.

If there is only one category, the optimal CT strategy is to break the oldest, since h_i decreases with time. This is reasonable since the longer a premises has been vacant, the more likely it is to be an abandoned one. When several categories are present, the effect of the parameters λ_i and q_i on the instantaneous reuse rate must be taken into account. The CT to break would be the oldest in its category, but the reuse probabilities for the oldest CT in each category must be compared to determine the lowest.

Figure 3 shows the optimal CT strategy as a function of the age of oldest CT in each of two categories for a case where Category I is more likely to be abandoned, but is also more likely to be reused sooner if it is not abandoned. In this hypothetical example, if the age of the oldest CT in Category I is small (less than 4 months), it is preferable to break even a new Category II CT. This occurs because the effects of abandonment are small relative to the effect of the λ_i for these values. For older Category I's (above 5.5 months), however, it may be preferable to break a newer Category I CT over an older CT in Category II.

3.4 Categorized beta type II with abandonment model

A further modification of the exponential model is to change the rate parameter, λ , from a known constant to a random variable with known

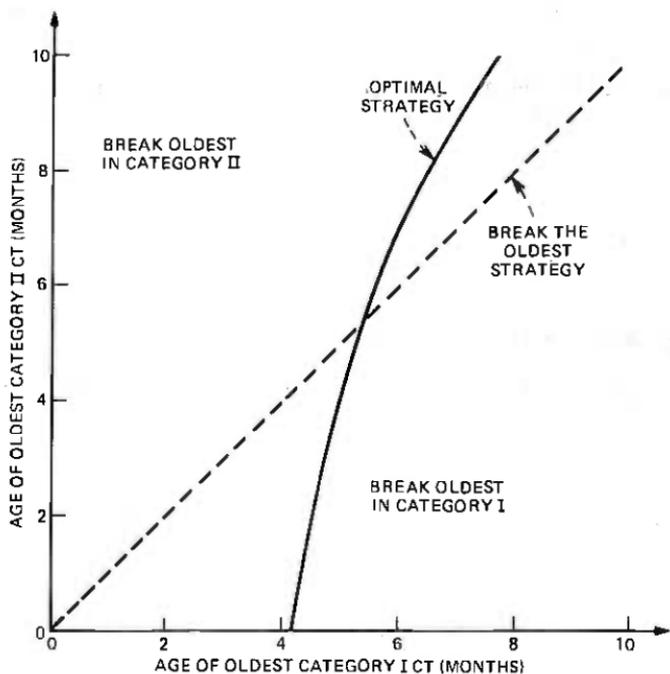


Fig. 3—CT strategy curves for hypothetical example. The parameter values used were $\lambda_1 = 0.25$, $\lambda_2 = 0.20$, $q_1 = 0.10$, $q_2 = 0.05$.

distribution. For example, any particular occupant of a premises may generate moves according to an exponential distribution with parameter which depends on the particular occupant. Then the premises would have an exponential distribution with a random parameter.

If a gamma distribution of scale parameter $1/d_i$ and shape parameter c_i is chosen to represent the known distribution of λ_i , the vacancy time distribution for a premises in category i becomes

$$f_i(t) = (1 - q_i)c_i d_i^{-1} (1 + t/d_i)^{-c_i - 1}$$

$$h_i(t) = (1 - q_i)c_i d_i^{-1} [q_i (1 + t/d_i)^{c_i + 1} + (1 - q_i)(1 + t/d_i)]^{-1} \quad (28)$$

The derivation of (28) is given in Mann et al.⁸ where it is called the exponential conditional failure distribution. Note that c_i/d_i is the expected value of λ_i and c_i/d_i^2 is its variance.

This distribution was used (under the name beta type II distribution) by Hoadley³ to model premise vacancy times based on the empirical observations of abandonment and of decreasing reuse probabilities with CT age. In particular, the empirical evidence showed that probability of reuse within the first few weeks is very high. Overall, 50 percent of the premises were reoccupied within 60 days, with some categories finding 80 percent reconnection within that time span. If such numbers are generally applicable, a high percentage of inward orders will result in reuses, so that the savings from using a good CT policy should be very high.

Although both the beta type II distribution and the categorized exponential with abandonment model give decreasing reuse probabilities over time when the abandonment probabilities are positive, only the former has this property when $q_i = 0$.

Under the assumption of beta type II distributed vacancy times, the optimal CT strategy when there is a single category is to break the oldest, since h decreases with time. For multiple categories, the reuse probabilities for the oldest in each category would have to be compared to find the lowest. Again, there will be cases where it is more advantageous to break a newer CT.

3.5 Summary

Although the exponential model is the simplest analytically, it is apparent that it does not account for empirical observations about vacancy time distributions. Both the categorized exponential with abandonment and the beta type II models are more realistic. Both have reuse probabilities decreasing with CT age, but of different functional form. The optimal strategy for breaking CTs under either model is to break one which is the oldest in its category, with the category determined by comparing the instantaneous reuse rates for the oldest in each category.

The use of an optimal strategy gives lower operating costs than the random strategy used in the relief timing derivations. This would have the effect of lowering the operating cost curve in Fig. 2, thus postponing the optimal relief time. Although analytic operating cost models to assess the exact extent of this effect have not been developed, typical serving areas were simulated under the various demand assumptions, using both a random breaking strategy and the policy of breaking the one with the minimum instantaneous reuse probability. The reduction in operating costs from using the optimal policy ranged up to twelve percent, depending on the demand parameters.

IV. CONCLUSIONS

In this paper, optimal procedures have been developed for two network operation decisions. The optimal time for relief of a serving area or allocation area is found as the time when operating costs (determined from a linear growth, birth-and-death demand model) exceed the levelized equivalent charges for relief. If the interface exhausts its spares and relief is not yet appropriate, CT pairs will have to be broken to provide service on some inward orders. The optimal CT to break is the one with the smallest instantaneous reuse probability; this will be one that is the oldest in its category, but the category will depend on the CT ages.

The question to be resolved before a model of this type can be implemented involve the data requirements and how to estimate the model parameters. Data (e.g., growth rates for working and CT pairs) may only be available at an aggregate level (e.g., by allocation area), so that a means of disaggregation may be required for these models. Although eq. (14) provided a means for estimating some of the model parameters, procedures for obtaining others (e.g., abandonment rates) remain to be developed.

The models developed here provide optimal operating policies for serving areas; however, serving areas constitute only a portion of the present loop plant. Extensions of these models to other loop network configurations is discussed by Koontz² elsewhere in this issue.

APPENDIX

Derivation of minimum cost CT breaking strategy

This appendix will derive the minimum present worth operating cost strategy for breaking CTs and show that it is approximately the same as minimizing the instantaneous reuse probability as given by the hazard function. The derivations are minor modifications of those originally developed by J. Freidenfelds in unpublished notes.

Assume that a CT pair has to be broken at time zero, and that the choice of which to break has been narrowed down to CT_1 and CT_2 (for example, by applying the derived results iteratively). Define

a_i = age of CT_i at time zero

T_i = random variable representing the time (relative to time zero) when a customer returns to the location of CT_i

$g_i(t)$ = p.d.f. for T_i

$G_i(t)$ = c.d.f. for T_i

$f_i(t)$ = vacancy time p.d.f. for location of CT_i

$F_i(t)$ = vacancy time c.d.f. for location of CT_i

Note that

$$g_i(t) = f_i(t)/[1 - F_i(a_i)]$$

and

$$G_i(t) = F_i(t)/[1 - F_i(a_i)]$$

Also define

Δ = time between breaking CTs at the interface

r = discounting rate, and

$$\theta_i(y) = E(e^{-rT_i}|y) = \frac{1}{1 - G_i(y)} \int_y^{\infty} e^{-rt} g_i(t) dt$$

Since, in addition to breaking a CT at time zero another one will have to be broken at time Δ , the options are to break CT_1 now and CT_2 at time Δ or CT_2 now and CT_1 at Δ . The cost, C_1 , of the former option is the sum of the present worths of the reconnection cost when customer 1 returns, the reconnection cost when customer 2 returns if he returns after Δ , and the reuse cost if he returns before Δ . Letting the reuse cost = 0, and the cost of reconnection relative to reuse = C_{REC} , then

$$\begin{aligned} C_1 &= C_{REC}[E(e^{-rT_1}|0) + E(e^{-rT_2}|\Delta)(1 - G_2(\Delta))] \\ &= C_{REC}[\theta_1(0) + \theta_2(\Delta)(1 - G_2(\Delta))] \end{aligned}$$

Similarly, the option of breaking CT_2 now costs

$$\begin{aligned} C_2 &= C_{REC}[E(e^{-rT_2}|0) + E(e^{-rT_1}|\Delta)(1 - G_1(\Delta))] \\ &= C_{REC}[\theta_2(0) + \theta_1(\Delta)(1 - G_1(\Delta))] \end{aligned}$$

Then CT_1 should be broken if $C_1 < C_2$, or

$$\theta_1(0) + \theta_2(\Delta)(1 - G_2(\Delta)) - \theta_2(0) - \theta_1(\Delta)(1 - G_1(\Delta)) < 0$$

Let

$$\mu_i = \theta_i(0) - \theta_i(\Delta)(1 - G_i(\Delta))$$

Then CT_1 should be broken if $\mu_1 - \mu_2 < 0$ which means the CT with the smaller μ_i should be broken.

In a serving area interface, Δ tends to be very small. Thus a valid ap-

proximation to μ_i is

$$\lim_{\Delta \rightarrow 0} \mu_i$$

Since this limit is easily seen to be zero for any μ_i , we need to look at

$$\lim_{\Delta \rightarrow 0} \mu_i / \Delta$$

to get a good approximation for small Δ . Applying l'Hôpital's rule gives:

$$\lim_{\Delta \rightarrow 0} \frac{\mu_i}{\Delta} = \lim_{\Delta \rightarrow 0} \frac{d\theta_i(0)}{d\Delta} - \left[(1 - G_i(\Delta)) \frac{d\theta_i(\Delta)}{d\Delta} - \theta_i(\Delta) \frac{dG_i(\Delta)}{d\Delta} \right]$$

Performing the differentiation gives:

$$\begin{aligned} \lim_{\Delta \rightarrow 0} \frac{\mu_i}{\Delta} &= \lim_{\Delta \rightarrow 0} \frac{-g_i(\Delta)}{1 - G_i(\Delta)} [G_i(\Delta)\theta_i(\Delta) - (1 - G_i(\Delta))e^{-r\Delta}] \\ &= g_i(0) \\ &= f_i(a_i) / [1 - F_i(a_i)] \end{aligned}$$

which is the hazard function of the vacancy time distribution. Thus we would break CT_1 if and only if its hazard function were smaller than that of CT_2 . In the general case, the CT with the smallest hazard function value should be chosen.

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Loop Plant Modeling:

Cost Models for Loop Plant Work Operations Using Semi-Markov Processes

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An important consideration in making economic evaluations of proposed loop plant relief, rearrangement and rehabilitation projects is the cost of loop plant work operations performed by splicers, installers, repair personnel, and support personnel. Using a semi-Markov process with states corresponding to activities performed during the work operation, probability distributions of the cost of work operations are obtained as a function of various plant conditions such as record error rates and defective pair rates. Transition probabilities and state delays are estimated using various plant reports and field data. A computer program calculates the distribution of the cost. A numerical example illustrates how the model can be used by determining the dependence of cable transfer costs on the number of pairs transferred and the percentage of working circuits transferred.

I. INTRODUCTION

An important consideration in making economic evaluations of proposed loop plant relief, rearrangement, and rehabilitation projects is the cost of loop plant work operations incurred to provide or maintain service. Such operations are performed by installers, cable maintenance and repair personnel, splicers, and support personnel including assignment and test bureau clerks, testers, frame personnel, and engineers.

The traditional method of determining these costs is by direct measurement. Direct measurement is usually limited, however, to a fixed set of conditions whereas a model can show how the costs vary with changing conditions. Therefore, a model can be more useful in estimating the change in costs caused by altering current work procedures, for ex-

ample, or by introducing some new technology. Also, a model can be tailored to fit local conditions such as a high defective pair rate, for example.

This article presents a method for obtaining cost distributions of loop plant work operations as a function of plant conditions such as record error rates and defective pair rates. The method views a work operation, such as a cable pair transfer or the completion of an inward service order, as a semi-Markov process with constant state delay times and an absorbing state corresponding to the completing step of the operation. The states of the process correspond to activities performed during the work operation. The constant state delay times, i.e., the times required to perform the activities, are defined to be the costs of the activities. The transition probabilities from one activity to the next are the probabilities of various contingencies that arise in the course of performing the activities. The transition probabilities and state delays are estimated using various plant reports such as the assignment pair change summary report and field data. A computer program calculates the distribution of the cost incurred to reach the final (absorbing) state of the process with a probability arbitrarily close to one.

The next section tells in more detail what a cable pair transfer is and why it is needed in the operation of the loop plant. Section 2.1 describes the method of application of the Markov model to the cable pair transfer. Section 2.2 gives a brief description of semi-Markov processes which highlights the properties relevant to the model. Section III contains numerical results illustrating the dependence of cable pair transfer costs on local conditions. In addition, cost estimates obtained using the semi-Markov model are compared to actual cost data. Finally, Section IV concludes that the model gives reasonable cost estimates under a wide variety of conditions.

II. WHAT IS A CABLE PAIR TRANSFER?

Throughout this paper, the particular work operation used to illustrate the method is the cable pair transfer. Cable pair transfers or cable throws are often used in the administration of Multiple Outside Plant (MOP) in conjunction with cable relief or in order to defer relief. Simply stated, a cable transfer involves changing the path by which a cable or portion of cable reaches from the central office to the customer. Two simple illustrations of transfers are given in Fig. 1. Figure 1a illustrates the use of a cable throw in conjunction with relief. Assuming that feeder cables and lateral cables are economically sized,¹ then, referring to Fig. 1a, the number of pairs in the laterals is approximately proportional to $\sqrt{g_1}$ and $\sqrt{g_2}$ where g_1 and g_2 are the lateral growth rates. Since the feeder growth rate is equal to $g_1 + g_2$, its size is proportional to $\sqrt{g_1 + g_2}$. The times to exhaustion are thus proportional to $1/\sqrt{g_1}$, $1/\sqrt{g_2}$, and $1/\sqrt{g_1 + g_2}$,

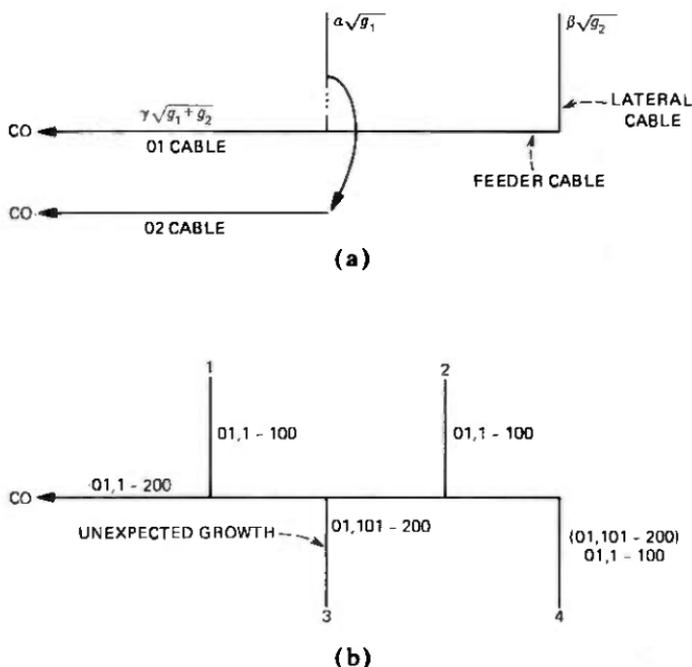


Fig. 1—(a) A cable throw in conjunction with relief. (b) A cable throw to defer relief.

for the two laterals and the feeder, respectively. Therefore, the feeder section (cable O1) exhausts before the lateral resulting in a mismatch. Relief of the feeder cable requires both placing new pairs and transferring some existing customers to the new cable. In Fig. 1a this is accomplished by transferring the pairs from one lateral to the new relief cable (cable O2). Transfers of this type are inherent in the MOP configuration and comprise a significant part of the cost of relief.

Cable throws are also used to defer relief in the event growth patterns are irregular and not as forecasted (see Ref. 2). This situation is illustrated in Fig. 1b. In this case, the feeder cable O1, pairs numbered 1–200, still has spare capacity, but due to an unforeseen growth spurt on lateral 3 the pairs of cable O1, numbered 101–200 have exhausted. In order to defer having to place new cable, lateral 4 is reconnected from cable O1, pairs 101–200 to O1, 1–100 so that pairs previously working in lateral 4 in count O1, 101–200 are now working in count O1, 1–100. This creates spare pairs in O1, 101–200 which can be used in lateral 3.

2.1 Defining the work operation in terms of a semi-Markov process

A semi-Markov process is a stochastic process which may be in any one of a set of states S_i , $i = 1, 2, \dots$. The process governing the transitions between states is Markov, but the length of stay or delay in any

given state is a continuous random variable. In general, the length of stay in any given state may depend on the state entered next in the process. In order to simplify exposition and parameter estimation, however, it will be assumed that the state delays are independent of the next state entered. Furthermore, for our purposes the state delays are assumed to be of fixed duration.

The goal of this work is to estimate the distribution of time required to perform a given work operation. To use the semi-Markov approach, it is necessary to define the steps of the process in such a way that the transition probabilities depend only on the present state of the process (the Markov property) and estimate the transition probabilities and state delays. The final step of the work operation is defined to be an absorbing state in the semi-Markov process, i.e., the probability of a transition from this state to any other state is zero. Therefore, the distribution of time to complete the required operation is the distribution of time to reach the final absorbing state. The required theory of semi-Markov processes³ is given in the appendix.

2.2 Method of application to a work operation

The approach used in developing cost models of loop plant work operations is first to define the basic tasks required of all departments involved in the operation and to determine the interrelationships between these tasks. The interrelationships between tasks and departments can be illustrated simply by use of a flow diagram of the entire operation.

An example of a flow chart for a portion of the cable pair transfer work operation is given in Fig. 2. Figure 2 shows the beginning of the splicing activity associated with a cable throw. This part of the operation begins with the identification of the "TO" count, i.e., determining (at the transfer location) the central office number of each pair in the new count which the cable pairs are to assume upon completion of the job. The task of identification involves transmitting a tone from the central office to the location of the transfer in the case of pulp-insulated cable and using color-code if PIC (polyethylene insulated conductor) cable. If a TO pair is defective and is to be part of a working circuit, then another pair in the TO count must be found to which the working circuit can be transferred. After identifying the TO pairs, the splicer proceeds to identify the "FROM" count, i.e., the pairs which are to be transferred to the new count (TO count). See Table I for definitions.

The flow-diagram (Fig. 2) consists of rectangular boxes which represent the steps in the work operation and diamond-shaped boxes referred to as decision diamonds that are used to represent the possible decisions that must be made at each step. Note that each step requires a given completion time that corresponds to a delay time.

At each decision diamond there are two possible paths by which the

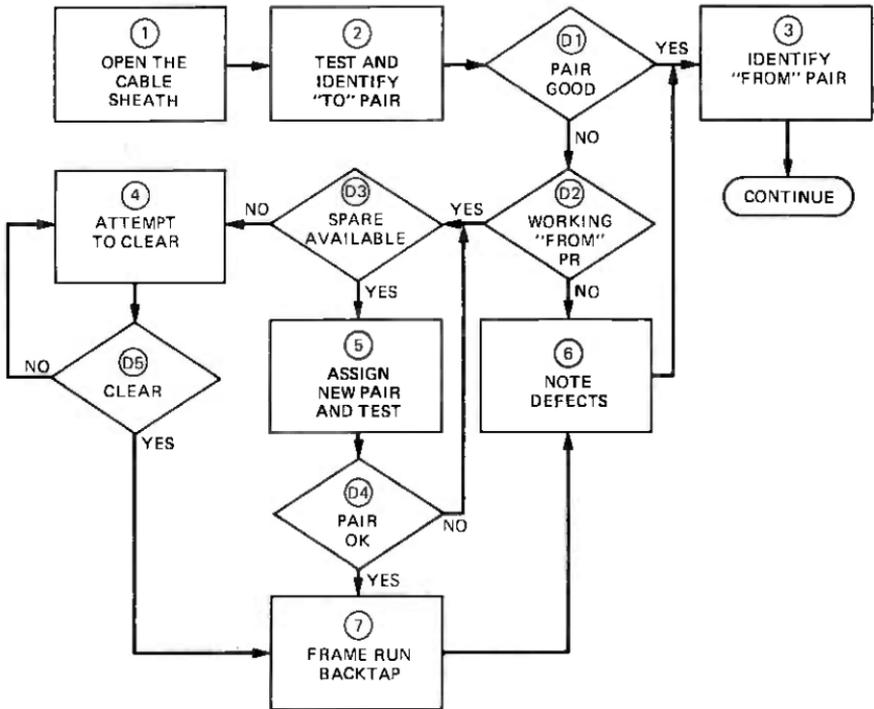


Fig. 2—Flow chart of splicing activity.

process can proceed. The probabilities of moving in the "YES" direction at each decision diamond are given in Table II. The parameters in Table II are defined in Table I. The probability of moving in the "NO" direction is, of course, one minus the probability of moving in the "YES" direction. Except for the steps which are connected directly (such as steps 1 and 2, for example), the paths from one step to another proceed by way of the decision diamonds. The probability of the process moving from one state to another by way of a given path is the probability that at each

Table I — Definitions associated with Fig. 2

$\gamma_t =$	$\begin{cases} 0, & \text{if the sum of the fractions of working and defective pairs in the FROM and TO counts is greater than or equal to one} \\ 1, & \text{otherwise} \end{cases}$
$\gamma_s =$	$\begin{cases} 0, & \text{if there are no known defective pairs} \\ 1, & \text{if known defective pair rate is non-zero} \end{cases}$
$\delta_s =$	fraction defective among spares in TO count (assumed to be 0.025 in examples)
$\delta_t =$	fractional rate of known defective pairs in TO count (assumed to be 0.05 in examples)
$W_f =$	fraction working in FROM pairs being transferred
$W_t =$	fraction working in TO pairs being transferred
$S_t =$	fraction spare in TO count = $1 - W_t - \delta_t$
$N =$	number of pairs being transferred

Table II — Decision probabilities

Decision diamond	Probability of "yes"
D1	$1 - \delta_s S_t$
D2	W_f
D3	γ_t
D4	$1 - \delta_s$
D5	γ_δ

decision diamond the process proceeds in the direction of the given path.

As an example, consider the path from step 2 to step 4 in Fig. 2. This path includes three decision diamonds. Let E_{24} denote the event that given the process is in step 2, then the process will proceed to step 4 next. It is assumed that at each decision diamond the probability of moving next along a particular path is independent of the paths chosen at previous decision boxes. Therefore, letting p_{24} denote the probability of event E_{24} , then, referring to Fig. 2 and the probabilities in Table II,

$$p_{24} = \delta_s S_t W_f (1 - \gamma_t)$$

where the variables are defined in Table I. The probability p_{24} is referred to as the transition probability between steps 2 and 4. In calculating p_{24} it has been implicitly assumed that the probability of proceeding to a given step depends only on the step in which the process resides at present and not on any step in the path which led to the present step, i.e., the process governing the transitions between steps is a Markov process (see Ref. 4). The remaining probabilities are calculated in a similar manner and are summarized in Table III.

In the preceding paragraph, the transition probabilities which govern the process of moving from one step to another were described. The time to move through the entire process, however, depends not only on the transition probabilities but also on the time delay at each step. As stated

Table III — Non-zero transition probabilities

Probability	Expression or value
p_{12}	1
p_{23}	$1 - \delta_s S_t$
p_{24}	$\delta_s S_t W_f (1 - \gamma_t)$
p_{25}	$\delta_s S_t W_f \gamma_t$
p_{26}	$\delta_s S_t (1 - W_f)$
p_{44}	$1 - \gamma_\delta$
p_{47}	γ_δ
p_{54}	$\delta_s (1 - \gamma_t)$
p_{55}	$\delta_s \gamma_t$
p_{57}	$1 - \delta_s$
p_{63}	1
p_{76}	1

previously it is assumed that there is a fixed delay t_i associated with each state. The procedure for estimating these delays is discussed in the next section.

To compute the average time (and subsequently the total cost) of a cable transfer, a set of interstep probabilities p_{ij} and delays t_i are calculated for the entire cable transfer process in the manner discussed in the preceding paragraphs. The probabilities and the delays estimated as described below can then be input to a computer program called MCHART that calculates the cost distribution using the equations given in the appendix.

2.3 Estimation of parameters

Estimates of the frequency and duration of each task have been obtained using several sources including time and motion studies conducted by personnel at Bell Laboratories and operating telephone companies, interviews with craftspeople in N.J. Bell Telephone Company, and from various operating company records. For example, in the cable pair transfer process, the times associated with tasks required by engineering, splicing and the test bureau are based primarily on field estimates. The slicing operations comprise the largest portion of the cost of a cable pair transfer, however, and actual data gathered on times to complete the splicing portion are consistent with model predictions as will be discussed in Section III. Estimates of time to complete the tasks required of assignment bureau personnel, repair clerks, and frame personnel were obtained from time and motion studies. Various plant statistics such as fills (i.e., percent of cable pairs in use) and defective pair rates were obtained from plant assignment sheets prepared at the time of the cable pair transfer.

Table IV gives the estimated delay times for the states in Fig. 2. It should be noted that these values may vary significantly depending on local conditions such as whether or not the plant is aerial or underground.

III. NUMERICAL RESULTS

As discussed in the preceding section, the semi-Markov model has been applied to develop a cost model of a cable pair transfer. A primary goal of this work is to be able to use the model to predict the average cable pair transfer cost as a function of local loop plant parameters. This would permit systems studies of the costs of various strategies for engineering the loop plant.

In the following examples normalized transfer times are used instead of costs since labor rates vary significantly throughout the Bell System. The major point illustrated is not the actual time required to make a cable pair transfer but rather the significant variation in time as a function of the various plant parameters.

Table IV — State delays

State	Task	Constr.	Delay (hrs/pair)		Test
			PAO	CO	
1	Open cable sheath—includes travel and any necessary preparation	$\frac{2.5}{N}$			
2	Identify pair to which transfer is to be made and check it	0.01			
3	Identify pair to be transferred	0.01			
4	Any action necessary to be able to use a pair currently classified as defective	2.0			0.5
5	Assignment office issues a new pair and test bureau tests it		0.2		0.2
6	Discovery of a defective pair requires that it be noted by splicer for entry into PAO records	0.02			
7	Frameman run a new backtap and splicer change drop wire to new pair	0.3		0.2	

Abbreviations

Constr.	construction or splicing force
PAO	plane assignment office personnel
CO	central office force
Test	test desk personnel

3.1 Example 1: transfer time as a function of pairs transferred

Figure 3 shows the model calculation of the normalized hours per pair transferred as a function of the number of pairs transferred. The times in Fig. 3 are average or expected times and unless otherwise stated, all estimates shown in this section are average times. The example assumes 25 percent of the pairs transferred are working circuits and the TO count has no working circuits. Note the sharp decrease in both total time and construction time per pair as the number of pairs transferred increases.

It is important to identify the percentage of working circuits involved in the transfer since work time increases as the number of such working circuits increases. For example, transferring a working circuit requires that backtaps be placed at the central office, old jumpers be removed after the throw, line cards updated, and the circuit verified by the splicer at the time of transfer. Furthermore, if the circuit fails when tested, then more time must be expended to fix the cause of the failure. Even when all goes well in the field, it is sometimes necessary to make a rearrangement in the network in order to avoid transferring a working circuit to a pair which is defective or contains another working circuit.

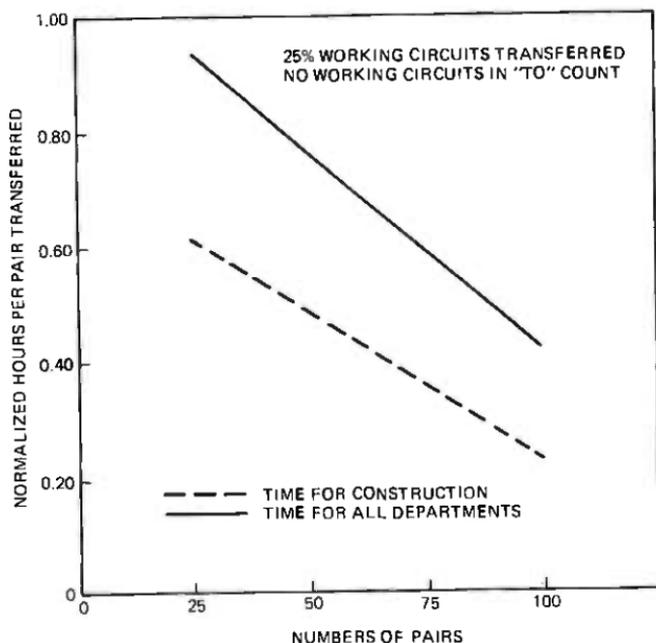


Fig. 3—Variation of cable pair transfer time as a function of the number of pairs transferred.

3.2 Example 2: transfer time as a function of working circuits

As an illustration of the increase in time when working circuits are involved, Fig. 4 shows the normalized construction time and total time for all departments for a 100 pair transfer as a function of the percentage of pairs transferred that contain working circuits. Note first the case in which there are no working pairs in the count to which the transfer is made. In this case it is seen that the construction hours increase as the percentage of working circuits increases but note the even sharper increase in the total hours. In the case where 20 percent of the count to which the pairs are thrown are working pairs, the increases are more rapid.

A major point illustrated by this example is that as the percentage of working circuits increases, the transfer costs increase significantly.

3.3 Prediction of time for the splicing force

Although normalized times have been used in the preceding examples, engineering studies in specific areas require actual times to be calculated. Consequently, it is important to know if the model can predict these times accurately.

The hours attributed to the assignment, repair, and frame forces are

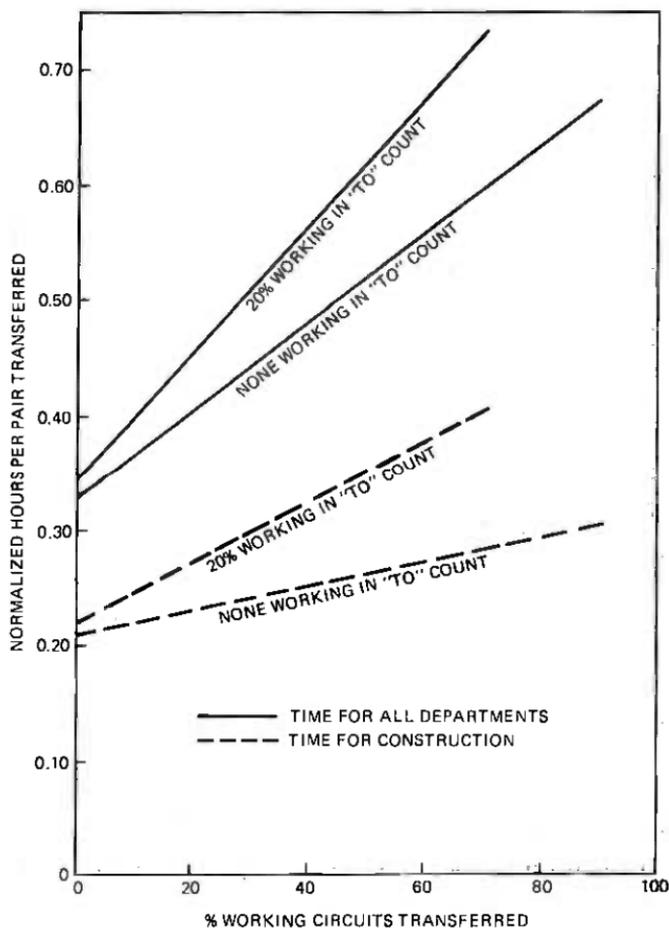


Fig. 4—Transfer time for a 100-pair cable throw as a function of the percentage of working circuits transferred.

based on actual time and motion studies. The times associated with the steps in the splicing operations, however, are based on interviews with splicers and splicing foremen and from unpublished studies and are therefore more subject to potential error. To check the accuracy of these times and the ability of the model to use them to estimate splicing work operation times, which comprise the major portion of the total time, data were gathered from a district in the New Jersey Bell Telephone Company on approximately 100 cable pair transfers involving 20–100 pairs each, the majority being 50 pairs each. The median splicing times are plotted with +’s in Fig. 5 as a function of the number of pairs transferred. The circled points represent the time estimates based on the semi-Markov model. The percentages of working circuits used in the model were taken to be equal to the median percentages in the data itself.

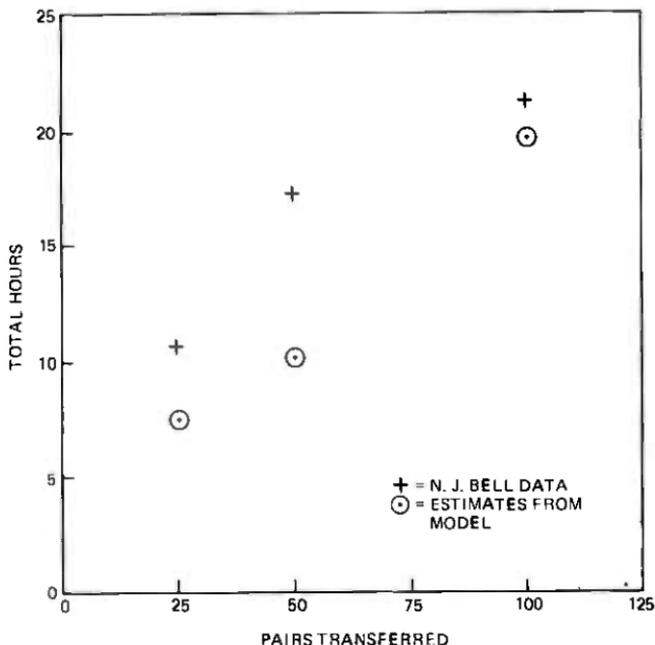


Fig. 5—Comparison of observed total splicing time and model estimates.

From Fig. 5 it appears that the model estimates reasonably well the rate of increases in splicing times with pairs transferred. The model does, however, seem to be biased low. This bias could be explained by underestimation of the time required to perform tasks which are not affected by the number of pairs transferred such as transportation time and opening and closing splice cases.

IV. SUMMARY AND CONCLUSIONS

This paper has presented a technique for applying semi-Markov processes to model the total costs of complex loop plant work operations. The use of such models permits cost studies which can consider the effect of a wide range of local plant conditions and designs. This is a distinct advantage over direct measurement of these work operations which apply only to operations carried out under the same conditions present when the measurements were made. Successful application of the semi-Markov approach to developing the cost of a cable pair transfer was discussed.

It is evident from the results presented that the semi-Markov model can provide reasonably accurate cost estimates of loop plant work operations. The estimates are necessary in order to carry out economic evaluations of different loop plant designs and methods of loop plant

administration. In addition, these models have the potential for use in predicting changes in work load requirements and productivity as a function of plant conditions.

ACKNOWLEDGMENT

The particular version of the MCHART program used to construct the numerical examples was written by J. S. Parsons. Mr. Parsons also applied the semi-Markov model to the telephone installation process in unpublished work. The original version of MCHART was designed and written by Mrs. N. L. Basford.

APPENDIX

In the text it was assumed that the states of the semi-Markov process are denoted by S_i , $i = 1, 2, \dots, n$, and the delays in each state by t_i , $i = 1, 2, \dots, n$. It is assumed that the final state is an absorbing state corresponding to the final step in the work operation. Therefore, the time to complete the work operation corresponds to the first-passage time to the final (absorbing) state.

To obtain the distribution of first-passage time to the absorbing date, let

$p_j(t)$ = probability of entering state j at time t

Assuming n states, the n th being the absorbing state, then

$$p_j(t) = \sum_{i=1}^{n-1} p_{ij} p_i(t - t_i) \quad (1)$$

where

p_{ij} = probability of a transition from state i to state j

and

t_i = delay in state i

In words, eq. (1) states that the probability of entering state j at time t is equal to the probability of entering some state i at time $t - t_i$, remaining in state i for the constant delay time t_i and then making the transition from state i to state j at time t . Note that

$p_n(t)$ = probability of being absorbed in state n at time t and corresponds to the probability of completing the given work operation at time t

To develop a computational algorithm for $p_j(t)$, $j = 1, 2, \dots, n$, it is assumed that

$$\begin{aligned} p_1(0) &= 1, \\ p_j(0) &= 0, \quad j = 2, 3, \dots, n, \end{aligned} \quad (2)$$

and that time t is counted in integer values. A computer program called MCHART has been developed which computes eq. (1) as a function of time. The maximum value of t is reached when $p_n(t)$ reaches a predetermined value arbitrarily close to one. The distribution of T_n , the time to reach state n , can be computed by noting that

$$P(T_n \leq T) = \sum_{t=0}^T p_n(t) \quad (3)$$

The moments of T_n are calculated by the formula

$$E[T_n^m] = \sum_{t=0}^T t^m p_n(t), \quad m = 1, 2, \dots \quad (4)$$

A typical work operation in the loop plant often involves participation by several different departments. Therefore, the distribution of total time required of each department is also of interest. Letting

$$\begin{aligned} X_j(t) &= 1, \text{ if the process enters state } j \text{ at time } t \\ &= 0, \text{ otherwise} \end{aligned}$$

then

$$v_j(T) = \sum_{t=0}^T X_j(t)$$

represents the number of visits to state j during $(0, T)$. Assuming constant delay times, $t_j v_j(T)$ is then equal to the total time spent in state j during $(0, T)$. The average time spent in state j during $(0, T)$ is then $t_j E[v_j(T)]$ where

$$\begin{aligned} E[v_j(T)] &= \sum_{t=0}^T E[X_j(t)] \\ &= \sum_{t=0}^T p_j(t) \end{aligned} \quad (5)$$

Now let t_{jk} denote the time required by department k while the process is in state j . Then in the same manner, the average time spent by department k in state j can be shown to be equal to

$$t_{jk} \sum_{t=0}^T p_j(t)$$

and thus the average time required of department k during $(0, T)$ is obtained by adding the time required in each state, i.e., by the sum

$$\sum_{j=1}^{n-1} t_{jk} \sum_{t=0}^T p_j(t)$$

Note that while the foregoing argument was stated in terms of time, the same argument applies to costs or any other quantity that can be expressed as a linear function of time.

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Loop Plant Modeling:

Economic Design of Distribution Cable Networks

By J. A. STILES

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Distribution plant under the Serving Area Concept (SAC) is the plant on the customer side of the Serving Area Interface. The major part of this plant is the cable network connecting each customer to the interface. Sizing of distribution cables involves a trade-off between current construction costs and future costs that may be incurred. Thus, providing more cable pairs initially costs more at the outset, but reduces relief and rearrangement costs in the future. A set of cost models is described which allows these trade-offs to be studied. These models are applied to examples of aerial cable plant to show how the best cable sizing may be determined.

I. INTRODUCTION AND SUMMARY

The overall structure of the loop plant, as well as the relevant terminology, is described by Long.¹ As described there, the complex cable network that makes up the plant is divided into feeder—the large cables emanating from the central office; and distribution—the finer cable branches ending in the customer's premises. Under the Serving Area Concept (SAC), to which attention is directed in this paper, distribution plant is that on the customer side of the Serving Area Interface. In the past, feeder plant has received considerable analytical attention,² distribution plant less so.

This paper describes a set of analytical models which are specifically tailored to the distribution plant, and which may be used for economic evaluation. In a later section, these models are applied to an example of aerial distribution plant to show how the best cable sizes may be obtained in that case.

The purpose of the models is to enable an economic trade-off to be made between current construction costs, and future costs, for distri-

bution plant. When new distribution plant is constructed, or existing plant is upgraded, a basic decision that must be made is how much cable to place. The more cable placed now, the less future costs will be, because there will be less future need to relieve (provide more cable) or rearrange the network.

In the past, in some networks employing multiple plant,* rearrangement costs have been high, giving a continuing operating cost which was burdensome. In contrast, new SAC plant as currently installed is sized so that *no* future relief or rearrangement should be necessary. The models described in this paper enable comparison of these and other alternatives, so that the optimum trade-off between current and future costs may be determined. The optimum situation is one in which the present worth of all costs is minimized.

Economic sizing of plant in this way is currently practiced in the feeder network.^{2,3} This paper extends the concept to the distribution plant.

The paper is organized as follows. The remainder of this section describes in more detail the problem under study and the approach taken: Section 1.1 details the sources of costs and the resulting cost models, Section 1.2 tells more about SAC design, and Section 1.3 describes the standard serving area used in all subsequent analysis. Details of the cost models then follow in Section II. In Section III these models are applied to examples of aerial cable plant.

1.1 Sources of costs; cost models

The most obvious cost for distribution plant is the *cost of current construction*. For a new serving area, or major upgrading of an existing one (such as conversion from multiple plant to SAC), this will be the major cost. What is modeled is the cost of material and installation for cables, terminals, and interface for a serving area. The interface connects the feeder and distribution cables, and the terminal connects the distribution cable to the service wire entering the customer's premises (see Fig. 1).

The size of distribution cabling is conventionally specified by the number of pairs provided per customer living unit[†] (pr/l.u.). As an example, SAC design generally specifies two pr/l.u. This means that for each living unit, two cable pairs are provided from the distribution terminal (or, in some cases, the customer's premises) back to the interface.

It is assumed that a primary line pair should be provided to each living unit. Extra pairs above this 1.0 pr/l.u. are available for *additional lines* (such as teenage or alarm lines) going to a living unit that already has a primary line. Additional line *penetration* is the number of additional

* See Ref. 1 for a description of the various types of distribution plant.

† A living unit is one customer's address—a house, apartment, etc. A living unit may require more than one telephone line (e.g., a primary and a teenage line).

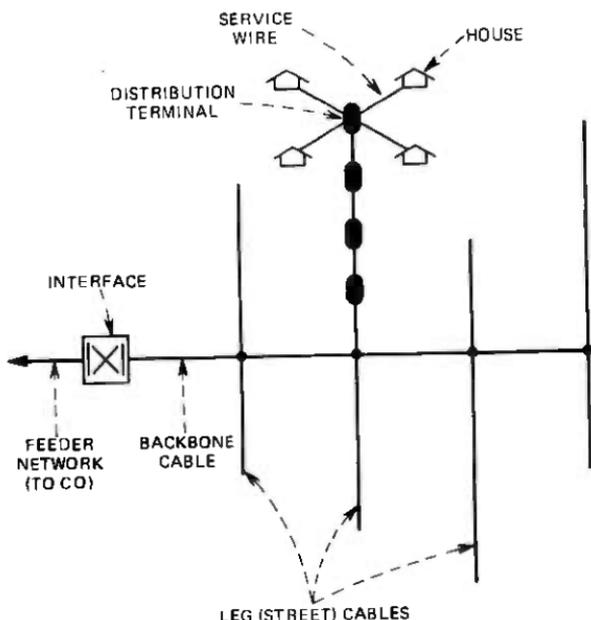


Fig. 1—Standard serving area configuration used in models.

lines per living unit in an area (usually expressed in percent). If fewer than two pr/l.u. are provided, each living unit cannot have a unique additional line pair. The available additional line pairs must then be shared among several living units.

Although additional line penetrations of only a few percent are common today, it is possible that they may increase considerably in the future. Also, unforeseen* growth in living units (due, for example, to subdivision of one-family houses into multifamily) has to be provided from the additional line pairs. To take care of these uncertainties, SAC design specifies a minimum of two pr/l.u. More are sometimes provided in localized areas.

If two or more pr/l.u. are provided, it is very unlikely that future relief cable will be required. On the other hand, if smaller distribution cables (i.e., fewer than two pr/l.u.) are provided initially, the possibility of future relief cannot be ignored. The *cable relief model* makes probabilistic calculations, based on additional line and living unit growth, to obtain the expected cost of future cable relief for a serving area.

If fewer than two pr/l.u. are provided in the distribution network, additional line pairs must be shared among the living units. Under this condition, there may be insufficient additional line pairs in a given terminal to serve the total additional line demand among the living units

* Attempts are, of course, made to forecast such increases and make provision for them.

served by that terminal. Such an event is known as a *blockage* and results in a cost penalty due to the construction or rearrangement activity required to provide the desired additional line. Models of blockage cost are discussed in this issue by Koontz⁴ and Freedman.⁵ A *terminal blockage* cost model, tailored specifically to a SAC distribution network, is developed in this paper.

Even if blockages do not occur, additional line demand can result in extra cost when fewer than two pr/l.u. are provided. This extra cost is the cost of disconnecting and reconnecting additional line pairs to living units as additional line demand moves from one house to another. The components of this cost are known as the *break connect-through* (BCT) cost and the *reterminate connection* (RTC) cost. These costs are modeled in a general context by Koontz⁴ and Freedman⁵ and in the SAC context here. Henceforth, we shall refer to the combined BCT/RTC costs as *break connect-through costs*.

The need to share additional pairs and change assignment of them can lead to complications in the plant assignment office, where the pair records are kept. The increased time of assignment leads to another cost model, for *assignment cost*.

To sum up, the five cost models are current construction cost, cable relief cost, terminal blockage cost, break connect-through cost, and assignment cost. Each of these models is described in detail in Section II and the appendices.

1.2 SAC design

Distribution design under the Serving Area Concept (SAC) is based on *ultimate* living units in a serving area. This is the maximum number of living units ever expected to exist in the area, taking account of future growth. Standard SAC design requires a minimum of two pairs for each ultimate living unit—one primary pair and one additional line pair. Since additional line penetration is unlikely to reach 100 percent, this design avoids future cable relief. Since every living unit can be given a specific additional line pair, terminal blockage and breaking of connect-throughs are also unlikely.* Assignment costs are low because the additional line pairs are always assigned to the same residence.

Under SAC design, each primary pair is *dedicated* to its living unit. That is, once assigned at an address, the pair cannot be reassigned elsewhere. Additional line pairs may be either dedicated or reassignable, depending on the local situation.

Pairs which are reassignable will often appear for use in several distribution terminals, in contrast to dedicated pairs, which appear in one terminal only. The purpose of this practice, called *multiplying* (see Ref.

* They are not impossible, though, as one living unit may require *more* than one additional line.

1), is to make the pairs more widely available. SAC allows multiplying, but does not rigidly specify the method.

Use of SAC design results in a network which needs relatively little attention once constructed. This was the intention of its creators, who were responding to operating problems encountered in multiple and dedicated plant, and the use or misuse of some particular terminal hardware. The price paid for this simplicity is increased initial construction cost. The models in this paper enable the initial construction cost to be balanced against future costs in a rational manner. In particular, the overall effect of installing fewer than two pairs per living unit can be evaluated.

1.3 The standard serving area

Actual configurations of distribution cable in serving areas vary widely according to geographical requirements and local practice. However, a few parameters serve to describe the salient features for purposes of these cost models. All but the most unusual areas can adequately be depicted as follows: a single *backbone cable* runs out from the interface, connected to a number of *street cables*, or *legs*. At each leg connection point two legs branch out, and the spacing of connection points is uniform along the backbone. Tapers (reductions in cable size) are allowed at various points along the backbone. Figure 1 gives an example of this configuration.

The leg cables may be various sizes and lengths. Spacing of terminals on the legs is uniform, and the same number of houses is served by each terminal (multifamily houses may contain various numbers of living units). If terminals are placed on the backbone, they, too, are assumed uniformly spaced. All terminals are assumed to be re-enterable, so that customer service wire connections in them can be changed.

Use of a standardized serving area of this form reduces the number of descriptors to a manageable level. Essentially, the serving area is specified by the number of legs, the leg cable sizes,* placement of backbone taper points, and terminal and leg spacing distances. Backbone cable size is not an independent variable, but a function of the leg cable sizes, since the backbone is assumed to be sized to connect all leg pairs back to the interface. This policy assumes at most one future relief of the backbone, to be done at the same time as leg relief.

II. COST MODELS

The cost models are of varying form and complexity. The initial construction model is straightforward, involving principally tallying and costing of the plant placed. On the other hand, the future cost models

* Leg cable sizes are determined by living units per leg times desired pairs per living unit.

involve probabilistic processes which occur over time, requiring integration to determine the overall expected costs. All the models have been computer implemented, so that a complete set of costs can be calculated for a given set of input parameters.

One set of inputs consists of the serving area parameters described in Section 1.3. Other principal sets include the parameters for living unit and line demand growth, and the component costs. The component costs are the actual installed first costs of the network components. These include both labor and material costs and are calculated from a number of sources. Hardware prices are combined with operating company estimates of labor times and costs and compared with so-called broad gauge costs* for verification.

Living units per house and line demand per living unit are both modeled as a class of saturating functions (this includes linear functions) whose parameters are program inputs. That is, the quantities tend to increase less rapidly as time goes on, or at most linearly. This ensures that the resulting integrals are bounded.

The output of each model is a cost represented as a present worth of annual charges (PWAC) (often shortened to "present worth"). In this way, all costs are referred to a common base at time zero and so may be compared. The present worth calculation can include inflation, if desired.

Section 2.1 describes the initial construction cost model. The future cost models follow. Section 2.2 covers the cable relief and terminal blockage models; Section 2.3, the break connect-through model; and Section 2.4, the assignment model. Further details of the future cost models appear in the appendices.

2.1 Initial construction model

The construction model calculates the cost of installing the serving area plant necessary to achieve a specified number of pairs per ultimate living unit. It incorporates models of living unit growth and of the hardware and connection costs for building the network. Costs are calculated for the backbone cable, leg cables, interface, and terminals. Additional costs for poles in aerial plant, or trenching in buried plant, are not included, as these are assumed to be the same whatever the cable sizes.

The living unit growth model assumes that the number of houses is fixed, but that one-family houses may divide into two- or three-family houses at specified rates. Each terminal is assumed to serve a fixed number of houses. Hence, the average number of living units in twenty years (the "ultimate") may be calculated for a street (leg) with N_T terminals. Multiplication by a specified number of pairs per living unit (say,

* Costs derived from the average costs of actual construction projects.

1.5) then gives the ultimate number of pairs needed on the street, and the leg cable is sized for this number. All legs with the same number of terminals, N_T , are sized identically by the algorithm.

An important aspect of leg cable sizing is that cable is available only in discrete sizes. For some N_T , the available leg cable size may be only slightly larger than the required numbers of pairs; for others, much larger. In the latter case, the actual number of pairs per living unit for the leg is greater than the specified value, and for such "oversized" legs the future costs (due to relief, etc.) will be less.

Example: Suppose a leg cable contains six terminals, each supplying four one-family houses. If a minimum of 1.5 pairs per living unit (pr/l.u.) were specified, the pairs required would be $6 \times 4 \times 1.5 = 36$. The next larger cable size, 50 pairs, would be installed, so the resulting available pr/l.u. would be $50/24 = 2.08$. The *same* cable would be installed for a minimum of 2.0 pr/l.u.

Separate treatment for each N_T also allows exact (pair-by-pair) specification of the terminal multiplying method; that is, the way multiplied pairs in a cable are shared between the terminals on the cable. The detail is needed by the terminal blocking model. Once the cost of each leg is established, a specified distribution of N_T is used to give the total leg costs for the serving area.

The backbone is then sized according to the aggregated pair demand. To reduce complexity, the backbone size is calculated from the mean aggregate pair demand summed over all legs. Taper points may be specified in a backbone. When a backbone is tapered, each section of cable is sized separately, to serve only the requirements of the legs feeding through it. Terminals may or may not be placed on the backbone. Both options can be evaluated.

2.2 Cable relief and terminal blockage

As line demand increases with time, for some legs the initially installed cable will eventually be too small, and relief will be required. Also, it may turn out that, although there are nonworking pairs in a leg cable, they are not accessible at particular terminals on the cable. These terminals are then said to be blocked,⁴ and action must be taken to give them access to the available pairs. Unblocking may involve pair rearrangement or new terminal addition.

Cable relief turns out to be the major future cost incurred in most cases. Terminal blockage, on the other hand, results in rather minor costs unless very small terminals are used. Both phenomena may be treated by the same model, a model that calculates the probability of relief or blocking at each future time, multiplies that by the cost of correction (with appropriate present worth factors), and integrates over time. A

mathematical description of the models is given in Appendix A. Here, it will be sufficient to sketch the approach taken.

Some simplifications in the mathematical treatment allow much more tractable models. First, the complex and fluctuating line demand process has been modeled by a simple growth process—the saturating functions referred to earlier.⁶ Second, the discrete demand process has been replaced by a continuous analog, and when this is done, it can be shown that line demand can be quite accurately approximated by a normal distribution. The probability of cable relief is then the probability that this normal line demand exceeds the installed cable size.* As time goes on, this probability increases, due to demand growth.

Cost of relief for a leg cable is calculated as that required to install a parallel leg cable complete with terminals, so as to give a designated total number of pairs per living unit (usually two). This method of relief is roughly equivalent in costs to other alternatives (such as throwing existing terminals). The backbone is assumed to be relieved at the same time that the first leg relief occurs. This may be somewhat conservative, as spare pairs existing in the backbone could sometimes accommodate initial leg relief.

Terminal blockage only occurs when some cable pairs are inaccessible in some terminals. Since we have assumed that each ultimate living unit is provided with a dedicated primary line, the blockage problem only applies to additional line pairs. It turns out that fewer than two pairs per living unit and 25-pair terminals allow the use of multiplying schemes which reduce blockage to quite low levels. In fact, for one family houses, the probability of blockage can often be reduced to zero, because all additional line pairs can be made available to all terminals on a leg.

Example: Suppose a leg contains eight terminals, each supplying four one-family houses, and that a minimum of 1.5 pr/l.u. is specified. Then a total of $8 \times 4 \times 1.5 = 48$ pairs is required, and a 50-pair cable would be used. In each terminal four primary pairs are terminated, leaving $25 - 4 = 21$ binding posts available for additional line pairs. These allow space for all 18 ($= 50 - 8 \times 4$) nonprimary line pairs in the cable to be terminated.

Because terminal blockages are unlikely, a simplified model can be used for them. It is assumed that a terminal blockage is cleared by throwing[†] pairs, or adding another terminal, rather than by piecemeal

* In practice, relief takes place when the cable is, say, 85 percent full. This is allowed for in the model.

† Connecting the terminal to a different set of pairs in the cable.

rearrangements. In other words, blockage is relieved in a lumped fashion, similar to cable relief, rather than in a continuous fashion. This simplifies the model and, in fact, allows use of the same model as was used for cable relief.

2.3 Break connect-throughs

When fewer than two pairs per living unit are provided, the additional line pairs must be shared between living units, and so must be transferred from one living unit to another as demand moves around. This gives rise to a cost called the break connect-through (BCT) cost. The source of this cost is the need for an installer to disconnect a service wire from a cable pair and reconnect a different service wire, possibly in a different terminal.

Primary line pairs, which are dedicated, are not included in this model. For additional line pairs, a connect-through (CT) policy^{4,5} is assumed, so that a service wire, once connected to a cable pair, is left connected (even though idle) until that pair is required elsewhere. At that time an installer changes the service wires.*

The model has to take account of various possible situations. For example, it may not always be necessary to break a connect-through to provide additional line service. There may be a connected-through pair already in place; or there may be pairs available not connected to any customer, which would be used in preference to a BCT. These *spare pairs* will gradually be connected, until all pairs are connected to service wires. It is necessary to model this process. As shown in Appendix B, this can be done by means of a differential equation.

When a connection has to be broken to provide service, that break may take place in the same terminal that provides the new service, or in another terminal. In the latter case, costs are higher because two terminals must be visited and opened. The model calculates the probabilities of these two situations and weights the costs accordingly. Note that costs of initial service wire installation are not included, as these are independent of the cable sizing or pair dedication policies being evaluated.

Appendix B provides the model details. The model is similar to the cable relief model, except that the BCT process is a continuous one, rather than a single event as in the case of cable relief. Hence it is necessary to integrate a product of present-worth-adjusted BCT cost and rate of BCTs. BCT rate, in turn, is the product of inward additional line service order rate and the probability that such an order requires a BCT. BCTs are

* Some connect-through policies specify a "reserve time" during which the connection cannot be broken. No such time is assumed in this model; if the pair is needed, it is used at any time.

assumed to stop if the cable is relieved, because then there are enough additional line pairs for each customer to have one.

2.4 Assignment costs

Sharing of additional line pairs leads to complications in the assignment process. The assigner can no longer look up the customer address and find the relevant additional line pair. Rather, a pair must be found from the available pool, possibly by breaking a connect-through, as described in the previous section. Estimates are available of the time taken to assign in each of these cases. Hence it is possible to ascribe a cost to the difference in the assignment process.

The overall assignment cost model is similar to, but simpler than, the BCT model. It is simpler because the extra cost is assumed to apply to all assignments, whether to spare pairs, connected-through pairs, or BCTs. The model is then an integral of the product of additional line service order rate and differential assignment cost.

III. COST RESULTS AND AERIAL PLANT EXAMPLE

All the models described have been computer implemented, so that it is possible to obtain the total cost (initial plus future) of a serving area constructed and operated with any desired set of input parameters. Input parameters include serving area geometry, growth rates for lines and living units, and minimum number of pairs specified per living unit.

The results of this section describe the application of the models to some typical cases of aerial plant. An aerial plant example was chosen because there were higher potential savings due to installing fewer than two pairs per living unit, and because it was easier to obtain accurate costs for cable relief. Relief in the case of buried plant would involve retrenching, an operation of high and uncertain costs. For this reason, use of fewer than two pairs per living unit would be expected to be more cost-effective in aerial plant than in buried plant. (Some further remarks on buried plant follow in Section 3.4.)

In the examples, variations are made in serving area size and housing configuration,* in additional line growth rates, and in minimum pairs per living unit (pr/l.u.). Lot size in the serving area is held constant at one-quarter acre, and four houses are assumed served by each distribution terminal. Twenty-five-pair distribution terminals are used.

The cost calculated in each case is the total cost: the sum of the present worths of the initial construction cost and future costs. For each set of input parameters, two such costs have been calculated: one for the designated minimum pr/l.u. (say, 1.3), and the other for 2.0 minimum pr/l.u. (Henceforth all pr/l.u. figures will be understood to be designated *minimum* values, unless otherwise specified.) The difference between

* "Configuration" primarily refers to housing type: one- versus two-family (see Section 3.2 for further explanation).

costs is expressed as a percentage of the costs for two pr/l.u. Hence the results represent percent cost saving (or cost increase) with respect to a network designed at two pr/l.u. Positive numbers mean that the network with the designated pr/l.u. has a lower present worth cost than that for two pr/l.u.

In what follows, Section 3.1 describes the cost results for one-family housing, and Section 3.2 extends the results to other housing situations. Section 3.3 discusses the effect of variations in the line growth rate, and Section 3.4 provides some comments on buried plant.

3.1 One-family houses

The first results to be presented involve serving areas containing only one-family houses. The housing is assumed to be *stable*, which means that the housing type is not changing. In particular, the houses are not subdividing into multifamily houses (this situation will be examined in Section 3.2). A typical set of cost results is shown in Table I. The parameters varied are serving area size (both backbone length and leg length, or number of terminals per leg), additional line growth, and designated pairs per living unit. Initial additional line penetration is five percent. Additional line growth is expressed by the additional line penetration in 20 years. Growth is assumed saturating; that is, pene-

Table I — Cost savings for one-family houses

b	t	LU	Percent savings versus 2 pr/l.u.			
			1.3 pr/l.u.		1.5 pr/l.u.	
			a = 10	a = 20	a = 10	a = 20
1500	6.5	208	13.3	9.8	10.7	10.3
	7.5	240	10.7	8.6	9.1	8.4
	8.5	272	8.8	6.0	2.8	2.6
2250	4.5	216	10.1	5.3	9.4	9.1
	5.5	264	10.9	9.7	10.9	9.7
	6.5	312	14.9	12.9	13.4	13.0
	7.5	360	13.2	11.2	11.6	11.0
	8.5	408	13.8	11.2	5.5	5.3
3000	4.5	288	7.5	2.8	1.0	0.8
	5.5	352	14.3	13.8	14.3	13.8
	6.5	416	16.6	13.4	7.1	6.7
	7.5	480	8.1	6.5	6.5	5.9
	8.5	544	8.8	6.6	5.4	4.8
3750	4.5	360	12.0	11.3	12.0	11.3
	5.5	440	15.7	9.1	7.9	7.5
	6.6	520	12.0	10.0	10.2	9.2
	7.5	600	9.0	0.8	5.4	4.3
Averages*			11.8	8.8	8.5	7.9
Standard deviations*			2.7	3.5	3.5	3.5

Initial additional line penetration = 5 percent

No terminals on backbone

b = backbone length (feet)

t = average number of terminals per leg (equivalent to leg length)

LU = average number of living units in serving area (at end of 20 years)

a = percent additional line penetration in 20 years

* Both calculated by assuming that all cases are equally likely.

tration increases most rapidly at first and slows down later. The effect of changing this assumption is examined in Section 3.3.

The most obvious feature of Table I is the considerable scatter of the results, also shown in Fig. 2 for the first column of results. This scatter is due principally to the fact that cables come in discrete sizes, so a small change in serving area parameters can cause a large change in cable sizing (this effect is most prominent in backbones). No obvious trend of the results with serving area size is evident, and this is generally true. Hence it is natural to express the results in terms of averages over the serving area size. These averages are shown in Table I, along with the associated standard deviations. Both the averages and the standard deviations were calculated by simply assuming that all cases were equally probable. The deviations show that while large fluctuations from the averages are possible, in the cases shown the savings will rarely become negative (more than two standard deviations).

Figure 3 extends the average values of Table I to a larger range of additional line growths. The average savings for 1.3 and 1.5 pr/l.u. are shown, together with a one standard deviation band for 1.3 pr/l.u. (to simplify the figure, the similar band for 1.5 pr/l.u. is omitted). It can be seen that, in this case, 1.3 pr/l.u. provides savings, even in the worst cases, up to about 20 percent penetration, but that 1.5 pr/l.u. is on the average better for penetrations greater than about 20 percent. Averaged results of this kind would allow establishment of a pair per living unit policy over a large geographical region containing diverse serving areas, when the appropriate line growth parameters were known.

To give an idea of the contributions of the various future costs to total cost, Fig. 4 shows the average* present worth of future costs as a percentage of total costs (initial construction plus future costs) for the case

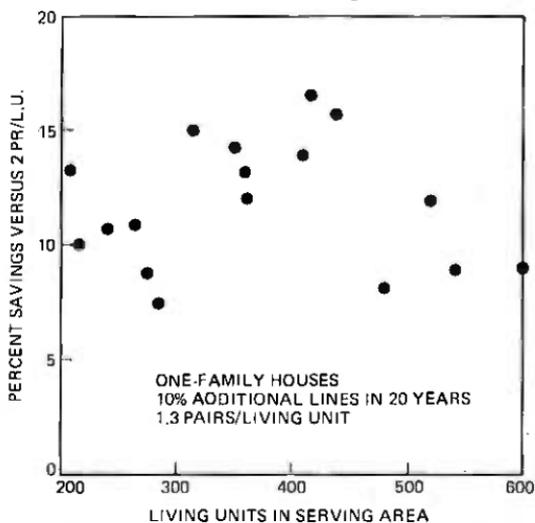


Fig. 2—Cost savings versus serving area size.

* Averaged over serving area size, as in Fig. 3.

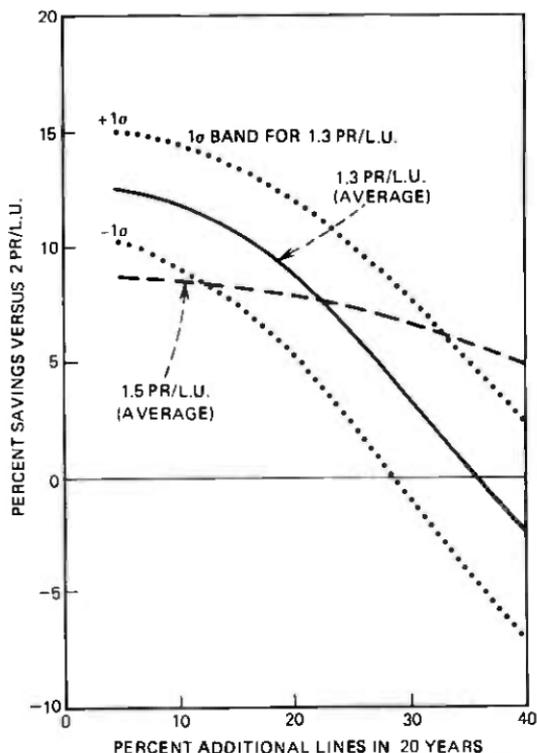


Fig. 3—Average cost savings for one-family houses.

of Table I, and 1.3 pr/l.u. The average percentage cost of cable relief only is also shown. It can be seen that the other future costs (terminal blockage, break-connect throughs, and assignment) never contribute more than a few percent to total cost in this example.

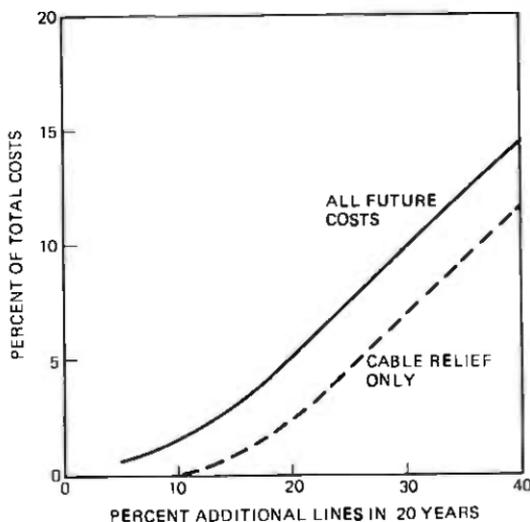


Fig. 4—Future costs as percent of total for one-family houses, 1.3 pr/l.u.

3.2 Other housing configurations

Although one-family stable neighborhoods are a common type, aerial plant tends to be used quite extensively in older, multifamily areas. Consequently, it is worthwhile examining the cost savings for different housing situations. Four configurations have been studied, as shown in Table II.

The first was the one-family stable configuration of Table I. Next, the effect of placing terminals on the backbone cable in the one-family stable case was investigated. The other two cases involved multifamily houses: the first with one-family houses progressively subdividing into two-family houses, and the second with a stable two-family situation.

Table II presents total cost savings averaged over serving area size, as in Table I. As can be seen, adding terminals to the backbone in the Table I situation does not change the results much. However, if the houses in the area are subdividing into two-family (Configuration III), greater savings are obtained by using less than two pairs per living unit. This apparently paradoxical result occurs because it is assumed that the growth is accurately predicted and, by designing for the ultimate living units, is allowed for. Thus the network, sized for the ultimate living units, is considerably oversized initially, reducing the probability of cable relief.

Also, fewer terminals are used to serve the ultimate living units. Thus a higher proportion of the total costs are cable costs, which is where savings are principally obtained by reducing pairs per living unit. This is also the reason for the higher savings with the two-family houses.

In all cases of Table II, savings for 1.3 pr/l.u. fall more rapidly than for 1.5 pr/l.u. as the 20-year additional line penetration increases. This is the same trend that was observed in Fig. 3.

If the plant costs were actually proportional to the minimum installed

Table II — Average savings for various housing configurations

Configuration	Initial housing	Division rate	Terminals on backbone	
I	1-family	Zero	No	
II	1-family	Zero	Yes	
III	1-family	5% per year*	No	
IV	2-family	Zero	No	

Configuration	Average percent savings versus 2 pr/l.u.			
	$a = 10^\dagger$	$a = 20$	$a = 10$	$a = 20$
I	11.8	8.8	8.5	7.9
II	10.2	6.7	7.4	6.7
III	15.0	14.1	12.0	11.8
IV	14.9	12.6	11.1	10.6

* Every year, 5 percent of the 1-family houses divide into 2-family. After 20 years, this increases the number of l.u. about 50 percent.

† a = percent additional line penetration in 20 years

pairs per living unit, much higher savings would be expected than have been evident in the examples so far. Thus, the percent savings in using 1.3 pr/l.u. instead of 2.0 would be $100 \times (2.0 - 1.3)/2.0$, or 35 percent. In fact, maximum values of 10–15 percent are observed. The reason for this is partly that there are fixed costs of construction (terminals, cable placement, etc.), and partly that discrete cable sizing causes larger average pair per living unit values than the minimum. Thus, instead of comparing 1.3 with 2.0, we should compare (say) 1.8 with 2.5.

3.3 Effect of line growth variations

Although ultimate (20 year) additional line penetration is the most significant line growth parameter, the initial rate of line growth is also important. This is shown in Fig. 5, which is drawn for a particular serving area configuration with high savings at 1.3 pr/l.u. (these are not averaged values, as in Table II and Fig. 3).

Three curves are shown. The first is the percent savings for the growth situation assumed so far: initial additional line penetration 5 percent, and saturating growth. The second curve shows the effect of decreasing initial growth rate so that the growth becomes linear throughout the 20-year period, while the third curve shows the effect of reducing the initial additional line penetration to zero, with saturating growth. In both these latter cases, savings are higher, as there are fewer additional lines at any given time, and so lower future costs.

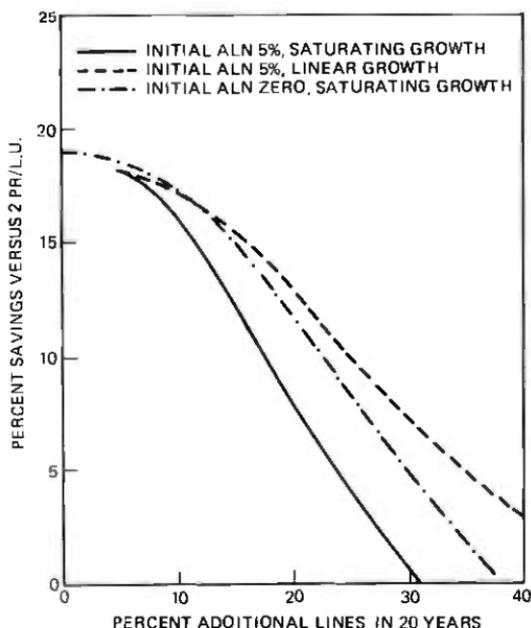


Fig. 5—Variation in cost savings with additional line (ALN) growth rate.

3.4 Buried plant

From Figs. 3 and 4, we can get an idea of the situation for buried plant. High trenching costs would probably increase the cost of cable relief severalfold. Other future costs would not be expected to increase if the terminals were pedestal-type and accessible; and it is unlikely that completely out-of-sight plant, with buried terminals, would be used with fewer than two pairs per living unit. However, since Fig. 4 shows that most future costs are for cable relief, the savings for buried plant would be expected to drop precipitously as soon as these relief costs became appreciable.

Figure 3 is appropriate since buried plant is most likely to be in stable one-family environments. This figure indicates that 1.3 pr/l.u. would probably not be satisfactory at all, but that 1.5 pr/l.u. might still provide useful savings for lower additional line penetrations. Actual results for buried plant can be computed, given the appropriate costs.

IV. CONCLUSIONS

Computer models have been developed to study the sizing of the distribution plant network. These include both initial construction costs and future costs which might be incurred for cable relief, terminal blockage, break connect-throughs, and assignment. The models are applicable to a wide variety of serving area parameters and additional line growth rates.

These models can be used as a flexible evaluation tool, allowing new or rehabilitated distribution plant to be sized appropriately, given the local conditions.

A set of examples of the application of the models to various aerial plant networks is presented. These show that, on the average, total cost savings in the 10–15 percent range can be obtained by using fewer than two distribution pairs per living unit, if future additional line penetration is less than about 20 percent. Examples are given of the effect of various parameter variations on these savings.

APPENDIX A

Cable Relief and Terminal Blockage Models

In both this appendix and the next, attention is focused on additional lines. For SAC distribution plant, it is here assumed that the ultimate required number of primary lines is provided (and dedicated, if desired). Hence all the future costs considered—cable relief, terminal blockage, break connect-through, and assignment costs—are due to inadequate provision for *additional* lines. Because of this, the emphasis and costs are somewhat different than those in Refs. 4 and 5, which consider primary lines as well, and which are also concerned with the feeder network.

In this appendix the cost of cable relief and terminal blockage is derived. These may be treated together because of the assumption made that terminal blockage is relieved in a lumped fashion by terminal throw or terminal addition, rather than by piecemeal rearrangements. This assumption is justified by the low level of terminal blockage costs. It allows us to treat terminal blockage as a single event, like cable relief. In what follows, we shall discuss cable relief. Exactly the same formulation and model apply to terminal blockage except that, instead of considering the whole cable, we consider a group of terminals in multiple.

A.1 The present worth integral

An assumption which allows simplification of the mathematics is that the additional line demand process may be treated as a pure growth process, and the effect of churning (turnover) may be ignored. This is supported by Ref. 6. Thus, it is the mean growth rather than the variability of the line demand process that is important.

Leg cable relief occurs when the line demand rises above a certain value* (say, primary plus additional line demand above 85 percent of the installed cable size). The above assumption means that the relief process can be depicted by the probability of the demand being above this value, rather than by using a rigorous stochastic process approach. This probability can be called the *instantaneous probability of cable relief*, $p_c(t)$, which is the probability that the cable first needs relief at time t .

The total present worth cost of relief for any cable may then be written as an integral over time of the product of $p_c(t)$ and the present worth adjusted cost of relief at time t . This latter quantity may be broken up into the product of the actual cost of relief at t and a present worth factor. Hence the overall *present worth integral* becomes

$$PW_{REL} = \int_0^T p_c(t) \cdot C_c(t) \cdot F_{pwc}(t) \cdot dt \quad (1)$$

where F_{pwc} is the present worth factor, C_c is the cost of relief, and T is the study period (here taken as 20 years).

C_c is assumed constant with time (inflation can be taken into account by adjusting the present worth factor). The present worth factor is

$$F_{pwc}(t) = \frac{A_c}{r} (e^{-rt} - e^{-rT}) \quad (2)$$

where A_c is the annual charge factor for the cable (30 year) account, r is the force of interest, and T is the 20-year study period.

Once the instantaneous relief probability $p_c(t)$ is known, eq. (1) may

* The backbone is assumed to be relieved when first leg relief occurs.

be integrated numerically to give the expected cost of relief of any specific cable. Appropriate addition gives the expected relief cost for the serving area. In what follows, we show how $p_c(t)$ is calculated.

A.2 Instantaneous relief probability

The mathematical simplification used to get $p_c(t)$ is the modeling of the line demand on the cable as a normal distribution. To do this, the Central Limit Theorem is invoked, as well as our previous assumption that we can consider line demand a smoothly growing function. The line demand for the cable is assumed to be the sum of independent demands from the houses supplied by the cable. Suppose there are N_T terminals on the cable, and H houses per terminal. If each house has a line demand distribution with mean μ_h and standard deviation σ_h , the corresponding mean and standard deviation for the cable are $\mu_c = HN_T\mu_h$ and $\sigma_c = \sqrt{HN_T}\sigma_h$. We shall return to the calculation of μ_h and σ_h in the next section.

The line demand for the cable is then assumed normal, with mean $\mu_c(t)$ and standard deviation $\sigma_c(t)$. Thus the probability that the cable has required relief by time t is the probability that this normal variate is greater than some value X (85 percent of the cable size); that is

$$\frac{1}{\sqrt{2\pi}} \int_{\beta(t)}^{\infty} e^{-x^2/2} dx \quad (3)$$

where

$$\beta(t) = \frac{X - \mu_c(t)}{\sigma_c(t)}$$

Equation (3) represents the probability that the cable was relieved at time t or before. The instantaneous relief probability $p_c(t)$ is obtained by differentiating eq. (3):

$$p_c(t) = -\frac{1}{\sqrt{2\pi}} \frac{d\beta(t)}{dt} e^{-\beta(t)^2/2} \quad (4)$$

A.3 House distribution parameters

The house distribution parameters μ_h and σ_h are obtained by considering the number of lines required by a house as the sum of the lines required by each living unit in the house. The number of lines required by a living unit and the number of living units are both random variables in the most general case. Assume they have means and standard deviations μ_L, σ_L (lines) and μ_U, σ_U (living units). Then for a house, standard probability theory gives

$$\left. \begin{aligned} \mu_h &= \bar{\mu}_L \mu_U \\ \sigma_h^2 &= \mu_U \sigma_L^2 + \mu_L^2 \sigma_U^2 \end{aligned} \right\} \quad (5)$$

Assume that there are a maximum of three lines per living unit (with penetrations $\alpha_1, \alpha_2, \alpha_3$ for first, second, and third), and three living units per house (with probabilities f_1, f_2, f_3 for one, two, and three). Then manipulation of eq. (5) produces

$$\left. \begin{aligned} \mu_h &= LF \\ \sigma_h^2 &= (\alpha_1 + 3\alpha_2 + 5\alpha_3)F - L^2\{(F-1)^2 + 1 - 2f_3\} \end{aligned} \right\} \quad (6)$$

where

$$\begin{aligned} L &= \alpha_1 + \alpha_2 + \alpha_3 \\ &= \text{expected lines per living unit} \\ F &= f_1 + 2f_2 + 3f_3 \\ &= \text{expected living units per house} \end{aligned}$$

As an example, in the case of all one-family houses ($F = 1, f_3 = 0$) with no third lines ($\alpha_3 = 0$), and 100 percent first line penetration ($\alpha_1 = 1$), eq. (6) reduces to

$$\begin{aligned} \mu_h &= 1 + \alpha_2 \\ \sigma_h^2 &= \alpha_2(1 - \alpha_2) \end{aligned}$$

APPENDIX B

Break Connect-Through and Assignment Models

In both the break connect-through (BCT) and assignment models, the present worth cost is obtained from an integral similar to eq. (1). The principal difference is that both these processes occur continuously with time, rather than once only as in the case of cable relief. Also, churn (turnover) now becomes an important factor, whereas for relief we considered growth only.

B.1 The present worth integral for BCTs

The present worth cost of BCTs is obtained by integrating over a product of three factors: the rate at which BCTs occur, R_b [replacing relief probability in eq. (1)], the cost of a BCT, C_b , and a present worth factor, F_{pwb} . All factors vary with time:

$$\text{PW}_{\text{BCT}} = \int_{T_s}^{T_M} R_b(t) \cdot C_b(t) \cdot F_{\text{pwb}}(t) \cdot dt \quad (7)$$

Integration extends up to a time T_M , which is the lesser of the expected time of relief and the study period T . (It is assumed that no BCTs occur

after relief). Integration does not start at time zero, but rather at a *spare exhaust time* T_s , at which all additional line pairs have service wires attached to them. Prior to this time, a pair with no service wire (spare pair) could be used to provide service if no connected-through pair existed at the service location.

For this calculation, terminals on a cable are assumed multiplied in groups, with the additional line pairs shared fully among terminals of one group, but no multiplying between groups (the group may often be all the terminals on the cable). The integral (7) is calculated for each terminal group, and the costs added for all groups to give the serving area BCT cost.

The full access provided by this multiplying ensures that any remaining spare pairs in the terminal group can always be used to fill an inward service order which is not CT. To relate this paper to the more comprehensive treatment of Ref. 4, note that no reterminated connections* can occur before spare exhaust. After spare exhaust, every reterminated connection is either a BCT, or the terminal group is blocked.

Four functions must be further specified in the integral (7): T_s , $R_b(t)$, $C_b(t)$ and $F_{pwb}(t)$. Of these, $F_{pwb}(t)$ is the simplest, though it is more complicated than the present worth factor for relief [eq. (2)] as the 10-year station account is involved. If r is the force of interest and A_s is the annual charge factor for the station account,

$$F_{pwb}(t) = \left. \begin{aligned} & \frac{A_s}{r} \{e^{-rt}(1 - e^{-10r})\} \quad \text{if } t \leq T_M - 10 \\ & \frac{A_s}{r} \{e^{-rt} - e^{-rT_M}\} \quad \text{if } t > T_M - 10 \end{aligned} \right\} \quad (8)$$

Before going on to detail the other functions $C_b(t)$, $R_b(t)$, and T_s , let us introduce some common notation. Let N_U be the number of living units under consideration; this will be the number of living units served by a single group of multiplied terminals. If the terminal group contains N_T terminals, the number of living units per terminal is $u = N_U/N_T$. To supply the N_U units, n additional line pairs are provided. At spare exhaust, all of these n pairs will have service wires attached. Finally, of the n pairs, $w(t) = \alpha_a(t) N_U$ are working at time t . The additional line penetration is $\alpha_a(t) = \alpha_2(t) + \alpha_3(t)$ (in the notation of Appendix A†).

B.2 Cost of breaking a connect-through

The cost of a BCT, $C_b(t)$, depends on whether one terminal must be visited (cost C_1) or two (cost C_2). Two must be visited if the terminal where service is desired does not contain a connected-through pair. Thus

* This term is used in Ref. 4 to describe the restoration of service to a location that has had a CT broken.

† Following eq. (5).

$$C_b(t) = C_1(1 - p_{nt}) + C_2 p_{nt} \quad (9)$$

where $p_{nt} = p_{nt}(t)$ is the probability of no connected-through pair in a terminal. Of the N_U living units, the total number which do not have a connected-through pair is $N_U - n + w$ (such living units are either working or have no connected service wire). The probability p_{nt} is determined by selecting at random a subset of size u from the N_U living units, and so by probability theory is given by

$$p_{nt} = \binom{N_U - n + w}{u} / \binom{N_U}{u} \quad (10)$$

B.3 Probability of a connect-through

The key to the determination of $R_b(t)$ and T_s is the calculation of the probability that a connected-through pair will be found at a location where additional line service is required. This probability, $p_{CT} = p_{CT}(t)$ can be simply modeled for one-family houses by assuming that additional line demand occurs at random—that is, all houses without a working additional line are equally likely to need one. In that case,

$$\begin{aligned} p_{CT} &= \Pr(\text{CT pair} \mid \text{pair is not working}) \\ &= (\text{number of CT pairs}) / (\text{number of nonworking pairs}) \\ &= \frac{n - w}{N_U - w} \\ &= \frac{n/N_U - \alpha_a}{1 - \alpha_a} \end{aligned} \quad (11)$$

For multifamily houses, however, a correction must be applied, because a living unit can also use a pair connected through to another living unit in the same house. Thus

$$p_{CT} = \frac{n/N_U - \alpha_a}{1 - \alpha_a} + p_{\text{corr}} \quad (12)$$

where

$$\begin{aligned} p_{\text{corr}} &= \Pr(\text{no direct CT}) \{ \Pr(\text{one neighbor l.u.}) \cdot \Pr(\text{neighbor CT}) \\ &\quad + \Pr(\text{two neighbor l.u.}) \cdot \Pr(\text{either is CT}) \} \\ &= \frac{1 - n/N_U}{1 - \alpha_a} \left\{ \frac{2f_2}{F} \cdot p_{nCT} + \frac{3f_3}{F} (2p_{nCT} - p_{nCT}^2) \right\} \end{aligned}$$

where

$$\begin{aligned} p_{nCT} &= \Pr(\text{neighbor is CT}) \\ &= \frac{n - \alpha_a N_U}{N_U - 1} \end{aligned}$$

and

$$F = f_1 + 2f_2 + 3f_3$$

as in Appendix A [following eq. (6)]. We neglect the "minus one" in the denominator of p_{nCT} , and obtain

$$p_{corr} = \frac{1 - n/N_U}{1 - \alpha_a} \cdot \frac{n/N_U - \alpha_a}{F} \{2f_2 + 3f_3(2 - n/N_U + \alpha_a)\}$$

From eq. (12), the probability of no CT may be written

$$1 - p_{CT} = \frac{1 - n/N_U}{1 - \alpha_a} \left[1 - \frac{n/N_U - \alpha_a}{F} \{2f_2 + 3f_3(2 - n/N_U + \alpha_a)\} \right] \quad (13)$$

B.4 BCT rate and spare exhaust time

The probability $1 - p_{CT}$ enables us to determine both $R_b(t)$, the rate of BCTs, and T_s , the time of spare exhaust. Once spare exhaust has occurred, each non-CT additional line order requires a BCT. Thus if $L_r(t)$ is the rate of inward additional line orders,

$$R_b(t) = L_r(t) \cdot (1 - p_{CT}(t)) \quad (14)$$

$L_r(t)$ is calculated using the quantity $\alpha_a F$, which is the expected number of additional lines per house:

$$L_r(t) = \left\{ \frac{\alpha_a F}{\tau_0} + \frac{d}{dt} (\alpha_a F) \right\} N_h \quad (15)$$

Here τ_0 is the mean occupancy time* for additional lines, and N_h is the number of houses served by the terminal group. In eq. (15), the first term represents additional line orders due to churn, and the second term those due to growth.

To determine T_s , we note that if spare exhaust has *not* occurred, each non-CT additional line order requires another pair to have a service wire connected. The analog of eq. (14) is therefore

$$\frac{da(t)}{dt} = L_r(t) \cdot (1 - p_{CT}(t)) \quad (16)$$

where $a(t)$ is the number of pairs with service wires connected (assigned pairs). T_s is obtained by numerical integration of eq. (16) from a specified initial value of a to the value $a = n$.

Having found T_s , overall cost is obtained by substituting eqs. (8), (9), (10), (14), (13), (15) in eq. (7) and integrating.

* The mean time that such a line is working.

B.5 Assignment costs

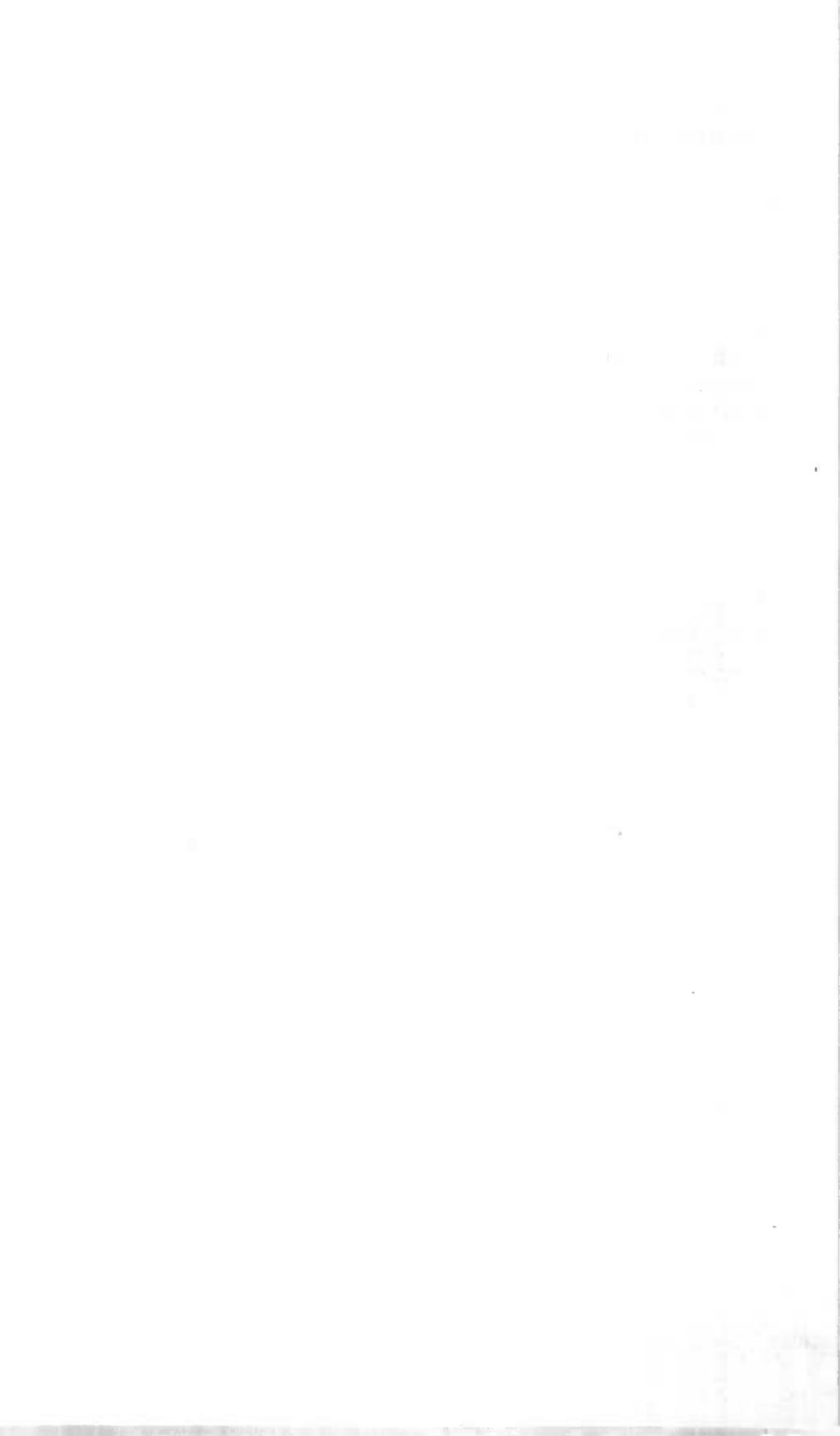
Assignment costs follow from the foregoing derivation of BCT costs. The basic integral (7) is replaced by

$$PW_{AST} = \int_0^T L_r(t) \cdot C_a \cdot F_{pwb}(t) \cdot dt \quad (17)$$

This integral extends from zero to the end of the study period T . In the integrand, $R_b(t)$ of (7) is replaced by $L_r(t)$, the inward rate of additional line service orders, since every order must be assigned, whether connected through or not. C_a is the assignment cost, a constant, representing the difference in cost of performing an assignment with the records appropriate to two pairs per living unit, and with those appropriate to less than two pairs per living unit. The present worth factor F_{pwb} is the same as in the BCT case [eq. (8)].

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Loop Plant Modeling:

Statistical Analyses of Costs in Loop Plant Operations

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The Serving Area Concept (SAC) involves a new procedure for the design and administration of the loop plant to reduce operating costs. Two major problems facing a loop plant engineer considering conversion to SAC are determining which areas should be converted (and in what order) and assessing the savings resulting from the conversion. This paper presents methodology and data analysis results useful for solving such problems. The data analyzed are from the Prototype District and measure a large number of facility related problems both before and after conversion to SAC. A cost penalty measure, based on observed facility problems, is calculated for a given area using data collected in that area over a certain period of time. The before conversion data are characterized and modeled in order to quantify the uncertainty, in the form of a confidence interval, associated with this cost penalty. Confidence intervals are useful to decide appropriate sizes for the data collection areas, appropriate lengths of time for data collection, as well as for comparing the results between two or more areas. The effect of conversion to SAC on the cost penalty measure is also examined. It is found that after conversion costs are much lower than before conversion costs, but that costs continue to decrease for at least 9 to 12 months after conversion takes place. The analysis and results presented here yield methods and guidelines to be used for data collection and analysis in other districts. These can help in reliably choosing areas for conversion to SAC which will maximize savings.

I. INTRODUCTION AND SUMMARY

Investment decisions in the loop plant, like most such investment decisions in the Bell System, are dependent on careful analyses and the data which underlie these analyses. This paper describes detailed studies of a large body of data measuring several kinds of loop plant operations and costs. The cost measures used are based on the *Facility Analysis Plan for Outside Plant* (FAP); this plan, described and discussed in Ref. 1, gives methods for managing the loop plant. The results of this paper contain guidelines for the use of certain FAP measures, as well as insights into related characteristics of the data.

The data analyzed here are from the Prototype District Project,² a major effort undertaken to analyze those operating costs of a district that can be controlled by changes in the design or administration of the loop network. This involved a nearly three year study of the Passaic District of New Jersey Bell Telephone Company. Passaic is an urban area with some small business, scattered apartments, and large old houses. Many sections were converting from single- to multiple-family dwellings. Much of the existing loop plant was congested and had maintenance problems. Thus, conversion to the Serving Area Concept (SAC)³ was considered appropriate for much of the district. This conversion involves departures from dedicated plant design and multiplied plant design.⁴ Serving area interfaces, which are basically large boxes containing cable pair interconnect points, are installed in appropriate places in the network. Then cable pairs are permanently connected from the interface to the customer, and complements of feeder pairs from the central office to the interface are supplied as needed. The Facility Analysis Plan, developed from the Prototype District Project, gives methods for determining when and where conversion to SAC is appropriate.

The Prototype District Data Base⁵ is the key to tracking district activities. Each month over 50,000 measurements of district operations involving facility related problems were recorded. (Many of these measurements were zero.) Data were retained by 50-pair complement by month for that part of the district undergoing extensive conversion to serving areas. Data are available from April 1973 through December 1975.

There are many procedures in the Facility Analysis Plan to aid in understanding costs and potential savings in the management of loop plant. Among the concepts involved are allocation areas,^{1,4} which are geographical regions used for tracking operating costs and cable usage. Allocation areas are also basic units of plant for planning additions or changes in the network such as conversion to SAC. Therefore, in order to trigger the need for treatment of the network these areas are initially ranked on the basis of facility problems in each area. This ranking is based on a weighted linear combination of facility problems normalized

by the number of assigned pairs in the area. The weights are costs associated with the individual problem items and together yield a "Cost Penalty Per Assigned Pair" (CPPAP). In Ref. 1, the Normalized Yearly Marginal Operating Cost, which is a generalization of CPPAP, is used as a basis for their discussion. Other cost calculations include the "Plant Stabilization Analysis Form" and the "CUCRIT" analysis to compute the rate of return associated with a given investment strategy. While these other cost calculations are important and relevant to FAP, the focus of this paper is on the CPPAP calculation and its component parts.

Three specific reasons motivate the choice of CPPAP for analysis here. First, it is the initial form used to analyze data in FAP and as such holds an important position. Second, the cost calculations for CPPAP are linear combinations of observed quantities and hence directly interpretable. Third, CPPAP does not require any special factors (e.g., "improvement factor") as are needed in most of the other measures.

The general purpose of this paper is to give insight into facets of these data relating to the conversion of selected allocation areas to SAC which took place during the Prototype District Project. Two important problems to the loop plant engineer are to determine which of the allocation areas should be converted (and in what order), and to assess the savings resulting from the conversion. The data analysis addresses these problems by modeling the variability of the FAP data. The uncertainty associated with projected savings is found to decrease as the serving areas become larger (in assigned pairs) and the data collection period increases.

An exploratory analysis of the before, during, and after conversion cost measure and its components (Section II) shows that the cost measure varies widely both across areas and time. Assignment changes, cable troubles, and defective pairs contribute the most to the level and variability. A detailed statistical analysis of the before conversion cost data in Section III is used as a basis to develop confidence intervals (Section IV) on the "true" cost penalty. These intervals quantify the uncertainty associated with an observed cost penalty for a given area. They are useful to decide appropriate sizes for the data collection areas, appropriate lengths of time for data collection, as well as for comparing the results between two or more areas. Moreover, confidence intervals show the trade-off between the size of the data collection area and the data collection period.

Finally, the effect of the conversion on the cost measure is examined in Section V. A regression equation is developed which models the after conversion costs in terms of before and during conversion variables as well as the time since conversion. The major result shows that costs continue to decrease after conversion takes place. In order to get an adequate measure of the savings associated with conversion to SAC, one must collect data for at least nine to twelve months after conversion.

It should be noted (before proceeding with the data analysis) that much of the work described was also performed on other savings measures including the rate of return. The same techniques which are shown for CPPAP were found useful, but for brevity their results are not shown.

II. GENERAL CHARACTERISTICS OF THE COST DATA

2.1. Introduction

The purpose of this section is to give some insight into the data used in the further analyses in this paper. As described above, the analysis focuses solely on the data in the CPPAP, which is calculated using the "Allocation Area Problem Ranking Worksheet."¹ This worksheet is shown in Fig. 1. Column B, the cost factors, are specific to the Prototype District, but they are also representative of other loop plant districts. Abbreviations used in Fig. 1 and throughout this section are as follows: LST—line and station transfer; WOL—wired out of limits; BCT—break connect-through; CDP—clear defective pair; BPC—break permanent connection; CIR—control point interconnection; RE—referred to engineer; RTC—reterminated connection; AC-SOD—assignment change because the originally assigned pair from a service order was found to be defective; AC-NS—non-service-order assignment change; AC-OTH—other assignment change; FCT-7AB—7A or 7B cable trouble associated respectively with splicing and terminating troubles; FCT-OTH—other cable trouble; DEF PRS—defective pairs. For definitions and discussion of these and other loop plant terms, see Ref. 4.

Two of the items on the worksheet were not measured directly in the data base. They are the BCT and RTC. However, based on engineering studies in the Prototype District⁶ it was determined that these could be adequately approximated for the Prototype District during the study period by a fraction of the total facilities assigned, which is measured in the data base. These studies determined that BCTs were 13 percent of the facilities assigned and that RTC were 35 percent of facilities assigned. Finally, the management of the loop plant used in the Prototype District was such that there were no CDP, BPC, or CIR. Therefore, in all further analyses these cost factors are ignored. All other variables, except the number of defective pairs, are available (monthly) in the data base. Defective pairs were entered annually from the district's yearly pair status report. This report gives the pair status (e.g., assigned, defective, etc.) as of January 1 and is used monthly for the twelve month period centered at January 1 (i.e., July through June). Thus, the data to be studied in this section are the monthly values of the CPPAP and the 11 sub-components of CPPAP that were either measured or estimated during the study.

		A	B	C
LINE #	ITEM	ENTRY	COST FACTOR	COST PENALTY
1	LST	_____ #/YR	X <u>17.52</u>	= _____
2	WOL	_____ #/YR	X <u>36.81</u>	= _____
3	BCT	_____ #/YR	X <u>7.64</u>	= _____
4	CDP	_____ #/YR	X <u>72.70</u>	= _____
5	BPC	_____ #/YR	X <u>24.64</u>	= _____
6	CIR	_____ #/YR	X <u>70.55</u>	= _____
7	RE	_____ #/YR	X <u>35.15</u>	= _____
8	RTC	_____ #/YR	X <u>9.48</u>	= _____
9	AC - S. O. Def	_____ #/YR	X <u>29.35</u>	= _____
10	AC - Non S. O. Def	_____ #/YR	X <u>68.14</u>	= _____
11	AC - Other	_____ #/YR	X <u>32.83</u>	= _____
12	FCT - 7A, B	_____ #/YR	X <u>83.32</u>	= _____
13	FCT - Other	_____ #/YR	DIST CO KFT X <u>109.00</u>	= _____
14	Def Prs	_____ #DEF Pr X _____	X <u>0.91</u>	= _____
15	TOTAL COST PENALTY (SUM 1 TO 14)			_____
16	COST PENALTY PER ASSIGNED PAIR _____ ÷ _____ # ASSIGNED PAIRS LINE 15			= <input type="text"/>

Fig. 1—Allocation area problem ranking worksheet.

2.2. Components of CPPAP

The CPPAP has 11 non-zero cost components. However, two of those variables are perfectly correlated since they are both proportions of the facilities assigned (i.e., BCT and RTC). Therefore, since both the cost factor (see Fig. 1) and the proportion of facilities assigned associated with the RTC is higher than that for BCT, it is the RTCs which will be used in the further analyses in this subsection. In later sections of the paper all components are used in the calculation of CPPAP.

A numerical summary of the level (mean) and variability (standard deviation) of the ten cost components for each of the three stages of area conversion is given by Table I. So that a few extreme data values do not overwhelm the rest of the data, the 25 percent trimmed mean and standard deviation were used. Thus these values are based on only the

Table I — CPPAP component costs for 10 converted areas

Variable	Trimmed mean			Trimmed std dev		
	Before	During	After	Before	During	After
LST	0.65	0.12	0.0	0.77	0.17	0.0
WOL	0.04	0.0	0.0	0.18	0.0	0.0
RE	0.77	0.04	0.0	1.20	0.19	0.0
RTC	0.53	0.40	0.21	0.22	0.14	0.09
AC-SOD	0.26	0.10	0.01	0.38	0.18	0.12
AC-NS	1.44	3.78	0.46	1.16	3.35	0.52
AC-OTH	1.12	1.47	0.44	0.56	1.41	0.41
FCT-7AB	2.30	4.97	0.21	1.36	4.50	0.61
FCT-OTH	0.19	0.13	0.0	0.82	0.39	0.0
DEF PRS	0.80	0.84	0.85	0.63	0.83	0.89

middle 50 percent of the data. First the trimmed mean across months for each area in each stage of conversion was computed; the tabled values are the trimmed mean and trimmed standard deviation of those values across the 10 converted areas. Focusing on the mean (level) values first, it is clear that the dollar costs shown in the table vary widely from component to component as well as for the stages of conversion. Perhaps the most remarkable change is in the non-service-order assignment change tickets (AC-NS) which go from \$1.44 before to \$3.78 during to \$0.46 after. However, considering the physical situation, this type of behavior is to be expected. During the conversion, many of the cable pairs are being handled by the nature of the design of an allocation area. This can cause many of the pairs to become defective and can cause an interruption in the customer's service. The service is restored either by changing the customer to a new pair (recorded as an AC-NS) or actually fixing the defective pair (recorded as an FCT-7AB). Note further that the occurrences of splicing and terminating cable troubles (FCT-7AB) also peak during conversion and fall to greatly reduced levels in the after period. However other cable troubles (FCT-OTH) contribute little to CPPAP. The category of assignment changes due to the originally assigned pair from a service order being defective (AC-SOD) drops to very nearly zero after conversion. Other assignment changes (AC-OTH) is a major contributor to CPPAP during all three periods of conversion. The LST, WOL, and RE after conversion all have zero trimmed mean and standard deviation. The category of defective pairs (DEF PRS) is interesting because its level stays the same from during to after, and its variability actually increases during this transition. However, since the defective pair data is only updated annually, these results should be considered preliminary. More detailed special studies of defective pair rates have been performed and are included in Ref. 2.

While the table is a helpful summary of overall behavior, it is not useful in trying to characterize the similarity and differences among the areas with regard to the components of CPPAP. Graphical displays of multivariate data are often useful for gaining insight into the basic structure

of data. However, they tend to become more complicated and less useful as the number of variables increases. Based on Table I, it seems fairly clear that most of the interesting (large and variable) dollar components of CPPAP are found in the assignment changes, the cable troubles, and the defective pairs. The costs associated with LSTs, WOLs, REs and RTCs tend to be both small and fairly stable. Therefore, in the graphical displays the focus will be on the six largest and most diverse cost components.

Figure 2 gives one example of a polygon plot⁷ for three of the converted areas and the mean converted area (i.e., the 25 percent trimmed mean of the converted areas). The polygon is formed by connecting the value of each variable plotted on its respective axis (see Fig. 2 key). By examining the polygons associated with different areas and stages of conversion it is possible to visually compare and contrast characteristics of the areas. Note the similarity of the areas for before, during, and to some extent after. The values in these plots are as in Table I, and show dollar amounts. The scaling is designed to show most of the variability in these data without being distorted by a few very large values. Although areas of a polygon do not directly correspond to the total cost associated with an allocation area, areas do give some idea of that sum. For example, it is clear that after conversion the cost penalty is very small compared with during and before. The anomalous large value of the non-service order assignment changes (mentioned earlier) is evident in the during period. The peak on the first axis from the vertical position is this large value.

2.3. Analysis of CPPAP

To achieve an initial feel for the nature of the CPPAP data, a plot of these values against time for the individual allocation areas is useful. Figure 3 shows a sequence of four allocation areas for their entire 33 month data history. Note that the vertical scales on the four plots, which show dollar cost penalties, are different. While such differences make across area comparisons difficult, the range of the data (particularly including converted and non-converted areas) is so large that using a single scale would obscure much of the available detail. Because there is a good deal of variability in the CPPAP measure, a non-linear (resistant) smoother is applied to the data and plotted (as the solid line) along with the raw values. The resistant smoother used is (3RSR), twice.⁸ Since this smoother is based on moving medians, rather abrupt changes may occur in the smoothed output. This smoother was selected for just this reason so that rapid changes in the level of the data (e.g., after conversion) would not be obscured.

Of these four allocation areas (212 through 215), two were eventually converted (213 and 214), while the other two were not. For those areas which have been converted, lines are drawn to indicate the end of the

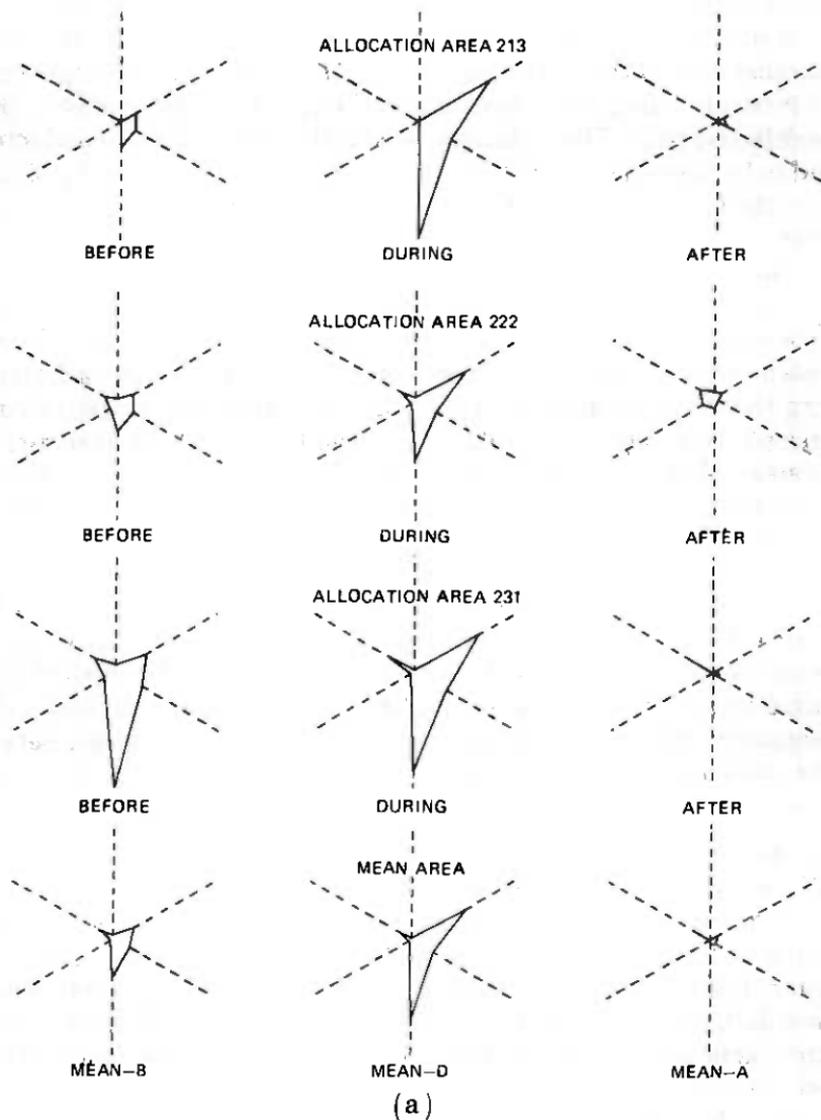


Fig. 2—(a) Components of CPPAP (radius length 7.12). (b) Key.

before conversion period, and the beginning of the after conversion period. Note that these vertical lines are drawn between actual monthly observations. The data accuracy only allows full month designations of before, during, or after. For example, in area 214 months 1–5 are before, months 6–13 are during, and months 14–33 are after conversion.

Analysis of this figure (and others) showing all the area-time histories gives a considerable amount of insight into the nature of the data.

(i) The CPPAP for the areas where there is no conversion tends to be more stable than for areas that undergo conversion.

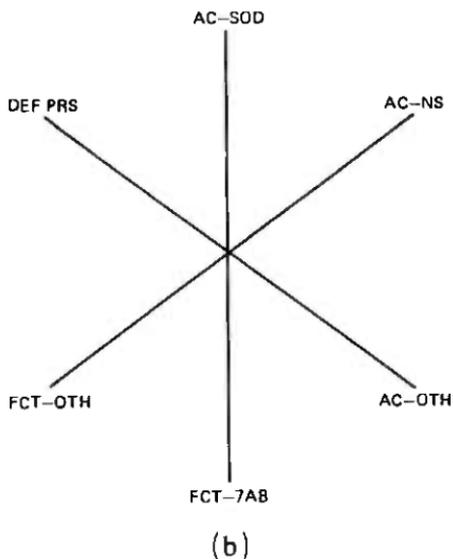


Fig. 2. (continued)

(ii) Fairly large excursions from a smooth value are evident for all areas. (Note that the resistant smoother is not affected by these unusual excursions.)

(iii) The level and variability of the before, during, and after may be quite different.

(iv) The after conversion behavior of these areas is quite different. For example, in area 213 the CPPAP drops quickly to a value near zero. In area 214 there is a slow but steady decline to a near zero value for CPPAP.

(v) No evident seasonal pattern is visible in this limited amount of data.

Table II shows a basic summary of the behavior of each of the 10 converted areas for before, during, and after conversion months. The 25 percent trimmed mean and standard deviation are used, as in Table I, so that the tabled values reflect the bulk of the data. Table II shows that both the level and variability change during the "life" of an area. The during period tends to have the highest levels. The after is the lowest (as would be both expected and presumed because the effect of conversion is to reduce the occurrence of the costly plant troubles) both in level and variability. The variability of the before conversion data is quite high and not uniform across areas.

In summary, based on these and similar displays, CPPAP values appear to vary quite widely both across allocation areas and stages of conversion. For those areas which were converted, the level and variability of the individual components of CPPAP tend to be concentrated in the assignment changes, cable troubles, and defective pairs.

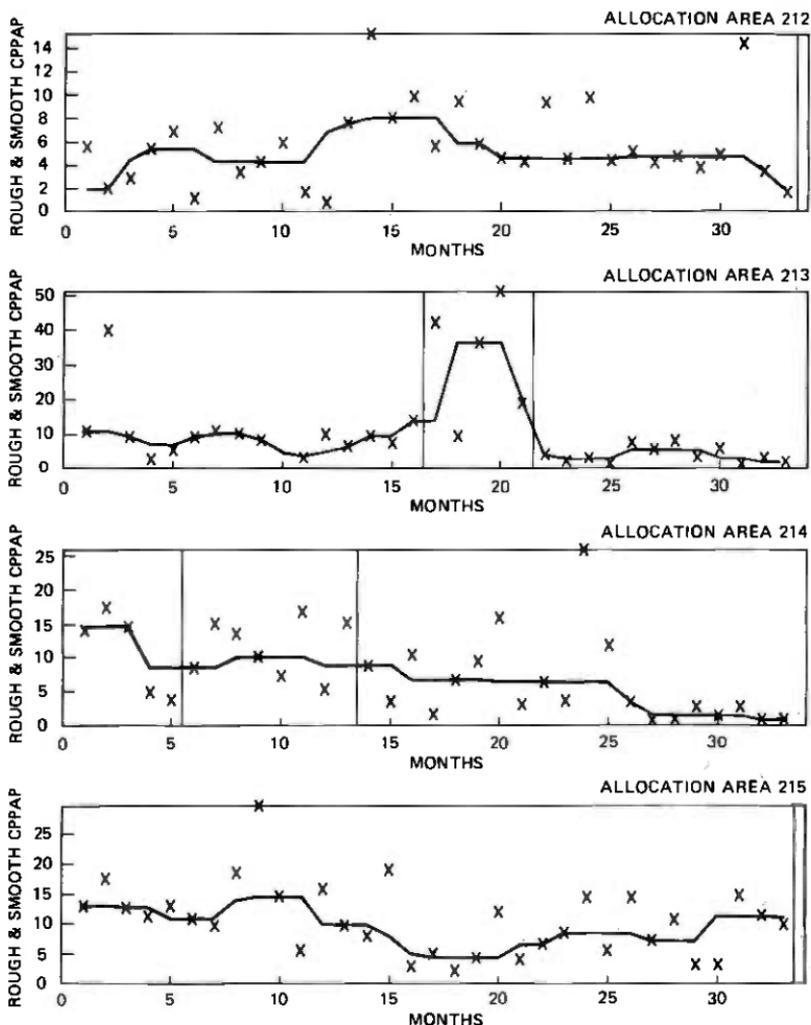


Table II — CPPAP for 10 converted areas

Area	Trimmed mean			Trimmed std dev		
	Before	During	After	Before	During	After
209	4.39	6.68	1.90	2.84	5.20	1.10
210	5.97	22.32	3.51	6.52	10.98	2.22
213	8.84	32.56	3.35	3.40	31.65	3.25
214	11.43	11.81	4.18	14.75	7.73	5.98
221	9.58	10.29	1.91	6.86	9.58	0.00
222	9.41	11.64	4.01	3.02	5.03	0.00
227	10.37	10.07	6.67	6.85	4.30	4.95
228	11.73	14.97	4.22	4.91	7.61	1.19
229	14.15	17.00	3.02	10.86	22.88	0.93
231	21.10	17.03	3.32	7.52	7.28	2.04

culated cost penalty across months and across areas can be large, as was seen in Section III. Thus, statistical methods are needed to help answer these questions. Since only before conversion data could be used to help in making decisions regarding conversion, only the before data from the data base are considered here. The analysis uses the cost measure CPPAP for reasons described in Section I.

The goal here is to examine the structure of the before conversion data so as to be led to reasonable methods of analysis (i.e., reasonable assumptions and models) to answer these questions. We concentrate on searching for and examining certain relationships by studying appropriate scatter plots. While certain numerical statistics are also useful for such purposes, an advantage of plots is that they are more exploratory in nature. Section IV then presents and uses a specific model, supported by the data, as a way of answering the questions in the previous paragraph.

3.2. Analyses

For the following plots, consider the cost penalty x_{ij} for area i and month j . The mean, \bar{x}_i , and standard deviation, s_i , of these values across months for each area were calculated. Only before conversion data were used, so the number of months differs from one area to another; however, recall that 13 of the 23 areas were never converted, so for these areas all 33 months are available. Figure 4 plots the standard deviation s_i vs. the average cost penalty \bar{x}_i for all 23 areas. A positive relationship between these two quantities is very clearly apparent. Such a relationship strongly violates assumptions that would be desirable and convenient to use.

Another look at this relationship can be obtained by considering the sizes of the 23 areas. Since the cost penalty x_{ij} is itself an average calculated over the number of pairs in the area (cost penalty per assigned pair), one might expect the standard deviation of these values, s_i , to be smaller the larger the size of the area. Figure 5 plots s_i vs. the number of assigned pairs in the i th area, p_i . From theoretical grounds one might expect the relationship between s and p to be of the form $s = \sigma/\sqrt{p}$, for

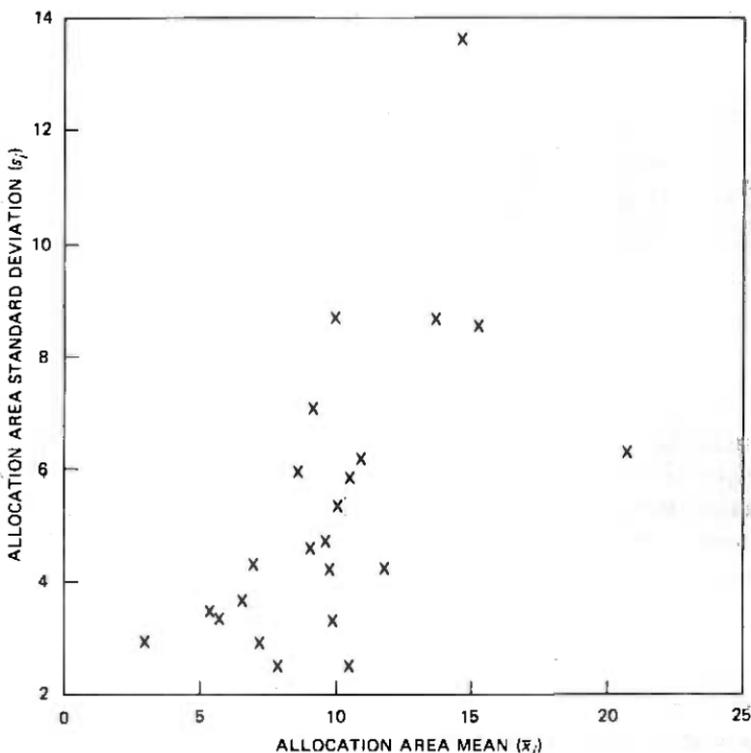


Fig. 4—CPPAP values before conversion.

some σ . The points in Fig. 5 look like they might generally follow a relationship like this, plus some scatter. Thus, we fit a curve $\hat{s} = \hat{\sigma}/\sqrt{\hat{p}}$ to these points using least squares* and then formed the residuals $(s_i - \hat{s}_i)$. Each residual is plotted against the corresponding \bar{x}_i in Fig. 6. Again a strong increasing relationship is apparent; the larger the average cost penalty \bar{x}_i for an area, the more likely it is that $(s_i - \hat{s}_i)$ is positive and large. Even after removing the effect of area size from the standard deviation s_i , higher area averages \bar{x}_i are associated with higher area standard deviations s_i .

One approach to answering the questions put forth in Section 3.1 would be to fit an appropriate linear statistical model to these data, and then make inferences from that model. However, one of the assumptions underlying the usual fitting of such a model is that of homogeneity of variance; i.e., the variance of the observations should be constant across different levels of other variables. Because of the relationships seen above, it is worthwhile also to consider transformations of CPPAP when exploring the before conversion data. Some transformed variable quite possibly could be generally appropriate for later, more formal analysis than would the raw CPPAP values.

* Weighted least squares were used, for reasons described below.

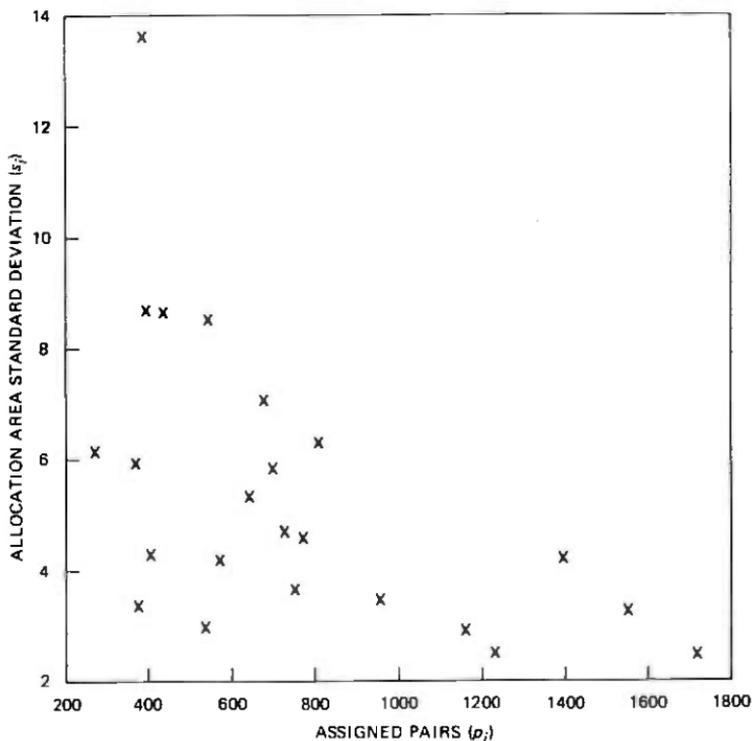


Fig. 5—CPPAP values before conversion.

Several transformations of the cost penalties within the family $y = (x + a)^b$, with a and b specified parameters, were calculated and studied. Considering the results as a whole, the most satisfactory and interesting properties appeared using the transformation $y = \ln(x + 1)$, which corresponds to $b = 0$, with x the CPPAP as before. Thus the following plots in this section were all constructed using this transformation.

Figure 7 plots the standard deviation $(s_y)_i$ vs. \bar{y}_i , with the plotting character showing the size of the area; "1" for areas with assigned pairs $p_i \leq 500$; "2" for $500 < p_i \leq 700$; "3" for $700 < p_i \leq 950$; "4" for $p_i > 950$. There appears to be no systematic relation between (s_y) and \bar{y} , although the two extreme (high and low) values on \bar{y} possibly suggest a decreasing trend; certainly there is nothing like the behavior in Fig. 4. Moreover, the higher number plotting characters tend to be at the bottom of the plot with the lower numbers at the top, implying that larger areas have smaller variability, apart from their average value. The area average \bar{y}_i is plotted against size p_i in Fig. 8; these quantities appear unrelated, so knowing *a priori* the size of an area does not enable one to say much about its expected average cost penalty.

Figure 9 shows the standard deviation $(s_y)_i$ plotted against size p_i . There is a downward trend, and one expects larger areas to have smaller

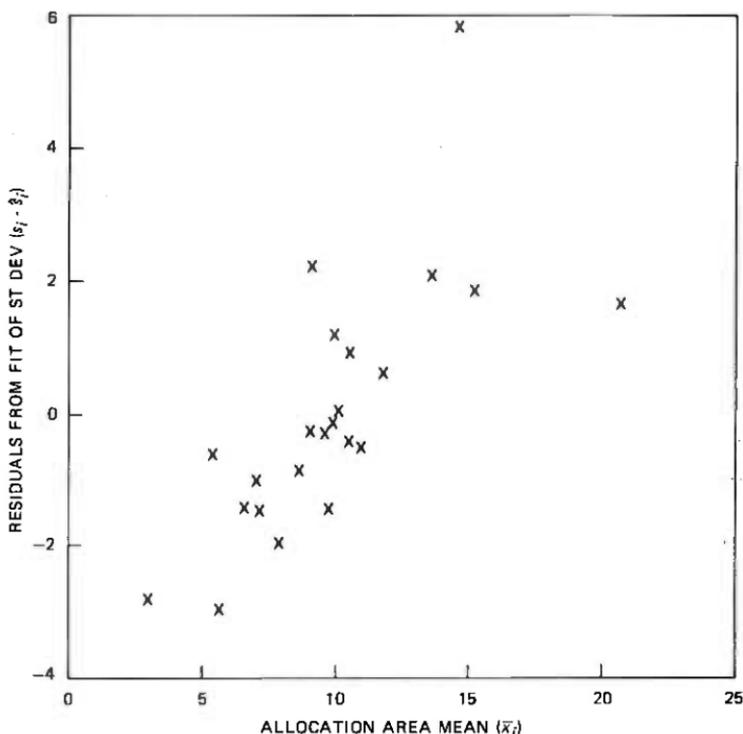


Fig. 6—CPPAP values before conversion.

variability. To see to what extent this trend is accounted for by a $sy = \sigma/\sqrt{p}$ relationship, $\hat{\sigma}$ was obtained by a weighted least squares regression of $(sy)_i$ on $1/\sqrt{p_i}$; the fitted curve is the solid line in Fig. 9. A weighted regression was used because the variances of the individual points $(sy)_i$ about their expectations $\tau_i = \sigma/\sqrt{p_i}$ depend on the values of τ_i and m_i , the number of months of before data for that area; assuming normality of the y 's, the variance is $0.5 \cdot \tau_i^2 / (m_i - 1)$. (This is derived from the χ^2 distribution associated with $(sy)^2$.) Thus, weights proportional to the reciprocal square roots of these variances were used, and the following three plots are the raw residuals multiplied by these weights.

The residuals $sy_i - \hat{s}_y$ are plotted against \bar{y}_i in Fig. 10. No strong relationship is apparent. Perhaps the points with extremely high and low \bar{y} suggest a downward trend, but if these single points are ignored no structure at all remains. Figure 11 plots each residual against m_i , the number of months of before data for that area. One would like to see a horizontal band, which would signify no relationship; indeed, the plot does not suggest any strong relationship. A normal quantile-quantile probability plot⁹ of the residuals is displayed in Fig. 12. This shows reasonably good normality of the residuals, although the largest value is somewhat larger than would be expected and there is some bunching of the residuals, for which we have no explanation.

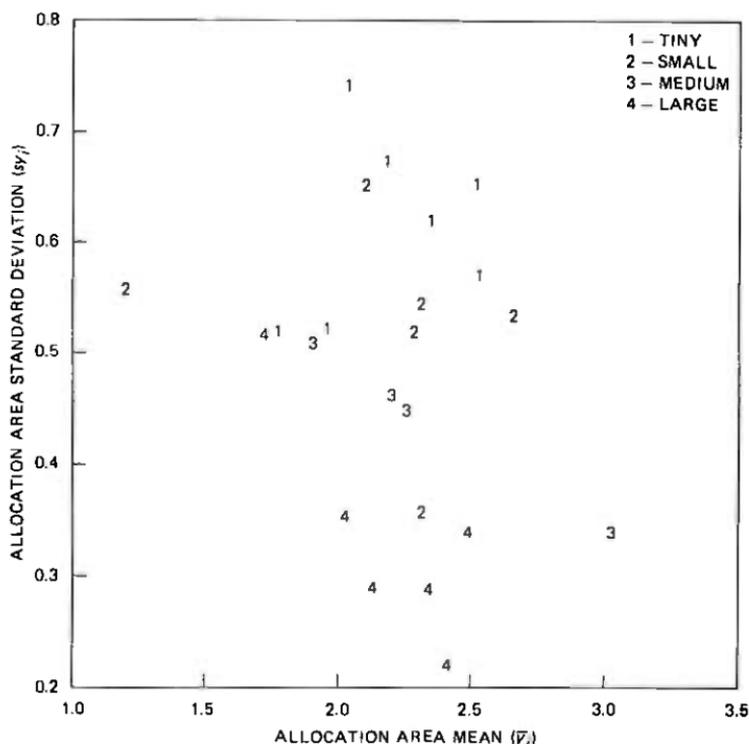


Fig. 7—Values of $\ln(\text{CPPAP} + 1)$ before conversion.

Thus, for the logarithmic transform of the original cost penalty nicer behavior results than with the raw variable. An area's standard deviation is unrelated to its level, but it is related to its size in a reasonable way; moreover, the residuals from this relationship have reasonable properties. A number of additional properties of these data were explored, but to conserve space only a few will be discussed in any detail.

For each month, the mean and standard deviation of the CPPAP values for all allocation areas for that month were calculated. Figure 13 plots the monthly standard deviations vs. the monthly average, again using $y = \ln(\text{CPPAP} + 1)$. There are 33 points in the plot, one for each month; of course the points from later months are based on successively fewer values as areas are converted. No relationship is apparent; this is consistent with the lack of relationship between standard deviation and mean as calculated for each area in Fig. 7. The monthly average vs. the month number and the smooth of these data [using $4(3\text{RSR})2$, twice, a non-linear smoother⁸], are shown in Fig. 14. This suggests somewhat of a cyclic behavior in the average cost penalty. Local peaks appear around months 1-2, 12-14, and 26-28. One might hypothesize the existence of a cyclic 12-month structure to these data due to seasonal local factors such as weather, churn, and inward and outward movement. However,

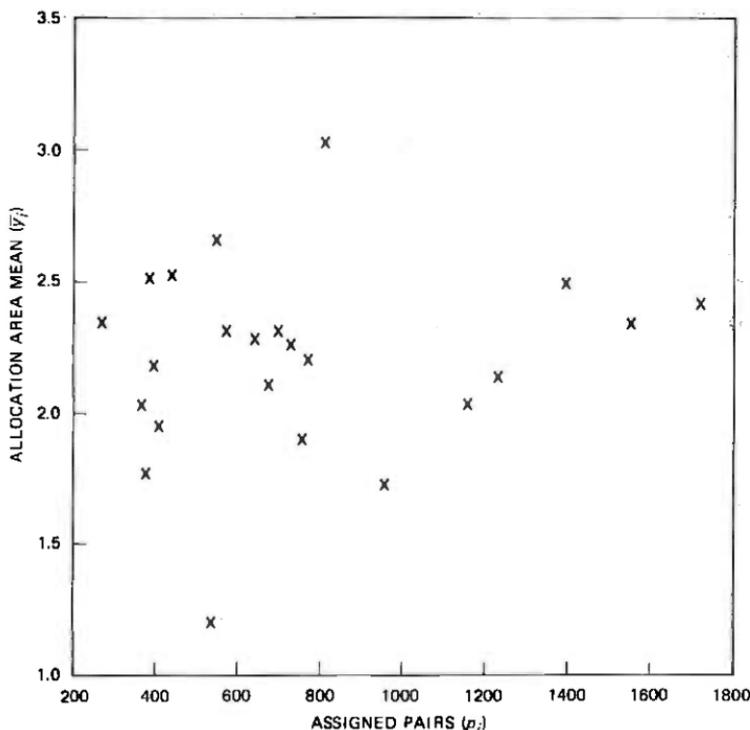


Fig. 8—Values of $\ln(\text{CPPAP} + 1)$ before conversion.

Fig. 14 does not show such clear behavior that one could extrapolate some fitted cycle with any confidence. Moreover, recall that the purpose of these analyses is to develop methods that could be used with (probably less extensive) data from other districts for decision making. We would not want to extrapolate a specific seasonal pattern from Fig. 14 to a new district without careful consideration of similarities and differences between the new district and the Prototype District. One might, though, wish to use 12 or 24 months data when arriving at decisions so as to remove seasonal effects. The possible seasonal factor is discussed further in reference to somewhat different purposes in Section V.

Distributional characteristics and the correlation structure of the transformed observations can also be of interest. Figure 15 gives a normal quantile-quantile plot of $(y_{ij} - \bar{y}_i) \cdot \sqrt{p_i}$ for all areas i and months j before conversion. This quantity is of interest because some differences between areas are expected, but can be removed by looking at the deviations $y_{ij} - \bar{y}_i$. No strong monthly effect was seen above, so that possibility is ignored here; and also it was found earlier that $\text{var}(y_{ij})$ is approximately σ^2/p_i , so the values $(y_{ij} - \bar{y}_i) \cdot \sqrt{p_i}$ should have approximately equal variance. Figure 15 shows that these values are distributed reasonably closely to the normal distribution.

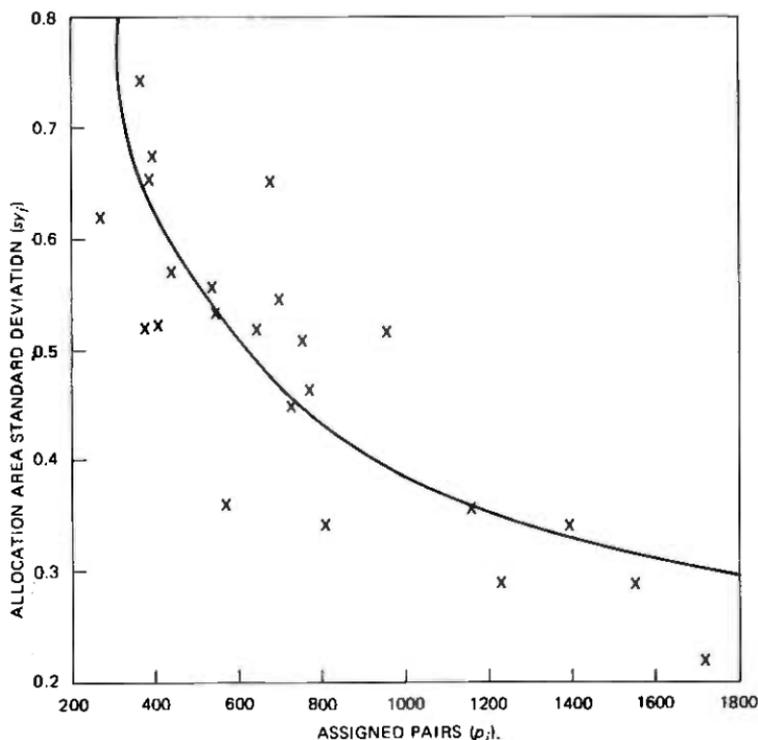


Fig. 9—Values of $\ln(\text{CPPAP} + 1)$ before conversion.

Turning to the possible relationships between areas, a different normal quantile-quantile plot, calculated from correlations in the following way, is given in Fig. 16. For each pair of areas k and l , the correlation between the above $(y_{ij} - \bar{y}_i) \cdot \sqrt{p_i}$, $i = k$ and l , was calculated over the before conversion months common for both areas. This gives 253 ($= 23 \cdot 22/2$) estimated correlations, and we would like to see to what extent these differ from a random sample of correlations where the true correlation coefficient is 0. Fisher's z transformation,

$$z = \frac{1}{2} \ln \left(\frac{1+r}{1-r} \right)$$

was used to achieve approximate normality. If the population correlation is 0, then mean (z) \approx 0,

$$\text{var}(z) \approx \frac{(n+1)}{(n-1)^2}$$

where n is the sample size and z is approximately normally distributed. For these data each z was divided by the standard deviation corresponding to the number of months n from which it was calculated, and Fig. 16 is a normal quantile-quantile plot of the standardized z 's. A

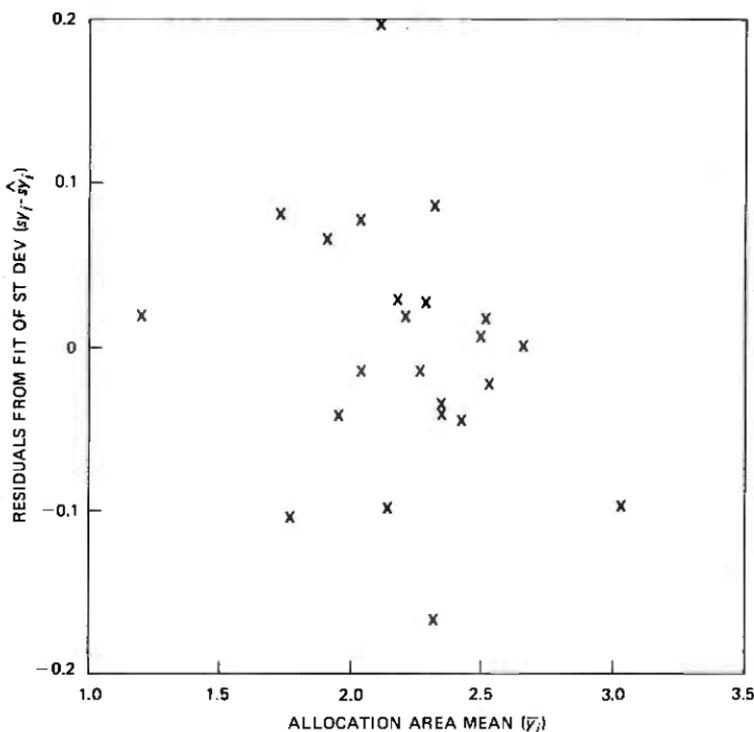


Fig. 10—Values of $\ln(\text{CPPAP} + 1)$ before conversion.

“perfect” result would have all points on the $y = x$ line, which is drawn on the plot. However, even if the true correlation were 0 one would not necessarily expect our standardized z 's to scatter exactly about this line since we do not have 253 correlation coefficients calculated independently of one another. Instead they are formed pairwise from 23 variables, implying some (complicated) structure among them. In Fig. 16 the points are uniformly above, but quite close to the $y = x$ line; the standardized z 's are slightly but consistently larger than would be expected if all true correlations were 0. The median of the standardized z 's corresponds to a population correlation of about 0.3. Thus there is evidence of a positive but not large correlation between the values in different areas at the same point in time. This result is not intuitively unexpected since geographic proximity is probably the cause. For example, a heavy rainstorm may increase cable troubles and hence larger values of CPPAP. A more exhaustive exploration of the correlation structure of these data could also consider correlations both between and within areas at different points in time, i.e., with leads and lags.

Another plot of some interest, Fig. 17, shows \bar{y}_i vs. the distance of each area from the central office, d_i . Although one might or might not expect such a relationship, the data strongly suggest that areas further from the

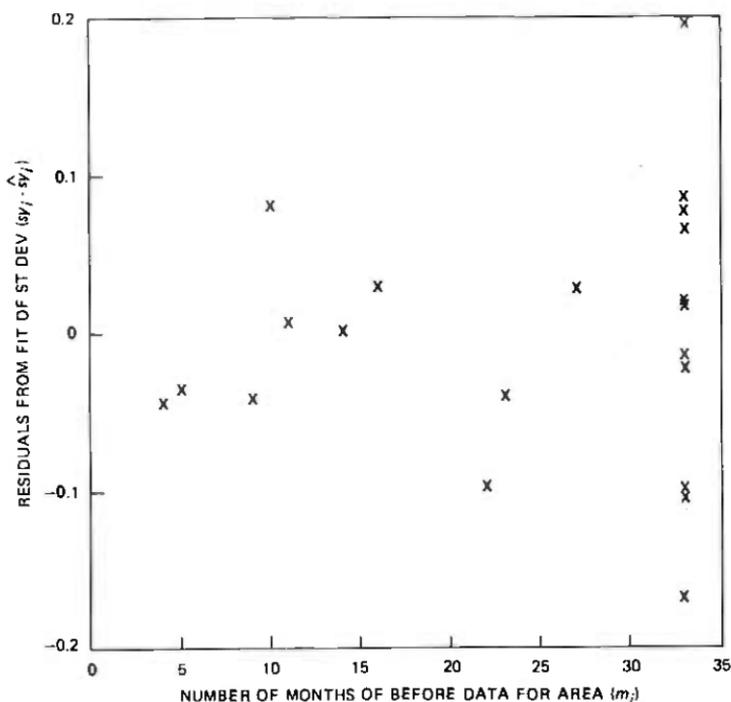


Fig. 11—Values of $\ln(\text{CPPAP} + 1)$ before conversion.

central office have higher cost penalties. It would be of interest to have explanations for this and to see if this relationship generalizes to other districts. Such investigations are in progress by the authors and others. However, as with the possible monthly cycle seen above, we would not necessarily want to extrapolate this in a straightforward way to other districts. It is also of interest to consider the plot of the weighted residual $(sy_i - \hat{sy}_i)$ vs. d_i , given in Fig. 18. Although the area average may be related to d_i , Fig. 18 shows that the part of the standard deviation not predicted from the size of the area does not seem related to d_i . This latter result fits in with the previous discovery that the standard deviations of the y 's do not appear to be systematically related to anything except the size of the area.

The entire set of plots and analyses described in this section were repeated using robust estimates of location and scale instead of the sample mean and standard deviation. The purpose was to see if a small number of deviant observations might be either causing, or hiding, the relationships considered above. However, there was no appreciable difference in the results. The results using the mean and standard deviation, rather than the more robust statistics, were presented above because of the widespread familiarity and use of these statistics.

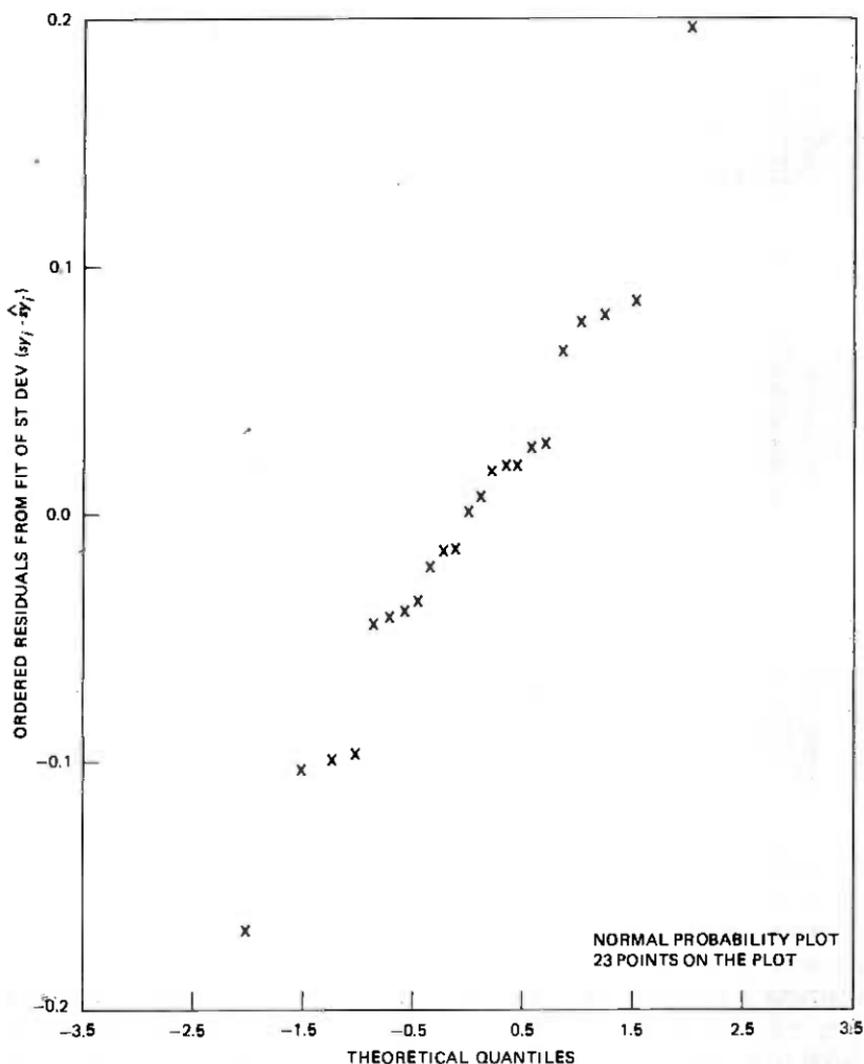


Fig. 12—Values of $\ln(\text{CPPAP} + 1)$ before conversion.

The analyses were also repeated using other cost measures. As in the case of CPPAP, for each of these measures some transformation of the original values was discovered which appeared more useful for interpretation and later analysis than was the raw cost measure.

IV. DATA COLLECTION GUIDELINES

4.1. General results

This section makes use of the results from the previous section to construct guidelines for the collection period and size of future allocation areas. These guidelines are in the form of confidence intervals for the

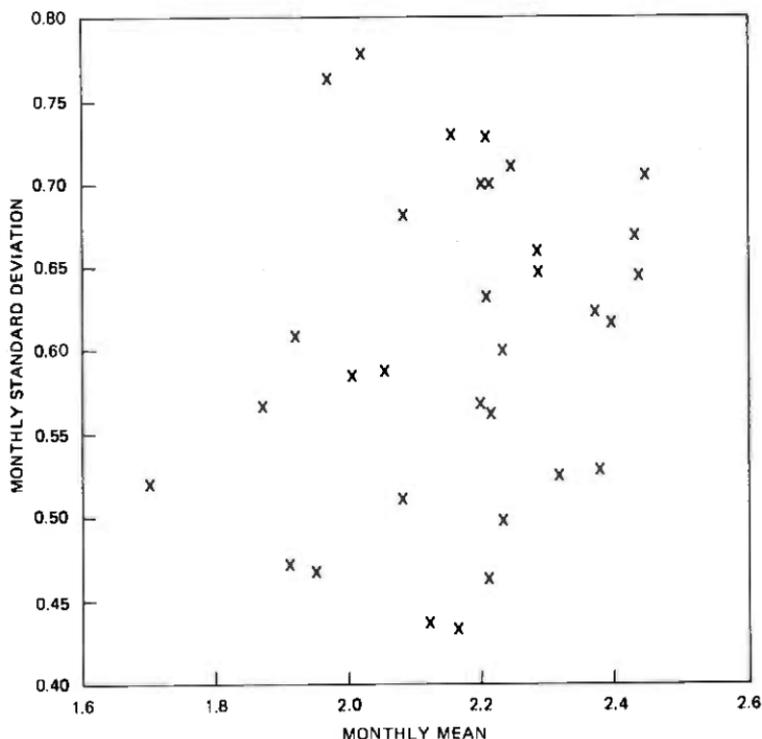


Fig. 13—Values of $\ln(\text{CPPAP} + 1)$ before conversion.

“true savings” given estimated savings, size of area, and the number of months of data collection. In addition, methods are presented for extending these results to local areas with characteristics different from those of the Prototype District.

Based upon the data analysis of Section III, it is reasonable to use the following model and analysis. Let $y_{ij} = \ln(\text{CPPAP} + 1)$ be the transformed cost measure for area i and month j . Express this as

$$y_{ij} = \mu_i + e_{ij} \quad (1)$$

where μ_i is the “true transformed CPPAP” for this area and e_{ij} is the “error” term corresponding to this month. We wish to make inferences about the area values μ_i and differences $\mu_i - \mu_j$.

Consider assumptions one can reasonably make concerning the e_{ij} . From theoretical grounds it is reasonable to assume that

$$\text{var}(e_{ij}) = \frac{\sigma^2}{p_i} \quad (2)$$

where p_i is the size, in assigned pairs, of the area. The quantity σ^2 can be interpreted as the inherent variability from one assigned pair in one month, and the error term e_{ij} results from averaging over p_i assigned

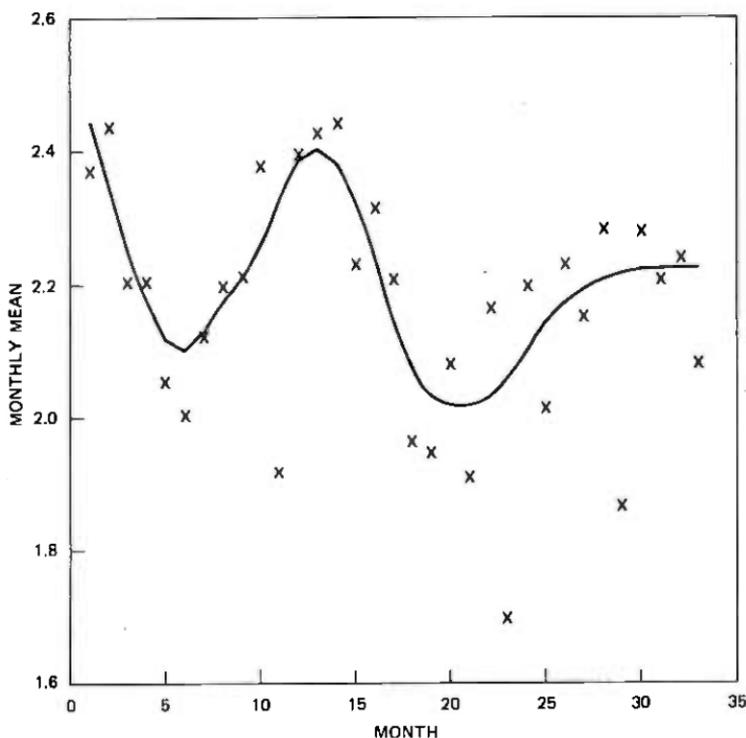


Fig. 14—Values of $\ln(\text{CPPAP} + 1)$ before conversion.

pairs. This assumption was supported by the analysis of Section III. Moreover, that analysis showed that the standard deviation (of the transformed CPPAP) does not seem to be related to any other available variable.

Considering further assumptions concerning the distribution of the e_{ij} , it would be convenient, natural, and relatively simple if we could assume that the e_{ij} are independently normally (Gaussian) distributed with 0 mean (and variance from eq. (2)). In support of these assumptions, it was shown in Section III that $\sqrt{p_i} \cdot$ the estimated e_{ij} (i.e., $(y_{ij} - \bar{y}_i) \cdot \sqrt{p_i}$) were normally distributed after transformation. As for the independence assumption, these values were found in Section III to have a positive, although not extremely large, correlation between areas. However, the independence assumption between areas is important mainly for the confidence interval comparison of two different areas, as in eq. (5) below, and a positive correlation implies that that interval would tend to be conservative, i.e. longer than necessary.

Thus, for purposes of the analysis we assume that the e_{ij} are independent normal $(0, \sigma^2/p_i)$. Thus the estimate $\hat{\mu}_i$ in eq. (1) is \bar{y}_i ; i.e., the "true transformed cost penalty" is simply estimated by the average of

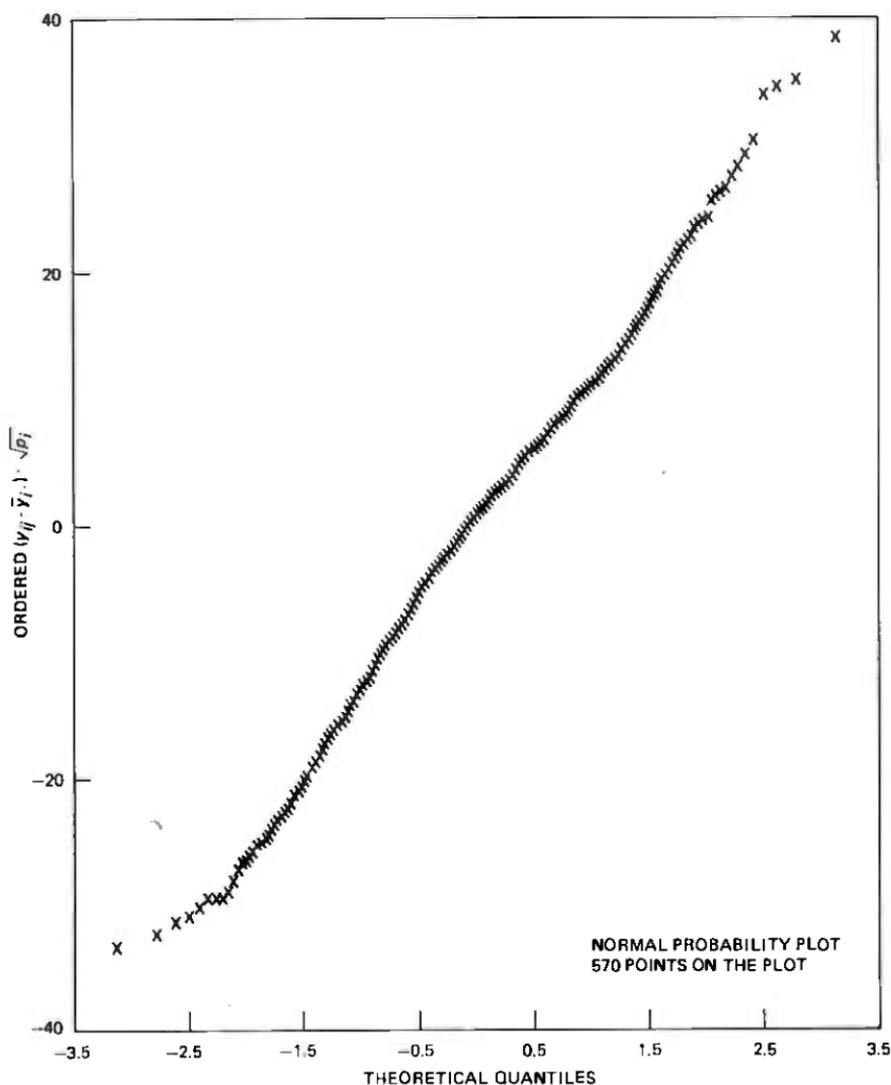


Fig. 15—Values of $\ln(\text{CPPAP} + 1)$ before conversion.

all observations for that area. Furthermore,

$$\text{var}(\bar{y}_i) = \frac{\sigma^2}{p_i \cdot m_i} \quad (3)$$

where m_i is the number of months of before conversion values available for area i . Confidence intervals for μ_i (or $\mu_i - \mu_k$) can be calculated using eq. (3) and standard normal theory. A $100(1 - \alpha)$ percent confidence interval for μ_i is

$$\bar{y}_i \pm z \cdot \frac{\hat{\sigma}}{\sqrt{p_i \cdot m_i}} \quad (4)$$

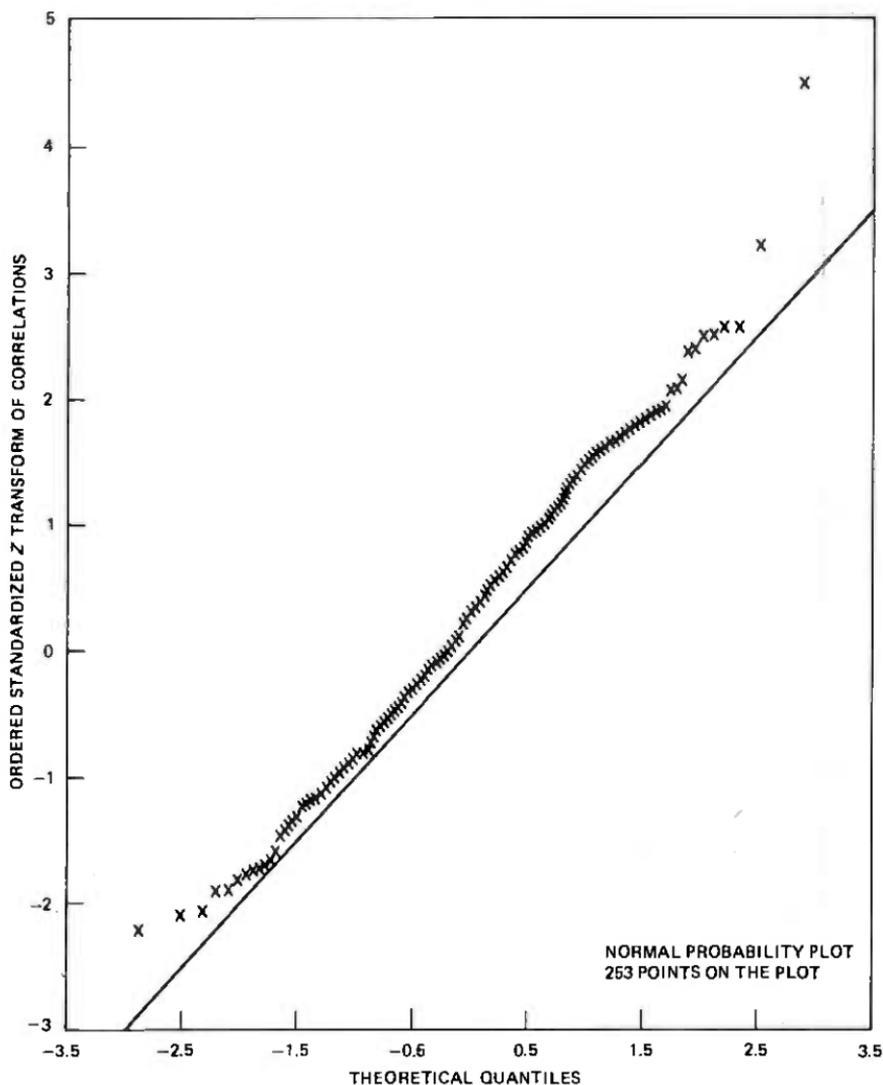


Fig. 16—Values of $\ln(\text{CPPAP} + 1)$ before conversion.

where z is the upper $1 - \alpha/2$ quantile of the standard normal distribution and $\hat{\sigma}$ is an estimate of σ described below. (Alternatively a t distribution could be used, but the degrees of freedom used in estimating $\hat{\sigma}$ should be large enough so that the difference in quantiles would be small.) Similarly, a confidence interval for the difference in “true” CPPAPs for two areas, $\mu_i - \mu_k$, is

$$(\bar{y}_i - \bar{y}_k) \pm z \cdot \hat{\sigma} \cdot \left(\frac{1}{p_i \cdot m_i} + \frac{1}{p_k \cdot m_k} \right)^{1/2} \quad (5)$$

The estimate of σ^2 , $\hat{\sigma}^2$, is obtained from the regression

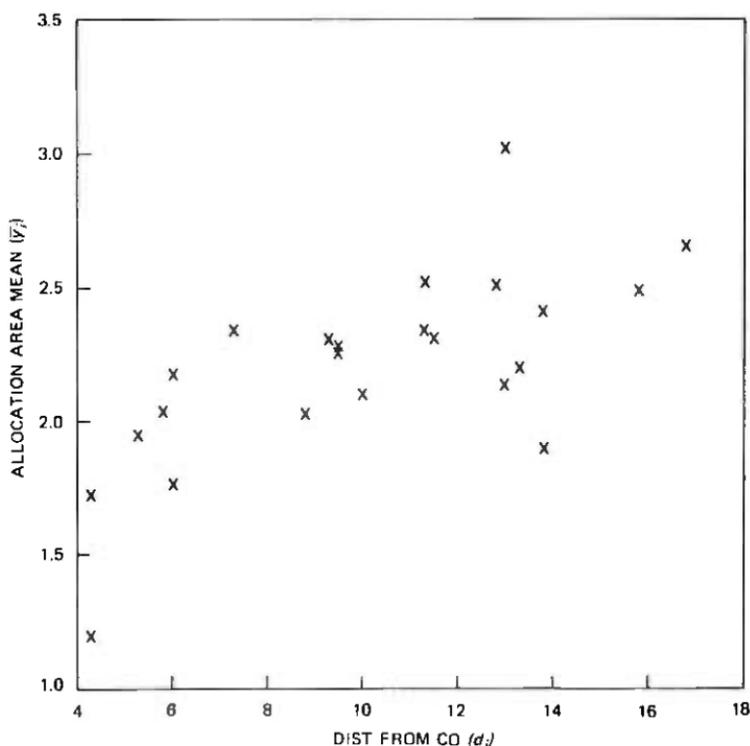


Fig. 17.—Values of $\ln(\text{CPPAP} + 1)$ before conversion.

$$(sy)_i = \frac{\hat{\sigma}}{\sqrt{P_i}} + \epsilon_i \quad (6)$$

where $(sy)_i$ is the observed standard deviation of the m_i values in area i , and ϵ_i is an error term reflecting the departure of the observed $(sy)_i$ from this model. Eq. (6) is obtained from eq. (2) and its use is supported by Fig. 9 and other analysis in Section III. The variance of ϵ_i , given in Section III, depends on i , so an iterated weighted regression is performed to obtain $\hat{\sigma}$. Our value is 12.40. Thus the variance is effectively estimated by pooling results across all areas, while allowing for the fact that different sized areas have different variance.

Up to this point all the work in this section has been on variables measured on the transformed scale, i.e., $\ln(\text{CPPAP} + 1)$. Recall this transformation was selected to reduce the dependence of the variability on the level of CPPAP and to improve normality. Therefore, confidence intervals are for parameters μ_i , μ_h which are also transformed. However, we are interested in having tables (for example) based on the original data (untransformed) and representing untransformed parameters. This is simply done by forming the confidence intervals on the transformed scale and then performing the inverse transformation $x = e^y - 1$.

Shown in Table III are the 95 percent confidence intervals for various

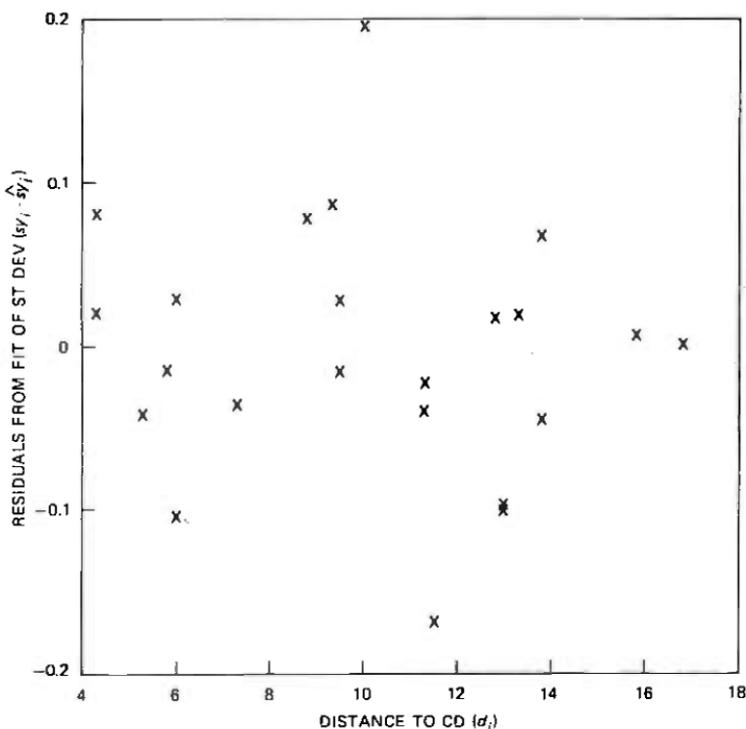


Fig. 18—Values of $\ln(\text{CPPAP} + 1)$ before conversion.

observed values of the CPPAP calculated using eq. (4) and $\hat{\sigma}$ estimated from the data. The time (in months) is the number of months used in forming the average value while the size is in pairs assigned. For example, suppose one has an area of 750 assigned pairs and has collected data for 12 months. If the computed average CPPAP is \$10, the confidence interval is from \$7.47 to \$13.29. If the computed CPPAP is \$30, the interval is \$22.87 to \$39.26. The interpretation is that 95 percent of the time, an observed CPPAP will be such that the associated interval covers the "true" CPPAP. Note that these intervals are not symmetric. On the transformed scale the assumptions yield a symmetric interval. However, when transforming back to the original scale, the nonlinearity of the exponentiation results in asymmetric intervals.

From the discussion of the variability of the average computed CPPAP it is clear that as the size of the area increases the variability decreases. Similarly, if the number of months used in computing the average CPPAP increases the variability of the estimate decreases. (In fact, based on eq. (4), and evident from Table III, the effects are symmetric.) To aid in assessing the magnitude of these effects Figs. 19 and 20 are provided. Figure 19 shows the upper and lower confidence limits for an observed CPPAP of \$20 formed by averaging over 12 months, for various values

Table III — 95% confidence intervals for:

Time (months)	Cost penalty per assigned pair of 10 dollars Size (in assigned pairs)											
	250	500	750	1000	1250	1500						
3	3.45	26.20	4.80	19.87	5.52	17.55	5.99	16.30	6.34	15.49	6.60	14.92
6	4.80	19.87	5.99	16.30	6.60	14.92	6.99	14.15	7.26	13.65	7.47	13.29
9	5.52	17.55	6.60	14.92	7.13	13.88	7.47	13.29	7.71	12.90	7.89	12.62
12	5.99	16.30	6.99	14.15	7.47	13.29	7.77	12.79	7.98	12.47	8.14	12.23
15	6.34	15.49	7.26	13.65	7.71	12.90	7.98	12.47	8.18	12.18	8.32	11.98
18	6.60	14.92	7.47	13.29	7.89	12.62	8.14	12.23	8.32	11.98	8.46	11.79
Cost penalty per assigned pair of 20 dollars												
3	7.49	50.94	10.07	38.84	11.45	34.42	12.35	32.02	13.01	30.48	13.51	29.39
6	10.07	38.84	12.35	32.02	13.51	29.39	14.25	27.92	14.77	26.96	15.17	26.27
9	11.45	34.42	13.51	29.39	14.53	27.40	15.17	26.27	15.62	25.53	16.15	25.00
12	12.35	32.02	14.25	27.92	15.17	26.27	15.75	25.33	16.15	24.71	16.46	24.26
15	13.01	30.48	14.77	26.96	15.62	25.53	16.15	24.71	16.52	24.17	16.80	23.78
18	13.51	29.39	15.17	26.27	15.96	25.00	16.46	24.26	16.80	23.78	17.06	23.42
Cost penalty per assigned pair of 30 dollars												
3	11.53	75.67	15.34	57.81	17.38	51.29	18.71	47.75	19.68	45.48	20.42	43.86
6	15.34	57.81	18.71	47.75	20.42	43.86	21.51	41.70	22.28	40.28	22.87	39.26
9	17.38	51.29	20.42	43.86	21.92	40.92	22.87	39.26	23.54	38.16	24.04	37.38
12	18.71	47.75	21.51	41.70	22.87	39.26	23.72	37.88	24.32	36.96	24.77	36.29
15	19.68	45.48	22.28	40.28	23.54	38.16	24.32	36.96	24.87	36.15	25.28	35.57
18	20.42	43.86	22.87	39.26	24.04	37.38	24.77	36.29	25.28	35.57	25.66	35.05

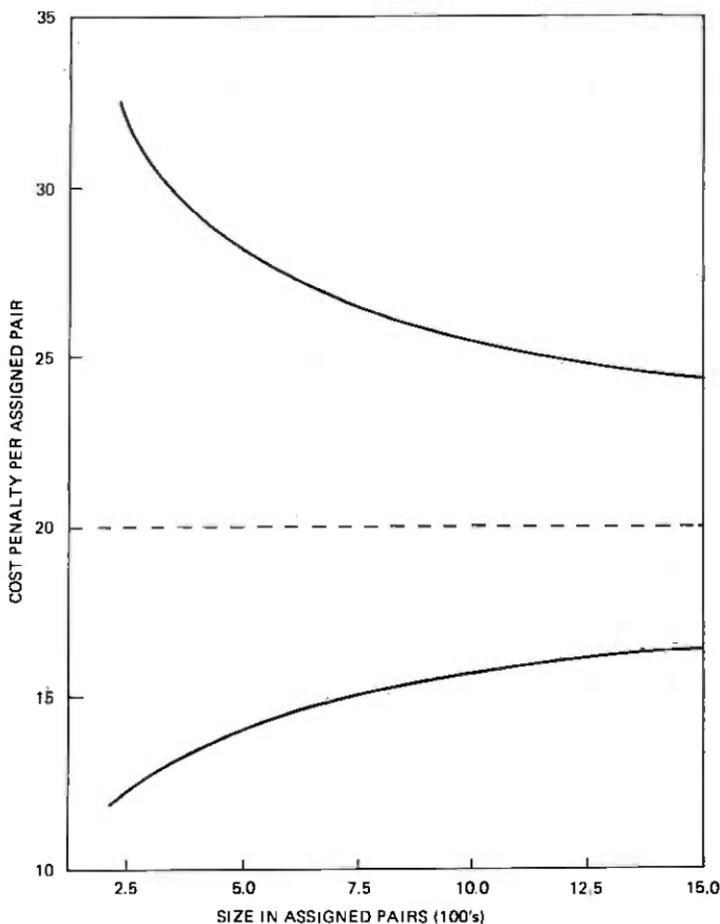


Fig. 19—Upper and lower confidence interval.

of the size. Both the asymmetry and the decrease in the size of the confidence interval are evident. Note that for the smaller areas the effect of the asymmetry is greater. Figure 20 is the same type of plot for an observed value of \$30 of CPPAP for an area with 250 assigned pairs for differing numbers of months. Note that for this very small area, the confidence limits are quite wide and the effect of the asymmetry is much greater than that seen in Fig. 19.

Table III can be used to help decide an appropriate size for allocation areas and an appropriate length of time for data collection. For a given size and time, the confidence intervals for various observed values of CPPAP can be read from Table III. For example, if allocation areas are created of size 500 assigned pairs or larger, and if data are collected for 12 months or longer, then an observed CPPAP of \$20 would give a confidence interval of \$14.25 to \$27.92—or a shorter interval if the area is

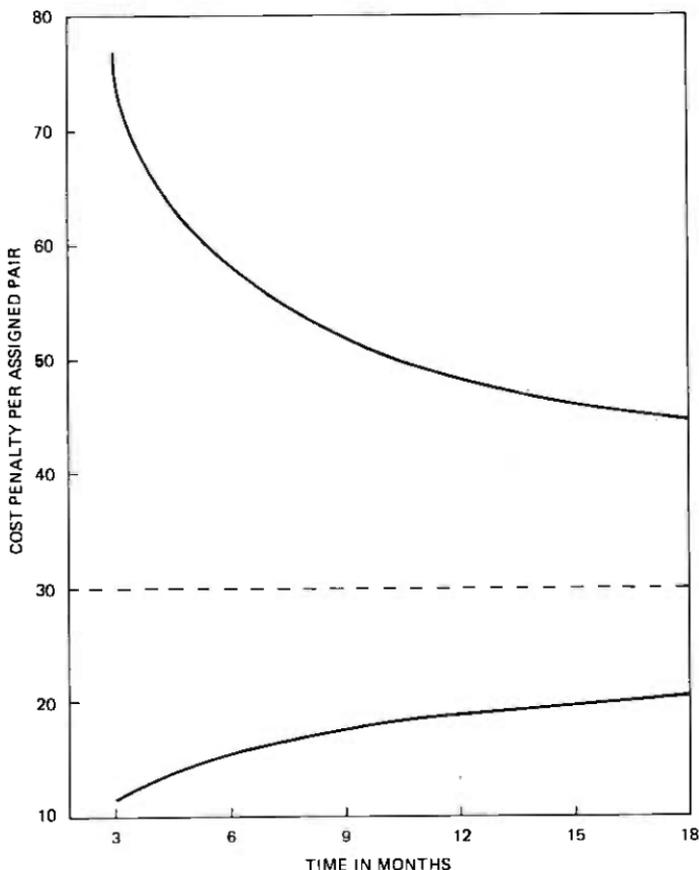


Fig. 20—Upper and lower confidence interval.

larger or the data collection period longer. If the uncertainty in the “true” CPPAP represented by this interval is acceptable, then allocation areas could be sized to a minimum of 500 pairs with data collection for a minimum of 12 months. The uncertainty resulting from alternative values of size and time can be checked in this way using Table III. When forming allocation areas in a district and determining the length of time for data collection, the minimum size and time should be chosen so as to produce results precise enough for the decision making needs of the district.

4.2. Extending results to individual areas

The basic results presented in Table III are given for only three values of the measured CPPAP, six different collection periods, and six area sizes. The first and most straightforward extension of this analysis to different areas and collection periods involves extending the tables using eq. (4) or by linear interpolation of the given table values. As can be seen from

Figs. 19 and 20, any linear interpolation is more valid for the range of the table associated with longer collection times and larger collection areas. This is simply because the effect of the transformation is more linear for this range of values.

In the event that users of CPPAP data feel that their areas are significantly different from the Prototype District, which is the basis of Table III, there are several ways in which this analysis can be modified. First, the constant associated with eq. (6) can be re-estimated using the techniques described in Section 4.1. While the estimation of the weights in the regression is somewhat more complicated than ordinary least squares, most commercially available statistical computation packages allow for this type of estimation. Having computed the constant which relates variability to size of area, it is a simple matter to generate tables analogous to Table III.

However, the logarithmic transformation of CPPAP used here for analysis might not always satisfy the desired assumptions. In this case a more exploratory analysis should be undertaken. Unfortunately, such an analysis will require additional statistical computation and display. The sequence of steps discussed in Section III can serve as a guide for the analysis, and for checking the appropriateness of various transformations. Finally, it is possible that no appropriate transformation will be found. Then the method of analysis employed in this section will not be adequate.

V. ANALYSIS OF AFTER CONVERSION DATA

5.1. Description of analysis

A major concern in the conversion of serving areas to SAC is whether or not the projected savings are being realized. To help answer this question the cost penalty data in the periods after conversion are examined. A regression equation is developed which models the after conversion costs in terms of before and during conversion variables as well as the time since conversion. The most important result shows that the cost penalty continues to decline for the period immediately following conversion. The implication of these findings on conversion analysis is that to adequately assess the effect of conversion, cost data must be collected for a period of nine to twelve months after conversion.

One might assume, *a priori*, that there will be differences in the converted areas but that such differences would not be related to the before or during conversion periods. These areas were all rehabilitated using the same FAP guidelines, so they should start off on the same footing. Differences might be related to installer productivity or activity, or geographic considerations of the areas. However, data on such variables are outside the scope of the Prototype District Data Base and are not currently available. It is of interest to know to what extent after con-

version behavior might be explained, and the analyses of this section are directed at using variables available in the data base to this end.

Since the logarithmic transformation of the before conversion data satisfied straightforward assumptions needed for analysis (see Section III), one might expect this transformation also to be reasonable for the after conversion data unless there are some "structural" changes in the after conversion period. Our analyses do not indicate any such change, so the quantity analyzed here is $y = \ln(\text{CPPAP} + 1)$. Ten of the 23 allocation areas were converted, and each of these areas has from 1 to 20 months of after conversion data. The total number of values (areas · months) is 100.

We search for a linear description of the 100 y 's of the form

$$y_{ij} = a_0 + a_1x_{1ij} + a_2x_{2ij} + \dots + a_\ell x_{\ell ij} + e_{ij} \quad (7)$$

where i denotes area; j denotes month; x_1 is some descriptive or explanatory variable with value x_{1ij} for the i th area and j th month; similarly for x_2, \dots, x_ℓ ; and e_{ij} is the residual which is unexplained, and which should not be related to any available variable. In accord with the analysis in Sections III and IV, we assume that $\text{var}(e_{ij}) = \sigma^2/p_i$. Thus, all regressions discussed here are weighted regressions with weights inversely proportional to the square roots of these variances. The problem is to find a good but parsimonious set of variables x_1, x_2, \dots, x_ℓ .

5.2. Fitted regression equation

Three classes of potential descriptive variables x are considered. First are variables which give some characteristics of areas, where these characteristics can be observed before the after conversion period. Such a variable has a fixed value for each area (i) across months (j). Examples include the distance of an area from the central office, the size of an area, and the average cost penalty for an area before conversion. The second class of variables concerns seasonal cycles across months. Such a variable has a value depending on the months (j) but is constant for each area (i). The third class consists of the single variable giving the number of months since conversion of that area; thus $x_{ij} = k$, where month j is k months past the conversion date of area i .

Consider the first class of variables. The most powerful such variables would be a set of 10, with each variable having some non-zero value in one area and the value zero in all other areas. This gives a one-way analysis of variance model, with the area corresponding to the treatment or groups. Doing this, one obtains an $R^2 = 0.28$. This means that 28 percent of the variation in the y 's can be explained by differences between the areas.

The fit is improved substantially ($R^2 = 0.37$) by adding to this model the variable which measures the number of months since conversion.

However, the further addition of variables allowing different values for different months—the seasonal or time effect variables—improves the fit only negligibly. Thus, use of all the variables available here would result in a model describing about 40 percent of the variability in the after conversion values. Although this is not a large percentage on an absolute basis, it is also not negligible, especially considering that this is variability over months and areas after conversion to SAC.

Now we would like to go further and discover specific characteristics of the ten areas and specific variables that would give a simpler but still relatively good descriptive model. The following eight variables measuring characteristics of the areas were considered: the size of an area, as measured by the number of assigned pairs; distance to central office along feeder cable, measured in kilofeet; area mean before conversion; area standard deviation before conversion; area mean during conversion; area standard deviation during conversion; number of months of before data available; and number of months during conversion. The above one-way analysis of variance implies that the maximum descriptive power of any subset, or transformations, of these variables is 28 percent.

In order to find a small but good set of variables and transformations, extensive regression analyses were done, including stepwise calculations and C_p analysis.¹⁰ As is often the case in such problems, no small set of variables clearly stands out as the unique “best” regression equation. Correlations between explanatory variables can permit several different sets of variables to fit the data approximately equally well. We will now discuss one simple model that does fit these data reasonably well.

Variables included in the model are the following: number of months since conversion; during conversion mean; during conversion standard deviation; and number of months before conversion. The fitted regression equation is summarized in Table IV, which gives the regression coefficients, the estimated standard errors, and the t -values for testing each coefficient equal to zero. The R^2 is 0.35 with residual standard error of 0.44, compared to a standard deviation of 0.54 for the dependent variable. Thus, use of only four variables gives a fit nearly as tight as can be obtained when using all possible explanatory variables available here.

Table IV — Fitted equation for after conversion data *

y_{ij} =	1.60	$-0.044x_{1ij}$	$+0.44x_{2i}$	$-1.13x_{3i}$	$-0.038x_{4i}$
Standard error	0.36	0.009	0.14	0.37	0.010
t -statistic	4.41	-4.89	3.19	-3.07	-3.80

* $y_{ij} = \ln(\text{CPPAP} + 1)$

x_{1ij} = number of months since conversion

x_{2i} = during conversion mean

x_{3i} = during conversion standard deviation

x_{4i} = number of months before conversion

(x_2 , x_3 , and x_4 are all the same over all months; hence, the time subscript j is omitted.)

No monthly time variable or cyclic time effect is included, since the analysis showed that they had no additional explanatory power.

Examination of various residual plots is important in determining the adequacy of this fit. Figure 21 gives a partial residual plot¹¹ for the number of months since cut-over (x_1) variable. The variable plotted on the vertical axis is the residual from the regression fit plus the contribution from this variable. Thus, one expects the points to scatter about a straight line with slope equal to the regression coefficient for x_1 , here -0.044 . This figure does not suggest any serious inadequacy in the fit as far as this variable is concerned. Partial residual plots and residual plots for the other variables, normal q-q plots, and various box plots of the data and residuals were also examined. They did not show anything particularly noteworthy.

Consider the interpretation of the variables in the fitted equation. For variable x_1 , the number of months since cut-over, it is not surprising that the level declines over time after the conversion is completed, since unknown cable troubles and defective pairs will be discovered and corrected. Figure 21, introduced above, shows graphically that there is a steadily decreasing trend as the number of months since cut-over increases. There is not an instantaneous decline to a low, constant level.

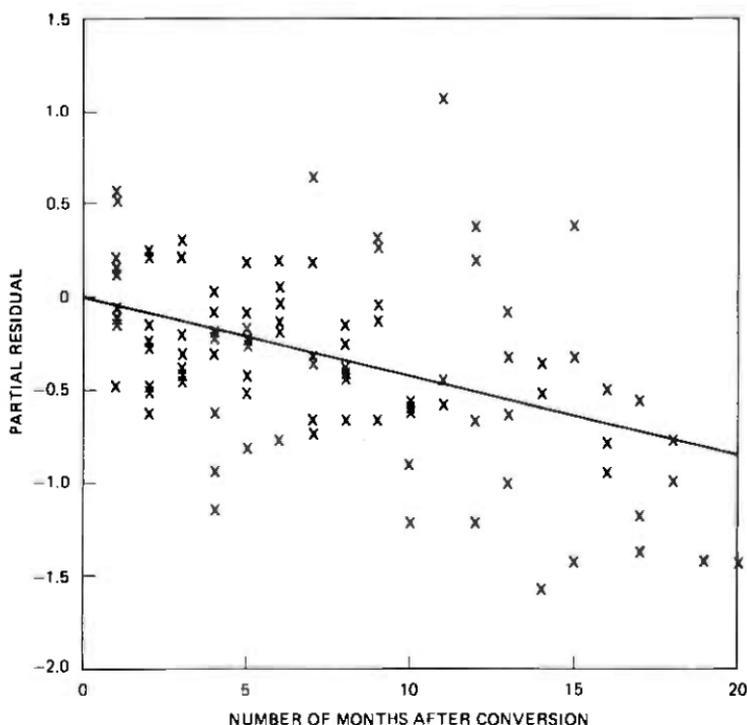


Fig. 21—Values of $\ln(\text{CPPAP} + 1)$ after conversion.

Moreover, this variable (x_1) appears with approximately the same negative coefficient in all "reasonably fitting" sets of variables, while other individual variables are not so strongly needed in order to obtain an adequate fit. For variable x_2 , the during conversion mean, it seems reasonable that a higher during conversion period (a proxy for the complexity of the conversion activity) will be associated with a larger after conversion level. However, the interpretations for the during conversion standard deviation (x_3) and the number of months before conversion (x_4) are not as straightforward. For example, one could speculate that areas with a high level of during variability have spots of local congestion causing occasional high costs (i.e., RE's LST's, WOL's, etc.). A large standard deviation implies that there are also months in which costs are low. It is just this type of area that can show large savings (and lower values of CPPAP) after conversion via FAP. The number of months before conversion could be a proxy for the ranking of the converted areas. Presumably, the worst areas would be converted earlier. Hence, the better areas are converted later and the post conversion costs of the better areas are lower (other things being equal).

VI. ACKNOWLEDGMENT

We are indebted to many members of Department 4511 for their time and effort in explaining concepts and issues pertaining to the loop plant. Special thanks are due Nancy Basford who never failed to respond helpfully to our many queries.

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Loop Plant Modeling:

The Facility Analysis Plan: New Methodology for Improving Loop Plant Operations

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The Facility Analysis Plan is a new methods system developed to reduce facility provisioning and maintenance costs in the loop plant. It addresses comprehensively the set of operational costs incurred by all work forces. The Plan consists of three components: (i) an information processing system, (ii) an engineering applications system, and (iii) a control system. The information processing system comprises a set of reporting procedures for data which portray the operation of the loop network within geographical regions called allocation areas, defined so that each represents a virtually independent segment of the existing network. The engineering applications system provides methods for using data from the information processing system to identify those allocation areas in which high operating costs are incurred, to determine the cause of the high costs, and to select and evaluate economical means of reducing these costs. The control system uses data from the other two component systems to ensure the validity of the economic evaluations and to verify that predicted cost reductions are actually achieved.

I. INTRODUCTION

The Facility Analysis Plan is a new methods system designed to improve loop plant operations. The objectives of loop plant operations are to provide telephone service to the customer on demand at the lowest possible cost and to maintain that service without interruption until the customer requests its termination. Meeting this objective requires the combined efforts of many work forces, each of which performs a distinct set of functions.

Prior to the development of the Facility Analysis Plan, each work force had separate plans to monitor and improve its effectiveness in providing service. While these plans involved collecting and processing large quantities of data concerning loop network operations, they were in many cases only partially effective because of three basic weaknesses. First, the data were not organized for easy use in identifying and correcting specific problems. Second, the plans usually focused on the operation of a single work force even though there are substantial interdependencies among the various forces. Third, the plans generally lacked the detailed and comprehensive procedures needed to make them an integral part of the normal work process.

The Facility Analysis Plan was developed in response to these identified weaknesses. It is designed to reduce facility provisioning and maintenance costs by addressing comprehensively the set of operational problems encountered by all work forces associated with the loop plant. The Plan has three components: (i) an information processing system, (ii) an engineering applications system, and (iii) a control system. These systems use data gathered by monitoring specific loop plant work operations and their associated costs. Section II describes the cost measures used in the Plan. Subsequent sections describe each component system in more detail. We have tried in our descriptions to avoid a surfeit of telephone company terminology which would obscure the essential concepts and underlying models.

II. COST MEASURES

Our objective is to minimize the aggregate cost of providing and maintaining loop facilities.

The *aggregate cost* is the total cost of providing the facilities needed to satisfy demands for service. This cost has four components—basic operating cost, basic scheduled cost, marginal operating cost, and marginal scheduled cost.

Basic operating cost is that incurred to satisfy a specific customer's immediate request for service. In the case where service to an address is being established initially, this component includes all, and only, those costs associated with the work operations required, given that a spare facility is available for use at the terminal nearest the customer's address. In the case where service was discontinued and is being reestablished, this component includes all, and only, those costs associated with the work operations required, given that the idle facility previously serving the address has remained connected from the central office to the customer's premises.

Basic scheduled cost is that incurred to make additions to or rearrange those parts of the cable network where current capacity can no longer satisfy forecasted service requirements. Both the basic operating and

basic scheduled costs are assumed to be fixed components of the aggregate cost throughout this paper.

Marginal operating cost (MOC) is that incurred in addition to the basic operating cost to satisfy a service request when the ideal facility conditions governing the basic operating cost are not met. This cost results from the additional work operations required to make a spare pair available to the customer's terminal and/or to reconnect the facility from the central office to the customer's premises. MOC is also incurred if the facility fails after service has been established, since work operations are then required to restore the service.

Marginal scheduled cost (MSC) is that incurred to make additions, rearrangements or other modifications to the network other than those associated with basic scheduled cost. Examples are the cost of replacing deteriorating facilities to provide more reliable service to existing customers, the cost of advancing cable relief, the cost of converting existing plant to the Serving Area Concept (see N. G. Long,¹ this issue), or the cost of initiating new administrative procedures such as the connect through plan or the conformance testing program.

MOC is strongly influenced by MSC. Often, by incurring an MSC it will be possible to decrease the MOC.

In order to minimize the aggregate cost, then, it is necessary to identify those parts of the network where incurring a particular choice of MSC will reduce the MOC by an amount which more than compensates for the MSC. This is accomplished by use of the engineering applications system, described in Section IV, which operates on measures of the MOC provided by the information processing system, detailed in Section III. In this section we describe those work operations which generate the MOC and define a cost factor for each work operation.

2.1 MOC work operations

There are two types of work operations which generate the MOC: service provisioning and service restoration. We give only a few examples, since a full listing of each type is not essential to the remainder of the paper.

2.1.1 MOC work operations associated with service provisioning

Consider the following example. Service is requested at a given address. The terminal which is designated to serve the address has no spare pairs. However, at a second terminal there is an idle connect-through pair connected to a vacant residence (see H. T. Freedman,² this issue). Access to that pair is also possible at the terminal where service is desired. Service can be provided by the additional work operation of breaking the connection at the second terminal, thereby creating a spare pair at the desired terminal. An MOC is incurred because of the time spent by

the assignment force in determining how to provide the spare pair at the desired terminal and by the installation force in breaking the connection at the second terminal according to instructions provided by assignment. This operation is called a break connect-through (BCT) pair. Other examples of this type of work operation are the line-and-station transfer (LST) and the wired out-of-limits (WOL) described by Koontz,³ this issue.

2.1.2 MOC work operations associated with service restoration

Assume that service to a customer is interrupted because of a faulty connection at a terminal. Service can be restored by the work operation of repairing the faulty connection. An MOC is incurred because of the time spent by the groups within the repair force to process the customer's report of service interruption, to determine the type and approximate location of the fault, and to physically make the repair. This example is called a found cable trouble. An alternative method of restoring service is to connect the customer to a different pair, provided that there is a spare pair at the customer's serving terminal. This second example is one of several types of operations that are called assignment changes.

2.2 Cost factors for MOC work operations

The cost factor for each work operation that results in an MOC is defined as the average cost to all force groups of an occurrence of the work operation. The cost factor K_i for work operation i is defined as

$$K_i = \sum_{j=1}^J t_{ij} l_j$$

where J = the number of different work forces involved in loop operations

t_{ij} = the average time spent by the j th work force on the i th work operation

l_j = cost of labor per unit of time for the j th work force

These cost factors will be used in subsequent sections to compute MOCs associated with portions of the loop network.

III. INFORMATION PROCESSING SYSTEM

Having discussed the cost measures and rationale to be used in minimizing the aggregate cost of providing and maintaining loop facilities, we next consider the information processing system. This system processes data on the occurrences of MOC work operations and produces outputs used by the engineering applications system and the control system. Outputs include an ordering of allocation areas by their normalized yearly MOCs and a history of monthly levels of work operations for each allocation area. Together these outputs are used to identify those

allocation areas where the aggregate cost can be reduced. The latter output is also used to verify that the cost is, in fact, reduced. The information processing system also allows organization of the data so that patterns of work operations become apparent. These patterns are used to determine the type and extent of the MSC to be applied within the allocation area.

3.1 Motivation for the allocation area

Individual work operations are essentially random occurrences. It was important, therefore, in structuring the information processing system to consider the size of the entity and time interval for which the MOC should be reported. As discussed in Ref. 4, the variability of any measure decreases with increases in the size of the area and the interval of time used. The statistical need for a large area and a long time interval must be balanced against the desire to quickly identify small portions of the network which exhibit a high MOC.

A suitable compromise is to measure the MOC for a period of one year in elemental geographic units called allocation areas (see N. G. Long,¹ this issue). Using twelve months' data is intuitively appealing, since seasonal variations will be effectively removed, but the interval is not so long as to mask actual changes occurring in the areas. Allocation areas must be large enough to give a statistically significant measure of cost for one year but not so large that actual concentrations of high cost are masked. Areas of 500 to 2000 assigned pairs are considered suitable. Allocation areas are fed by groups of 50 feeder pairs (called complements). The term "allocation" is used because the area is also the basic geographic unit to which feeder pairs are allocated (see B. L. Marsh,⁵ this issue). Allocation areas are also defined so as to minimize the number of feeder pairs terminating in more than one area. This ensures that data collected by complement (see Section 3.2) are associated with the proper allocation area and that an MSC applied in one allocation area will not affect any others.

3.2 Data organization

The data to be collected are the number of occurrences of MOC work operations. For each complement, the number of monthly occurrences of each type of service provisioning work operation and those service restoration work operations known as assignment changes is recorded. These "initial data records" are retained for later use in the engineering applications system. They are also summarized by allocation area each month, by applying a transformation which maps each complement to a particular allocation area.

For each of the remaining service restoration work operations (those known as found cable troubles), the allocation area and the address of

the trouble within the allocation area are recorded. These "initial data records" are retained for later use in the engineering applications system. They are also summarized by allocation area each month.

3.3 Allocation area data reporting

The data on monthly occurrences of each type of MOC work operation are presented in a historical report for each allocation area. The report for a given allocation area allows comparisons of monthly levels of work operations over as long as a two-year period in order to detect trends.

Semiannually, a report is generated listing the allocation areas in order of decreasing normalized yearly MOC. The normalized yearly MOC for an allocation area is obtained by dividing the yearly MOC (the total over the past year) by the size of the allocation area as measured by assigned pairs. The normalized yearly MOC is referred to in Ref. 4 as the cost penalty per assigned pair (CPPAP). The term "cost penalty" is used there in the sense that the MOC is a penalty over the basic operating cost. This semiannual report also shows for each allocation area the yearly MOC and the effective cable fill (number of assigned and defective pairs divided by the number of available pairs in complements feeding the allocation area).

The use of the historical report and the ordering report is described in Section 4.1. The further use of the "initial data records" is described in Section 4.2.

IV. ENGINEERING APPLICATIONS SYSTEM

The data processed and output by the information processing system are used primarily by members of the engineering work force to make decisions to incur an MSC so as to reduce the aggregate cost of the loop network. There are three basic processes involved. The first is to identify those allocation areas with cost reduction potential—usually those with a high MOC. The second is to determine the physical conditions causing the high MOC and the type of MSC that will significantly reduce it. The third is to predict the magnitude of the expected cost reduction and decide on a course of action.

The models governing these processes describe relationships that hold in general, but that are not always sufficient to describe specific situations. Their successful use requires engineers who are familiar with the loop environment. For this reason we shall generally refrain from citing specific applications or procedural techniques, but shall instead discuss, in abstract terms, basic relationships defined by the models.

4.1 Data screening process

The initial application of the data obtained from the information processing system is to identify allocation areas in which there is good potential for reducing the aggregate cost. As was described in Section II, the means to achieve such a cost reduction is by incurring an MSC that will be smaller than the resulting decrease in the MOC in the allocation area.

Recall that the information processing system only provides measurements of the MOC in each allocation area. The magnitude of the MSC required to reduce the MOC is not known a priori. This is a function of a large set of variables and network characteristics and can only be determined from a thorough study by an engineer familiar with the allocation area. The time involved precludes studying every allocation area in this manner. For this reason, a process has been developed to screen the data so as to identify those allocation areas with the greatest expected cost reduction potential.

This screening process involves comparing the allocation areas on the basis of three measures. Generally, the higher the level of a given measure, and the larger the number of measures at a high level, the greater the expected cost reduction potential in an allocation area. The three measures are:

(i) The normalized yearly MOC. This is the most substantive of the three measures. As described in Section 3.3, the information processing system provides an ordering of the allocation areas according to this measure. Further refinement of this ordering can be achieved by examining, on the historical report for each allocation area, the month-to-month trends of the occurrences of MOC work operations. Thus, two areas which are in statistically close proximity (see D. M. Dunn and J. M. Landwehr,⁴ this issue) in the ordering can be differentiated by comparing their trends. An area with an increasing trend has greater expected cost reduction potential than one with a flat trend. Furthermore, an area with a distinctly decreasing trend has little or no expected potential because such a trend pattern usually indicates that MSCs have already been applied to tap a previous cost reduction potential.

(ii) The effective cable fill. High values of this measure indicate that the number of cable pairs available to meet future service requests is limited. This condition usually leads directly to an increase in marginal operating costs (see W. L. G. Koontz,³ this issue). An ordering of the allocation areas based on this measure can be refined by considering the growth rate of the areas. If two areas have similar fill levels, the one with the higher growth rate has the greater expected cost reduction potential.

(iii) The yearly MOC. Allocation areas with very high yearly MOC (relative to other allocation areas in the district), regardless of the nor-

malized value, may also have a high cost reduction potential. This is because the conditions causing the high costs may be concentrated in such a way as to enable substantial reductions in these costs with a modest MSC investment.

The validity of these measures as independent indicators of the relative expected cost reduction potential in allocation areas has been demonstrated empirically, and they have been applied successfully in the data screening process. However, we have not yet derived a specific quantitative relationship among these three measures, nor have we identified such desirable characteristics as "threshold" levels, i.e., absolute values of the measures above or below which allocation areas could be classified as possessing or not possessing cost reduction potential. More information from field locations using the Facility Analysis Plan is needed before work in this area can proceed.

4.2 Data Interpretation process

An allocation area that has been identified as possessing a high expected cost reduction potential is examined in greater detail to determine the type of MSC required to reduce the MOC.

4.2.1 Establishing cause from effect

We have identified twelve specific network conditions that may cause the work operations which are the source of the MOC. Examples of these are insufficient spare pairs, imbalances in the network, rapid customer movement in and out of the allocation area, old and deteriorating cables and terminals, inadequately maintained records and unexpected growth of customer demand. Each of these network conditions creates the need for a certain type (or types) of work operation to provide or restore service. For this reason the network conditions in a given allocation area can usually be identified by noting which type(s) of work operation occurred during the previous year.

To facilitate identification, a matrix has been developed to illustrate the cause and effect relationship between the network conditions and work operations (see Fig. 1). The various work operations are grouped into 11 categories on the left. Each category comprises from one to seven different work operations. Those operations in a given category are considered indistinguishable in that any one or more of them may be caused by a particular category of network condition. The 12 network conditions are grouped into eight categories along the top of the matrix. Each of these categories comprises from one to four different network conditions. Any one of the network conditions in a given category may cause a particular category of work operation.

An "x" in the matrix indicates that the category of network condition

		CATEGORY OF NETWORK CONDITION							
		A	B	C	D	E	F	G	H
CATEGORY OF WORK OPERATION	1	X							
	2	X	X						
	3		X						
	4		X	X					
	5			X	X	X			
	6			X		X			
	7				X	X			
	8					X			
	9						X		
	10							X	
	11								X

Fig. 1—Matrix to identify cause and effect relationship between network conditions and work operations.

listed in that column can be expected to cause occurrences of the category of work operation listed in the associated row. Thus network condition B (high inward/outward subscriber movement) often causes work operations 2 (facility modifications such as BCT, LST, and WOL), 3 (reterminating service connections at established customer locations), and 4 (repair of faulty connections in terminals). By using the matrix in reverse, the observed set of work operations identifies the category of network condition likely to be present in the allocation area of interest. For example:

(i) If work operations in categories 5 and 7 are observed, a network condition in category D is probably present.

(ii) If, however, work operations in categories 5, 6, 7, and 8 are observed, a network condition in category E is most likely to be present and one in category D may be present as well.

(iii) Finally, if work operations in categories 5, 7, and 9 are observed, a network condition in both categories D and F are probably present.

While the large majority of work operations appear in the patterns shown in Fig. 1, exceptions do occur occasionally. In such cases the engineer examining the allocation area must draw on a personal knowledge of the conditions in the area to determine the cause of the observed operations.

Given the general category of network condition present in an allocation area, it is usually a simple matter to examine other outputs from the information processing system (e.g., effective cable fills, defective pair rates) to further delineate the specific network condition.

4.2.2 Determining impact level

A network condition may impact at one of three levels. It may affect (i) all parts of the allocation area, (ii) all parts of one distribution area (see N. G. Long,¹ this issue) within the allocation area, or (iii) only certain distribution cable areas within a distribution area. A distribution cable area is a small geographic region served by a single complement or unique set of complements. The cable comprising this (set of) complement(s) is usually referred to as a cable "leg." For a given network condition, the appropriate type of MSC is normally different for each impact level.

The impact level can be determined from information on the "initial data records" compiled by the information processing system (see Section 3.2). For certain work operations (those dealing with service provisioning as well as assignment changes), this record shows the number of occurrences within each complement. For the remaining operations (all types of found cable troubles), the record shows the street address of each occurrence. The data are organized so that information pertaining to complements or addresses within a given allocation area can easily be extracted.

To use these data, the complements serving an allocation area must be partitioned into groups and subgroups such that a group contains those complements which serve a particular distribution area and a subgroup, a particular distribution cable area. This allows work operations recorded by complement to be mapped geographically and therefore to be combined with those operations recorded by address.

The resulting data patterns can then be observed. If the work operations are distributed rather uniformly across the groups, the impact level is the allocation area. If they are concentrated within one group but distributed uniformly across the subgroups, the impact level is the distribution area. If they are concentrated within specific subgroups, the impact level is the distribution cable area.

Where more than one network condition is present, the process must be applied separately to the work operations caused by each condition. The result may be that the conditions are independent (at different impact levels or in different groups or subgroups within the allocation area) or dependent (same impact level and same group association). For those that are independent, separate MSCs should be applied. For those that are dependent, a type of MSC must be found that will reduce the combined set of MOCs.

4.2.3 Identifying network enhancements

The last step of the data interpretation process is to find the most effective means of reducing the aggregate operating costs in a given allocation area. An MSC is applied by making one or more planned enhancements to the network which are designed to correct the condition(s)

identified as causing the observed MOC. Seventeen specific types of network enhancements have been defined. These include clearing defective pairs, relieving the terminals or cables, adopting connect-through administrative procedures, instituting preventive maintenance techniques, and converting part or all of an allocation area to the serving area concept (see N. G. Long,¹ this issue, for description). Each of these enhancements is effective for a specific set of network conditions at a particular level. The objective is to identify the least costly enhancement that will correct the network conditions.

For this purpose a three-tiered matrix has been developed (see Fig. 2). Each tier corresponds to one of the three impact levels. Within each tier the rows correspond to the 12 network conditions and the columns correspond to the 17 network enhancements. The enhancements are arranged from left to right in order of increasing complexity and expected cost. An "x" in the matrix indicates that the enhancement listed in that column will correct the network condition listed in the associated row at the impact level of the corresponding tier. Thus enhancement E_5 will correct network conditions C_1 and C_6 at impact level L_3 , condition C_6 only at impact level L_2 , and is not applicable at impact level L_1 .

The matrix is used by locating the row and tier associated with the network condition and its impact level, and then moving across the columns from left to right to the first column that contains an "x." The enhancement identified with this column will generally result in the greatest reduction in the aggregate cost of the allocation area. For example, condition C_4 at impact level L_2 should be corrected by enhancement E_6 .

If a set of dependent network conditions is present, an enhancement must be found that will correct all the conditions. This can be done by locating the corresponding set of rows on the proper tier and moving along these rows to the first column that contains an "x" in each. For example, network conditions C_1 , C_2 , and C_6 at impact level L_3 should be corrected by enhancement E_8 . If no single enhancement can be found that will correct all the network conditions, the combination of enhancements requiring the smallest expected scheduled cost should be chosen. For example, for network conditions C_7 and C_8 at impact level L_1 , enhancements E_{14} and E_{15} should provide the most cost effective solution.

There are also cases where, even if there is a single enhancement that will correct all conditions, a combination of enhancements may be more economical. Where such a possibility exists, both choices should be analyzed according to the procedures in Section 4.3 to determine which is more economical.

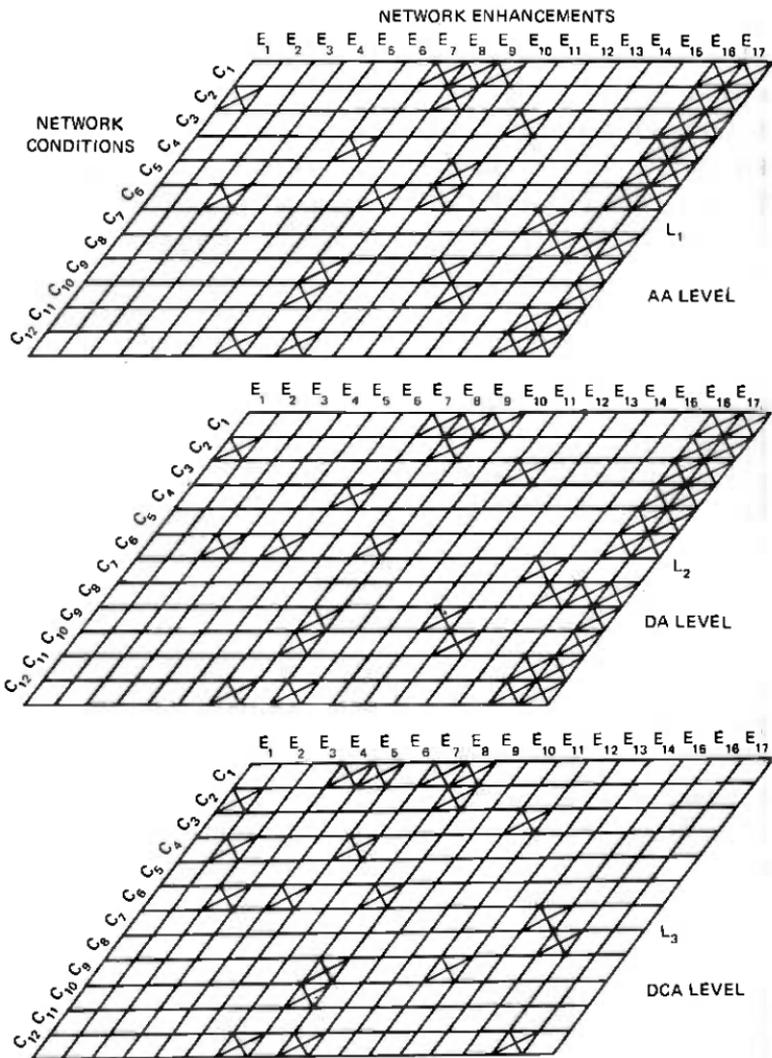


Fig. 2—Matrix to identify relationship between network enhancements and network conditions.

4.3 Economic analysis process

The network enhancement(s) derived from the data interpretation process are those with the greatest possibility of reducing the aggregate costs in an allocation area. Whether such a reduction can in fact be achieved, however, can only be determined by numerically comparing the MSC to the expected reduction in the MOC. If the former is smaller than the latter, the aggregate cost can be expected to decrease, and the enhancements should be applied. If not, the aggregate cost may already be at its minimum level, or another, less extensive enhancement may prove cost effective.

There are two economic analysis techniques used to make the cost comparison. One has been developed expressly for cases in which the network enhancement involves conversion to the serving area concept (SAC). This enhancement is often the most desirable of the available choices because it results in permanent and stable distribution plant which virtually eliminates the need for future enhancements (see N. G. Long,¹ this issue, and also J. O. Bergholm and P. P. Koliss⁶). It also requires the largest commitment of MSC. For these reasons a complete analysis is required before this enhancement is applied. The other analysis technique is used for the non-SAC enhancements and, at present, makes use of a more rudimentary model. The non-SAC technique is described first since the concepts employed are fundamental and also apply to the more complex model of the serving area concept.

The two techniques are conceptually alike in that both are designed to estimate accurately the incremental change, A , in the aggregate cost of an allocation area resulting from an MSC of size S that reduces the MOC by amount O . Both techniques make use of empirical models for estimating the decrease in work operations that result when network enhancements are made. Work on theoretical models of these effects is described by W. L. G. Koontz³ and H. T. Freedman² in this issue.

In the discussions to follow, all three costs will be expressed in present worth dollars, so that

$$A = S - O \quad (1)$$

In order to compute the value of A , it will be necessary to develop expressions for S and O in terms of parameters whose values are readily obtained. Acceptable parameters include those whose values are:

(i) Set at the corporate level for the purposes of economic studies, such as the cost of money and the costs of the various work operations.

(ii) Measured or forecast for individual allocation areas, such as the number of occurrences of particular work operations and the growth rate of assigned pairs.

(iii) Estimated for the particular allocation area under study, such as the costs of the proposed enhancement and the expected reduction in work operations resulting from the enhancement.

4.3.1 Analyzing non-SAC enhancements

In this section we describe a general model which is applicable to the analysis of any non-SAC enhancement. We also present two simplified versions of the model that can be applied to the most common of the enhancements.

To determine the value S of the MSC, we first define the term σ_* to be the levelized equivalent annual cost (LEAC) of an expenditure, $*$, which

contains a capital and associated expense element, c_* and x_* , and which is applied according to the repeated plant assumption (see J. Freidenfelds,⁷ Appendix, this issue). All expenditures used in the derivation of S are of this type. Thus

$$\sigma_* = \alpha_* c_* + \gamma_* x_* \quad (2)$$

where α_* is the annual charge factor that applies to the type of plant placed by the capital expenditure, and γ_* is defined so that the present worth of a constant annuity of $\gamma_* x_*$ dollars over the life of the plant placed equals the present worth of x_* . The present worth of annual charges (PWAC) of expenditure $*$ is then the present worth of the series of σ_* dollars applied from the time the expenditure is incurred, τ_* , until the end of the study period, T (see J. Freidenfelds,⁷ Appendix, this issue). Thus

$$\begin{aligned} \text{PWAC (expenditure } *) &= \int_{\tau_*}^T \sigma_* e^{-rt} dt \\ &= \sigma_* (e^{-r\tau_*} - e^{-rT})/r \end{aligned} \quad (3)$$

where r is the force of interest [$r = \ln(1 + \text{cost of money})$].

In the most general case, the total cost of a network enhancement includes certain basic scheduled costs—typically for cable relief—that would normally be expended at time τ_b in the future. The PWAC of these future costs is therefore deducted from the PWAC of the total cost of the enhancement in order to determine the value of the MSC.

Let σ_e be the LEAC of the total enhancement cost, which is incurred at time $\tau_e (= 0)$, and let σ_b be the LEAC of the future basic scheduled cost, incurred at time τ_b . Then the value of the MSC is

$S = \text{PWAC (enhancement cost)} - \text{PWAC (future basic scheduled cost)}$
which can be expressed, from eq. (3), as

$$S = [\sigma_e(1 - e^{-rT}) - \sigma_b(e^{-r\tau_b} - e^{-rT})]/r \quad (4)$$

To determine the value of the MOC reduction, O in eq. (1), we first compute the expected reduction, ω , in the annual MOC. For work operation i , let w_i be the number of annual occurrences, and let ρ_i be the fraction of these which the MSC is expected to eliminate. w_i is taken from the information processing system and ρ_i is estimated based on the empirical knowledge of the engineer designing the enhancement. More sophisticated estimates of ρ_i may result from the theoretical work described by H. T. Freedman² and W. L. G. Koontz³ in this issue. Then $\rho_i w_i$ is the expected reduction in the annual number of work operation i . Furthermore, if K_i is the cost of work operation i (see Section 2.2), then define k_i to be

$$k_i = \begin{cases} K_i, & \text{if the cost of operation } i \text{ is an expense} \\ \text{PWAC}(K_i), & \text{if the cost of operation } i \text{ is capitalized} \end{cases} \quad (5)$$

where

$$\text{PWAC}(K_i) = \int_0^{L_k} \alpha_k K_i e^{-rt} dt = \alpha_k K_i (1 - e^{-rL_k})/r \quad (6)$$

and where α_k is the annual charge rate and L_k is the lifetime of the capitalized cost. $\text{PWAC}(K_i)$ is computed only over the life of the investment because the repeated plant assumption does not apply in this case. These costs are derived from work operations associated with providing service to a customer, which represents a commitment of capital only to the point in time at which the customer requests that the service be disconnected. The average service life, L_k years, is generally less than the length of the study.

Then $\rho_i w_i k_i$ is the expected reduction in the annual cost of operation i , and the expected reduction in the total annual MOC of the allocation area is

$$\omega = \sum_i \rho_i w_i k_i$$

In many cases, incurring the basic scheduled cost would probably have achieved a portion of the annual cost reduction, ω , starting at time τ_b . Let ω' represent the part of ω attributable to the basic scheduled cost. The result of incurring the MSC then is to reduce annual costs by amount ω until time τ_b and by amount $\omega - \omega'$ from time τ_b to T .

The total reduction in the MOC is therefore

$$\begin{aligned} O &= \int_0^{\tau_b} \omega e^{-rt} dt + \int_{\tau_b}^T (\omega - \omega') e^{-rt} dt \\ &= [\omega(1 - e^{-rT}) - \omega'(e^{-r\tau_b} - e^{-rT})]/r \end{aligned} \quad (7)$$

From eqs. (1), (4), and (7), we have the following general expression for A , the change in the aggregate cost of the allocation area:

$$A = [(\sigma_e - \omega)(1 - e^{-rT}) - (\sigma_b - \omega')(e^{-r\tau_b} - e^{-rT})]/r \quad (8)$$

Equation (8) reduces to a simpler form for the two most common MSC applications (other than conversion to SAC, discussed in the next section). The first is where the MSC represents simply the advancement of future basic costs. In this case $\sigma_b = \sigma_e$ and $\omega' = \omega$. Equation (8) reduces to

$$A = (\sigma_e - \omega)(1 - e^{-r\tau_b}) \quad (9)$$

The second application is where there is no basic scheduled cost component in the cost of the enhancement. In this case $\sigma_b = 0$ and $\omega' = 0$. Equation (8) reduces to

$$A = (\sigma_e - \omega)(1 - e^{-rT}) \quad (10)$$

If the value of A is negative, the aggregate cost will be reduced, and

the network enhancement is justified. If A is positive, however, the enhancement should not be made. In either case if other reasonable enhancements exist, they should also be investigated and the best one chosen if it pays.

4.3.2 Analyzing conversion to SAC

This model and the analysis techniques built around it apply exclusively to the case where the network enhancement involves converting parts of an allocation area to the serving area concept (SAC). Refer to the article by N. G. Long¹ in this issue for a definition of SAC.

The simplest form of conversion to SAC is called "stabilization." This involves placing a serving area interface between the feeder and distribution networks, breaking all multipling (see N. G. Long,¹ this issue) within the distribution network behind the interface, and providing enough distribution pairs to permanently connect one pair to each existing living unit and supply sufficient additional pairs to satisfy service demands for at least two years. In order to satisfy these criteria, it is usually necessary to add cables to the distribution backbone (the main cable paths extending out from the interface) and to rearrange the connections between the "leg" cables (the small cables containing the terminals serving the customers' premises) and the backbone cable (see Fig. 3). The network design changes affected by stabilization eliminate virtually all future work operations associated with providing service (see Section 2.1.1) as well as some operations associated with service restoration (see Section 2.1.2).

A more complex form of conversion to SAC involves replacing certain of the existing leg cables and terminals in the area *in addition to* doing the stabilization work described above. This *additional* work is simply called "replacement." The effect of replacement is to eliminate most of the remaining work operations associated with service restoration.

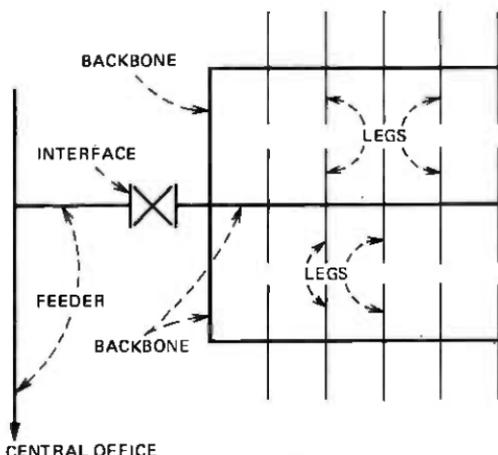


Fig. 3—Example of feeder and distribution (backbone and leg) cables.

The choice of the conversion design—either pure stabilization or some combination of stabilization and replacement—depends upon the type of work operations generating the MOC in the area and the location of the network conditions causing the operations. If a design combining stabilization and replacement is chosen, both parts of the enhancement must be shown to decrease the aggregate cost of the allocation area. If stabilization pays, but replacement does not, conversion to SAC is justified, but only if the replacement work is modified or eliminated. If replacement pays, but stabilization does not, then conversion to SAC is not justified and some other enhancement—such as merely replacing selected troublesome cables—should be examined.

Let A_S and A_R be the incremental changes in the aggregate cost of the allocation area that result from incurring MSCs S_S , the stabilization cost, and S_R , the replacement cost, respectively. Also denote by O_S and O_R the reductions in MOCs resulting from stabilization and replacement, respectively. As before, all costs will be expressed in present worth dollars, so

$$A_S = S_S - O_S, \quad A_R = S_R - O_R \quad (11)$$

4.3.2.1 Costs of stabilization and replacement. In this section we derive expressions for S_S and S_R . We begin with the following definition of the stabilization cost:

$$S_S = I + B - E - D \quad (12)$$

where

(i) I is the PWAC of the interface cost. This is the cost of placing the serving area interface between the feeder and distribution parts of the cable network.

(ii) B is the present worth of the cost of advancing backbone cable relief. The relief is needed in order to enable the elimination of multiplying conditions and provide enough pairs to satisfy the SAC distribution design criteria (see description of stabilization in Section 4.3.2). We only include the advancement cost in this case because the relief would ordinarily have been done at some future date.

(iii) E is the present worth of the cost of future cable pair transfers that are eliminated by the stabilization work. Since this cost would otherwise be part of the future basic scheduled costs of the allocation area, it is credited to the stabilization work.

(iv) D is the present worth of the deferred cost of feeder relief resulting from the stabilization work. Because SAC provides improved efficiencies in the utilization of feeder cables and also makes available certain previously unusable feeder pairs, future relief of the feeder network is deferred for a period of time. The value of deferring this basic scheduled cost is therefore credited to the stabilization work.

The interface cost includes both capital and expense components. Let σ_I be the LEAC of this cost as defined in eq. (2). Then we have

$$\begin{aligned} I &= \text{PWAC (interface cost)} \\ &= \sigma_I(1 - e^{-rT})/r \end{aligned} \quad (13)$$

The relief of backbone cables is actually a basic scheduled cost that would have been required at time τ_B in the future. For this reason only the advancement, B , of the backbone cable costs is included in the cost of stabilization. Letting σ_B be the LEAC of the backbone cost, B is given by

$$\begin{aligned} B &= \text{PWAC (backbone cost at time 0)} \\ &\quad - \text{PWAC (backbone cost at time } \tau_B) \\ &= \sigma_B(1 - e^{-r\tau_B})/r \end{aligned} \quad (14)$$

The value of τ_B may be calculated from parameters of the allocation area. τ_B is the time at which the assigned pair fill (assigned pairs/available pairs) of the backbone cables reaches the level at which relief is normally provided. This level is called the nominal fill-at-relief and is denoted f' . The current fill of the backbone cables is denoted f_B . If, for example, demand for additional pairs in the allocation area grows exponentially at the rate g ,

$$f' = f_B e^{g\tau_B} \quad (15)$$

Solving eq. (15) for τ_B gives

$$\tau_B = \ln(f'/f_B)/g \quad (16)$$

A cable pair transfer—sometimes called a cable throw—is the process of rearranging the physical cable pair connections within the network. This is done frequently in areas with multiple plant design. Specifically in such areas, as cable relief is provided, distribution pairs are transferred in order to maintain a balanced multiplying arrangement between the distribution and feeder pairs. Assuming such a balanced arrangement exists initially, this means that the average number of pairs transferred each year must equal the number of distribution pairs grown during the year. Of course distribution pairs are actually added as relief is needed, and transfers are made in groups, many at the time of relief. We assume, however, that the cost of transfers can adequately be estimated using a continuous rate of transfers. Therefore, assuming again that the allocation area grows exponentially at rate g , the number of distribution pairs grown—and the number of pairs transferred—in year t is

$$de^{gt} - de^{g(t-1)} = d(1 - e^{-g})e^{gt}$$

where d is the number of distribution pairs at time zero. If x_E is the cost of a cable pair transfer—an expense—then the present worth of the cost of all transfers made during the study is

$$\begin{aligned}
 E &= \int_0^T x_E d(1 - e^{-g})e^{gt}e^{-rt} dt \\
 &= x_E d(1 - e^{-g})(1 - e^{-(r-g)T})/(r - g) \quad (17)
 \end{aligned}$$

Since the need for these transfers is eliminated by conversion to the SAC design, the stabilization cost is reduced by amount E .

The deferral of feeder relief results from two effects of stabilization. First, under SAC design the feeder cables can be worked to a higher assigned pair fill before requiring relief than under multiple plant design. This is because SAC design provides greater access to the feeder network. The first line to a customer's premises remains assigned as either a dedicated or CT pair (see H. T. Freedman,² this issue) after service is discontinued, and requests for second lines or service at new premises can be satisfied by any spare feeder pair in the interface. Thus if δ is the fractional increase in the fill at relief afforded by SAC and a is the total number of available pairs in the area to be converted to SAC, then δa additional feeder pairs are available for use before relief is required. Second, when the interface is placed between the feeder and distribution networks, feeder pairs that were previously unusable, because of defects in the distribution pairs to which they were connected, now become available for use. Let b be the number of pairs recovered in this way. The total effect of placing the interface, therefore, is to increase by $\delta a + b$ the number of feeder pairs available and thus to postpone the time when each successive future basic scheduled cost for feeder relief must be incurred. The general form of the expression for D is

$$D = \text{PWAC}(\text{future feeder relief costs})(1 - e^{-r \cdot \text{deferral interval}})$$

However, calculating values for feeder relief costs and the deferral interval are beyond the scope of the Facility Analysis Plan, because they are functions of parameters such as the spare pair levels, cable gauge and structure requirements, and growth rates of each section of the feeder route. For this reason a standard cost factor, x_D (based on average feeder route conditions), is used to approximate the value of feeder deferral. Specifically, x_D represents the present worth value per unit length of one feeder pair gained through stabilization. The expression for D is therefore

$$D = (\delta a + b)lx_D \quad (18)$$

where l is the length of the pairs (the distance from the central office to the location of the interface). Since D represents a reduction in basic scheduled costs afforded by conversion to SAC, the stabilization cost is reduced by amount D .

This completes the stabilization cost terms [eqs. (13), (14), (17), and (18)]. We turn next to the replacement costs.

The cable replacement cost, S_R , is associated with replacing those

"leg" cables in the allocation area which experience recurring work operations for service restoral purposes.

This cost would normally be expended at some point τ_R in the future when relief of the leg cables is needed. If we let f_R be the assigned pair fill of the cables to be replaced, then

$$\tau_R = \ln(f'/f_R)/g \quad (19)$$

where f' is the nominal fill-at-relief and g is the exponential growth rate [see eqs. (15) and (16)].

The replacement cost, S_R , then is taken to be the cost of advancing this relief τ_R years:

$$S_R = \sigma_B(1 - e^{-r\tau_R})/r \quad (20)$$

where σ_B is the LEAC of the relief cost.

4.3.2.2. Reductions in marginal operating costs. In this section we derive expressions for O_S and O_R , the reductions in the MOC resulting from stabilization and replacement respectively. Recall that the MOC accrues from the occurrence of the work operations discussed in Section 2.1. Since these operations are affected in several distinctly different ways by conversion to SAC, we shall classify them into four groups for the purposes of this discussion.

Group 1 contains operations such as BCTs, LSTs and WOLs (see Section 2.1.1), which are caused by facility shortages and network imbalances. Since these conditions are corrected by stabilization, reductions in Group 1 operations are credited solely to the stabilization part of the conversion. The specific reduction factors applied to these operations are constants derived from studies of numerous conversion jobs in several operating telephone companies.

Group 2 contains operations caused primarily by activity in terminals and cable splices resulting from customer movement. Included in this group are cable troubles found in terminals and splices (see Section 2.1.2) as well as assignment changes made when a defective pair is encountered while installing service. The network activity causing these operations is substantially reduced, but not eliminated, by stabilization. For example, the activity due to reinstalling service at an existing customer's premises is largely or totally (depending on the choice of SAC design) eliminated. On the other hand the activity due to installing service to a new customer's premises is not reduced at all. The reduction factors, based on stabilization, that are applied to the operations in this group are variables. Their values are calculated based on the levels of network activity, the mix of subscriber demand (new vs. reinstallation) and the choice of SAC design.

Those Group 2 operations which are not directly affected by stabilization are eliminated, however, wherever the leg cables and terminals

are replaced in the serving area. The reduction factors based on replacement are therefore the unit complements of the stabilization factors.

Group 3 contains the operations associated with types of cable troubles other than those in Group 2. These are caused primarily by old and deteriorating cables (usually those with lead sheaths). Since this condition is not related to the network design or activity, these operations are reduced only where the cables are replaced. The reduction factors for this group are constants which reflect the vast improvement in the integrity of modern plastic sheathing materials as compared to lead.

Group 4 contains one type of operation—the assignment change made to restore service to a customer (described in Section 2.1.2). This operation is caused by the same conditions that cause both Group 2 and Group 3 operations. The reduction in this operation is therefore expressed as a weighted average—based on the relative numbers of Group 2 and Group 3 operations—of the reductions in the other two groups.

If unchecked by conversion, the operations in Groups 1 and 2 are assumed to increase over time at the same rate as the growth in assigned pairs. This assumption is made because these operations are related to the size and movement of the subscriber population. The operations in Group 3 on the other hand are assumed to remain constant since they are not affected by these factors. The Group 4 operation will be treated in two parts, one growing and the other remaining constant because of the composite nature of its causes.

Expressions for the marginal operating cost reductions for the operations in each of the four groups are developed below. For the purposes of this discussion, we define the following parameters:

k_i \equiv PWAC cost of work operation i as defined in eq. (5)

w_i \equiv number of annual occurrences of work operation i in the area to be converted (from the information processing system)

w_i^* \equiv number of annual occurrences of work operation i in the parts of the converted area to be replaced (from the information processing system)

ρ_i \equiv fractional reduction in work operation i resulting from stabilization (fixed or computed as described above)

ρ_i^* \equiv fractional reduction in work operation i resulting from replacement (fixed or computed as described above)

ω_{Sj} \equiv expected annual reduction in the MOC for Group j work operations resulting from stabilization

ω_{Sj}^* \equiv expected annual reduction in the MOC for Group j work operations resulting from replacement at the normal relief time and thus credited to stabilization

ω_{Rj} \equiv expected annual reduction in the MOC for Group j work

operations resulting from replacement at the time of conversion

$O_{sj} \equiv$ present worth of total reduction in the MOC for Group j work operations resulting from stabilization

$O_{Rj} \equiv$ present worth of total reduction in the MOC for Group j work operations resulting from replacement

Reductions in Group 1 operations are assumed to hold for the length of the study, T . While some reductions might coincide with regular reliefs (in which case our assumption may overestimate SAC related reductions), these would only be temporary if the basic design of the network is not changed. On the other hand, if the network is not currently in need of relief, operations can be expected to increase as relief nears (in which case our assumption may underestimate SAC related reductions). Improvements in this assumption may be possible in the future as a result of theoretical models currently under investigation (see W. L. G. Koontz,³ this issue). Based on the present assumption,

$$\omega_{S1} = \sum_{Gp1} k_i \rho_i w_i$$

as in Section 4.3.1, and since the exponential growth rate, g , applies in this case,

$$\begin{aligned} O_{S1} &= \int_0^T \omega_{S1} e^{gt} e^{-rt} dt \\ &= \omega_{S1} (1 - e^{-(r-g)T}) / (r - g) \end{aligned} \quad (20)$$

Since reductions in these operations are all due to stabilization,

$$O_{R1} = 0 \quad (21)$$

Reductions in Group 2 operations result from both stabilization and replacement. The stabilization component can be viewed as consisting of two parts. The first applies to the entire study period:

$$\omega_{S2} = \sum_{Gp2} k_i \rho_i w_i$$

The second part is a further reduction which begins at time τ_R when leg cables would have normally been replaced:

$$\omega_{S2}^* = \sum_{Gp2} k_i \rho_i^* w_i$$

The MOC reduction due to stabilization then is

$$\begin{aligned} O_{S2} &= \int_0^T \omega_{S2} e^{gt} e^{-rt} dt + \int_{\tau_R}^T \omega_{S2}^* e^{gt} e^{-rt} dt \\ &= \omega_{S2} (1 - e^{-(r-g)T}) / (r - g) + \omega_{S2}^* (e^{-(r-g)\tau_R} - e^{-(r-g)T}) / (r - g) \end{aligned} \quad (22)$$

For those leg cables which are replaced now (advanced from τ_R), a reduction of

$$\omega_{R2} = \sum_{Gp2} k_i \rho_i^* w_i^*$$

is obtained from now until year τ_R . This is the MOC reduction due to replacement:

$$\begin{aligned} O_{R2} &= \int_0^{\tau_R} \omega_{R2} e^{gt} e^{-rt} dt \\ &= \omega_{R2} (1 - e^{-(r-g)\tau_R}) / (r - g) \end{aligned} \quad (23)$$

Reductions in Group 3 operations result solely from replacement. In this case

$$\omega_{R3} = \sum_{Gp3} k_i \rho_i^* w_i^*$$

As for Group 2, these replacement reductions are only obtained from now until year τ_R . No stabilization reductions exist here because these effects are independent of the network design. The above cost reductions would be realized, beginning in year τ_R , even without SAC design. The MOCs for Group 3 (remember that growth does not apply here) are therefore

$$\begin{aligned} O_{R3} &= \int_0^{\tau_R} \omega_{R3} e^{-rt} dt \\ &= \omega_{R3} (1 - e^{-r\tau_R}) / r \end{aligned} \quad (24)$$

and

$$O_{S3} = 0 \quad (25)$$

Reductions in the Group 4 operation are expressed as weighted averages of the reductions in the Group 2 and 3 operations, and the costs are prorated accordingly. Thus we have four components of the annual reduction in the MOC for the Group 4 operation. These are $\omega_{S4}(2)$, $\omega_{S4}^*(2)$, $\omega_{R4}(2)$ and $\omega_{R4}(3)$, and they correspond respectively to ω_{S2} , ω_{S2}^* , ω_{R2} (from Group 2) and ω_{R3} (from Group 3). The MOC reductions for Group 4 are therefore

$$\begin{aligned} O_{S4} &= \int_0^T \omega_{S4}(2) e^{gt} e^{-rt} dt + \int_{\tau_R}^T \omega_{S4}^*(2) e^{gt} e^{-rt} dt \\ &= \omega_{S4}(2) (1 - e^{-(r-g)T}) / (r - g) \\ &\quad + \omega_{S4}^*(2) (e^{-(r-g)\tau_R} - e^{-(r-g)T}) / (r - g) \end{aligned} \quad (26)$$

and

$$O_{R4} = \int_0^{\tau_R} \omega_{R4}(2) e^{gt} e^{-rt} dt + \int_0^{\tau_R} \omega_{R4}(3) e^{-rt} dt$$

$$= \omega_{R4}(2)(1 - e^{-(r-g)\tau_R})/(r - g) + \omega_{R4}(3)(1 - e^{-r\tau_R})/r \quad (27)$$

The total marginal operating cost reductions due to stabilization and replacement are

$$O_S = \sum_{j=1}^4 O_{Sj} \quad \text{and} \quad O_R = \sum_{j=1}^4 O_{Rj} \quad (28)$$

where the O_{Sj} and O_{Rj} are given by eqs. (20) through (27).

4.3.2.3 Changes in aggregate costs. Expressions for the changes in the aggregate cost due to conversion, A_S and A_R , can be derived by substituting into eq. (11) the expressions derived for S_S , S_R , O_S , and O_R . If the value of A_S is positive, the aggregate cost due to stabilization will not be reduced, and the area should not be converted to SAC. An alternative, less costly network enhancement should be sought to correct the most serious of the network conditions.

If A_S is negative, stabilization can be expected to reduce the aggregate cost of the allocation area. If A_R is also negative, then the conversion work may be undertaken as designed. However, if A_R is not negative, then more selective (or no) replacement work should be investigated until a negative or zero value of A_R is achieved. The resulting design may be adopted for the conversion in this case.

V. CONTROL SYSTEM

5.1 Feedback

The control system is essential to achieving a reduction in the aggregate cost. Data from the information processing system are used in the control system as feedback to verify that the reduction in aggregate cost, predicted by the engineering applications system, is actually achieved.

In considering whether a particular enhancement is economical, the engineering applications system utilizes reduction factors for each type of MOC work operation. These reduction factors are now used to set an objective level for occurrences in the allocation area of each type of work operation. The objective level is computed by multiplying the reduction factor by the semiannual level of occurrences which existed in the portion of the allocation area to be affected by the MSC in the period prior to incurring the MSC and then adding the result to the semiannual level of occurrences in the portion of the allocation area that will not be affected by incurring the MSC.

By using data on the historical report for the six-month period after the work associated with the MSC has been completed, the actual levels of occurrences of each MOC work operation are compared with the objective. If the objectives are met and continue to be met, the aggregate

cost has been reduced. If not, the techniques of the engineering applications systems may need to be reapplied to determine what is preventing the cost reduction. Work along the feeder, for instance, may be causing service interruptions that increase the number of work operations. Another possible explanation is that the level of occurrences of MOC have increased in the portion of the allocation area not treated. It may now be desirable in that portion of the allocation area to incur an additional MSC that will reduce the aggregate cost.

Field experience has shown that objectives are usually met. When they are not, the cause can usually be determined and successful corrective action taken.

5.2 Management procedures

Other elements of the control system are management procedures. One is that use of the engineering applications systems be required for approval of all MSC items. Another is a forum in which managers of the various forces involved in loop provisioning and maintenance regularly discuss the status of high cost allocation areas and agree on the MSCs which will reduce the aggregate cost. In some cases the areas treated are different than if the costs to only one work force are considered. Such cases demonstrate the importance of considering costs to all forces when deciding where to incur MSCs.

VI. APPLICATION

The Facility Analysis Plan is being applied successfully in several telephone companies. In most it has been introduced with manual data manipulation procedures. In one company, the plan has been introduced using prototype computerized versions of both the information processing system and the economic analysis segment of the engineering applications system. The benefits of computerization have been lower cost, improved accuracy, and the ability to produce report formats that are not practical to produce manually. One such report makes it possible for the company to allocate money for MSCs to its operating divisions by depicting the distribution of the normalized yearly MOC in the allocation areas of each division.

Experience from these applications is being used to further refine the models and processes in the plan, and to expand its application to other companies. In all cases, application of the plan has reduced the aggregate cost of providing and maintaining loop facilities by much more than the relatively small cost of applying the plan, while at the same time improving service to the customer.

VII. ACKNOWLEDGMENT

Many people contributed to the development of the Facility Analysis Plan. Some have authored other papers in this issue. The authors are grateful to all, but wish in particular to acknowledge the constant inspiration of Joseph O. Bergholm, without whom the plan would never have been developed.

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Loop Plant Electronics:

Overview

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The dramatic improvement of semiconductors and other electronic components has yielded the low costs and high reliability long sought in electronic loop systems. The scope of existing systems includes range extension, analog and digital carrier, and loop switching systems. Use is accelerating and procedures are evolving to make such systems a fully accepted substitute for traditional cable pairs in telephone company operations.

The past ten years has been a period of intensive work in the application of electronics to the subscriber loop plant. The percentage of subscriber loops with some form of electronic augmentation is on a steep upward slope as a result of the combination of several major factors.

Most obvious is the fact that the costs of electronic alternatives to physical loop plant are coming down with each successive design generation. This, of course, parallels what is happening throughout the electronic equipment business, most obviously in the consumer market. At the same time, the costs of cable and associated construction and installation have been rising due to the pressure of material and labor cost increases.

The basic technological advance most responsible for lower cost electronics is the integration of analog and digital circuit functions into silicon semiconductor devices. The scores of discrete components formerly required to implement a function, such as companding, have been replaced with a single silicon chip with appropriate diffusion and metallization patterns. The attendant reduction in the number of components not only yields economies but greatly increased reliability as well. This is in striking contrast to the situation as recently as 15 years ago.

I. HISTORY OF LOOP ELECTRONICS

Looking back over the history of efforts to introduce electronics into the loop plant, one sees the recurring and allied problems of cost and reliability. The loop environment is much more challenging than the trunk environment in both respects, with the obvious exception of reliability requirements on underseas cables. For this reason, the loop network has not yielded to the application of electronics nearly as readily as the exchange and long-haul trunk network.

The most obvious difference between the loop and trunk situations is circuit length. Only since the early 1960s have carrier systems on exchange and toll trunks begun to prove in at distances less than ten miles. The length distribution of trunks is such as to offer a sizeable market for carrier beyond this prove-in distance and this has led to a long series of successful trunk carrier developments. On the other hand, the typical loop is about one or two miles in length and only about four percent of loops are longer than six miles. The capital expenditures that can be saved by eliminating copper pairs have not justified the cost of multiplexing except at the long distance extreme of the loop length distribution.

Rural loops long enough to support the cost of multiplexing to save pairs generally occur in small cross-sections. There is, therefore, little opportunity for achieving the economies of scale which have driven down long-haul transmission costs.

Any pair gain system used to serve subscribers must have one end located remotely from the central office. The cost of installation and operation of remote terminals must be taken into account in comparing such systems with the copper pair alternative. Reliability is an important consideration in loops where system failures mean loss of a customer's telephone service. Technology quite acceptable for trunks with both ends in central offices and with paralleling alternate routes may not be appropriate for loops. For example, electron tubes, which provided the basis for early carrier systems, were never suitable for large scale use in subscriber systems because of problems of limited life.

In spite of these difficulties, there have been continuing efforts to innovate in the loop plant, because the stakes were recognized as being very high. Much of the groundwork for loop systems was established in the years immediately after World War II but these systems were not really viable with the then available technology.

One of the earliest efforts involved the use of electric power distribution systems as the transmission medium for reaching remote rural customers. The M carrier system was introduced in 1945 and found limited use for this purpose.¹ The costs of getting carrier signals on and off high-voltage AC conductors and the difficulty of maintaining the integrity of the transmission path in the face of power company rearrangements caused this system to fall into disuse. Another early effort

was the use of radio. This approach has been used since 1946 to serve remote customers. Costs, even today, are too high to support large scale use of radio, although this approach is under continuing review.

The next major effort to use the techniques of long haul transmission in loops came after the invention of the transistor. P carrier was the first system to use transistor amplifiers and was an early attempt to reduce the cost of serving rural customers.² This system was introduced in 1956 but found only limited use because of problems of costs, reliability of early vintage transistors and other newly developed components and the rigors of a hostile environment. In many ways, it served as the prototype for station carrier systems introduced by several manufacturers some 15 years later with a more mature semiconductor technology. The latest in analog loop carrier systems are described in this issue.³ These are a single channel system for urban use (the *SLC*[™]-1 system) and a multichannel system, similar to P carrier, for rural use (the *SLC*-8 system).

At the same time that P carrier was being introduced, interest was turning to the newer field of digital transmission. Carrier terminals implemented by pulse code modulation techniques were found to be lower in cost than the then-existing analog carrier terminals. However, a much wider bandwidth was required to transmit digitally encoded speech signals. For example, a 24-channel PCM bank produces a 1.544 megabit digital signal for transmission between terminals to convey a total speech bandwidth of less than 100 kHz.

This apparently unfavorable bandwidth tradeoff is more than offset by the increased immunity to noise and distortion resulting from the use of low cost digital repeaters. Provided that the signal is fully regenerated at regular distances, typically 6000 feet on 22-gauge cable pairs, impairments do not accumulate enough to cause errors. There is, therefore, little degradation of transmission with length.

The T1 digital repeater, designed to receive and regenerate bipolar pulses at a 1.544 megabit rate, made possible low cost carrier transmission over cable pairs in existing exchange trunk cables. Digital transmission on these exchange trunk cables turned out to be very robust. It was an obvious step to consider the use of the same repeaters as the basis for systems on loop cables. The first such system (the *SLM*[™] system) was introduced in 1972^{4,5} and has since been superseded by the *SLC*-40 system, a more cost-effective second-generation system.⁵ Digital transmission has now been firmly established as a viable technique in loop as well as trunk cables, opening up the future possibility of end-to-end digital transmission.

In parallel with early efforts to exploit carrier transmission techniques, switching solutions were also being studied. The most straightforward method is to remotely concentrate the traffic from nearby customers on

a smaller number of trunks back to the serving central office. It is also possible to complete connections within the remote concentrator, but this is cost effective only when there is a high community of interest among the customers served.

Research of the late 1940s and early 1950s led to exploratory development attempting to use electronic techniques to build such remote concentration systems. These techniques had not matured sufficiently in the mid-1950s to provide an economically viable approach. A system based on electromechanical elements, designated the 1A concentrator, was introduced in 1962.⁶ These concentrators found use primarily for the temporary deferral of the installation of cable in urban and suburban areas. However, they lacked the reliability, maintainability, and traffic administration features required for wide-scale use.

Exploratory development has continued in an effort to produce lower cost, more reliable systems that are easier to administer from a traffic standpoint. At the present cost of electronics, the use of such concentrators as permanent plant is viable, particularly in rural situations, and a modern system, the LSS, is described in this issue.⁷

This work on the application of transmission and switching techniques to gain the equivalent of more cable pairs in loop cables took place over a period of 25 years against a background of work to reduce the costs of cable pairs themselves. Lower costs were achieved not only through improved cable design and installation methods, but by the introduction of electronic techniques to permit the use of finer gauge wires.

Two broad system approaches have been introduced to reduce the required wire gauges through electronics. The Unigauge approach allows the use of 26 gauge pairs for all subscribers out to a 30 kft limit. Unigauge was implemented by range extenders behind the first stages of switching in No. 5 crossbar in 1969 and in No. 2 ESS in 1972.⁹

The second approach, Long Route Design, permits the use of 22 gauge and finer cable for loops as long as 82 kft. The introduction of Long Route Design coincided with the introduction of miniature Dial-Long-Line equipment and the 2A range extender, first manufactured in 1969. These miniature Dial-Long-Line equipments have since been largely superseded by a family of range extenders with gain (REG), introduced in 1972 to achieve lower overall costs of range extension.¹⁰ The basic principles in REG have been subject to further refinement to reduce costs and simplify installation procedures and about one-half million have been delivered to the operating telephone companies. A concentrated version called CREG will supersede the Unigauge range extenders in ESS.¹¹

II. EFFECT OF TECHNOLOGY IMPROVEMENTS

As stated in the introduction, no real progress was made in the introduction of electronics to subscriber loops before the advent of the

transistor. Actually, it was an improvement of the whole family of electronic components including coils, capacitors and resistors as well as transistors which made possible the introduction of electronics in loops. However, had component technology not gone beyond discrete transistors and passive components, progress would have been very limited.

Today, the use of small-, medium-, and mainly large-scale integrated circuits pervades loop electronics products. The most dramatic effects of this IC technology is to be found in pair gain systems, whether analog and digital carrier or loop switching.

In the case of analog carrier, the pacing technology is bipolar custom linear integrated circuits. These ICs, typically involving 60×60 mil silicon chips, are capable of performing whole circuit functions; for example, modulation, demodulation, or companding. The single chips are the equivalent of hundreds of discrete transistors. Optimum ICs are not just discrete component circuit designs mapped into integrated circuit topology. Rather, the whole circuit design approach is changed to take advantage of completely different tradeoffs between the costs of passive circuit elements and transistor junctions. Furthermore, the junctions can be carefully matched in their basic characteristics through manufacture and they can be made to track very closely with the effects of temperature. This is fundamental to the improvement of performance over the earlier discrete component designs of functional circuits.

In the case of the companding function, a single chip, implementing a novel circuit approach only practical with integrated circuits, performs the functions of compression or expansion of speech signals depending on an external connection option. This IC compandor, used in both the *SLC-1* and *SLC-8* systems, goes well beyond the long-established performance objective for trunks with regard to signal distortion, control of channel loss, and speed of response. And yet, the two silicon chips and a number of discrete resistors and capacitors now replace a complete circuit pack in N2 carrier. The N2 system was introduced in 1962 on the basis of discrete transistors and passive components.

Digital carrier systems, too, are based on the extensive use of custom ICs. In fact, such applications occurred first because the digital IC technology was generally well ahead of analog IC technology.

The SLM system, 1972 vintage, was based on the use of custom MOS devices combined with thin film resistors on ceramic substrates. Today, digital bipolar devices with large scale integration as well as MOS devices are found extensively in loop pair gain systems. In fact, such devices are so solidly entrenched and accepted for these digital applications that little is said explicitly about the device technology in the system descriptions found in this issue. The use of ICs in digital systems has ceased to be at issue.

The LSS system includes a functional unit only possible with today's IC technology and this is worth special emphasis. That functional unit is a microprocessor. In LSS, the microprocessor is used to implement stored program control of the system at the central office terminal. The microprocessor used is PROCON, manufactured by the Western Electric Company. This particular microprocessor uses 8-bit data and 24-bit instructions and has a two microsecond cycle and instruction execution time. To control LSS, 5700 words of read-only memory (ROM) and 512 words of random-access memory (RAM) are required.

Of the 5700 words of ROM, only 2000 are used for call processing. The remaining two-thirds are required for automatic trouble location, manual testing, alarms, and traffic measurements. These are functions which would be hardly practical without the stored program control approach. With electronic sophistication, it is possible to make today's systems fit much better into the telephone company environment. LSS succeeds where earlier line concentrators failed in this regard.

III. FITTING THE ENVIRONMENT

Important work has been done to better fit the physical environment of the loop distribution network.¹² While much clever design has been carried out to reduce the costs and improve the appearance of the central-office-mounted parts of the product-line, the primary challenge is at the remote terminals.

Physical designers have had to cope with the harsh environmental extremes of North America for products ranging from a 2 cubic inch isolation filter to a 16 cubic foot SLC-40 remote terminal. Because of these great differences in size and differences in internal heat dissipation, there is no universal solution to the design of outside housings.

The SLC-8 system, having a small modular size and no batteries and battery ventilation, has offered the opportunity for the most innovative approach. In this system, plastic moldings enclosing 8 subscriber channel units nest into standard outside plant closures. The need for unique construction procedures for installing the system are avoided, a major step toward eliminating the special nature of loop electronics.

Until the recent generation of systems, loop electronics fit only into very special situations: on very long routes, at locations of rapid growth of service demand, or in areas of extremely high construction costs. There was no great difficulty in identifying applications in trailer camps and at river crossings, and the economic advantages were usually quite obvious. It was quite practical to install and maintain systems on a special engineering and maintenance basis when quantities were limited.

Today's lower costs offer the potential of much greater penetration and more widespread savings. However, this is not going to happen unless

these systems enter into the main stream of telephone company engineering and operations.

First, and most obvious, is the fact that the systems must have the right features and parameters to fit the telephone company needs. This is particularly important from the standpoint of the craft personnel who must operate and maintain loop facilities. Equally important is the development of fully documented methods for planning, engineering, installation, and maintenance.

While much remains to be done to achieve full compatibility with the management of more traditional telephone plant, significant progress has been made in cooperation with AT&T and the operating telephone companies to establish workable procedures for planning, engineering, maintenance and administration.^{13,14} Most of the work so far has been in the context of rural applications where loop electronics has found its first application because of the high cost of traditional plant construction methods.

The planning and engineering process begins with the recognition of a need to satisfy a service demand either present or forecast. Once this need and the limitations of the existing plant have been characterized, broad guidelines can be used to determine what alternatives, from the wide range of systems available, are worthy of more detailed studies. Depending on the complexity of the particular problem and local preferences, these detailed studies can be carried out either manually with well-established step-by-step procedures or by time-shared or batch computer programs. The net result of this work is a fundamental plan for satisfying the service demand in an optimum way.

These methods are well advanced for the analysis of rural applications, though work continues on unification and simplification of procedures. The developers of these methods have had available a good characterization of the rural environment in the long route data base assembled by AT&T. Furthermore, in these rural applications, capital cost is a dominant factor and the necessary cost parameters are relatively easy to obtain. Capital cost is also a dominant factor for temporary applications where major plant construction can be deferred in suburban areas. Achieving a similar capability in the optimum use of loop systems in suburban areas is the subject of on-going studies. Here, the impact of the systems on the cost of operations is a much more important factor.

In examining the maintenance and administrative aspects, it is helpful to compare loop electronics carrier channels with the feeder cable pairs which they replace. Methods and procedures being recommended are gradually leading to a full acceptance of that approach.

An example of a recommended procedure is the trouble-shooting of a single channel failure in a subscriber carrier system. In the event that

such a failure occurs in a *SLC-40* system, an installer-repairperson dealing with the trouble should transfer service to a spare channel in the *SLC-40* just as he or she would transfer to a spare feeder cable pair. The change-out of a faulty *SLC-40* plug-in should be the responsibility of craft more experienced in the maintenance of the carrier system. This is analogous to feeder pairs being fixed by cable repair forces, not by installer-repairpersons.

Admittedly, much about the operation of loop electronics is still considered special and dealt with by methods outside normal procedures. To a large extent, this is a holdover from past practices which have not yet been changed. For example, it has long been customary for electronic equipment of any kind to terminate on the horizontal terminal blocks of the main distributing frame. On the other hand, if subscriber carrier channels are to appear like feeder cable pairs, they should terminate on the vertical side along with the feeder cable pairs. Through cooperative efforts of Bell Labs, AT&T, and the operating telephone companies evolutionary problems like these will be solved. It is both timely and necessary to make these changes because the costs and reliability of loop electronics today support use of these systems as a substitute for cable pairs on a wide scale basis.

IV. PRODUCT LINE CONSIDERATIONS

The equipments described in this series on loop electronics constitute a complete product line. That is, all the known and significant systems approaches for loops are matched by members of this set of products or combinations thereof. Each product has areas of application where it is more effective than the other products in reducing capital costs. These primary areas of application are summarized in Table I.

Of course, these categories may appear more disjoint than they really are. Economic studies of the kind previously discussed can resolve most issues of application, but "gray" areas will still exist. For example, a cluster of six *SLC-8* systems will give a cost per pair gain comparable to a *SLC-40*. In cases like this, choices will have to be made on the factors other than cost. Some important factors are listed in Table II for *SLC-8*, *SLC-40*, and *LSS*. Different companies are likely to give different weight to these factors based on local service conditions.

Table I — Primary applications

System	Primary application areas
REG	Fringe suburban and rural areas
CREG	High-growth suburban areas
<i>SLC</i> TM -1	Mature, but changing, urban and suburban neighborhoods
<i>SLC-8</i>	Distant, low-density rural areas
<i>SLC-40</i>	Distant, clustered rural areas
<i>LSS</i>	Suburban areas with high construction costs and rural areas with low pair gain ratio requirements

Table II — Factors affecting choice of systems

	<i>SLC</i> TM -8	<i>SLC</i> -40	LSS
Transmission	Analog	Digital	Voice Frequency
Repeater spacing	4 miles	1 mile	None
Channel/trunk test	Loop-back	None	Automatic
Drop test	None	None	Unlimited
Drop length	400 ohms	900 ohms*	1700 ohms [†]
Power reserve	Powered from CO	8 hrs	Powered from CO [‡]
Pairs gained	7	36	82
Pairs required	1	4	34
Pair gain ratio	7	9	1.82

* 1600 ohms under special circumstances.

[†] The total resistance from the central office to the customer cannot be more than 2800 ohms, with range extension applied to the trunks for resistance greater than 1600 ohms.

[‡] Pair resistance greater than 2800 ohms from the central office to the remote terminal requires a remote power feed.

For example, consider an area where there is heavy emphasis on the ability to apply all existing methods for testing metallic loops all the way to the customer's ringer. Consider a further need to survive power blackouts longer than the nominal 8-hour reserve of back-up batteries in remote terminals. LSS is the obvious choice provided the existing cable cross-section is adequate for the application, and it is not necessary to gain the transmission improvement of analog or digital carrier. *SLC*-8 permits convenient modular growth and is completely powered from the central office. On the other hand, *SLC*-8 has a more limited drop range than *SLC*-40, and it uses analog line transmission, an approach less compatible with the long-term trend to a digital network.

This discussion simply illustrates some of the hard choices to be made. It is recognized that local preferences and the desire to standardize on a subset of available systems in a given operating area may cause the bending of strictly economic decisions. We continue to strive for a single, unified system approach with the best features of the present diverse product line. Unfortunately, that perfect system has not yet arrived and cannot be firmly predicted. There are, however, some very clear trends for the future.

V. FUTURE TRENDS

The future will see the introduction of pair gain systems far cheaper than anything available today. For example, it is likely that by 1979 the effective installed cost will go down by about 2:1. There is a further prospect of reduction in cost through integration of digital loop systems with an overall digital network plan. This will continue the very steep downward trend that has been experienced over the past two years.

One of the consequences of lower costs will be a greatly increased penetration of electronics into the loop distribution network. Today, loop electronics of all kinds are applied to about 2 percent of Bell System

growth lines. If the kinds of cost reductions predicted above are actually realized, this penetration of growth lines will increase tenfold. The use of such large quantities of electronics in the loop plant will place increased emphasis on size and power reductions. This large penetration also means that the cross-sections of digital transmission over the three or four major feeder routes of a wire center will become very large, in excess of several thousand speech channels. This may well afford the best opportunity for introducing optical fiber transmission into the loop plant. Optical fibers, with their high speed capability over long repeaterless spans, may be the only viable way to deal with this greatly increased penetration of digital transmission. Of course, once fibers are introduced to support existing telephone and high speed digital services, there will be the further possibility of adding video bandwidth services in a very graceful manner. New services are likely to be a very important factor in the future of loop electronics development. A similar review ten years from now will most probably present an impressive picture of expanding services as well as reduced costs.

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Loop Plant Electronics:

Planning for Loop Electronic Systems

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Loop electronics planning is important to the overall telephone loop planning process. It is important to the operating companies because they must determine the most economical solution to each specific facility relief problem. In addition, it is important for Bell Laboratories to understand the economics of loop electronics systems in order to define and evaluate the potential impact of new loop electronic system designs on the Bell System, to develop new planning techniques, and to assist, along with AT&T, the operating companies with specific applications. This paper describes the development planning process used at Bell Labs to define new loop electronics systems and applications. It also describes the planning techniques used in the operating companies to identify and evaluate applications of loop electronics. The interrelationships of these two activities are also discussed.

I. INTRODUCTION

Planning for loop electronics in the Bell System responds to at least two distinct, but interrelated, areas of need. The operating telephone companies plan for the use of loop electronics as part of their annual construction program, and Bell Laboratories, together with AT&T, plans the development of new systems and new applications for existing systems. These two planning functions, occurring at two different levels throughout the entire Bell System, are interrelated.

This paper describes the planning process for loop electronics now recommended for use in the Bell System companies as well as the development planning process in use at Bell Labs and AT&T. The interrelationships and interdependencies of these two processes are outlined in Fig. 1.

Development planning for loop electronics is needed to guide devel-

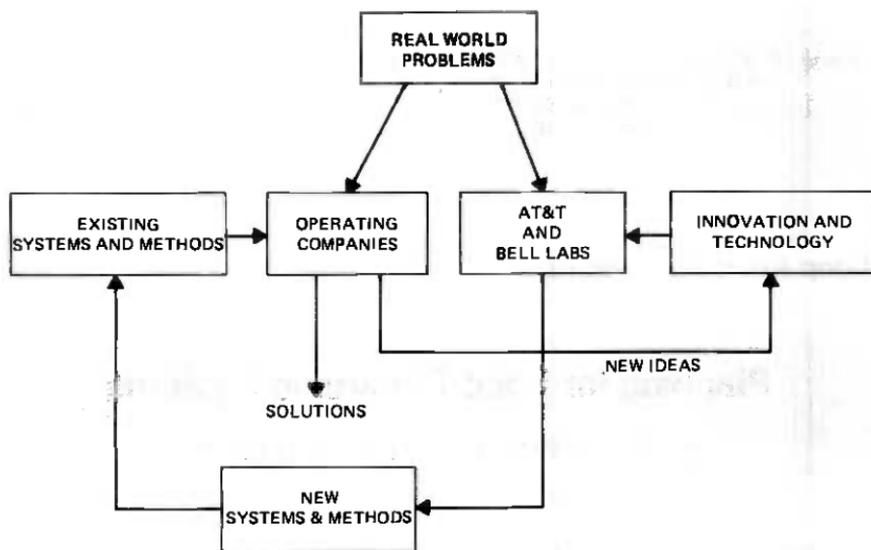


Fig. 1—Interrelated planning process.

opment of new systems, to identify new applications for both new and existing systems, and to estimate the economic benefits of these actions across the Bell System. This planning represents a means of providing for present and future needs of the operating companies and allows for the timely introduction of new systems and new applications in anticipation of those needs. This permits the operating companies to achieve the maximum economic benefits with a minimal amount of delay.

Bell Laboratories development planning is based on the use of a "bottom up" study technique, that is, loop electronic systems are considered as solutions in a small but carefully selected set of facility relief problems. These results are then used iteratively to reconfigure the proposed systems or techniques under consideration in order to maximize savings. The results of this analysis are then extrapolated to the whole Bell System by conducting surveys in the operating telephone companies.

The operating companies, on the other hand, must evaluate the alternatives available to provide outside plant facilities for specific projects. The problem has become more complex with the introduction of loop electronics. Planning must now include the decision on whether or not to use loop electronics on a project, as well as decisions on which system to use, how many to use, and when and where to install them. Planning enables the operating company to identify the specific projects for which loop electronics can be economical and to evaluate the magnitude of those benefits.

Section II describes the development planning process used at Bell Labs and AT&T to determine future generations of loop electronics

systems. The methods used to hypothesize new system configurations and new applications for existing systems are discussed, along with the techniques used to test these hypotheses against the economics and constraints of the real world. The interrelationships among Bell Labs, AT&T, and the operating companies are also described.

Section III describes the operating company planning process. This includes a discussion of screening guidelines, practical considerations, and study tools. The methods have been available to the operating companies for almost two years and are being implemented through the Bell System.

Section IV gives an illustrative example, describing the results of both the operating company planning process and the Bell Labs planning process. The example demonstrates how operating company planning can lead to innovative applications and how the identification of these applications can stimulate the study of new system configurations.

II. LOOP ELECTRONICS DEVELOPMENT PLANNING

2.1 *The need for the detailed study*

Many attempts have been made to simplify the planning associated with the application of loop electronic systems. Some of these simplifications, however, can result in misleading conclusions. For example, one of these approaches compares the average installed first cost of a loop (as a function of the cable distance between the central office and the customer's premises) with the installed first cost per pair gained on a loop pair gain system. This latter quantity is a standard figure of merit for pair gain devices and is defined as the total installed first cost of a system divided by the number of incremental communication channels provided. Thus, a 40-channel carrier system that uses four cable pairs and costs \$3600 has a cost per pair gained of $3600/(40 - 4) = \$100$. Based on the cable-electronics cost comparison illustrated in Fig. 2, it might be concluded that it is economical to use pair gain systems to derive all loops whose length is greater than L . This comparison is misleading for the following reasons:

(i) The first cost of outside plant facilities is generally capitalized. Long term studies must reflect the amortization and income tax effects that result from this capitalization. The present worth of annual charge (PWAC) technique that includes these effects is, therefore, a better method of comparison than installed first cost.

(ii) Because of the economies of scale associated with cable placement, low cost feeder route expansion strategies dictate the placement of larger size cables than are required to satisfy short term subscriber demand forecasts. Hence, at a given instant of time, there may be excess cable facilities available on a given cable section. Because of this in-

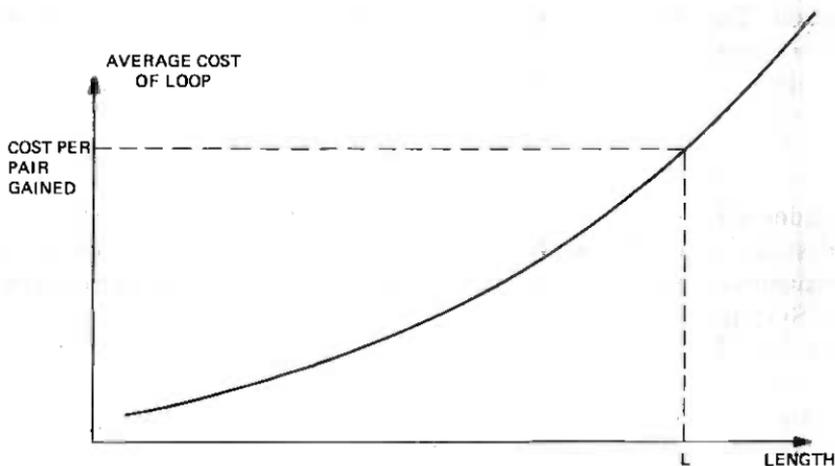


Fig. 2—Electronics—cable cost comparison.

ventory, the provision of a loop of length L may require the addition of only $L_1 < L$ feet of cable.

(iii) Pair gain systems, as well as cable, come in discrete sizes. Thus, while a specific system may have an extremely low cost per pair gained, it may also provide a large and unneeded number of communication channels.

This example suggests that, unless the detailed mechanics of the facility expansion process are examined in conjunction with exact pair gain system configurations, faulty conclusions can be reached:

2.2 The Bell System application study

Because of their high cost, early loop electronic systems could not be considered as potential universal alternatives to cable. Rather, their cost tended to justify their use only in the very long loop rural areas. As new, lower cost systems have been developed, however, the list of generic applications in which they could be used economically expanded to include their temporary and permanent placement in suburban areas. The use of this generic applications list somewhat simplifies the process of operating company planning by automatically eliminating areas where electronics should not normally be considered as alternatives to cable.

Detailed feeder route studies must be conducted in order to both identify new generic applications for existing or hypothetical systems and to evaluate the impact that hypothetical systems could have when used in generic applications. Thus, at the heart of the development planning process for loop electronics is the aptly named "application study." This type of study should not be confused with an operating

company planning study which evaluates the suitability of using loop electronic devices to solve a particular facility relief problem.

As is shown in Fig. 3, the motivation for a particular application may come from the following four different areas:

(i) Identification by the operating companies or AT&T of a particular type of facility relief problem, for example, the extremely high cost of providing service to subscribers who live in remote rural areas.

(ii) Anticipation of future problems by Bell Labs, for example, the facility problems that will arise from a large growth in the demand for wideband data services.

(iii) Development of a new technology that might aid in solving an existing problem, for example, the use of optical fibers as a loop transmission medium.

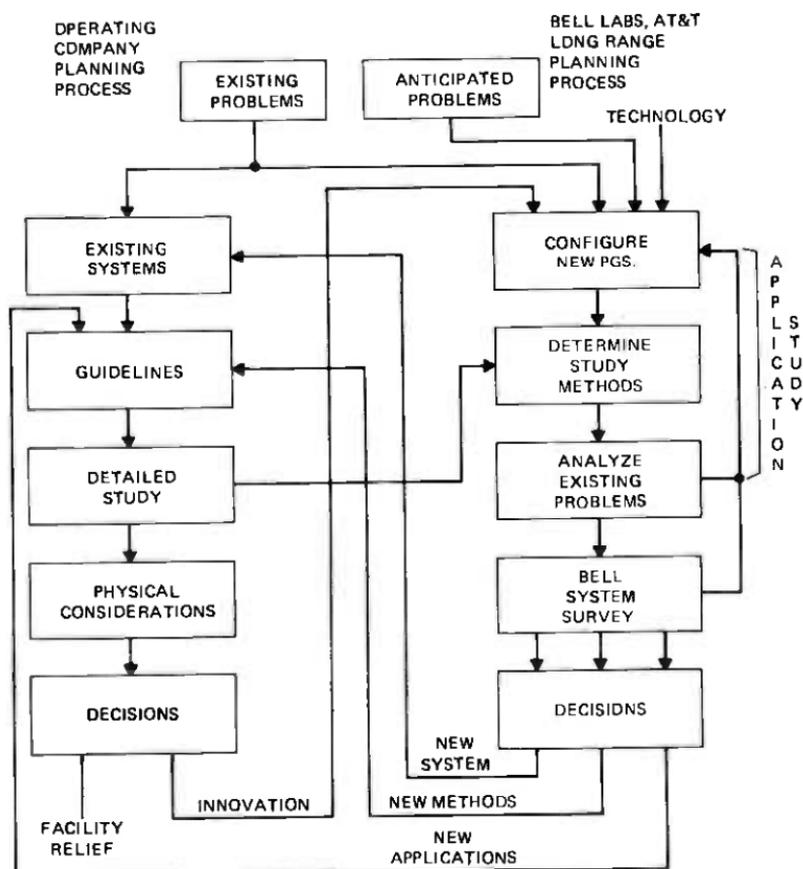


Fig. 3—Development planning flow chart.

(iv) Innovative use by the operating companies of existing systems to solve new problems. An example of this kind of motivation, which involved an operating company's use of the *SLM*TM system to defer large suburban conduit additions, is described in Section IV.

Based on these motivations, AT&T may commission an applications study in a single Bell System operating company. As Fig. 4 indicates, the telephone company has the responsibility for supplying the description of the feeder routes to be considered. This includes providing information on route topology; existing cable facilities; forecasts of subscriber

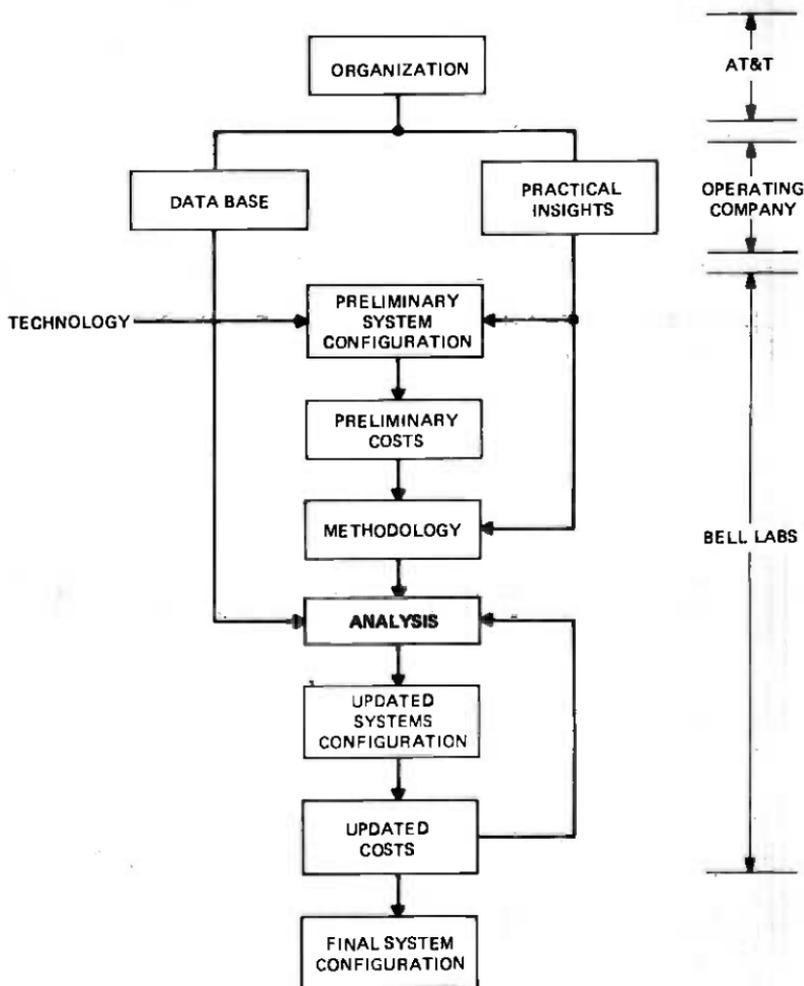


Fig. 4—Application study flow chart.

demand; local installed costs for such things as cable, conduit, pole lines; as well as outside plant maintenance and rearrangement costs. Operating companies also provide practical insights that can be used in the development of systems concepts. For example, local land usage may make it necessary to design a new pair gain device for only underground mounting. In addition, because of their familiarity with their own problems and their previous use of loop electronic systems, operating companies can also provide valuable advice on the assumptions and approaches to be used in the actual economic analysis.

Primary responsibility for the generation of new loop electronic system concepts and the estimation of their preliminary costs is that of Bell Laboratories. These concepts are influenced both by the characteristics of the problem under consideration and the current state of technology. A list of preliminary pair gain system parameters that might be identified at the beginning of an application study is contained in Table I.

Economic analyses are used to evaluate these preliminary configurations as alternatives to cable. In such analyses, an optimum or sub-optimum feeder relief plan is identified that minimizes the PWAC required to provide service over the study period. The actual methods used to obtain this plan vary greatly. In some cases, standard Western Electric computer programs such as EFRAP¹, LFRAP² or LCAP², which utilize branch and bound optimization algorithms, are employed. In other instances, variations of the guidelines approach discussed in Section III or the analytical minimization techniques discussed in Ref. 3 are found appropriate. A typical analysis approach that is used by Bell Labs is described in Ref. 4.

Analysis of the preliminary system generally indicates that the PWAC savings attributable to the use of a device can be increased by changing several of its key parameters. As is illustrated in Section IV, system and module sizes are typical parameters that are adjusted. At this early

Table I — Preliminary pair gain system design parameters

Size:
System size
Module size
Transmission:
Type: analog, digital or baseband
Maximum resistance of cable between RT and customers' premises
Transmission line repeater spacing
Mechanical:
RT mounting configuration
Traffic:
Maximum customer traffic handling capability
Features:
Testing
Maintenance
Powering

stage of the development planning process, an estimate of the price at which this reconfigured system might be made available to the operating companies is also provided. This is a critical step. It is the point where the possible problems and resultant costs associated with manufacturing a new technology system are included in the planning process.

The steps just outlined are designed to yield the configuration of new loop electronic systems. If existing systems rather than hypothetical configurations are used, however, the same process provides an evaluation of the suitability of using these systems in new generic applications.

The analysis methodology used by Bell Labs during the applications study may also be suitable for use by the operating companies in their loop electronic planning process. The LFRAP program is an example of standard Bell System planning software that had its genesis in a Bell Labs applications study.

2.3 Bell System survey

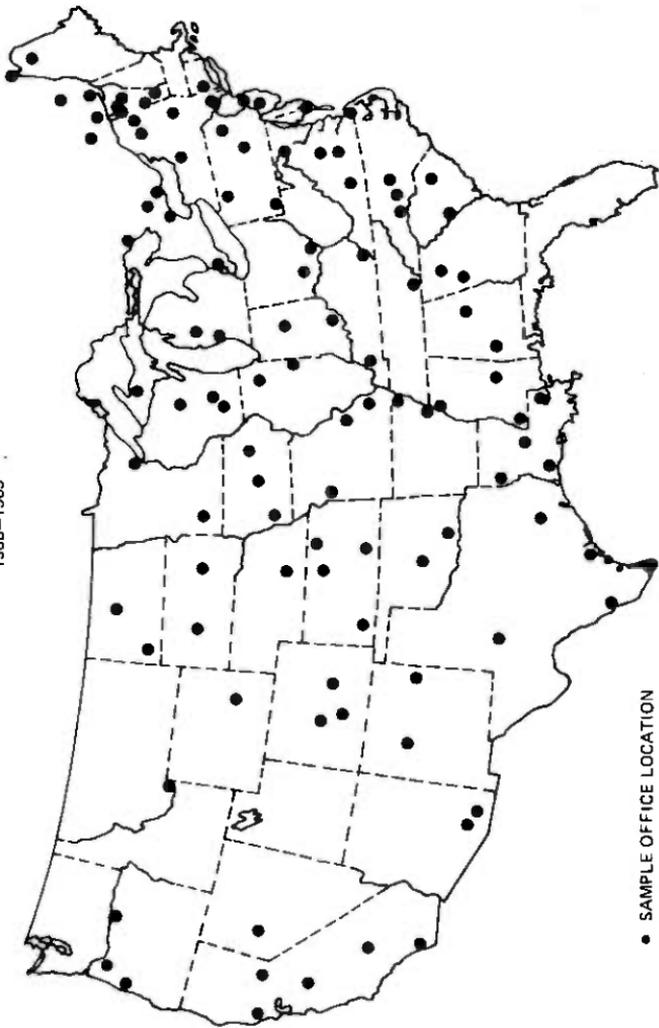
The results of an application study apply only to the small number of feeder routes actually considered. To estimate the number of routes in the Bell System that have the same characteristics, surveys must be conducted in other operating companies. These surveys are then used to extrapolate to the total Bell System the applicability of the systems, applications and methods identified in the application study.

The collection of Bell System surveys and data bases is generally conducted by AT&T, with technical support from Bell Labs. One of the best examples of such an activity is the Long Route Data Base,⁴ compiled in 1968. This survey represents a 2 percent sample of all Bell System wire centers that have at least one loop whose length exceeds 40 kft. A map indicating the location of the 110 wire centers in the survey is shown in Fig. 5.

The Long Route Data Base contains information that describes in detail the 363 feeder routes in the sampled centers. A partial listing of the data available is contained in Table II. This information was used to evaluate the impact that the *SLC*TM-8, *SLC*TM-40 and the Loop Switching System (LSS) might have in Bell System rural areas.

Frequently, however, the results of these surveys may indicate that in a given configuration, a proposed system or technique is not broadly applicable to all operating companies. The process of reconfiguration and analysis can then be repeated to increase the potential usefulness of the proposed system.

1968-1969



• SAMPLE OFFICE LOCATION

Fig. 5—1968 long route survey.

Table II — Partial listing of information in 1968 Long Route Data Base

Central office data:
Name
Location
Type of switching machine
Route data:
Route identification
Length of longest loop
Number of sections on route
Section data:
Section identification
Length
Identification of preceding section
Type of cable or wire on section and the number of pairs provided
Existing telephone subscriber demand
Forecast subscriber demand

2.4 Decisions

Bell System surveys provide a means for estimating the effect that new loop electronic systems, application strategies, and analysis techniques might have on all the operating companies. If the economic savings are sufficient, these new approaches are developed and ultimately appear as inputs into the operating company planning process. It is important to note, however, that since the Bell Labs development planning process is motivated by real problems, the outputs of the process are in fact tailored to the needs of the operating companies. The extent to which these new approaches can impact on operating company problems is, therefore, optimized. A description of how operating companies actually use new electronic systems, planning techniques, and application strategies is described in the next section.

III. THE OPERATING COMPANY PLANNING PROCESS

3.1 Specific relief problems and available solutions

The operating company outside plant planning engineer is continually faced with the problem of planning cable relief projects for congested cable routes. These relief requirements are usually identified through the records of existing facilities and the evaluation of new forecast information. Often, the engineer uses qualitative forecast information gathered from contacts in the area, including real estate developers and planning boards, to identify potential facility congestion-areas.

The addition of cable facilities to an existing feeder route can be extremely costly. Preparation of a long term facility addition plan is complicated, not only by the economies of scale associated with placing cable, but also by the discrete sizes in which cable is manufactured. Optimization programs, such as the Exchange Feeder Route Analysis Program (EFRAP) are, therefore, often used to prepare a basic long range facility addition plan for individual feeder routes.

Electronic systems can also be used to provide loop services, often at a total cost lower than that of the cable alternative. Since electronic systems increase the number of alternatives to be considered, the complexity of the facility expansion problem is increased. The increased complexity emphasizes the importance of systematic and effective methods for planning relief with electronics.

Planning methods that deal with relief using cable and structure facilities are well established. They include sizing guidelines and mechanized study procedures that assist the engineer in timing, sizing and pricing the relief alternatives. These techniques are continually being improved and modernized.^{1,5}

Planning methods that deal with the application of pair gain systems to provide facility relief are newer. These methods include both guidelines and detailed economic study techniques. The guidelines quickly indicate whether further consideration of pair gain systems is worthwhile. The detailed economic study methods, both mechanized and manual, help the engineer develop and cost the pair gain relief plan.⁶ Since the planning engineer is usually limited to the use of existing systems in standard modes of application, both the guidelines and detailed study methods assist the engineer in the evaluation of the applications of a specific pair gain system to a specific relief problem. This systematic planning procedure is illustrated in the left hand side of Fig. 3.

The following sections will describe the pair gain system planning methods now available for use in the Bell System. These methods are not tied to specific hardware, but can be easily adapted to the evaluation of any pair gain system. They can, therefore, be used to identify and study the vast majority of pair gain applications.

3.1.1 A rule of thumb

The first step in the study process, once the congestion problem is identified, is to decide whether or not pair gain systems should be considered, and whether they should be considered as a permanent or temporary alternative. A permanent application is defined as one in which the pair gain system is used until the economic life of the equipment is over, at which time it is replaced with like equipment. Temporary applications are defined as those in which the intent is to remove the pair gain system, and perhaps reuse it, in a time period less than the economic life of the alternative cable or conduit. Typical temporary applications last from two to five years. A more detailed discussion of the economics of temporary applications can be found in Ref. 4.

The operating company engineer uses previous experience and knowledge of typical economic pair gain applications to decide whether or not pair gain devices should even be considered. The engineer then

decides to consider either a permanent or temporary application by using the following rule of thumb: A pair gain application should be permanent unless (i) route parameters may change significantly in the next few years, or (ii) the growth is so high that too many pair gain systems (physical or practical limits) would be needed to provide relief over the study period. Condition (i) refers to events, such as a future route rearrangement necessitated by the construction of a new highway or wire center. Condition (ii) usually applies in suburban and light urban areas, where growth can be high and space for remote terminal (RT) sites limited. Since this decision is based on a rule of thumb, it is by no means irrevocable. There are several points in the process where the engineer can reconsider this decision if the situation warrants it.

3.1.2 Economic guidelines

Once the preliminary decision to consider either a permanent or temporary application is made, the engineer applies the appropriate pair gain systems guidelines. The form of these guidelines differs somewhat for permanent and for temporary applications, but similar functions are performed in either case.

The guidelines enable the engineer to determine quickly whether or not the pair gain systems being considered have potential economic benefit. If they do not, the engineer can immediately proceed to study the cable alternatives without further consideration of pair gain. If a potential economic benefit is indicated, the guidelines can assist in rank ordering the systems by their potential economic savings.

3.2 Study methods for rural permanent pair gain applications

3.2.1 Economic guidelines for rural permanent applications

Guideline curves for rural permanent applications are break-even PWAC curves of a pair gain solution versus a cable solution on a model of a route. The model route is characterized by its length, called weighted loop length (WLL), and by the size of cable needed to satisfy 15 years of growth, called weighted cable size (WCS). WLL and WCS are present worth equivalents of route length and cable shortages. Present worth equivalents are used because cable relief requirements are spread out over time. The PWAC of placing a cable of size WCS and length WLL is compared to the PWAC of placing a series of installations of the particular pair gain system to satisfy the same growth over the study period. The resulting break-even curve is dependent on the getting-started and per-line installed costs (commonly referred to as A and B costs, respectively) of both cable and pair gain, their associated annual charge rates, and on the size and pair gain of the system being considered. Telephone

company staffs can create and publish series of curves for the systems they use and for local variations in both cable and pair gain installed costs, using calculation methods provided by AT&T. An example of such a curve is shown in Fig. 6a.

To use these curves, an engineer in the field need compute only the WLL and WCS equivalents of each section of the route under study and then plot the cumulative results on the appropriate break-even curves. These computations can often be done in less than 15 minutes. The resulting plots can indicate which routes are definitely not economic for pair gain applications and, for those that may have savings, which system may have the largest savings. An example of a plot for the route in Fig. 7 is shown in Fig. 8. The numbered segments in Fig. 8 correspond to the cumulative WLL and WCS equivalents of the corresponding section numbers in Fig. 7. For example, the point at the end of 3 in Fig. 8 corresponds to the WLL and WCS of Sections 1 through 3 in Fig. 7. The plot in Fig. 8 indicates that the sample system would probably be economic if the RT were placed beyond the end of Section 5, or approximately 63 kft from the CO. This does not lead to the conclusion that the proposed pair gain system should be used. It does, however, indicate that it is worthwhile to make a detailed economic study comparing the use of this pair gain system with the cable alternative.

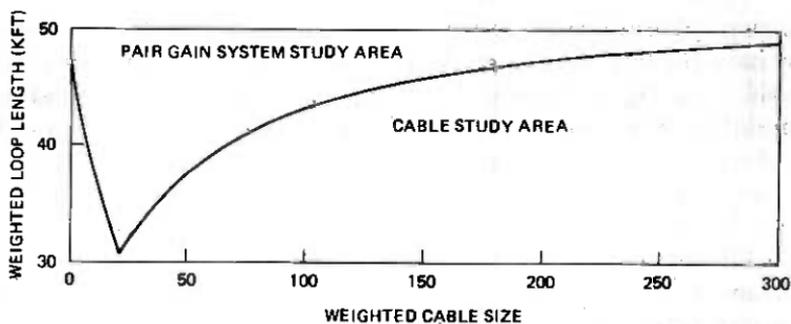
These guidelines resolve many of the inaccuracies that arise from the use of the approach described in Section 2.1. The validity of the permanent applications guidelines has been established by testing their effectiveness on the 363 routes of the Long Route Data Base.

3.2.2 The system checklist—considering other factors

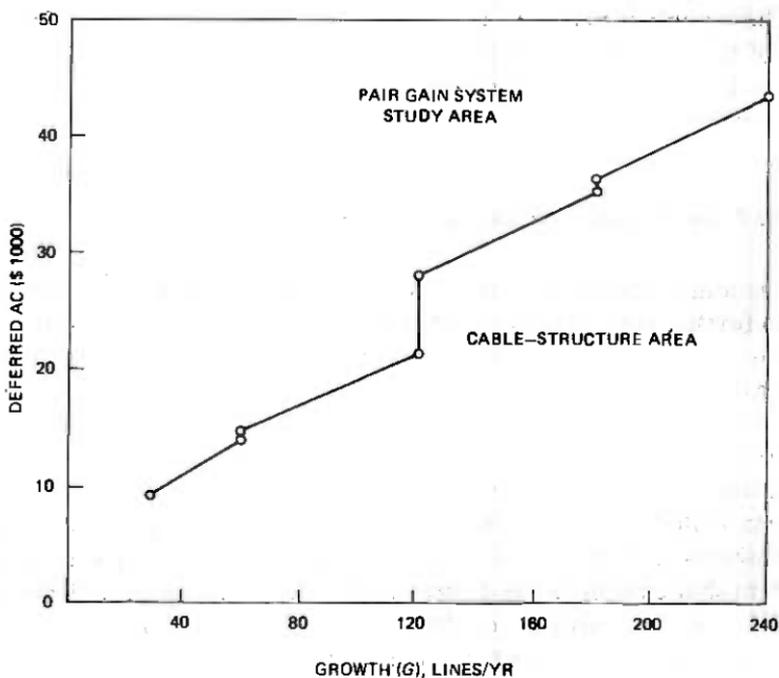
Economic guidelines can assist the engineer in eliminating systems from further consideration and in ranking, with a minimal effort, the remaining systems in order of their potential economic savings. Factors other than economics must be considered, however, such as the physical, service, and technical features of the proposed pair gain system. The operating companies have been provided with a standard format for creating comparison charts of these features for each system. These charts, called system checklists, are usually provided with the curves themselves and are used to guide the engineer in choosing the pair gain system that combines the desired and required features with good potential economic savings for the specific problem at hand.

In this way, the engineer can make the decision to eliminate pair gain systems from further study or to choose one system to study, with a minimal effort for each relief problem. The speed of this method allows the engineer to consider pair gain solutions for relief problems on many

routes, without spending a lot of time. The engineer can then efficiently select the alternatives to be studied in detail with a high degree of confidence that these alternatives will result in maximum savings.



(a)



(b)

Fig. 6—(a) Permanent guidelines (sample system). (b) Temporary guidelines (sample system).

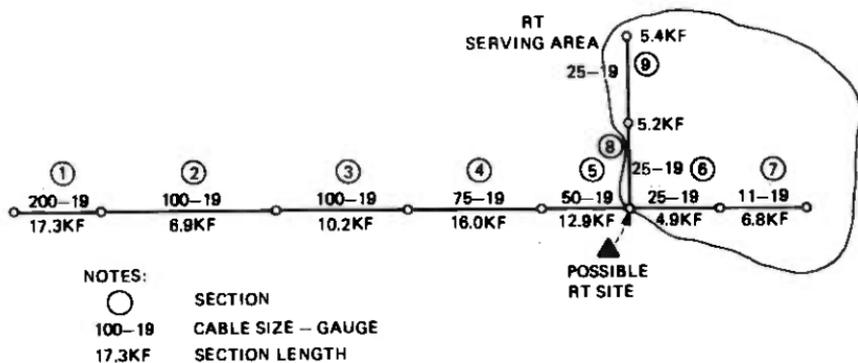


Fig. 7—Route example.

3.2.3 Detailed economic studies of permanent appreciations

When the use of the guidelines of 3.2.1 results in a decision to study a particular pair gain system for a particular relief project, the engineer uses one of a family of available study techniques. These include fully

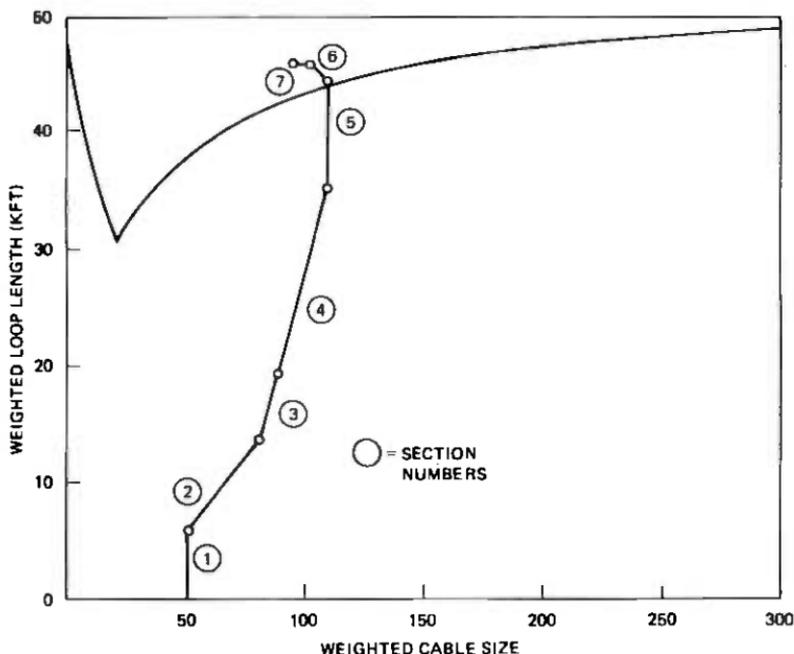


Fig. 8—Permanent economic guidelines plot of example route.

mechanized computer techniques such as LCAP (Loop Carrier Analysis Program) for distributed⁷ systems or LFRAP (Long Feeder Route Analysis Program) for lumped⁷ systems. These programs, for use on rural routes, provide the engineer with a computer-generated relief plan that, with minimum PWAC, satisfies all facility shortages over the entire study interval, using a mix of cable and pair gain and then compares that solution to an all-cable solution. The all-cable solution may include the use of voice frequency range extension devices, if desired. The engineer uses these computer results and practical knowledge of the specific project to develop a realizable and economic relief plan.

As an alternative to LCAP/LFRAP, the engineer can develop relief plans manually and test the PWAC of each with any one of a variety of cost analysis programs available to Bell System companies. These include TOPPS (Time Share Outside Plant PWAC Studies), EASOP (Economic Alternative Selection for Outside Plant) and CUCRIT (Capital Utilization Criteria). With the manual approach, the engineer has more control over the design of the relief plan, but the plan may not achieve the optimal PWAC savings obtained via LCAP/LFRAP. In either case, the engineer must insure that, in addition to economic savings, the relief plan meets all physical, service and technical requirements for the project.

3.3 Study methods for temporary applications of pair gain systems

3.3.1 Economic guidelines for temporary pair gain applications

Guideline curves for temporary applications are break-even annual charge curves that compare the annual charge of deferring a construction project for one year with the annual charges of the construction project itself. Deferrals that cannot save money for one year are usually uneconomic for longer periods. The guidelines, therefore, can eliminate pair gain systems from further consideration by indicating negative savings in the first year. These temporary application break-even curves do not depend on a model of the route, but are rather a direct plot of the annual charges associated with accommodating a particular growth rate through use of the pair gain system. [See Fig. 6(b).] As with permanent curves, temporary guideline curves can be created and published by telephone company staffs for any pair gain system eligible for use and for local variations in installed costs.

To use these curves, the engineer determines both the annual charge of the project to be deferred and the associated growth and then plots the associated point on the graph. The difference between this point and the break-even curve is the estimated saving for one year. Points above the curve indicate positive savings. The engineer can easily investigate

the effect of deferring parts of a project as well as the difference in savings because of the use of different systems. As with permanent guidelines, this procedure usually takes 15 minutes or less.

The system checklist, described in 3.2.2, is also used in temporary studies to insure that any pair gain system selected from the guidelines would meet all route requirements.

3.3.2 Detailed economic studies of temporary appreciations

The currently recommended approach to temporary studies is to manually develop relief plans and to test the PWAC of each plan with a cost analysis program as described in 3.2.3. This manual effort is complicated by the fact that the extra degree of freedom added to temporary plans by the unknown deferral interval makes temporary plans more difficult to optimize than permanent plans.

The guidelines process includes a method for estimating the deferral interval that maximizes PWAC savings. This estimate can be used as a starting point in manually generating relief plans for computerized cost analysis with TOPPS, EASOP, or CUCRIT.

The manual development of relief plans for temporary applications allows the engineer to include physical design considerations as an intrinsic part of the plan. Typical physical design considerations are discussed in 3.4.2. This takes on more significance in temporary studies than in permanent studies because temporary applications are often found in suburban or light urban areas where physical design considerations can be quite constraining. Manual plan development for temporary applications offers the engineer the fine control needed to apply pair gain systems in these areas.

3.4 Relief decisions

The engineer uses the results of the studies described above to develop a relief plan. This plan is based on both economic and practical parameters. These decision criteria are described in more detail below.

3.4.1 Economic decision criteria

The basic economic criterion on which relief decisions are based is PWAC. This allows the engineer to include the long term effects of the plan as well as differences in annual expenses, such as maintenance and administrative costs. The consideration of annual costs allows the engineer to account for differences among capital cost intensive plans that result in different depreciation schedules, tax credits or relief timing. Minimum PWAC is the optimization criterion used in LFRAP and should be the prime economic factor in any relief decision.

Another important economic criterion is "early" IFC (Installed First Cost), that is, the capital costs associated with the plan in the first few years. This can be important because of construction budget constraints that may limit the amount of capital available in any year. The effect of such a constraint may be the choice of a pair gain plan over a cable plan, even when the long-term PWAC of the cable plan is lower. This could occur if the size of the pair gain system in the plan allows the placing of relief in smaller line increments than the cable in the plan. Similarly, budget constraints may tend to favor the placing of smaller pair gain systems in order to more closely match the facility requirements and keep first costs low.

When budget constraints are important, but not controlling, combinations of PWAC and IFC can be used as decision criteria. This allows the engineer to consider the long term costs and varying expenses between plans, using PWAC, while also accounting for total capital costs, and with the early years emphasized.

Since local constraints must always be considered, the economic decision criteria to be used are a local decision. In all cases, however, the effects of today's decisions on future decisions should be weighed.

3.4.2 Physical decision criteria

The physical constraints of the relief problem are as important as the economic criteria described above. The feasibility of a pair gain solution is usually insured by the use of the guidelines and system checklist. However, physical constraints can have a significant impact on the details of the relief plan.

One important physical constraint applies to the location of the remote terminals of the pair gain system. The economically optimum site location may not be practical because of too few assignable customers beyond the site or because of insufficient or unsuitable trunk facilities to the site. In the first instance, there may be too few customers available in any one year to sufficiently fill the system to utilize the available pair gain. In the second instance, existing facilities up to the proposed RT site may not be sufficient to allow a smooth cutover to the system without temporarily taking customers out of service or the facilities may be in such poor condition that transmission would be unreliable. In either case, a new RT site may have to be chosen, changing the economics.

Physical factors such as right-of-way, available space, building codes or highway safety regulations may also affect RT placement. In addition, the administration of the RT serving area (see Fig. 7), in both the short and long term, must be considered. RT placements can affect the transmission design of cable beyond the RT, which in turn can have both administrative and capital cost effects not previously considered.

In each of these instances, the economic plan must be adjusted and

reevaluated to assess the impact of satisfying these constraints. The engineer has the same tools available for restudy as for the initial study, but because of limited time, often uses abbreviated methods to evaluate differences in costs. Fortunately, the economic impact of meeting the reality of the physical world, while not insignificant, is often not decisive. While details may change, the most economic relief plan usually remains relatively unchanged.

3.5 Unusual circumstances and innovative solutions

The preceding discussion was based on the engineer's encountering problems amenable to standard solutions and study techniques. Although the possible combinations of cable and pair gain systems provide great flexibility in solving problems and, although the study techniques are general in nature, the engineer is often faced with atypical problems. When this is recognized, the engineer must innovate. Today's innovations, of course, may well become tomorrow's standard applications.

Examples of these situations, by their very nature, cannot readily be hypothesized. Existing examples can be described, however. For example, when Mountain Bell was faced with the need to convert PBX facilities to centrex CO at a large Idaho installation, they chose to use digital carrier (*SLC-40*) to provide the additional circuits between customer premises and the serving central office. In another instance, Bell of Pennsylvania deferred a wire center in the Pocono Mountain resort area by using *SLM* and, later, *SLC-40*. Since these installations were planned, the innovative use of pair gain to provide centrex CO service or to defer wire centers has become more commonplace and is now considered a standard application.

IV. ILLUSTRATIVE PLANNING EXAMPLE

Because of the high inflation rate in the early 1970s, the cost of installing conduit in suburban areas was becoming prohibitive. Several operating companies began using the 80-line *SLM* system. This system was originally configured for rural application and was not ideally suited in terms of cost and module size for widespread permanent or temporary suburban use.

As a consequence, at least one operating company contacted AT&T and Bell Labs on the possibility of designing a new pair gain system whose characteristics would be better suited for suburban use. In response, AT&T commissioned a joint application study with this operating company. Bell Labs analyzed the use of a new 96-line, 32 trunk, subscriber loop concentrator for temporarily deferring conduit additions on sixteen selected feeder routes. This preliminary analysis showed that, for maximum economic savings, the concentrator should be composed of not one but two 96-line modules. A subsequent economic analysis

confirmed that this postulated two-module system would indeed be more effective in deferring conduit additions.

This conclusion was reached through an analysis of feeder routes from a single operating company. To extrapolate this conclusion to other operating companies, AT&T conducted a survey of five additional companies. That survey substantiated the proposal that a 192-line loop concentrator could generate large PWAC savings when used on suburban feeder routes. This system has now been developed by Bell Labs and is currently being manufactured by Western Electric. It is the Loop Switching System that is described in this issue of the Bell System Technical Journal.⁸

The application study mentioned above resulted not only in the development of the LSS, but also indicated that other pair gain systems, such as the *SLM* and *SLC-40* systems, could also be used in conduit deferral applications. The study also resulted in the development of the temporary application guideline curves that were described in Section 3.3.1.

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Loop Plant Electronics:

Maintenance and Administration of Loop Electronics

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The economic advantages of loop electronic pair gain systems are clear; however, we must also be concerned about the impact of these systems on service reliability. This paper reviews the maintenance philosophy, procedures and features for the SLC™-40, SLC™-8, and LSS systems. It is shown that these systems are designed to have a minimal impact on customer trouble report rates and operating company maintenance functions. Recent field data on SLC-40 trouble report rates is presented to demonstrate the viability of the maintenance plans. In addition, the administrative procedures for pair gain systems are discussed.

I. INTRODUCTION

A watchword in our telephone system today is reliability. Since the loop is an element in every customer's telephone circuit, its reliable performance is essential. However, when loop electronic devices such as pair gain systems are employed, equipment is added to the loop and the potential for failure is increased. It is one purpose of this paper to present the maintenance philosophy, procedures and features for the SLC™-40, SLC™-8, and LSS systems. The other purpose is to describe the administration of such loop pair gain systems.

Since pair gain systems encompass only a small percentage of loops in any operating company district, maintenance procedures must be designed to fit within the existing operational structure. Therefore, it is appropriate to briefly define the current loop operations environment.

1.1 Loop operations environment

In the loop area the Customer Services Department is responsible for assignment, installation and maintenance of telephone service. The assignment of a particular facility to a customer's order for telephone service is accomplished by the Assignment Center. Assigners use records of cables, crossconnection boxes, and distribution cable terminals to select a spare facility from the customer's home back to the serving central office. Their facility assignment is then forwarded to the installation forces for completion.

The installation forces are responsible for installing the customer's station sets, wiring his home, installing drop wires and interconnecting cable pairs in crossconnection boxes. The station repair forces usually respond to customer trouble reports, and are responsible for maintenance of station sets and household wiring.

The RSB (Repair Service Bureau) has the responsibility for receiving and acting on customer trouble reports. Customer trouble reports are received by the repair attendant who records the customer's complaint, retrieves the customer's line record card and passes the trouble report to a dispatch clerk if the fault is obvious (such as a worn station set cord). Otherwise, the trouble reports go to the test desk where the customer's line is tested to determine if the dispatch of a station repairman or some other craft is necessary. In cases where the test desk finds no trouble, the complaining customer is contacted, and if agreed to by the customer, no further action is taken. The dispatch group functions as the interface between the repair forces and the RSB. They dispatch trouble reports to the station repairmen, receive the results of the repair visits, close out the trouble report, and post the line card records with this trouble report information.

1.2 Maintenance and service objectives for loop electronics

Our objective is to develop a maintenance plan for loop electronics that is compatible with the present operational environment. To achieve this goal, the assignment of maintenance forces to loop electronics must be carefully considered. One must weigh the craft skill level against the availability of the craft. For example, switchmen may not always be equipped for travel to remote terminal sites and PBX repairmen may not exist in some rural areas. In short, the assignment of maintenance responsibilities is partly based on the local environment.

One major objective of the Customer Services Department is to insure that reliable service is provided to telephone customers. As a measurement of service reliability, customer trouble report rates are carefully monitored in the RSB. Although customer trouble report rates are a complex function of environment, cable condition, customer attitudes and many other parameters, it is a major index that is used to judge the performance of the Customer Services Department.

Since subscriber loop electronics adds equipment to a customer's loop, a degradation in overall loop reliability is possible. It is our objective to offer reliability on electronically provided loops that is virtually equivalent to the reliability of loops provided by cable pairs. It is also our intent that this equivalent reliability results in an equivalent customer trouble report rate. In order to accomplish our goal, the inherent reliability of the electronic equipment must not only be high, but maintenance procedures and features must be carefully developed to minimize the customer trouble report rate.

It is one purpose of this paper to show how the maintenance plans for our pair gain systems (*SLC-40*, *SLC-8* and *LSS*) accomplish this goal. The discussion of specific systems is contained in Sections II through IV. In addition, results of a customer trouble report rate study on the *SLC-40* system are presented in Section V. Section VI covers the administrative plan for pair gain systems.

II. *SLC-40* MAINTENANCE PLAN

The *SLC-40* system is a 40-channel digital subscriber carrier system, described in a companion article.¹ For the purpose of discussing the *SLC-40* maintenance plan, refer to the simplified block diagram in Fig. 1. In this figure the central office terminal (COT) and remote terminal (RT) each consist of 40 channel units and common equipment. The channel units in the actual *SLC-40* system are individual plug-in circuit boards that encode and decode speech signals to and from digital form. These units also provide the necessary signaling functions. The common equipment consists of plug-in circuit boards and power supplies. Its primary function is to transmit the channel unit signals over the repeatered line. The T1-type digital repeatered line includes a spare re-

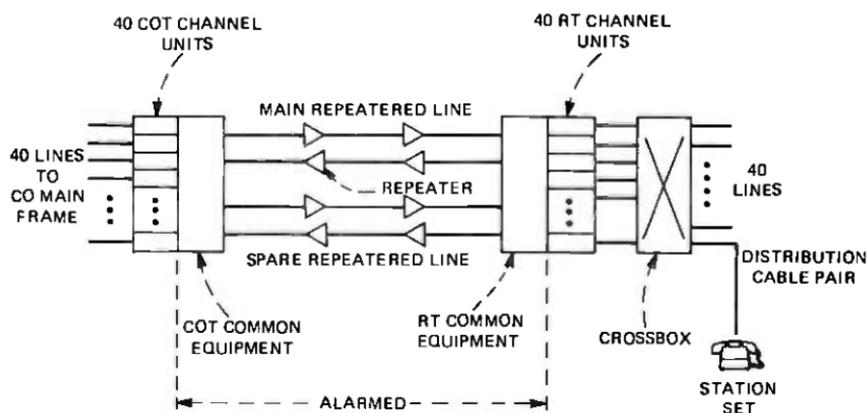


Fig. 1—*SLC*[™]-40 system block diagram.

peated line that is automatically switched in case of a main line failure. Both the main and spare repeated lines require a total of four cable pairs.

The remote terminal (RT) is placed somewhere along the cable route in the area needing additional loop facilities. At the RT is a crossbox which allows the installers to interconnect any *SLC-40* channel with any distribution cable pair. As will be described, the crossbox is also used for fault sectionalization.

As shown in Fig. 1, the loop circuit provided to a customer via a *SLC-40* system consists of the following major elements:

- (i) Central office main frame wiring
- (ii) *SLC-40* COT channel unit
- (iii) *SLC-40* COT common equipment
- (iv) *SLC-40* repeated line (repeaters and cable)
- (v) *SLC-40* RT common equipment
- (vi) *SLC-40* RT channel unit
- (vii) Crossbox
- (viii) Distribution cable pair
- (ix) Station set and household wiring

The above items are classified into two major failure modes—common system and single line failures. The common system failures are associated with the COT or RT common equipment and the repeated line. A failure in any of these elements affects the service provided to all or many of the customers on a *SLC-40* system. On the other hand, the remaining circuit elements only affect one customer's loop when they fail. If the *SLC-40* derived loop is compared to the usual voice frequency (VF) loop design, one sees that a VF loop has mainly single line failure modes. The only equivalent to a common system failure is a massive cable failure, such as a severance.

2.1 Common equipment maintenance

When a common system failure does occur it is essential that the outage time be minimized. The *SLC-40* common equipment and repeated line are fully monitored, and a failure causes an alarm to be displayed. The *SLC-40* alarms are interfaced with the standard central office alarm system, so that CO maintenance personnel are immediately notified of system failures. These alarms are classified as either major or minor. A major alarm indicates a disruption in service to a large group of customers on the system, while a minor alarm indicates a nonservice affecting fault, such as a switch to the spare repeated line.

When a major alarm occurs on the system, it is essential that the RSB be immediately notified by the CO of any *SLC-40* major alarm so that they can intelligently handle customer trouble reports. The flow diagram

in Fig. 2 summarizes the fault localization and repair process. Trouble isolation is begun at the COT where interpretation of the alarm display lamps (see Fig. 3), fuse conditions and power supply voltages can quickly indicate many COT failures. Further diagnostic information is obtained by "looping-back" the COT, that is, connecting its transmitting T1 rate port to its receiving port. In this mode the COT is isolated from the repeatered line and the RT, and it is tested for proper operation. If the performance of the COT in the "loop-back" mode is normal, the trouble is in the RT or repeatered line. The responsible maintenance craft must now visit the RT to determine if the fault is in the remote terminal or the repeatered line. If the fault is in the RT, it is repaired by a plug-in unit change. An alarm display at the RT (see Fig. 3) allows confirmation of proper system operation after replacement of a common plug-in unit. When the failure is located in the repeatered line, appropriate troubleshooting procedures are followed which identify the faulty repeater or cable section.

The maintenance of the COT will be the responsibility of the central office switchmen, and in many areas this craft would also be responsible for the RT and repeatered line maintenance. However, if the switchmen do not have access to company vehicles, PBX repairmen are assigned to RT and repeatered line maintenance. The PBX repairmen will be dispatched by the RSB after the switchmen have determined that the failure is not in the COT.

2.2 Single line failure maintenance

Unlike the common system failure, single line failures are not alarmed and are only detected through customer trouble reports received in the

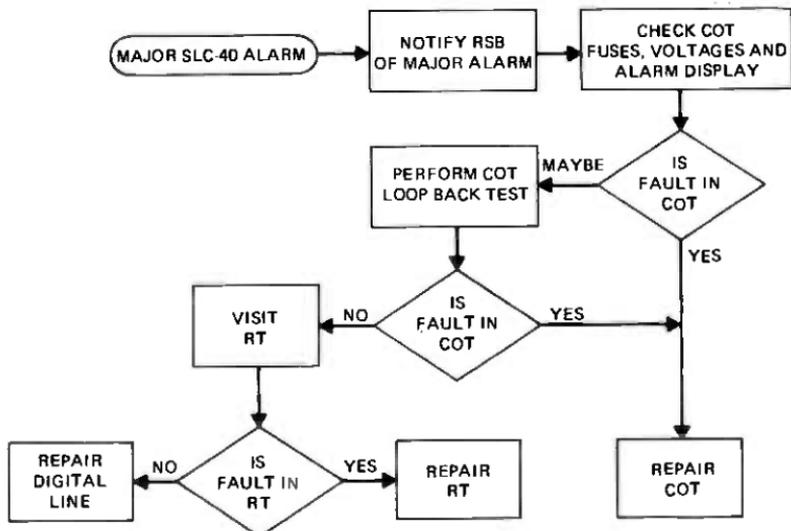


Fig. 2—SLC™-40 major alarm fault localization and repair.

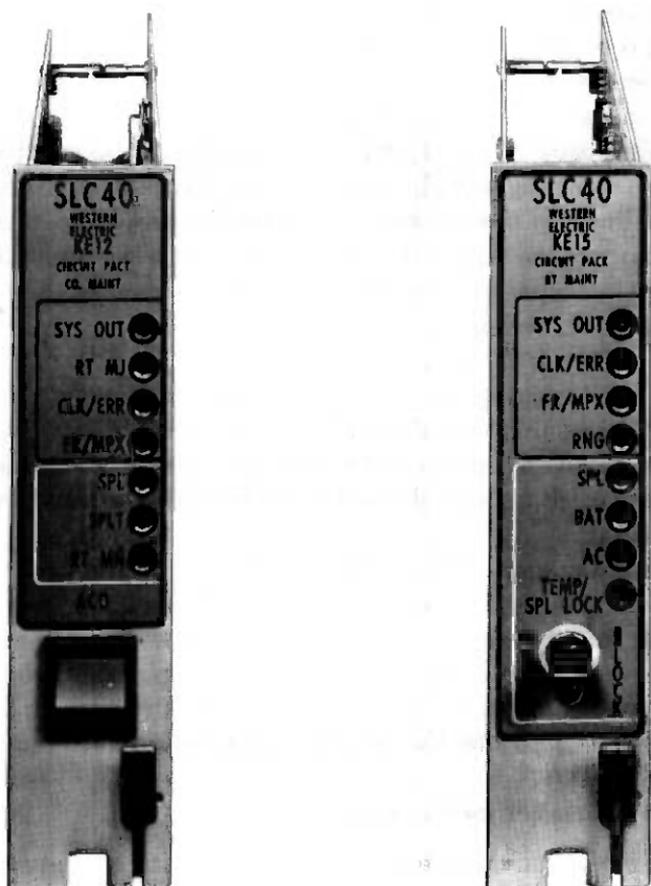


Fig. 3—*SLC*[™]-40 alarm displays.

RSB. When the trouble reports are received, the RSB first checks for alarms reported by the CO. If there are no alarms, the report is assumed to be related to a single line failure. Since the *SLC*-40 COT and RT channel units are very reliable, troubles are most likely in the station set, household wiring or distribution cable. Central office troubles can be easily detected through standard test desk procedures. However, the test desk cannot “see” any further than the COT. Through analysis of the reported trouble, customer’s trouble history and possibly conversations with the customer, the decision to dispatch a station repairman is usually made. We have found that the thorough system alarms and spare repeated line obviates some of the need for RSB testing, because a portion of the loop circuit is continuously being monitored.*

The station repairman treats this trouble report like any other. Generally he will first visit the customer’s premises, and if the trouble is

* Although the *SLC*-40 system does not have RSB line testing features, technology has advanced to the point where these features are economical and will be provided in future digital subscriber carrier systems.

confirmed, attempt to isolate the trouble by testing at the crossbox. If the trouble is not present on the *SLC-40* channel feeding into the crossbox, then the trouble is in the distribution cable pair. Conversely, trouble in the *SLC-40* channel is cleared by transferring the customer to an alternative facility or by having the proper craft replace the faulty channel unit. The station repairman does not replace plug-in units in the RT. He treats this trouble just as he would if the customer was entirely served by a cable pair. In this manner, training costs for *SLC-40* system maintenance are minimized.

III. LOOP SWITCHING SYSTEM MAINTENANCE PLAN

The Loop Switching System (LSS) is a 192-line concentrator consisting of two 96-line concentration systems. The LSS is described in more detail in a companion article,² but a basic block diagram is shown in Fig. 4. For purposes of discussion, only the 96-line system modules will be shown. The 96 lines are concentrated onto 32 trunks (usually VF cable pairs), by the switching network that is composed of twelve plug-in circuit boards called line units. Analogous to the *SLC-40* channel unit failure, an LSS line unit failure affects service for only a few customers (at most eight). The common control in the COT operates the COT switching network directly and the RT network through the data link (employing two pairs). Through the use of a microprocessor, numerous diagnostic capabilities have been incorporated into the LSS. In addition, the fully metallic switching network allows for complete loop testing from the RSB test desk.

The LSS routinely tests the trunk and switching path for continuity, leakage and insertion loss prior to establishing a customer's connection. This test routine virtually eliminates any customer trouble reports due to individual trunk pair failures. It is only in the rare event of a major cable fault that customers' service would be disrupted.

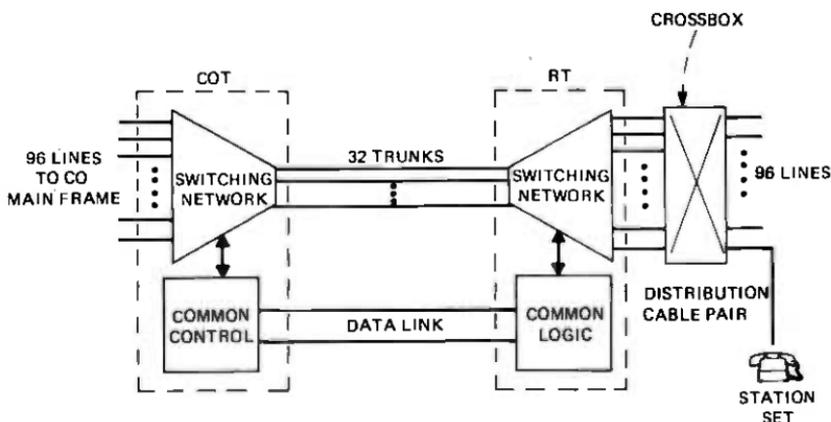


Fig. 4—LSS block diagram.

3.1 Common system maintenance

Naturally a failure of the common control units in the COT or the common logic in the RT can cause an entire system failure. Many data link failures are avoided by using two of the least used trunks as a spare data link. When a fault is detected, a switch will automatically occur. The LSS contains many diagnostic routines that result in a very detailed COT alarm display (see Fig. 5). For example, an alarm is present for each of the COT common control plug-in units. In many cases, the system can also determine whether the trouble is in the COT or RT. In addition, the test and display unit (Fig. 5) allows the craftsman to manually initiate trouble-shooting routines.

As with the SLC-40 system, both major and minor LSS alarms are interfaced with the CO alarm system so that switchmen are immediately notified of its failure. The LSS alarm status can be remotely monitored through a dial up alarm query feature. When dialed, this feature transmits audio tones indicating major or minor alarm status and location (COT or RT).

The diagnosis of system failures at either the COT or RT involves checking fuse conditions and power supply voltages and analyzing alarm conditions. The defective plug-in unit can be rapidly identified, changed,

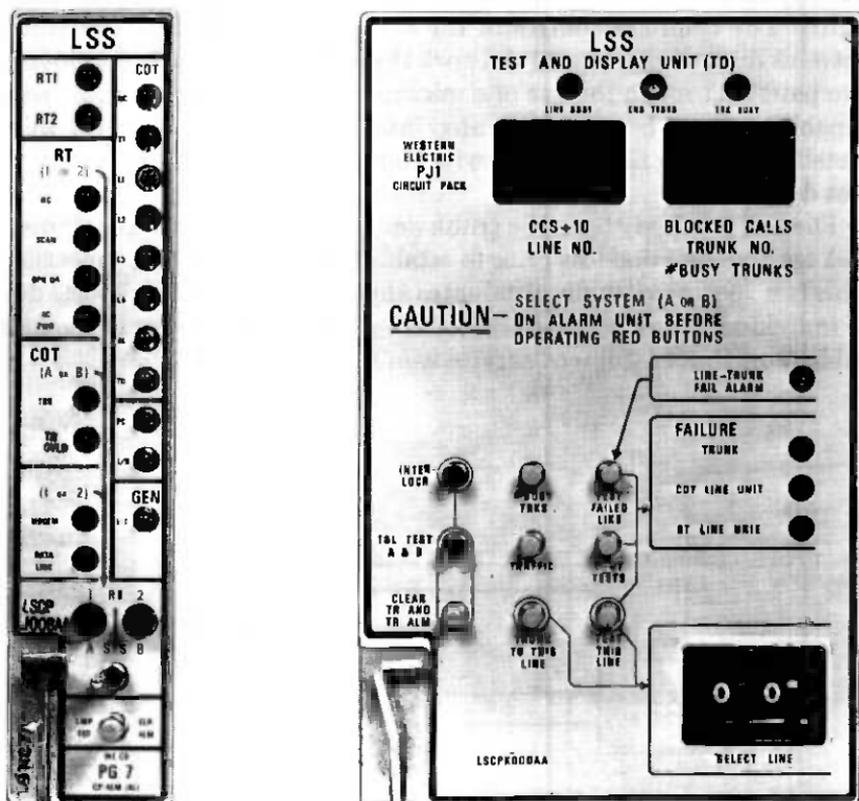


Fig. 5—LSS COT maintenance units.

and proper operation confirmed. There is a small alarm display in the RT maintenance unit to aid in confirmation of system operation (see Fig. 6). This maintenance unit also allows the repair personnel at the RT to initiate the automatic line-trunk testing routine. In this manner, the operation of individual line units and the total system can be very quickly confirmed.

Like the *SLC-40* system, the COT is always maintained by CO switchmen, and the RT may be maintained by switchmen or PBX repairmen.

3.2 Single line failures

Single (or multiple) line failures in the switching network or trunks



Fig. 6—LSS RT maintenance unit.

will almost always be detected by the LSS and result in a minor alarm. The test and display unit allows these network or trunk failures to be read out of the microprocessor's memory. This unit also enables a rapid retest of stored failures by manual commands.

The system will attempt to maintain the customer's telephone service by selecting good trunks or network paths, but in some cases customer trouble reports will result. Also trouble reports will be due to failures in the crossbox, distribution cable pairs or station set. However, these faults can be detected by the test desk and the proper maintenance craft dispatched. As with the *SLC-40* system, very few customer trouble reports are expected to be due to LSS failures, and dispatch of the station repairman is most likely.

When major system failures do occur, the CO is instructed to notify the RSB. However, if this notification does not occur, the test desk would not "see" the loop beyond the COT. This clue indicates a system failure that can be confirmed through the alarm query feature.

IV. *SLC-8* MAINTENANCE PLAN

The *SLC-8* system is an eight-channel analog subscriber carrier system. Through frequency division multiplexing, eight two-way speech channels are transmitted on a single cable pair. A companion paper describes the system in more detail.³

Two versions of the *SLC-8* remote terminal are available—lumped and distributed. The lumped version contains all RT equipment for one system in a single cabinet, while the distributed system allows the RT equipment for each channel to be placed separately along the carrier frequency pair.

Figure 7 shows a block diagram of the lumped *SLC-8* system. The COT consists of eight channel units and one common plug-in circuit board.

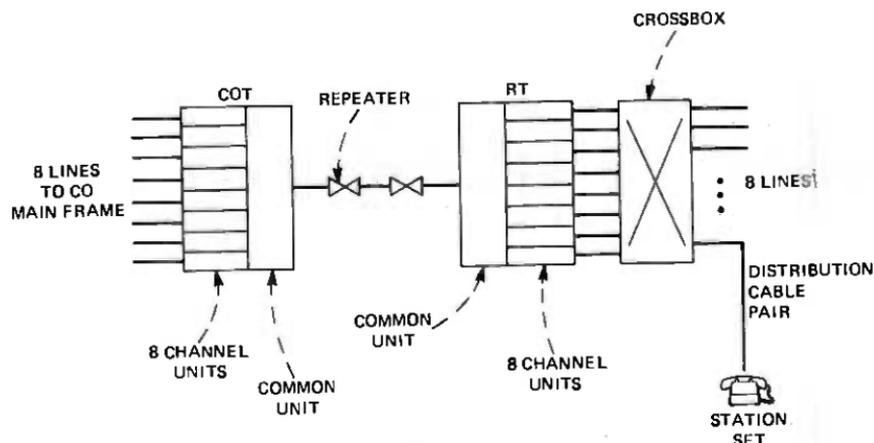


Fig. 7.—Lumped *SLC*[™]-8 block diagram.

The common unit provides power to the repeaters and RT, and also serves as the interface between the channel units and the cable pair. The RT consists of an arrangement of plug-in circuit boards that perform a function very similar to the COT. The bidirectional repeaters are located periodically along the carrier pair. As with the *SLC-40* and *LSS*, the lumped RT interconnects to the distribution cable pairs through a crossbox.

The distributed *SLC-8* system, shown in Fig. 8, allows the channel units to be individually placed at the location of the customer to be served. The channels do not work through a crossbox or distribution cable pair, but rather connect directly to the customer's drop wire. Each channel unit and a power unit are placed in a closure. This combination is referred to as a distributed RT.

The distributed and lumped *SLC-8* systems are maintained in similar manners. Therefore, the following description of maintenance operations applies to both versions.

The *SLC-8* maintenance plan differs somewhat from the *SLC-40* and *LSS* plan, largely because a comprehensive set of alarms are not economically feasible on analog subscriber carrier systems. Also since these systems are small in line size, they are usually installed in a scattered manner along sparse rural routes, and are most efficiently maintained by station repairmen in the field. CO switchmen will maintain the central office terminal.

The *SLC-8* has two alarms—one due to COT power supply failure and the other due to carrier line short circuits. However, most *SLC-8* failures will be identified through customer trouble reports. The *SLC-8* system has a feature that allows the operation of individual channels to be checked from the RSB test desk. If a channel fault is detected, other channels can be tested to determine if a system failure has occurred. Essentially, the channel test feature is used on *SLC-8* to gain the same information provided on *SLC-40* by its alarms.

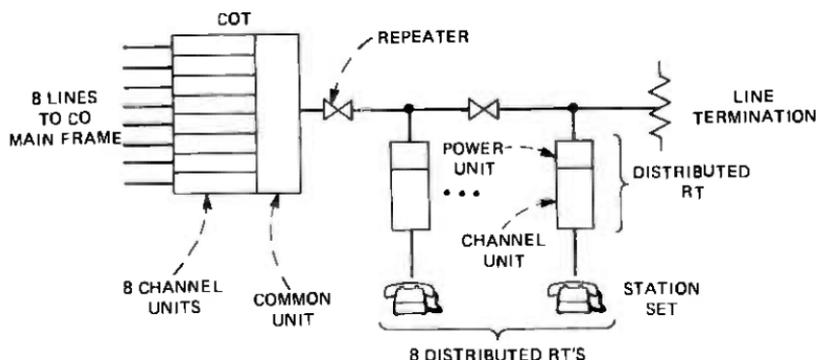


Fig. 8—Distributed *SLC-8* block diagram.

Many of the *SLC-8* failures will involve replacing COT or RT plug-in units; however, failures will also occur in the carrier pair or repeaters. These troubles are located by using a distributed RT as a test set. The distributed RT is placed at various points along the carrier line to isolate the fault location. In cases of more obscure troubles, a frequency selective voltmeter may also be used. Since repeaters are single plug-in units, they are easily replaced when defective.

V. FIELD PERFORMANCE DATA

In order to demonstrate that a maintenance plan is viable, actual field performance data must be analyzed. The *SLC-40* system will be used as an example of the ability of pair gain systems to provide telephone service as reliable as that provided only over cable pairs. Although "black box" reliability estimates for the *SLC-40* system indicated that the customer trouble report rate performance would be acceptable, studies in an actual operating environment provide conclusive results.

The *SLC-40* customer trouble report rate data was taken on the first seven systems placed in service. The data gathering began a few months after the systems were installed and continued for one year. To evaluate the *SLC-40* customer trouble report rate, a control group of voice frequency (VF) lines was also studied. The VF control group was selected from lines in the same area of the same cable route as the *SLC-40* customers. In all ways the control group customers were as similar as possible to the *SLC-40* customers.

Table I gives a comparison of the customer trouble report rate for the VF control group and the *SLC-40* group. The *SLC-40* report rate appears slightly higher, but this difference is not statistically significant. Essentially the *SLC-40* customers were provided equivalent service to the VF control group customers. Customer trouble reports associated with *SLC-40* equipment and feeder cable failures are explicitly shown. In the other trouble report categories, the rate in both groups is roughly equal since these reports are not related to the carrier system performance. These categories include such things as station set, distribution cable and central office failures.

Since the beginning of this study a few minor design and documen-

Table I — Observed data, *SLC*TM-40 customer trouble report rate (customer trouble reports/100 stations/month)

	VF	<i>SLC-40</i>
Feeder cable	1.0	0.1
<i>SLC-40</i> COT	—	0.7
<i>SLC-40</i> RT	—	0.9
Repeaters	—	0.1
Other	4.3	3.9
Total	5.3	5.7

tation changes were made in the *SLC-40* system. These changes resulted in improved reliability and more efficient system repair. Although the attendant reduction in customer trouble report rate is small, the data was adjusted to account for these changes. The VF rate was also adjusted downward to account for an inequality in the distribution cable trouble report rates that was due to slight environmental differences between the control and *SLC-40* groups. Table II shows the adjusted comparison between customer trouble report rate of the VF control group and the *SLC-40* group.

It is interesting to note in Tables I and II that the feeder cable trouble report rate for the *SLC-40* group is much lower than in the VF group. The feeder cable portion of the *SLC-40* loop circuit is used for the T1 carrier line and has a redundant path (spare line) that is automatically switched to in the event of a main line failure. Therefore, many cable failures that affect only a small number of pairs (usually one) will not result in the interruption of a customer's service. The same type of failure in the VF control group does result in an outage, hence, the higher feeder cable report rate in the VF group.

This reduction in the feeder cable trouble report rate offsets the *SLC-40* equipment related trouble report rate, and results in an overall report rate that is essentially equal. The slightly lower *SLC-40* trouble report rate in Table II is not statistically significant at a 95 percent confidence level.

We expect the data in Table II to be representative of the current *SLC-40* product. When analyzed in detail, the data not only show an equivalent level of customer service, but also demonstrate the ability of telephone company personnel to maintain the system.

VI. ADMINISTRATION

As mentioned in Section I, it is the Assignment Center that administers the loop facilities. Since the Assignment Center employs well established standard procedures and records, it is the objective of this section to show how subscriber pair gain systems fit within the existing structure. In order to describe the function of the various assignment records, we will first trace through the assignment process for the usual cable pair facility.

Table II — Adjusted data, *SLC*TM-40 customer trouble report rate (customer trouble reports/100 stations/month)

	VF	<i>SLC-40</i>
Feeder cable	1.0	0.1
<i>SLC-40</i> CO ^T	—	0.2
<i>SLC-40</i> RT	—	0.4
Repeaters	—	0.1
Other	<u>3.9</u>	<u>3.9</u>
Total	<u>4.9</u>	<u>4.7</u>

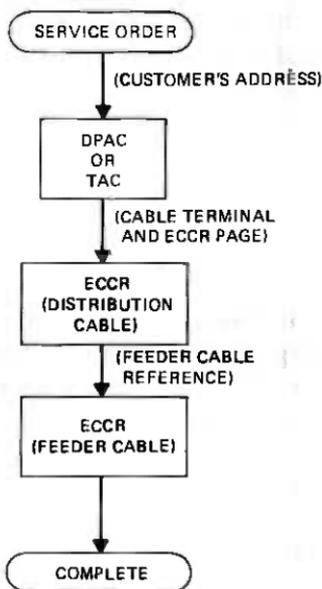


Fig. 9—Conventional assignment process.

Figure 9 illustrates the assignment process, which has been simplified to convey the basic concepts. The facility being assigned is shown in Fig. 10. The customer's service order must first be associated with a cable terminal and distribution cable. Using the customer's address, the DPAC (Dedicated Plant Assignment Card) or TAC (Terminal Address Card) provides the identifying number of the closest cable terminal. The cable terminal is the means by which the customer's residence (or business) is connected to the distribution cable via a drop wire. Also provided by the DPAC or TAC is the page reference for the distribution cable ECCR (Exchange Customer Cable Record). The ECCR is the record of cable pairs, and the page referred to by the DPAC or TAC shows the association of cable pairs to the cable terminal of interest. From the ECCR a distribution cable pair is selected and assigned to the customer.

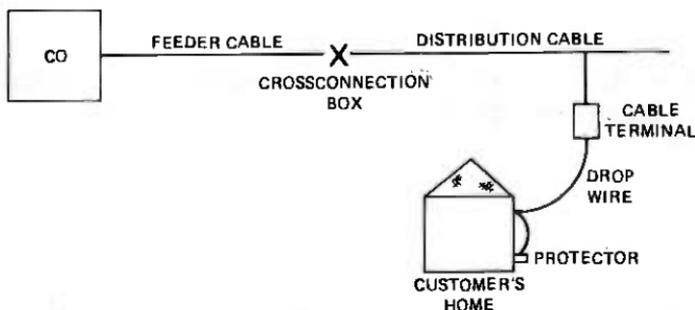


Fig. 10—Assignment example.

In this example, we are assuming that the distribution cable must be interconnected to the feeder cable at the crossconnection box. Therefore, the distribution cable ECCR provides a reference to a group of feeder cable pairs that "feed" the crossconnection box. The feeder cable ECCR is referred to for selection of a vacant feeder cable pair. The assignment of the distribution and feeder cable pairs uniquely defines the interconnection wiring that must be performed by the installer in the crossconnection box.

Now let us assume that we are dealing with assignment of a lumped pair gain system (such as an *SLC-40* system). Since lumped pair gain systems always have their remote terminals placed in association with a crossconnection box, their assignment is analogous to feeder cable. The pair gain system's line or channel provides the facility between the CO and the crossconnection box. In fact, the distribution cable ECCR will refer to the pair gain system as one of a number of feeder facilities. However, instead of using an ECCR, the pair gain system is administered with a Miscellaneous Central Office Facilities Record (MCOFR). The MCOFR performs the same function for pair gain systems that the ECCR performs for feeder cable pairs. In addition, it allows easy administration of the pair system's plug-in channel or line units. Therefore, the assignment flow for lumped pair gain systems, shown in Fig. 11, is very similar to the normal process.

For systems like the distributed *SLC-8*, the remote terminal equipment is associated with distribution cable terminals. Therefore, the above procedures cannot be used for distributed pair gain systems. In this case more specialized procedures must be employed that essentially treat all customers on the distributed system as if they were all assigned to a multiparty line. Although this represents a departure from con-

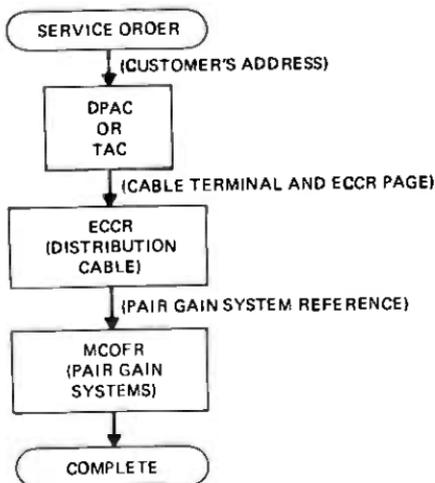


Fig. 11—Lumped pair gain system assignment process.

ventional assignment procedures, distributed systems are only used in very sparse rural areas. Hence, their penetration in any Assignment Center is not high.

VII. CONCLUSIONS

This paper has presented the maintenance plans for *SLC-40*, *LSS*, and *SLC-8* systems. These plans are designed to insure customer trouble report rates equivalent to that provided by VF cable pairs, and to fit easily into the existing loop operations environment.

Through actual field performance data on *SLC-40* systems, it has been demonstrated that our maintenance goals for that system have been met.

VIII. ACKNOWLEDGMENTS

I wish to acknowledge my coworkers at Bell Laboratories and AT&T who have contributed to the *SLC-40*, *LSS*, and *SLC-8* maintenance plans. Special recognition also goes to W. P. Arvidson of Bell Laboratories who provided the *SLC-40* field performance data.

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Loop Plant Electronics:

Voice Frequency Electronics for Loop Applications

By G. T. HAWLEY and K. E. STIEFEL

(Manuscript received October 21, 1977)

Over the past 100 years the telephone loop connecting central office switching equipment and the telephone customer's premises has evolved from aerial open wire to buried plastic-insulated twisted pairs. In the last 20 years the use of electronic circuits for range extension has become popular. These circuits allow use of higher resistance loops (finer gauge wires) than central offices are normally designed to accept. This paper reviews some of this history, describes rural and suburban systems of voice frequency range extension, presents requirements and circuit design considerations, and compares several electronic range extender circuits currently used to implement these systems.

I. GENERAL

The term voice frequency loop electronics refers to a family of equipment, usually central-office-mounted, intended to aid the operation of subscriber loops having much higher resistance than provided for in standard switching equipment designs of the past. This allows telephone companies to reduce outside plant expenditures through the use of cable of finer gauge than might otherwise be used. Such circuits are generally referred to as range extenders or loop extenders, indicating the function of extending central office loop resistance range by aiding signaling, supervision, and transmission functions. The use of such circuits dedicated to individual loops is particularly appropriate in rural areas where cable routes of 5 to 20 miles in length are common. By integrating range extension capability into the switching system, so that several lines share

a single range extender, voice frequency loop electronics can be economically applied in many suburban areas, as well.

These loop designs allow relatively simple prescription of electronic treatment for high-resistance residential facilities. They do not in general apply to special business services like PBX trunks or to coin station lines of high resistance. These latter facilities usually require engineering design attention to satisfy precision loss or complex signaling requirements.

This article gives some of the history of voice frequency loop electronics in the Bell System, reviews rural and suburban range extension systems, outlines the requirements that range extenders must satisfy, and describes in detail some Bell System circuits tailored to the rural and suburban loop environments.

II. HISTORY OF VOICE FREQUENCY LOOP ELECTRONICS

2.1 Rural areas

Within a few years of the invention of the telephone, the open wire pair became the dominant medium for telephone transmission. The rapid growth of telephone demand in urban areas quickly caused the problem of open wire congestion in cities, fostering the development of cables. Improvements in relay sensitivity and cable insulation made it possible to use cable finer than the original 18 gauge that was intended for universal use. The high cost of copper encouraged the use of 22 gauge, 24 gauge, and finally 26 gauge cable as early as the 1920s in the cities.¹ A four gauge family (including 19 gauge) is almost universal in loop applications to this day, although aluminum cables have been used to a small extent in recent years. The trend toward finer gauges is illustrated in Fig. 1.

In rural areas demand for telephone service generally lagged behind that in urban areas until well after World War II with open wire continuing to be used widely as the principal loop transmission medium. Open wire with its low resistance and low capacitance per unit length has been a very efficient loop facility not ordinarily requiring range extension. In the 1950s increased rural demand for service coincided with the introduction of polyethylene insulation offering the prospect of water-resistant wire and cable needed to obtain low maintenance costs on long cable routes.² As cables and plastic-insulated wire began to replace open wire, it became apparent that even 19 gauge copper wire, the coarsest gauge commonly available in cable, limited signaling, supervisory, and transmission range of existing central offices to a maximum distance of 50,000 to 80,000 feet depending on the vintage of switching equipment. To serve longer loops required the continued use of open wire.

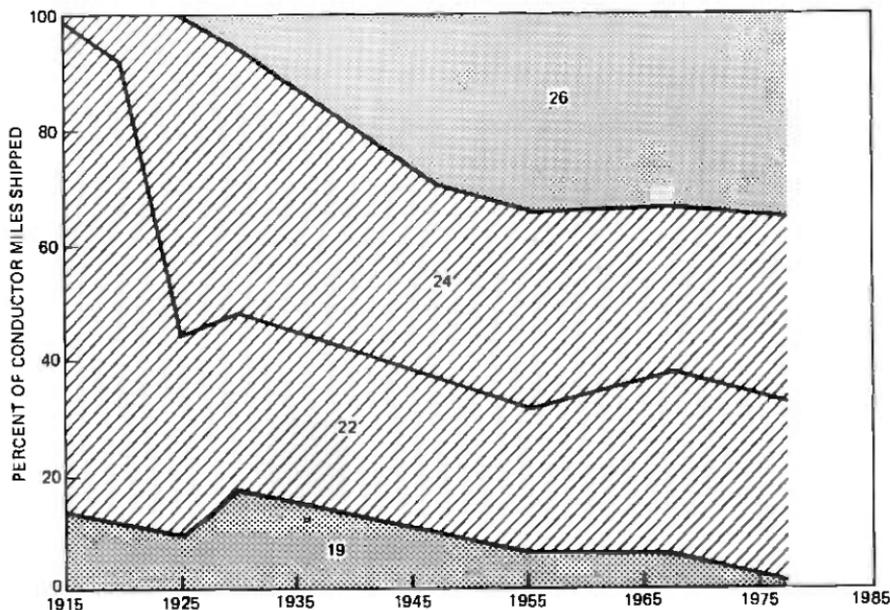


Fig. 1—Recent shipments of exchange cable by Western Electric by cable gauge.

To be able to remove open wire altogether, it was necessary to improve central office switching system performance, first by increasing the sensitivity of relays that sense the state of the telephone switch-hook on originating calls and terminating calls and during dial pulsing. Rather than overhaul an entire switching system, sensitive circuits called "dial long lines" (DLL) were interposed between the loop termination on the main distributing frame and the central office line equipment as required. These DLL circuits incorporated repeat coil transformer isolation between loop and central office equipment. Relay designs, generally more sensitive (and more expensive) than those in the switching system were used to sense telephone set switch-hook states. The DLL supplied dc voltage for supervisory, signaling, and talking current. This voltage was usually higher than the normal 48-V central office battery voltage. Figure 2 is a block diagram that illustrates this principle. Most step-by-step switching systems, No. 1 ESS and some No. 5 crossbar systems can signal and supervise to a maximum conductor loop resistance* of

* Conductor loop resistance includes an allowance for central office wiring and station drop wire as well as the resistance of the outside plant cable from the central office to the drop wire terminal. Total loop resistance is the conductor loop resistance plus the resistance of station equipment and wiring (usually 200 ohms or less).

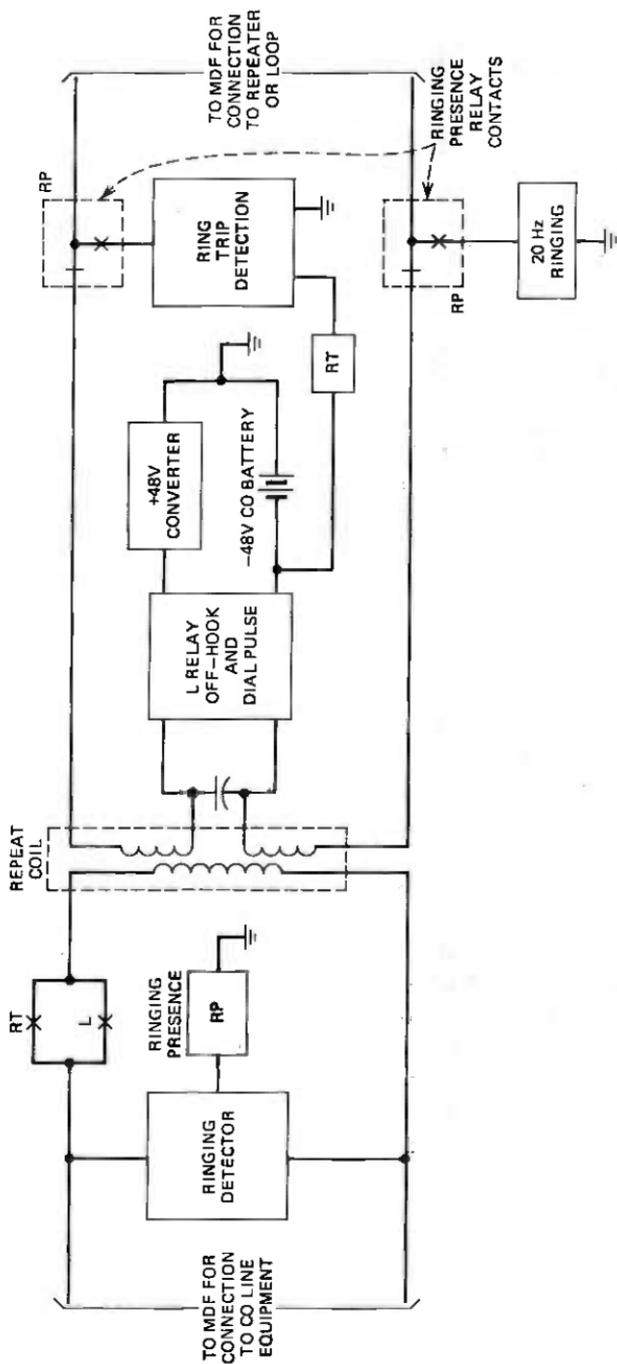


Fig. 2—Typical Dial Long Lines (DLL) configuration (simplified).

1300 ohms without range extension. Newer No. 5 crossbar offices can operate to 1540 ohms and No. 3, No. 2, and new No. 1 ESS offices have a 1600-ohm range, unaided. DLL circuits have been used to extend these ranges to more than 3600 ohms of conductor loop resistance when located in the central office and to 5200 ohms when a second DLL is placed along a cable route.

Twisted cable pairs have much higher voice frequency attenuation than open wire pairs, due to the higher capacitance per unit length. The use of loading coils was adopted by the Bell System early in the 20th century to improve transmission on long cable circuits. Today the H-88 loading plan is used in the Bell System on all loops longer than 18,000 feet. This plan requires 88 mH inductors to be placed at 6000 foot intervals, beginning 3000 feet from the central office.

Loops with resistance greater than 1600 ohms also require amplifiers to aid transmission. The application of solid state circuitry to voice frequency repeater design in the late 1950s for precision equalization of trunk and special business circuits provided an economical subscriber loop repeater as well.³ The E-6 negative impedance repeaters⁴ were the most popular type used in the 1950s and 1960s for loops even though they required more precise adjustments than necessary for residential and single line business service. Figure 3 shows how this repeater can be connected with DLL circuits to provide both range extension and amplification (gain) functions on long loops. The E-6 repeater was particularly convenient to use since its series negative impedance elements were inductively coupled to ring and tip, allowing dc supervision and signaling and 20-Hz ringing currents from the DLL circuit to pass through to the loop.

In the 1950s independent telephone companies and Bell System

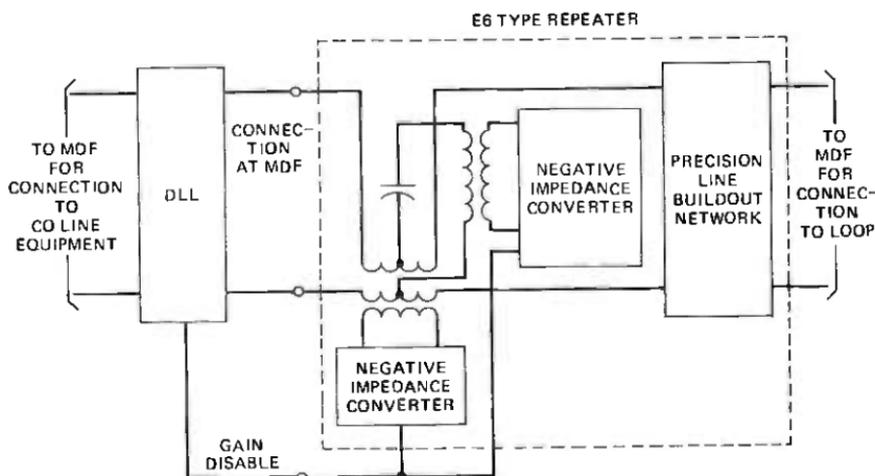


Fig. 3—DLL and E-6 type repeater providing central office range extension and gain.

companies serving rural areas began actively seeking ways in which to reduce the capital expense of replacing open wire and serving new growth in demand with cable. Early loop carrier systems like the M and P1 systems⁵ were used as alternatives to cable in some instances, but the greater part of telephone company effort went into the application of voice frequency electronics to effect gauge savings.

A knowledgeable telephone company engineer could select range extension equipment and repeaters for each rural customer service order that was received. The E-6 repeater in such an instance required a relatively complex analysis to determine the positions of approximately 36 screw switches needed to attain precision gain and line-build-out settings. Whenever party line customers were to be added or reassociated, an engineer was required to survey the request to determine when changes in central office circuits and customer loaded cable end-sections were necessary to provide satisfactory service.

While copper savings were obtained in this way, it became apparent that a significant amount of engineering effort was needed to make such a system work. Furthermore, individually tailoring each loop led to a very complex loop network administration. In the 1960s the Rural Electrification Association staff developed a simpler system⁶ for the use of electronics by prescription while Bell System companies like Southern Bell were pioneering a similar approach to the use of voice frequency loop electronics. The approach included designing groups (referred to as complements) of loops to serve designated areas (zones) along a rural cable route, prescribing repeater settings for each such group in advance and limiting party line associations to selected pairs available for use only in certain terminals (preferred or restricted pair counts). With this approach, operating personnel in the assignment and installation departments could provide service and reassociate party line customers without engineering participation so long as facilities were available in appropriate customer cable terminals. Prescription design is used in most rural areas of the Bell System today with telephone companies all using a form of the AT&T "Long Route Design" system.⁷

2.2 Suburban areas

In the late 1950s and early 1960s growth of demand in suburban areas served by the Bell System created the need for economy similar to that in rural areas. However, the suburban cable routes were generally shorter (usually less than 50,000 feet) and more densely populated than rural routes. Range extension and amplification applied on a per-line basis in suburban areas are not so economical as for rural areas, since the distances are shorter and the gauges used are 26 and 24 gauge in place of 22 gauge. In rural areas the substitution is typically 22 gauge for 19 gauge or about twice the copper savings per unit length as for suburban routes.

This problem and the growing use of No. 5 crossbar switching equipment with a single line appearance for terminating and originating calls made possible the invention of switched range extension⁸ and the Unigaugue system.⁹

Switched range extension is the concept of building the extended loop resistance range capability into the switching system and wiring repeaters behind a stage of switching concentration as illustrated in Fig. 4. With such a system the cost of a repeater is divided by the effective* concentration ratio, typically about 3 to 1, allowing economical range extension to be applied to the longer suburban cable routes.

The switched range extension concept was first introduced as part of the Unigaugue system in 1966 in No. 5 crossbar. The first application was in Rockford, Illinois. In 1971 this feature was made available in No. 2 ESS¹⁰ and was first used in North Madison, Connecticut. The Unigaugue system is intended to maximize the use of 26 gauge cable. The suggested outside plant design is shown in Fig. 5. Note that while other Bell System loop designs call for the use of H88 loaded cable (first load coil 3 kf from the central office with subsequent coils at 6 kf intervals), Unigaugue requires only partial loading on loops longer than 24 kf, reducing the number of load coils required.

The Unigaugue design has been used to advantage in new wire centers coincident with large-scale additions to the cable plant. However, it has been difficult to use Unigaugue in replacement central offices where there are large amounts of existing cable other than 26 gauge. In such cases buffer cable of 26 gauge is required for the Unigaugue repeaters to operate with margin against unwanted self-oscillation (singing).

III. RANGE EXTENSION TODAY AND TOMORROW

3.1 Long Route Design

3.1.1 Long Route Design—description and procedures

The Long Route Design system came into use beginning in late 1971 after an extensive study of the rural cable provisioning problem in the Bell System. Concurrently an economical circuit called the Range Extender with Gain (REG) circuit was introduced by Western Electric Company.¹¹ This circuit, described in detail later, combines the dc signaling and supervisory range extension functions of the DLL with the transmission gain of a negative impedance repeater in a single plug-in unit. The REG circuit is prescribed for use in loops of several resistance ranges or zones as illustrated in Fig. 6. All Long Route Design loops use full H88 loading with 12,000 feet maximum customer end section plus bridged tap.

* This is the achieved ratio of range extended loops in service to repeaters including spare capacity as opposed to the theoretical ratio (m/k , in Fig. 4).

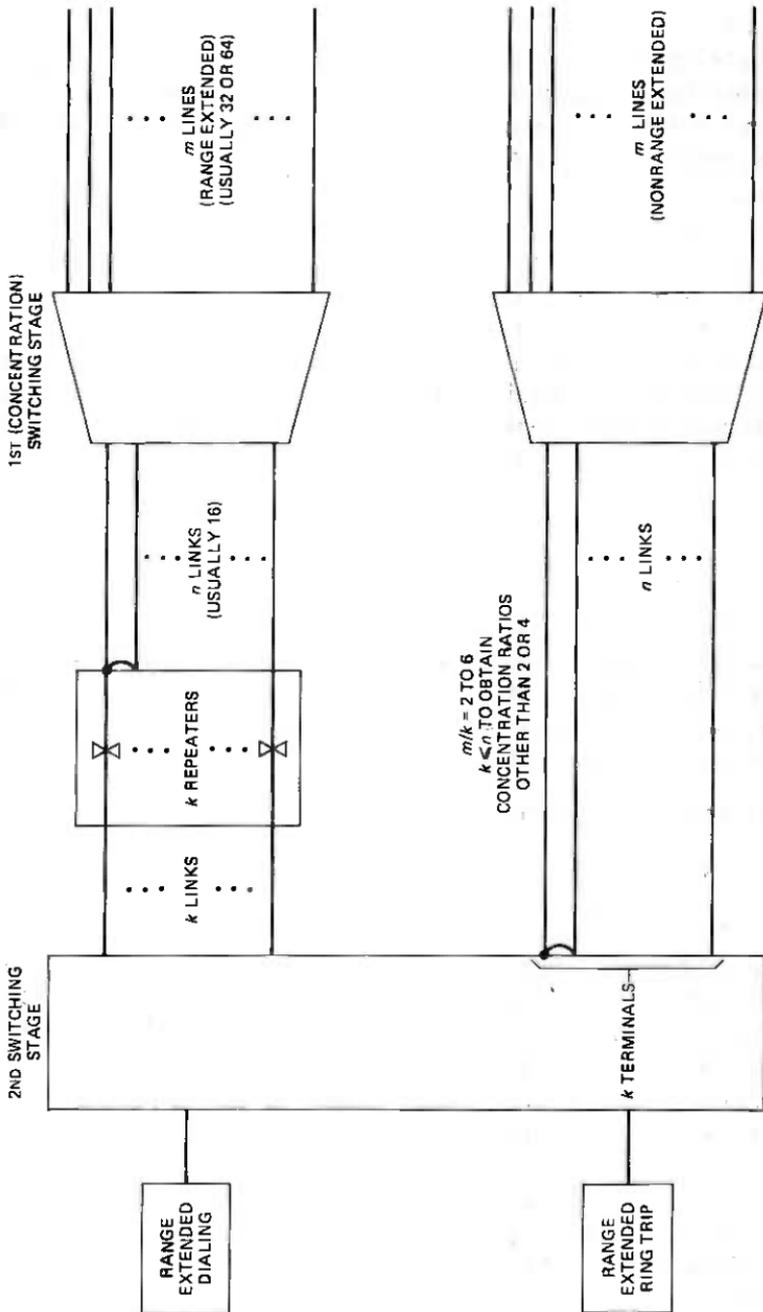


Fig. 4—Generalized switched range extension concept (varies among different switching system designs).

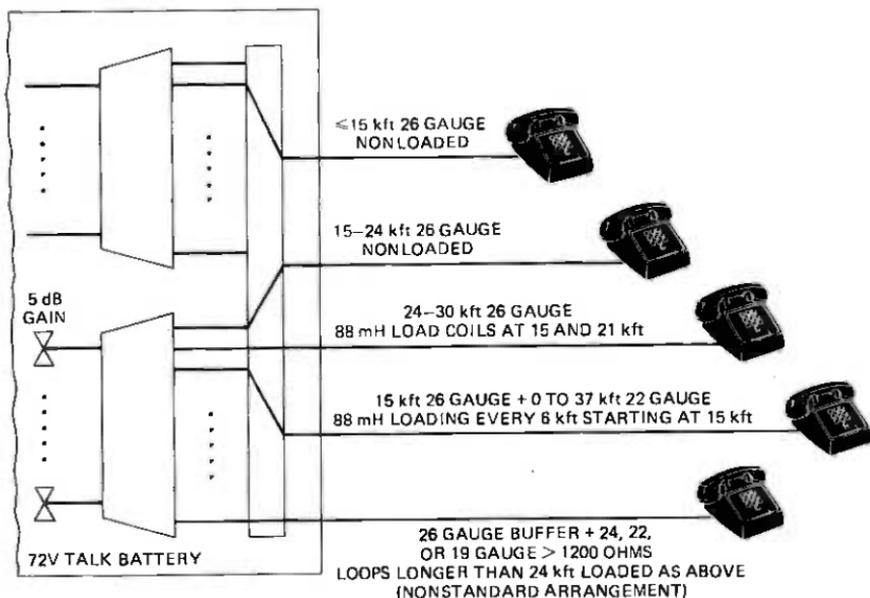
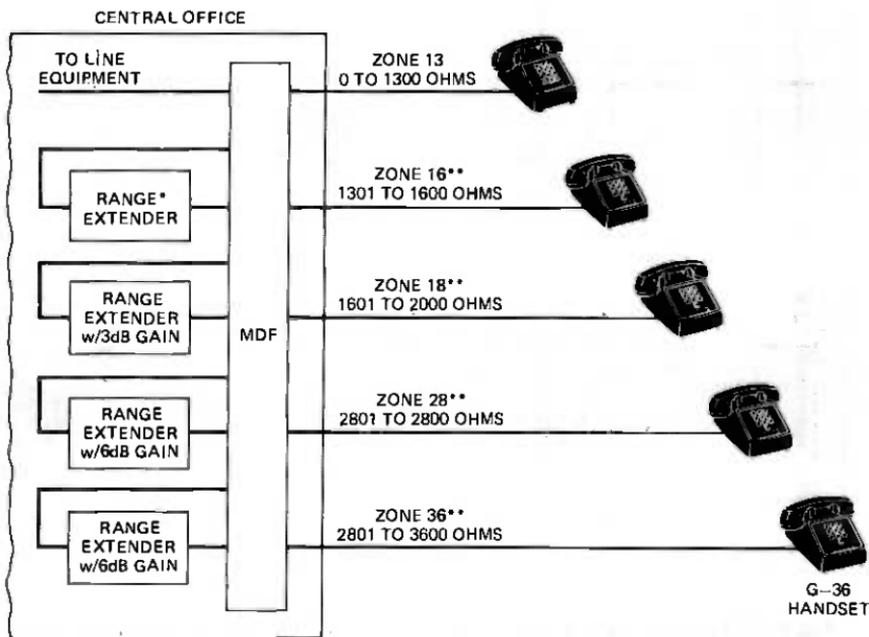


Fig. 5—Unigauged switched range extension system with standard and buffered, non-standard loops.

Telephone company engineers in applying Long Route Design take into account existing cables, forecast growth in demand, and route length. Several plans may be formulated resulting from different gauging strategies. Typically, some alternative plans will include carrier or concentrator systems as well as voice frequency electronics. The new cable and range extension equipment are evaluated for each plan and a present worth of annual charges (PWAC) comparison of the plans is made. Generally, the plan with the lowest PWAC value over a 20-year study period is selected, subject to local capital constraints. This process results in the selection of gauges for new cable additions which then define the location of resistance zones.

As each cable addition is completed, the plant operations personnel receive the list of available cable pairs in each resistance zone and the record of added central office range extension capacity. When a customer requests service that requires the assignment of a new loop facility, the plant assignment person correlates the customer's premises location with a nearby cable terminal. This is most easily accomplished if the customer has a street address. This is not always possible in rural areas. Sometimes a plant visit is necessary to locate the terminal nearest the customer. The assignment person then searches the cable record for the preferred count in the terminal. If a cable pair is available, it is assigned by notation in the cable record. The cable record also gives the resistance zone number for the terminal from which the pair is assigned. The Resistance Zone Key Record is examined to translate the resistance zone number into



- * REQUIRED ONLY IN OLDER ELECTROMECHANICAL OFFICES
- ** FULL H8B LOADED LOOPS REQUIRE—ANY GAUGE MIX

Fig. 6—Present Long Route Design system.

an equipment type. A Miscellaneous Central Office Equipment Record lists the range extender plug-in slots available for assignment. The assignment clerk transcribes the cable pair identification, zone number and range extension equipment number onto the service order. Central office craftspeople connect the cable pair to the correct range extender and the range extender to the central office line equipment with all connections made at the main distributing frame. In older installations repeater settings were required to adapt the repeater gain to the resistance zone and the repeater line build-out network to the cable pair impedance. As discussed below, with more recent circuits these adjustments are no longer necessary.

In this way all range-extended loops in a route are engineered at one time and, through an orderly administrative procedure, clerical personnel are able to prescribe the electronically augmented facility. This procedure saves engineering time, speeds service to the customer, produces a more orderly loop design over a span of time, but adds some complexity to the assignment and installation functions.

3.1.2 Long Route Design—transmission plan

Before the release of Long Route Design to the operating telephone companies, a very careful analysis was made of the transmission quality

as a function of cost. The goal was to provide satisfactory quality while achieving a low-cost system for rural areas. It was determined as a result of this study¹² that a maximum loop insertion loss (at 1 kHz referred to 900 ohms) of 8 dB would guarantee satisfactory service at a reasonable cost for long rural loops. A 10-dB loss design would increase the savings to the telephone company in electronics and cable costs but would cause a disproportionate degradation of transmission quality. A 6-dB design would improve transmission quality but with a substantial increase in system cost. The loss profile of an all 22-gauge cable route using Long Route Design is illustrated in Fig. 7.

While Fig. 7 illustrates a pessimistic 1-kHz loss characteristic along a long cable route using Long Route Design, Fig. 8 shows the loss versus frequency of a 2800-ohm loop with a 6-dB REG repeater compared to that of the highest resistance (1300 ohms) nonrange-extended loop designs. The 2800-ohm loop with repeater is about 1 dB lossier than the 1300-ohm loaded loop at 1 kHz and 2.5 kHz and about 3 dB lossier at 300 Hz. On the other hand the 2800-ohm loop is about 2 dB less lossy than the highest loss nonloaded 1300-ohm loop at 1 kHz and 6 dB better at 2 kHz. This 2800-ohm loop represents a satisfactory design for residential telephone service as indicated by a transmission grade of service study and in fact represents an improvement over the 1300-ohm nonloaded loops that have been standard in the Bell System for the past 20 years.

Transmission grade of service is the expected value of one of two quantities: the percentage of calls "rated" good or excellent or, alternatively, the percentage of calls "rated" poor or unsatisfactory. In either

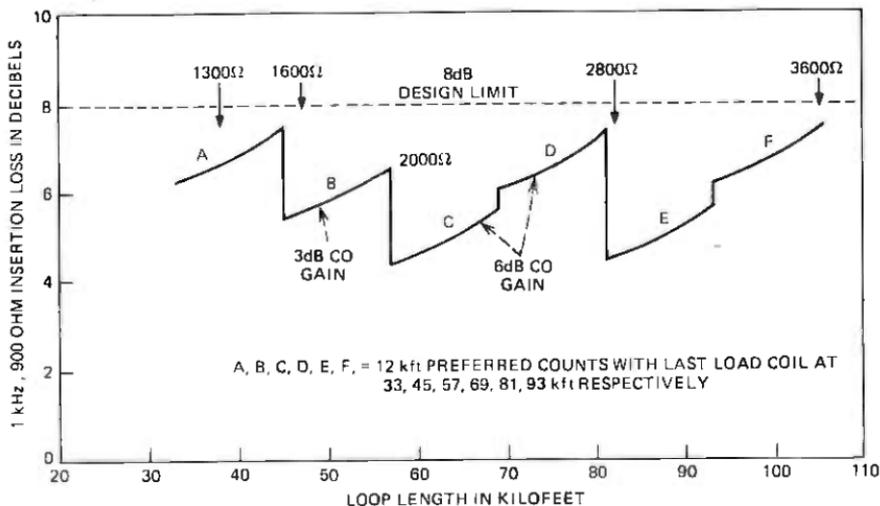


Fig. 7—Long Route Design 1-kHz loss profile—22 gauge route laid out in successive 12 kf (maximum length) preferred counts.

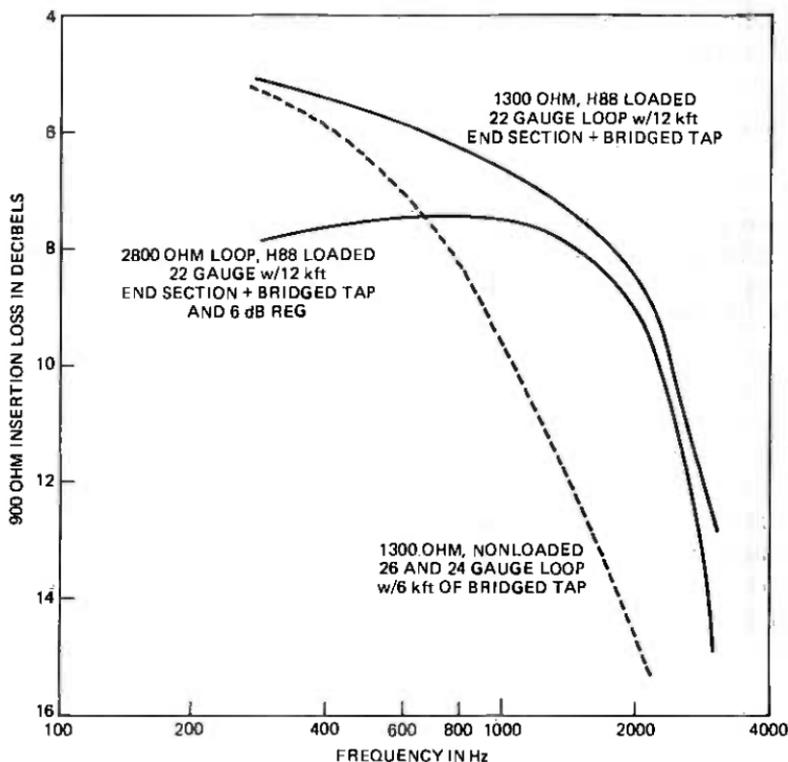
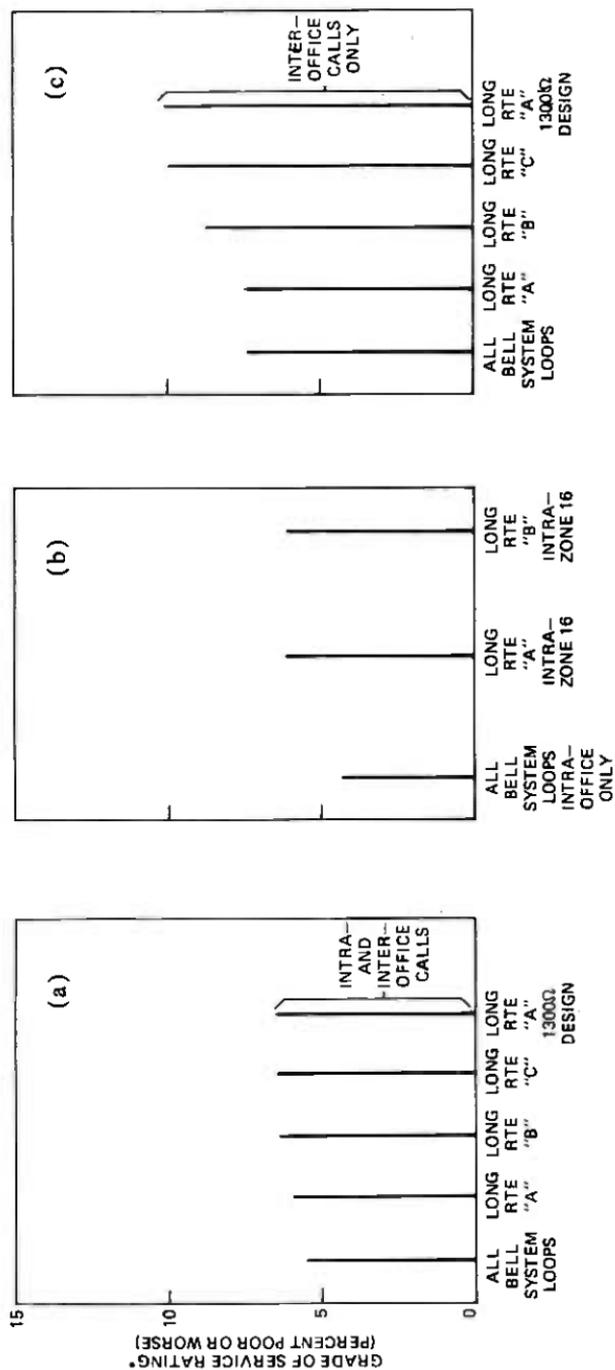


Fig. 8—Loss vs. frequency of maximum resistance range-extended and nonrange-extended loops.

case the rating of telephone connections is based on the results of Bell Laboratories studies whereby people participating in experiments rate the quality of connections on which noise and loss were varied from call to call and does not necessarily represent the absolute ratings that might be achieved in practice. These subjective ratings have been reduced to probability distributions with parameters defined as a function of loop loss and noise.

In the case of Long Route Design, telephone company engineered routes with 20 year forecasts of growth in demand were used as a basis for simulation. Customers were located at random along the route so that each section of the route would have the forecast demand. Each route realization was then used as a data base for a transmission simulation. Each simulation consisted of 1000 connections of five types: community of interest calls (local calls from one Long Route Design customer to another), intraoffice calls and short, medium, and long toll calls to customers drawn at random from a 1973 Bell System loop survey.¹³ The expected frequencies of the various types of call were set at approximately 0.20, 0.46, 0.29, 0.04, 0.01, respectively.

The results in Fig. 9a show the overall grade of service performance



* Grade of service in this context is not a prediction of absolute performance but is intended to be a reliable indicator of the relative performance of two or more comparative systems.

Fig. 9—(a) Overall Long Route Design grade of service. (b) community of interest calls. (c) toll call performance.

for several study routes comparing Long Route Design results to those of a random sample of 1100 Bell System loops. In Fig. 9b the community of interest call performance for zone 16 is shown for two routes and compared to overall Bell System intraoffice grade of service. In Fig. 9c the performance on toll calls is shown for long routes and compared to the Bell System norm. Also shown in Fig. 9c is the toll call performance for one of the long routes designed, using 19 gauge cable, to a maximum resistance of 1300 ohms requiring no range extension, this being the voice frequency alternative to Long Route Design.

It is seen from these results that overall transmission performance of Long Route Design loops is comparable to the general Bell System performance but that toll call performance is somewhat degraded. This is due to the fact that general Bell System loops have losses ranging from 0 to 10 dB with an average of 4 dB. Long Route Design loops range from about 4-dB to 8-dB loss with an average of 6 dB. The trunking loss in toll connections tends to reduce echo effects and at an average of 6 to 7 dB is enough to affect the quality of calls involving Long Route Design loops. However, in Fig. 9c it is apparent that long route A experiences about the same toll transmission performance, whether designed to 1300 ohms, using costly 19 gauge cable, or designed to 2800 ohms with half the copper, using 22 gauge cable and voice frequency electronics. This example illustrates that when loops are sufficiently long to place them at the high end of the Bell System loaded loop loss distribution under the 1300-ohm standard maximum loop resistance design, service will be as good or better with the more economical 2800-ohm design.

These results have led to the conclusion that the 8-dB maximum loss design with 6-dB average loss is satisfactory for all long route applications.

3.1.3 Long Route Design—other transmission factors

In addition to the loss plan, the Long Route Design loops must satisfy other transmission objectives: crosstalk, noise, delay distortion, and return loss.

The Bell System objective for noise at the terminals of the customer's service drop is 20 dBrc. It is generally recognized that noise performance on loops is dominated by induced harmonics of 60-Hz power transmission and by the electrical balance of the telephone physical wire pair and its terminations. The induction of 60 Hz and harmonics as a common mode signal into paired telephone cable is enhanced by increasing the length of the exposure to power line coupling. Long rural cable routes, therefore, have a greater likelihood of experiencing voltage above any threshold value than shorter urban routes. The Long Route Design transmission plan assumes a level of 25 dBrc present at the subscriber terminals for this reason. Grade of service studies assumed this noise

level on Long Route Design loops. Loop noise is also a function of terminal balance at both ends of the loop. Figure 10 gives the requirement for range extension circuit balance and shows the test circuit used to verify performance. Experience has shown that longitudinal noise on long loops can exceed 50 V rms. The balance requirement is necessary to attain less than 25 dBrc of unwanted metallic circuit noise. The telephone set floats with respect to ground at the station end of the loop on individual line service with bridged ringers and is not a significant factor in common mode conversion. However, many (several million) rural loops continue to serve party lines and require grounded ringer operation. In the same category are the affluent long route customers with 4 or 5 ringing telephones, whose telephones must be rung through a connection to ground to increase the power delivered to the ringers on high resistance loops.¹⁴ Connecting the ringers to ground at the station creates an electrical unbalance and increases unwanted longitudinal-to-metallic conversion. The solution to this problem has been to use ringer isolation circuits that provide a low impedance path from ringer to ground when ringing is present but isolate the ringer from ground when ringing is not present, maintaining station balance during the conversation.

A second degradation is crosstalk. Long Route Design loops have increased susceptibility to the near-end crosstalk phenomenon, illustrated in Fig. 11. Consider the case where two active Long Route Design loops are connected to two short loops. The signal level arriving at the central office from the short loop station is relatively high and is amplified by the range extension repeater. Crosstalk occurs within the cable, dominated by coupling between pairs near the central office, and experiences low propagation loss back to the CO on the other long route cable pair. The crosstalk is amplified again in the opposite direction by another repeater and experiences low loss to the disturbed customer station. For every added decibel of repeater gain, near-end crosstalk levels are increased by two decibels. The currently accepted objective for crosstalk allowed is 0.1 percent chance of hearing intelligible crosstalk in any given conversation. A repeater with more than 6 dB of gain at 1 kHz exceeds that 0.1 percent objective in today's loop plant.¹⁵ This fact along with the 8-dB maximum loop loss establishes 2800 ohms as the highest-resistance H88 loaded loop that can be served with central office gain alone. It is possible to extend loops to 3600 ohms with 3.5 dB of transmit and receive gain at the station, as is done with the G-36 handset, without violating the more stringent crosstalk design limit at the station end of the loop.

Talker echo is a very annoying transmission degradation on long toll calls. This echo is dependent on obtaining matched impedances at trunk terminations. The impedance of a loop terminated in an off-hook tele-

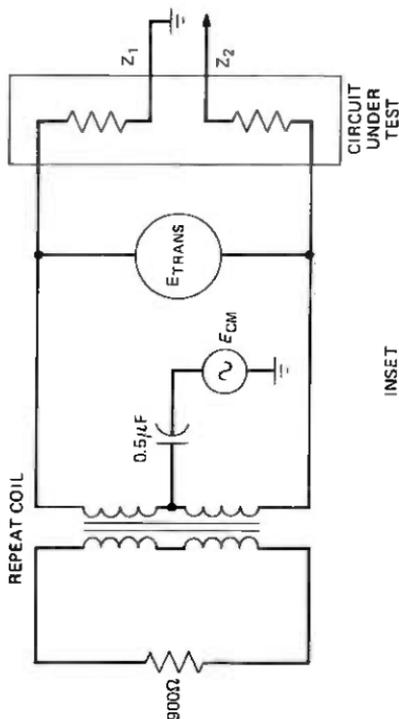
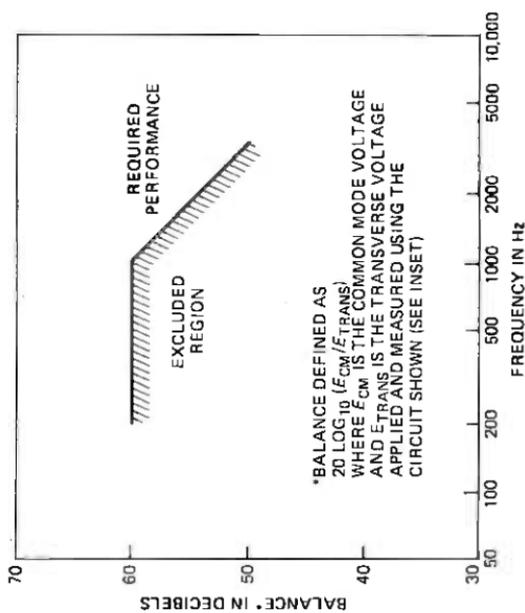


Fig. 10—Range extension circuit terminal common mode balance requirement.

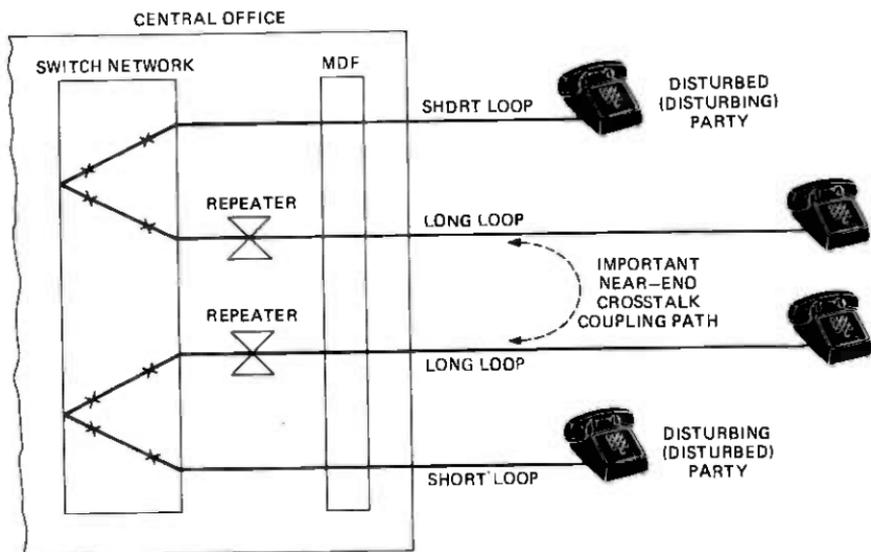


Fig. 11—Range extension repeater near-end crosstalk problem.

phone set is quite variable due to the large range of loop lengths and the intermixing of four wire gauges. This impedance is related to echo by the equation for return loss:

$$\text{return loss} = 20 \log_{10} \left| \frac{Z_{\text{Loop}} + Z_0}{Z_{\text{Loop}} - Z_0} \right|$$

where Z_{Loop} is the impedance of the loop measured at the central office with an off-hook telephone set and Z_0 is the reference impedance. In this case Z_0 has traditionally been taken to be 900 ohms in series with 2.16 microfarads. Echo return loss is defined as a weighted average of return loss between the frequencies of 500 and 2500 Hz. Bell System loops have evolved with an average echo return loss of 11 dB with a standard deviation of 3 dB. Repeaters that are part of range extension circuits should and typically do exhibit better than average performance in this respect.

A degradation related to echo is the near-singing or "rain barrel" effect that is sometimes experienced. This is controlled by placing an objective on singing return loss, the lowest single frequency value of return loss in the range of 250 to 3200 Hz. This objective is related to an existing Bell System average value for loops of 6 dB, standard deviation of 2 dB referred to 900 ohms in series with 2.16 microfarads. The objective for voice frequency loop repeaters is that their use makes long loop performance no worse than Bell System overall performance.

A method of controlling return loss other than by impedance matching is voice-switched gain, used in echo suppressor circuits. These circuits control echo by sensing the direction of transmission in the four-wire

path on long toll connections and then introducing high loss in the opposite direction. This technique is also used in *SPEAKERPHONE*® design to reduce acoustic feedback. It is possible to use voice-switched gain in loop repeaters. Voice-switched gain circuits must by necessity respond to the initial spoken syllables. The switching action occurs at this time and may impair transmission. Examples of possible impairments are loss due to false switching, increased break-in difficulty, and speech clipping. It is a current Bell System recommendation that voice frequency repeaters for loop range extension not use voice-switched gain to control return loss.

Data transmission has increased in the loop plant and requires control of phase distortion as well as loss. This equalization is currently custom-engineered by the telephone companies and not provided by range extension circuits. As the incidence of data transmission in the long loop areas grows, there may be an impetus to provide prescription equalization for range extended loops.

Voice frequency repeaters must not distort signals of reasonable amplitude. The recommended distortion requirement is less than 0.5-dB compression of any single frequency in the range of 300 to 3300 Hz at a peak level of +6 dBm into 900 ohms. As an objective, repeaters should be linear in the 300 to 3300 Hz range up to signal levels of +10 dBm.

3.2 Future direction

Further evolution of Bell System outside plant voice frequency loop design is possible. As seen in Fig. 8 nonloaded 1300-ohm loops have higher loss over much of the voice band than 2800-ohm rural loops with 6-dB repeater gain. It is also the case that with the introduction of No. 1, No. 2, and No. 3 ESS offices in the past 13 years, central office supervisory range is not necessarily limited to 1300 ohms. The No. 2 and No. 3 ESS systems have circuit options built in for 1600 ohms of conductor loop resistance and the No. 1 ESS has recently added such a feature. Appropriately equipped existing No. 1 ESS and No. 5 crossbar systems can operate to 1500 ohms of conductor loop resistance. New cable, placed in the past decade, has largely been buried or in underground ducts, reducing the resistance margin necessary to allow for aerial cable in hot weather. These facts suggest a new standard design for nonrange-extended loops of 1500 ohms resistance instead of 1300 ohms. Such an increase would provide gauge savings and, if REG circuits are used on loops over 1500 ohms, the elimination of resistance Zone 16 as an administrative entity in all but older step-by-step switching systems. To improve transmission, loading can be used on loops longer than 15,000 ft instead of 18,000 ft from the central office. This will eliminate the longest nonloaded loops and their high loss, while not affecting average loop loss. Such a plan would also remove the lossiest Zone 16 loops.

To help make this plan a reality, a concentrated range extender is being designed for use in No. 1 and No. 2 ESS offices to be compatible with fully loaded loops of any gauge mix so that urban (1500-ohm maximum loops), suburban (2800-ohm maximum loops, concentrated range extension), and rural (2800-ohm maximum loops, per line REG) switching centers can have a unified loop design with maximum range extension economy consistent with good transmission and high reliability and relatively simple administration.

IV. RANGE EXTENSION FUNCTIONS

As just discussed, unaided loop telephone service becomes marginal when loop cable resistance exceeds about 1300 ohms. To some extent this threshold is a function of switching system type. Beyond 1300 ohms, deficiencies occur in both signaling and transmission. More specifically, the functions needing enhancement for long loops are origination, dial pulsing, ring trip, telephone set transmitter current, voice frequency transmission, and ringing.

ORIGINATION. Office line circuits require a minimum of 10 to 16 mA to detect a call origination. Loops of 2800 ohms resistance, the maximum standard range-extended loop, draw a minimum of about 10 mA. Some older small step-by-step offices with high-resistance line circuits provide only 7 mA. Therefore all step-by-step and most crossbar offices require assistance in detecting call origination on loops between 1300 and 2800 ohms.

DIAL PULSING. Office circuit resistances are lower during dial pulsing than during origination; however, office circuit thresholds are higher and the relation between loop currents and the threshold is more critical to preserve dial pulse fidelity. Consequently, long loops for all offices require dial pulsing assistance.

RING TRIP. Loop currents when a called party answers during ringing are greater than those during origination; however, office circuit thresholds are also higher and again all offices require assistance when serving long loops.

TELEPHONE SET CURRENT. The telephone set is designed to perform acceptably over a 20- to 80-mA range of loop currents. Below 20 mA *TOUCH-TONE*® dials become marginal at low temperatures and the set transmit gain function suffers. All range extenders provide a form of boosted battery to maintain a minimum of 20-mA loop current.

VOICE FREQUENCY TRANSMISSION. Long loops in the Bell System are always loaded (H88 type). For these loops the loss is proportional to resistance (Fig. 12) and at 1600 ohms approaches 8.0 dB worst case, the maximum loss limit for Long Route Design loops. Beyond 1600 ohms supplementary bilateral gain is required.

RINGING. Ringing does not normally require enhancement to operate

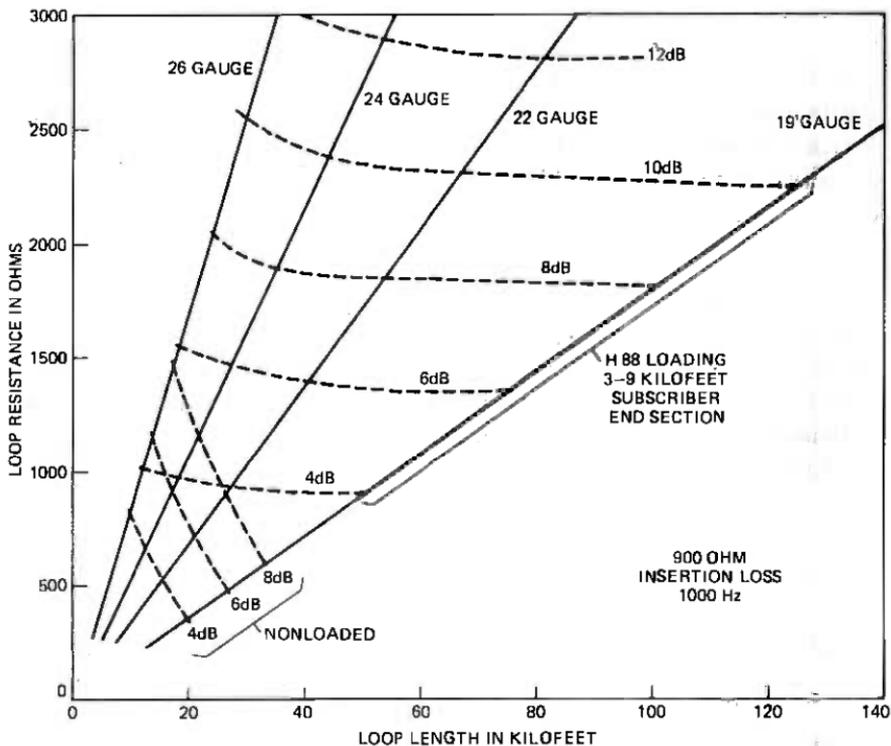


Fig. 12—Loop cable resistance and loss vs. loop length. Present Long Route Design requires H88 loading beyond 18,000 feet and range extension beyond 1300 ohms.

three bridged ringers at 2800 ohms. Ringing range can be a significant problem when more than three ringers are required. A ringer isolation device is normally required for party line service where ringers must be rung to ground but otherwise isolated to prevent loop imbalance and enhanced sensitivity to 60-Hz longitudinal induced currents. Ringing aid devices are, therefore, common on long routes and will be discussed as part of this article.

V. THE RANGE EXTENSION ENVIRONMENT

Before discussing range extension circuits in the next section, it will be helpful to discuss briefly the range extension environment including cable and office characteristics.

Long route cables differ from those within the normal office limits in that they have higher resistance and loss. The prime purpose of range extension devices is to compensate for this increased resistance and loss, to bring them within acceptable limits. Other long route cable characteristics are impedance and cable irregularities. These characteristics do not differ significantly from loaded short cables; however, they do influence range extension circuit design considerably.

The range extenders under discussion here are located in the central office and are terminated on one port by the office switch. This interface is a complex one when considering the variety of switching machines, switching circuits, and operating modes. A general-purpose range extender should be as broadly compatible as possible. A few of the dominant office features influencing range extender design are also presented here.

5.1 Cable Impedance and stability considerations

The voice frequency impedance of the cable pair is important because it has a strong bearing on amplifier stability and also because it influences power transfer into the cable. Range extension amplifiers (commonly known as repeaters) are bilateral gain devices and consequently their gain and stability depends on the terminating impedances at both ports, i.e., that of the loop pair and the office termination.¹⁶ As the office impedance is constrained only to being positive real in a practical sense, then it falls on the loop impedance to determine the maximum practical amplifier gain. If this impedance is too variable then the amplifier configuration must be customized around each of several cable impedance subsets. Even if this customization is achieved by screw adjustments on a prescription basis, there is a significant administrative penalty. For POTS service, freedom from all field adjustments is an important goal.

Figure 13 shows typical impedances for Bell System loops over 1300 ohms. The impedances are shown on a Smith chart impedance grid normalized to 900 ohms. When scaled in this manner, distances between impedances tend to correlate, with the difficulty in matching a repeater to these impedances in a stable, non-oscillatory manner. It is obvious that loaded loops form a much more constrained set of impedances than nonloaded loops. This fact, in addition to the "gain" provided by the load coils, makes loaded loops definitely preferable for long routes. With a nonadjustable amplifier, 6 dB of bilateral repeater gain is practical for Bell System H88 loaded loops. With nonloaded loops only about 5 dB at 1-kHz fixed gain is practical and then only if the impedance spread is constrained by a 3000- to 5500-foot section of 26 gauge buffer cable at the office end of the loop. This latter design is the basis of the Uni-gauge loop network used in a few Bell System wire centers.

The H88 loaded impedances of Fig. 13 are for "good" loops. Good loops are ones with 3000-foot office end sections and are either long loops or, if short, have 3000-foot subscriber end sections. Range extenders must also work on minimum length loops with 12,000-foot subscriber end sections, and on loops with some load coil placement deviations. Figure 14 shows the impedance of a pair of worst case allowable Long Route Design loops giving a hint of the variations possible.

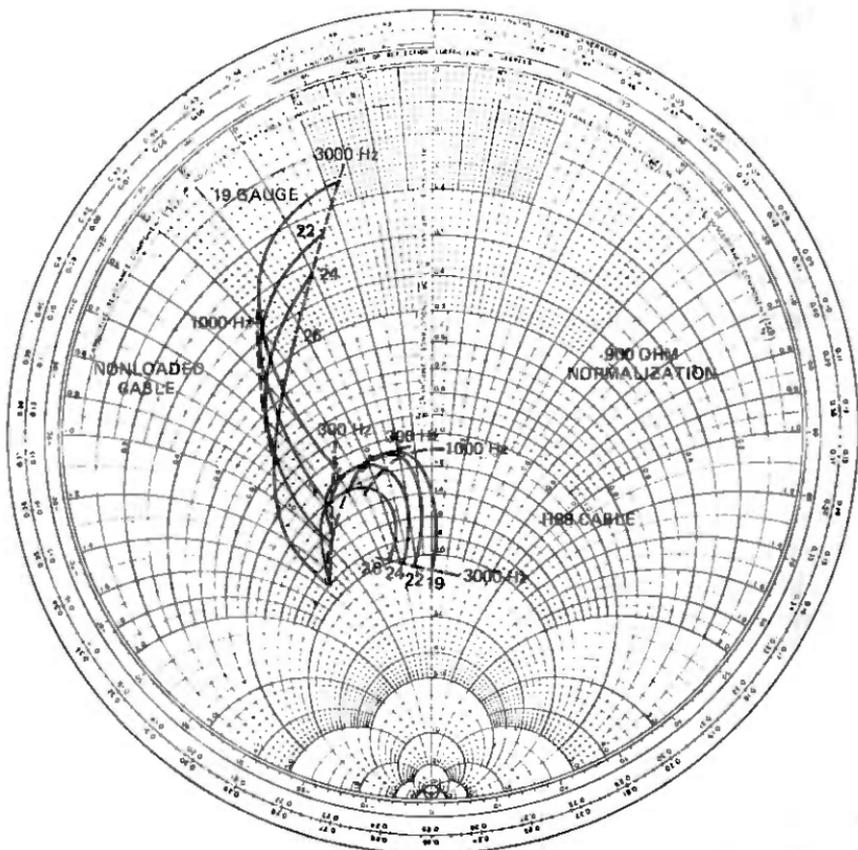


Fig. 13—Loop cable impedance vs. frequency for electrically long H88 and nonloaded loops.

An attempt has been made to define all allowable Long Route Design loops and to determine their impedances at critical frequencies. Figure 15 shows this impedance spread for the midband 1000-Hz frequency and the band edge frequencies of 300 Hz and 3000 Hz. Proper matching network design can transform these sets of impedances to a nominal 900 ohms. Their spread is sufficiently small to allow 6 dB amplifier gain, an unconstrained positive real office impedance, and still have unconditional amplifier stability.

H88 loading is specified by 88-mH load coils, spaced 6000 ± 120 feet apart, starting 3000 feet from the office. The subscriber end section plus bridged tap limits are 3000 to 12,000 feet and neither loaded bridged tap nor bridged tap between loads is allowed in Long Route Design. Unfortunately plant deviations are common and a reasonable tolerance to these is desired. One approach is to group network irregularities into two categories, minor and major.

A minor irregularity is an error in load coil location of 500 feet or a

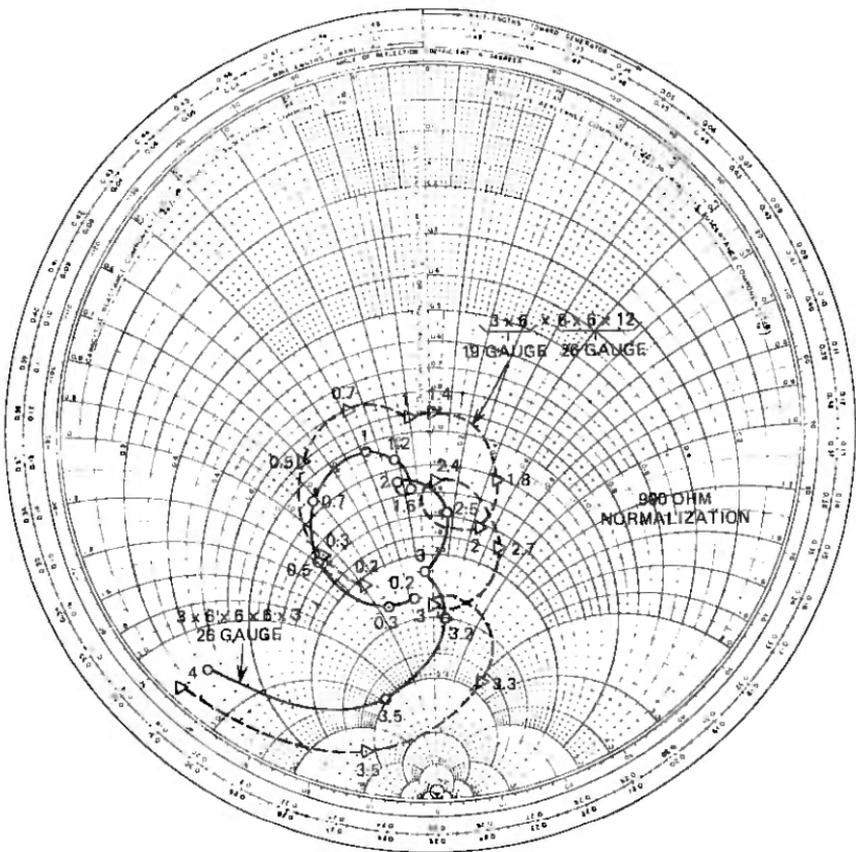


Fig. 14—Impedance vs. frequency for two worst-case Long Route Design loops.

500-foot bridge tap, within the first five load sections. Beyond about five sections errors can be several times greater. The effect of these on loss is limited to a few decibels at most, and as an objective any associated repeater should be stable.

Major irregularities include missing or doubled load coils, spacing errors of greater than 500 feet in the first five load sections, and loaded bridged tap. In this category transmission loss can be affected by many decibels and even if such a loop is stable it may be service-affecting and should get priority maintenance attention. Repeater stability on such loops is not expected and is not a design objective. Rarely do offices have more than a few percent of loops in this category.

5.2 Office response times

A finite delay exists between an off-hook telephone set signaling call

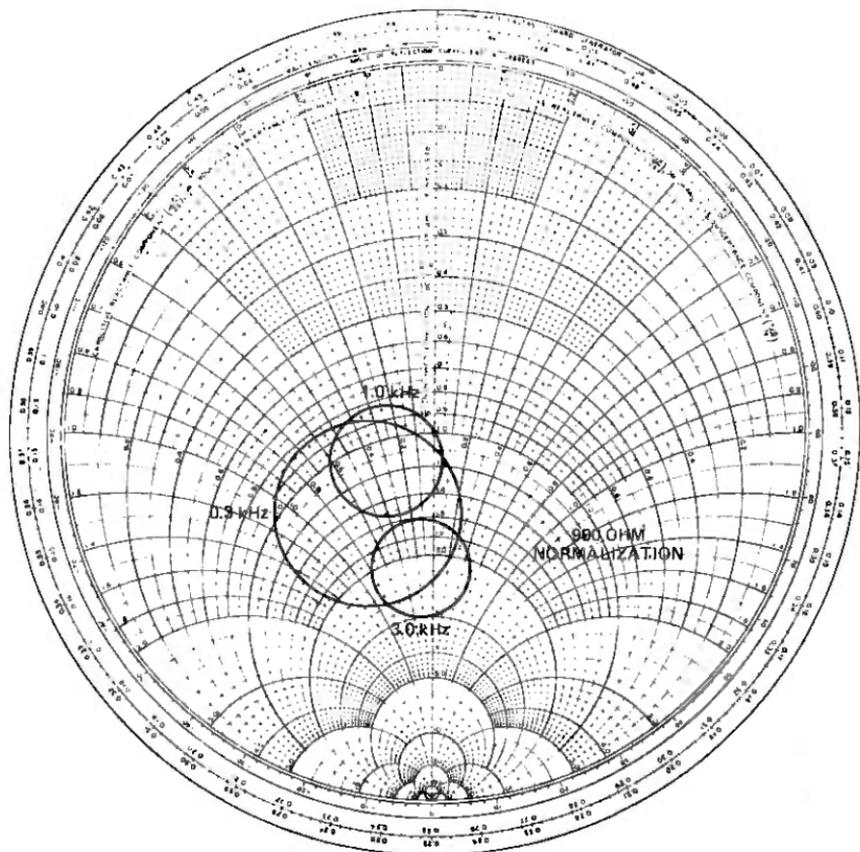


Fig. 15—Impedance boundaries at band center and band edges for all allowed Long Route Design H88 loops.

origination and office switch response. These delays range from about 50 msec to several hundred milliseconds or longer during high traffic delays. The switching network is reconfigured, in general, several times in establishing a talking path. Loop current is broken during these operations, affecting range extender loop current sensing. After network reconnection, different delays occur before subsequent switching circuit response. Throughout these sequences the range extender must work compatibly. A sufficient design is one that responds faster on resumption of loop current than the associated switching circuit. In this mode the range extender appears transparent to the switch and compatibility is assured. Range extender design is frequently simplified if slow response times are implemented, but the interface with the switch then has a new dimension, that of time, and the interface is appreciably complicated.

5.3 Switching circuit source impedance

In general, standardization of source impedance exists among office types for a given operating mode. In existing offices, for example, talking circuits are invariably 400-ohms dc resistance, ringing circuits are under 400 ohms, 20-Hz impedance, and dial pulsing circuits are about 400 ohms resistance. Line circuits that detect call origination, however, have greater variation, ranging from 500 ohms to 1300 ohms typically but reaching 3200 ohms for a particular widespread step-by-step circuit. This latter circuit is especially annoying in the design of range extenders. Switching systems under development but not yet in service are tending toward high impedance, constant current battery feed. These circuits are generally compatible with existing range extender designs.

5.4 Tip party identification circuits

In the Bell System the calling customer for two-party service can be automatically identified. When off-hook the tip party customer's set connects a 2650-ohm resistance to earth ground. The office senses this path by breaking loop current and attempting to force a current from tip to ground. Repeat coil type range extenders, to be discussed later, break this current path and are, therefore, basically incompatible.

5.5 Office talking circuit balance

As discussed earlier a loop cable pair is balanced with respect to ground. Office and telephone set circuits are also balanced, thereby offering high immunity to longitudinal currents induced by nearby power lines. Office talking circuit balance ranges from a minimum of 40 dB for some step-by-step offices to over 60 dB for ESS offices. As long loops tend to have greater induced currents, it is imperative that range extenders for step-by-step and crossbar offices provide an improved balance to the loop to effect the requirements of Fig. 10.

5.6 Ringing waveform crest factor

Essentially all loop ringing in the Bell System is 20 Hz superimposed on a dc trip bias battery. As discussed earlier, ringing aids are frequently used on long routes. These circuits must detect the presence of ringing voltage. As most of these circuits are peak voltage sensors the ringing waveform crest factor can significantly affect ringing aid operation. Crest factors range from 1.2 to over 1.6 as a function of ringing plant design. This factor is a significant variable affecting ringing aid operation. An example is the cold-cathode gas tube used for selective ringing detection where low crest factors can cause marginal triggering.

5.7 Testing circuit compatibility

Bell System offices have a variety of circuits for testing for loop faults. The first requirement for testing is to provide loop access. Range extenders should have a reliable dc path through the circuit; otherwise office personnel must manually connect a test shoe to gain loop access. The lack of a "test-through" capability can be a liability, particularly when the office is equipped for routine automatic loop testing.

A second requirement imposed by test needs is to minimize leakage currents created by range extender circuits. These conductive paths mask cable faults in that the office test circuits cannot distinguish differences. Requirements on range extenders to assure compatibility are for tip-to-ring and tip- or ring-to-ground resistances to have minimum values on the 1- to 3-megohm range; higher values are desirable.

VI. RANGE EXTENSION CIRCUITS

Attention can now be focused on loop range extension circuits that have evolved in the last few years.

6.1 Dial long line circuits and E6 repeaters

In the late sixties long loops were treated using dial long line circuits (DLLs) and E6 repeaters (Fig. 16). The DLLs by then had evolved into plug-in assemblies for compactness and ease of maintenance. The DLLs were sometimes used alone where telephone companies allowed loop designs with more than 8-dB loss. These DLLs were generally characterized by:

- Individual shelf mounting, 12 circuit capacity
- 72- or 96-V boosted loop battery
- 97-Vac repeated loop ringing
- Repeat coil with good balance
- Repeated selective ringing
- Repeated dial pulsing
- Repeated ringing trip
- No test-through arrangement
- No tip party identification circuitry
- A family of units for different ringing services
- Ballast lamps for short circuit protection

Some of these characteristics are shown in Fig. 2. Boosted loop ringing was employed to provide some ringing range extension especially when driving gas-tube-isolated ringers. Boosted loop battery provided 20-mA current over 3600-ohm loops in addition to the E6 repeater resistance. In the last few years the DLL approach has lost favor primarily because of no test-through capability, no tip party identification, and the size, cost, and multiplicity of units required for party-line ringing options.

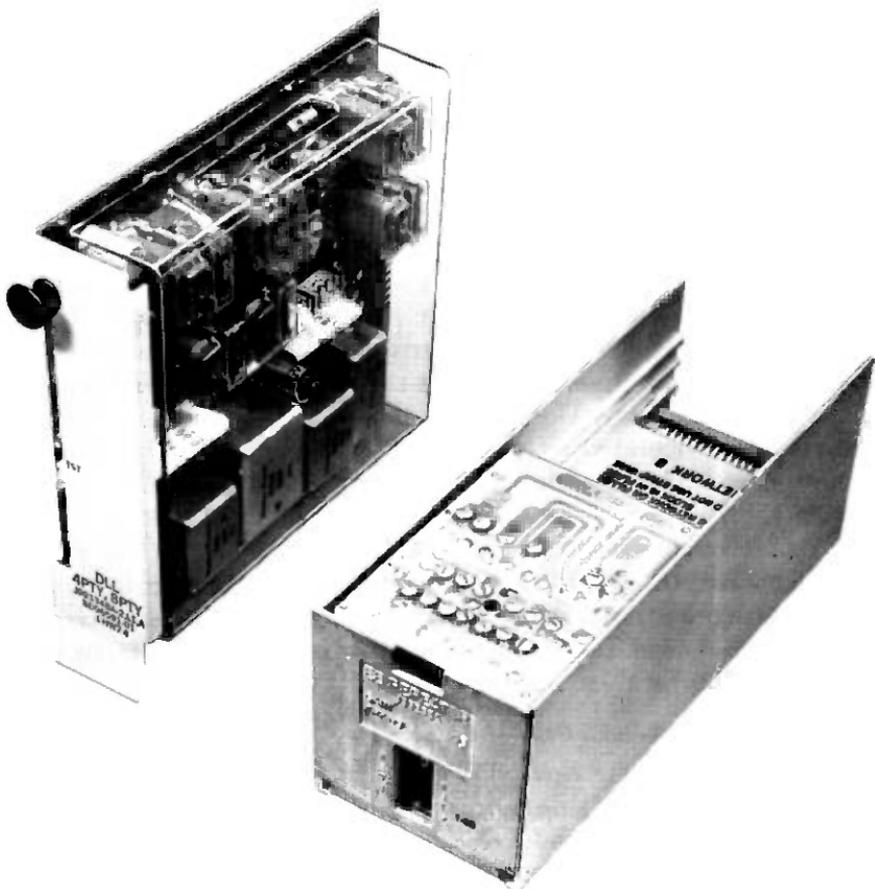


Fig. 16—Dial long line (left) and E6 repeater (right) plug-in units.

Voice frequency gain was provided by the E6 repeater, a two-wire negative impedance amplifier as previously mentioned.⁴ It could be connected in tandem with the DLL circuit, preserving dc continuity for loop powering and signaling while adding up to 13-dB gain (Fig. 3). Optional adjustable matching networks were available for either port to match resistance, nonloaded cable, or loaded cable terminations. Other characteristics were:

Separate shelf mounting, six across

Negative impedance repeater with shunt and series elements

Broad matching capability with many screw adjustments

Gain to 13 dB in 0.1-dB steps through many screw adjustments

The E6 repeater was designed for exchange trunk and special services applications where its performance and circuit flexibility were required. For loop service the cost of its separate mounting and complexity of adjustments were liabilities.

6.2 2A range extender

One of the first devices to provide signaling range extension in a more attractive form was the 2A range extender.¹⁷ Step-by-step and crossbar offices both employ relays for supervision and line monitoring in the on-hook state. The single most limiting aspect of these offices in serving long loops is the current required to pull in these relays upon call origination. Maximum ranges of 1300-ohms loop resistance are typical. However, once the line relay is operated the current drawn by loops over 1800 ohms is sufficient to hold the relay operated. This results from the well-known hysteresis characteristic of standard relay designs. The 2A range extender exploits this characteristic by placing a shunt across, say, an 1800-ohm loop upon origination to make it appear less than 1300 ohms to the line relay. The 2A shunt is removed in 0.5 second, after the line relay has operated, so as not to affect subsequent voice frequency transmission. The 2A shunt is operated by a sensitive loop current detector.

The 2A is a small solid state circuit, as shown in Fig. 17, that mounts on the main distributing frame. With attributes of minimum installation effort and a price that is about one-sixth of a DLL, the 2A has successfully met a special need. Introduced in 1967, it is still in manufacture and 500,000 are currently in service.

6.3 Range Extender with Gain—REG

About 1970 it was recognized that significant improvements could and should be made in the loop range extender art. Loop loss control was of increasing importance and a maximum 1000-Hz loss of 8.0 dB was an accepted design limit. This limit meant that gain was necessary for all loops over 1600 ohms. With long range extended loops becoming more common (a few offices had over a thousand DLLs and E6s), size and cost were important. The need for a test-through capability was also evident.

To fill these needs a combination Range Extender with Gain (REG) was devised by J. L. Henry and L. G. Schimpf of Bell Laboratories.¹⁸ This REG List 1 was a single solid state plug-in assembly, Fig. 18, that mounted 93 to a 7-foot bay. The bay also contained a common bay power supply for the boosted battery. The installed cost was about half that of the DLL-E6 combination it replaced. Major characteristics were:

- One unit served all single and multiparty service
- Compatible with office tip party identification
- Idle state bypass allowing test-through
- No adjustable relays requiring maintenance
- 1600- to 2800-ohm range
- Gain of 4 or 6 dB set by screw adjustments
- Cable matching network with five screw adjustments

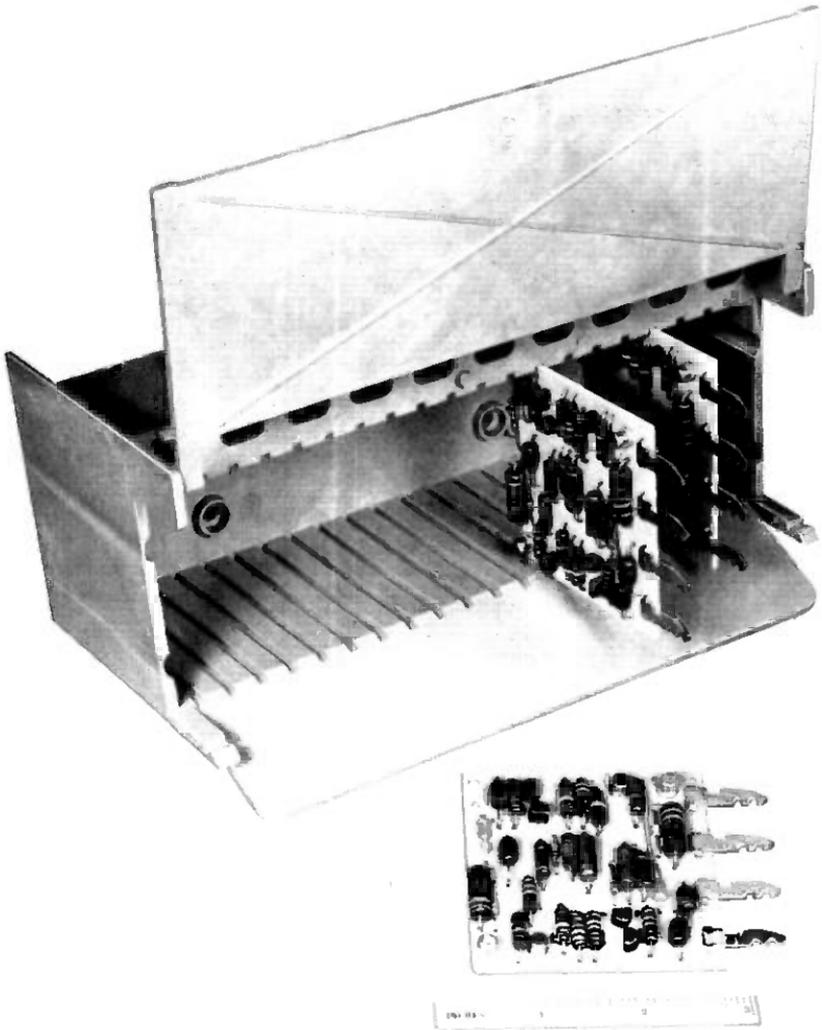


Fig. 17—2A range extender and distributing frame apparatus mount.

- All adjustments set by prescription based on resistance zones and cable characteristics
- Solid state devices providing short circuit protection
- Repeat coil for good balance
- Fast-responding range extension circuit
- Negative impedance amplifier

This first REG List 1 offered in 1971 was a success and was followed in 1973 by a reduced-cost version known as REG List 2. Cost was reduced by simplification of the signaling and logic circuitry, allowing elimination of one printed circuit board. One consequence of simplification was that the circuit now operated more slowly than some office switching machine

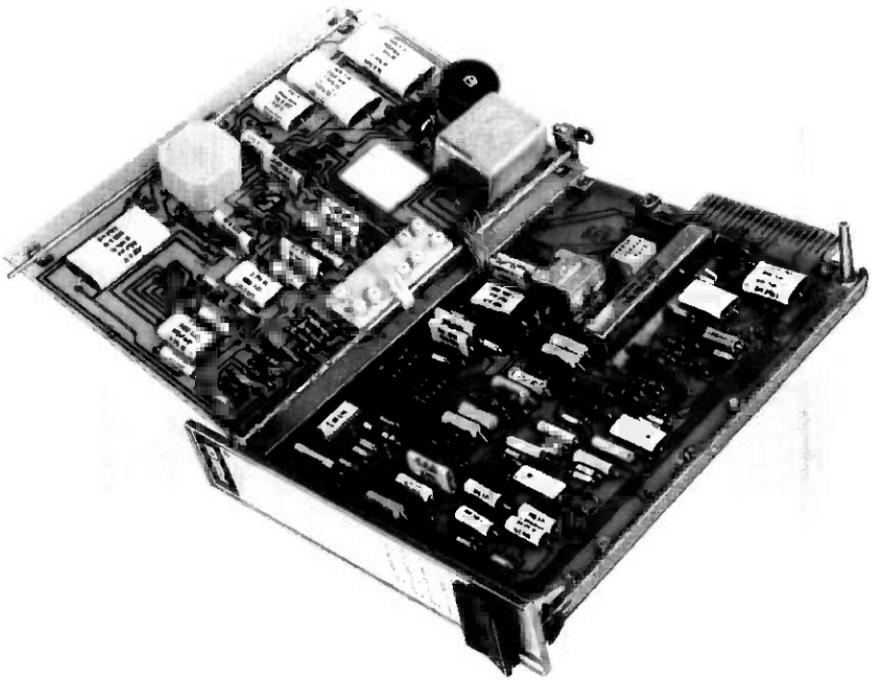


Fig. 18—Range Extender with Gain (REG) List 1 showing swing-out daughter board.

functions. A program patch was required in No. 1 ESS to achieve compatibility.

In 1975 the REG List 3 was offered. It was part of a Zone 36 package and differed from REG List 2 only in that it had an additional screw adjustment allowing a lower loop current threshold for the more distant zone.

6.4 5A REG

In 1976 the REG family was further improved with the introduction of the 5A REG. While retaining the same plug-in format of previous REGs it offered a 24 percent cost reduction and the following features:

- 3 or 6 dB gain automatically controlled by loop resistance
- Single line matching network for all H88 loaded cable
- Elimination of all adjustments
- Operation on loops as short as 1000 ohms
- Improved reliability
- Improved stability

The 5A unit is shown in Fig. 19. The 5A REG has been well received by the operating companies, not only because of the cost reduction but also because the elimination of all adjustments allows simple administration.

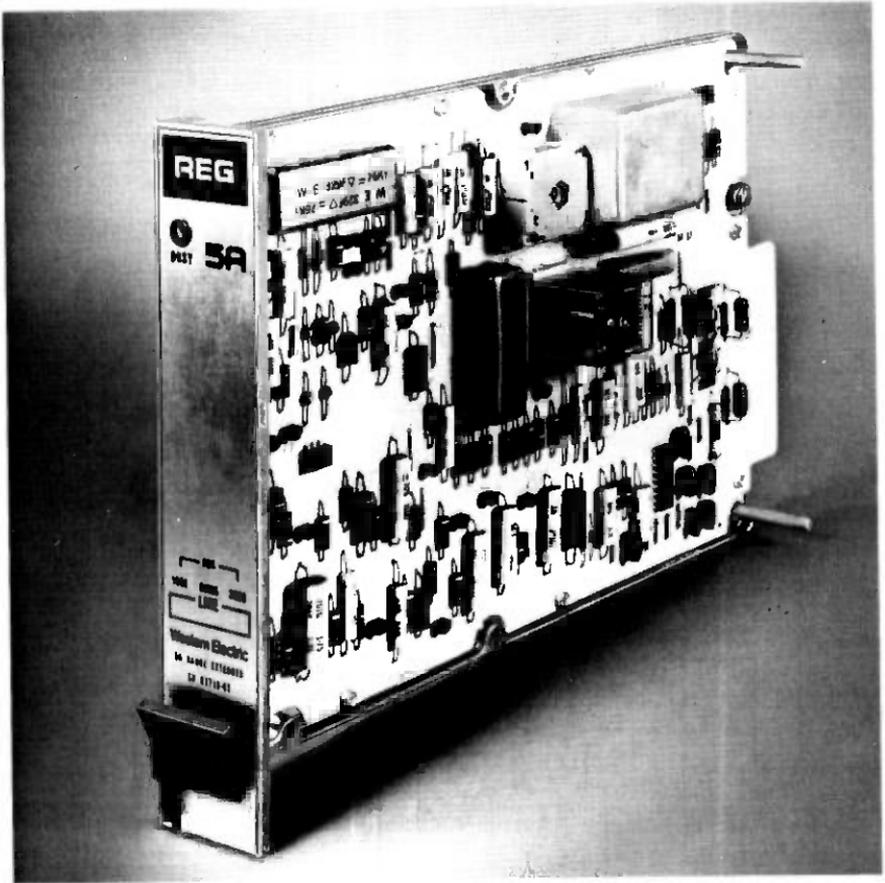


Fig. 19—5A REG plug-in assembly.

Besides use in Zones 18 and 28, it is seeing significant use in Zone 16 for ESS offices where transmission enhancement is wanted and in older No. 1 ESS systems where signaling enhancement is needed. Occasionally the REG is used in Zone 16 for crossbar systems where standardization on a single range extender is considered more important than the lower price of the 2A range extender that the REG supplants.

Operation of the circuit can be understood by reference to Fig. 20. The circuit configuration is controlled by two relays. The RO relay when operated places the amplifier and repeat coil in the loop. Otherwise the circuit is in a bypass mode with only a pair of 100-ohm resistors in series with the loop. The L relay repeats dial pulses around the repeat coil. The loop current detector is a high-impedance balanced bridge circuit that is basically immune to longitudinal loop currents. Upon call origination it senses the loop current in about 100 milliseconds, causes the L relay to operate, initially placing the 1000-ohm shunt across the loop, and then operating the RO relay to reconfigure the unit into the active state. The

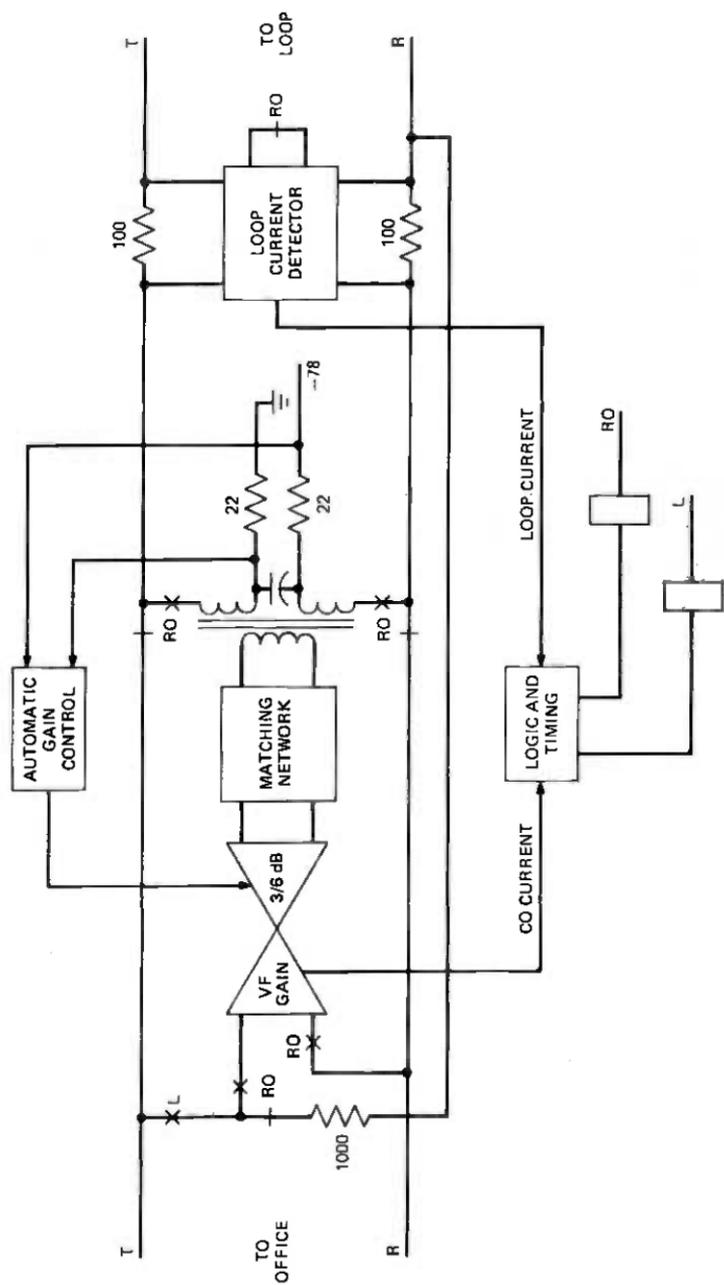


Fig. 20—5A REG simplified schematic.

office sees the low resistance of the REG amplifier and starts the origination sequence. RO relay operation also applies a 78-V boosted battery to the loop to aid dial pulsing and talking. Dial pulsing occurs in the active state and is repeated by the L relay. The logic and timing circuit regenerates the dial pulses and removes some distortion. A terminating call rings through the REG in its idle state. When idle, the loop current detector contains a low pass filter to inhibit pretrip which would otherwise occur due to the effects of ringing current on the detector.

When the office switch makes a tip party identification test, it breaks loop current. This break is sensed by an optical coupler in the amplifier which causes the RO relay to revert to the unoperated state. The L relay, however, stays up and completion of the tip-party test is sensed when returning office current flows through the 1000-ohm resistor and one side of the loop current detector. The RO relay is then re-energized, restoring the active state. The tip-party test is not aided by the REG, which is in the passive state during the test. All office circuits have the ability to perform this test unaided over 2800-ohm loops.

The amplifier gain is set by the AGC circuit. Loop resistance is determined by a comparison of loop voltage and loop current sensed at the 22-ohm series resistor. The amplifier is maintained in the 3-dB state for loops under 2000 ohms and during dialing and other transient states. The matching network maintains acceptable stability for all Bell System H88 loaded cables. Echo return loss is typically 15 dB, and worst case about 7 dB for loops such as those of Fig. 14. Singing return loss at the worst single frequency is typically 8 dB, and worst case about 2 dB. These values are better than for nonrange-extended loops.

VII. RINGING EXTENSION DEVICES

Boosting ringing voltage in the central office is not a practical way to extend ringing range significantly. The main problem is that voltages sufficiently high to appreciably boost ringing range are considered hazardous to personnel and would undoubtedly require costly changes in outside plant design and maintenance procedures. Fortunately other means exist to accomplish ringing range extension. A view of the problem is seen in Fig. 21. Here a ringer is connected between the ring lead and ground with the switch portion of a ringer isolator in series. A grounded ringer connection is shown, as this is necessary when more than three single party ringers are required or when serving party lines. The ringer isolator is shown generically and is used to represent a variety of Bell System devices including conventional cold-cathode gas tubes. Its function is to preserve line balance by keeping the ringer isolated from the line except when ringing voltages are present.

There are three parameters of this circuit under design control that affect ringing range. These are:

- (i) The ringer isolator switch impedance R_s . For gas tubes this is

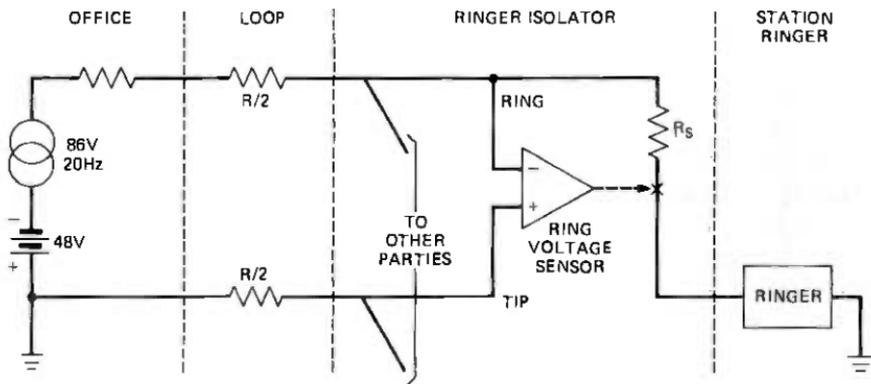


Fig. 21—Typical grounded ringing circuit.

a nonlinear impedance creating an essentially constant voltage drop of 70 volts. This is so great that a special low impedance ringer connection is required in order to preserve any significant ringing range.¹⁴

(ii) The ringer sensitivity. Ringers have a sensitivity adjustment. In low notch position a ringer will typically ring on about 35 volts. In high notch position the ringer requires about 50 volts. The high position must frequently be used to prevent crossing, the undesirable ringing of another party's phone. This phenomenon could occur, in Fig. 21, if the office were ringing the tip conductor and the tip party went off-hook during the ringing interval. The low resistance of the tip party's set would effectively short the tip and ring conductors, placing about 40 volts of ringing on the ring conductor and causing the ring party to ring until ringing was tripped at the office.

(iii) The ring voltage sensor threshold. The voltage across the ring voltage sensor is the sum of the ringer drop and the voltage drop across R_s . The sensor threshold must be below the sum of these voltages or it will limit the ringing range. With a low notch ringer setting and a small R_s the ring voltage sensor should threshold below 35 volts. Typical thresholds for ringer isolators vary from about 5 volts to roughly 65 volts.

An optimum ringer isolator is, therefore, one that has a negligible R_s , a ring voltage sensor threshold below 35 volts, and somehow allows ringers to be placed in the low notch without crossing. This latter objective can be met by a high R_s value which desensitizes the ringer but conflicts with the first two objectives. This approach is common in practice. A more elegant solution is to make the ring voltage sensor dc polarity sensitive. In the aforementioned example ringing the negative tip party will reverse bias the negative ring party's sensor and prevent

Table I — Ringer isolator performance comparison

Ringer isolator device	R_s or equivalent voltage drop	Ring voltage sensor threshold	Polarity sensitive	Ringer capacity @ 2800 ohms
Capacitor	0	No sensor	No	2*
D180036 Kit	2V	5V	No	2*
Gas Tube	65V	75V	Yes	1
11A Extender	10V	40V	Yes	4
28A Isolator	2V	30V	Yes	5

* Assumes ringer bias spring in stiff notch to prevent crossing on party lines.

crossing. Note the polarity sensitive sensor is used here only to prevent crossing and not in the usual sense to provide full selective ringing.

The degree to which existing devices meet the above objectives is indicated in Table I. The first three devices are older designs that mount inside the phone set.¹⁹ Their ringing capabilities are limited because of deficiencies in one or more of the three critical design parameters. The last two devices, the 11A extender¹⁴ and the 28A isolator have been made available in the past 4 years. Their improved performance is evident in Table I. Because of polarity sensitivity they can also be used for full selective ringing, thereby making them good general-purpose devices.

7.1 28A ringer isolator

The 28A isolator supersedes the 11A extender and is the current choice for Bell System party lines or long route single-party service requiring multiple ringers. It is shown in Fig. 22 in a typical mounting adjoining a protector block outside a house. The 28A isolator features:

- Reliable solid state design
- Mounting outside house
- One unit serves all phones at station
- Rings five phones at 2800 ohms
- Polarity sensitive for selective party lines
- Tolerates 150-V, 60-Hz induction
- Causes no radio or TV interference
- Unaffected by ringing voltage crest factor
- Compatible with three-conductor house wiring
- Compatible with 97-volt ringing
- Will not pretrip ringing on short loops
- Allows loop continuity testing without test desk modification
- Small size—9 cubic inches
- Low price—about half that of the 11A extender

VIII. CONCLUSIONS

The voice frequency range extension field extends several decades into the past and includes a large variety of range extension devices working

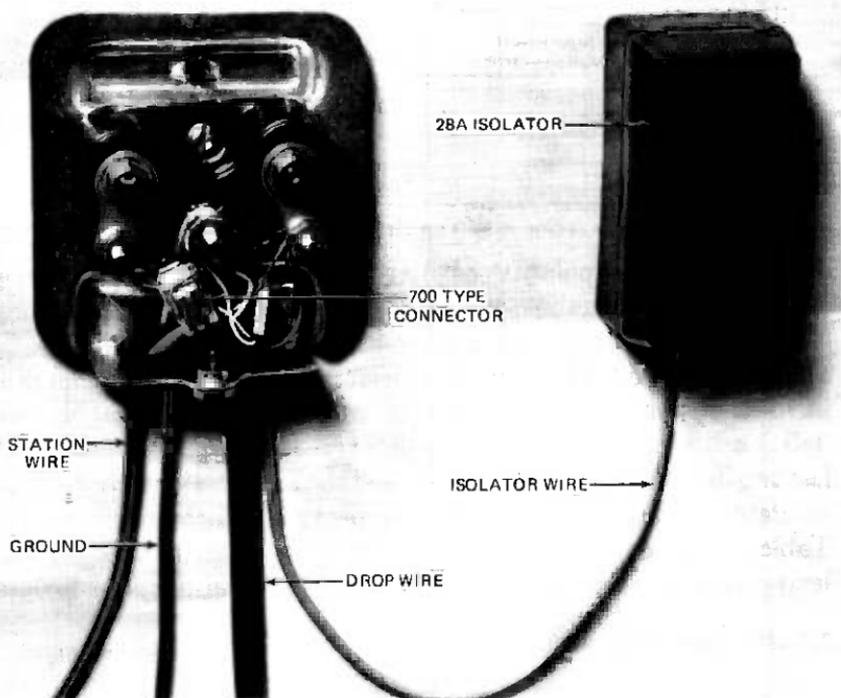


Fig. 22—28A ringer isolator mounted with station protector.

compatibly within the spectrum of Bell System central offices. In the last six years Bell Laboratories has mounted a concerted effort to define objectives for this range-extended plant, to design attractive low-cost range extension devices, and to provide features minimizing administrative and maintenance efforts. The 5A REG and the 28A isolator are the evolutionary products of this effort. Together they provide low-cost quality service to 2800-ohm loop resistance.

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Loop Plant Electronics:

Analog Loop Carrier Systems

By T. N. RAO

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Analog loop carrier systems have several distinct advantages which allow these systems to be economical even for as few as one to eight channels per cable pair. These advantages arise from their insensitivity to near-end and far-end crosstalk between cable pairs and the lack of complex common equipment at the terminals. Until recently, the analog systems have not benefited from the low hardware costs possible with the use of custom linear integrated circuits which can make the cost of companding, modulation, and demodulation inexpensive compared to the costs of powering and interface circuitry. Once this has been achieved as in the SLCTM-1 and SLCTM-8 systems, the choice between digital and analog systems can no longer be based on hardware costs alone at full fill, but must also be based on such other differences as engineering and installation costs, flexibility, maintainability, and range of application.

I. INTRODUCTION

The analog carrier systems used in the loop plant fall into two distinct groups: the single-channel systems that piggyback a carrier channel on top of an existing voice channel and the multichannel systems where four to eight voice channels are multiplexed onto a single pair of wires. The single-channel systems piggyback a carrier channel on top of an existing voice channel without disturbing the baseband channel in any appreciable manner. These systems are limited in their use to urban and suburban areas where no intermediate repeaters or pair conditioning, like removal of load coils, is necessary. In their range of application, they provide the most economic or frequently the most expeditious pair gain technique. The multichannel systems, on the other hand, tend to be used on longer routes with repeaters. These systems, because of having to

deload the line for carrier frequency transmission and difficulties in handling baseband signals at the repeaters, do not attempt to preserve the normal baseband transmission on the wire pair and all channels are derived via carrier techniques. In both the single and multichannel systems double-sideband AM with transmitted carrier is popular for several reasons. Wideband FM,¹ even though practical for single-channel systems with some advantages over DSB-AM, has limited use for multichannel systems because of the large bandwidth requirements. The desirable compatibility between single and multichannel systems used in the same cable usually forces both systems to use the same or similar modulation technique and consequently wideband FM is not common for single channel systems. Narrowband FM does not have large bandwidth requirements but suffers from low noise immunity as compared to AM. SSB is very acceptable from the bandwidth point of view and, in addition, has several transmission advantages like insensitivity to phase distortions on the carrier line and decreased levels on the carrier line. However, at the present time, the greater complexity of the SSB modems as compared to DSB modems and the inability to use the extra channels possible in a given bandwidth due to the limited number of channels that can be powered from the central office without local power at the remote terminal, favors the DSB systems. DSB-AM systems, having their beginnings in the trunk plant, have in the loop plant progressed from the early vacuum tube versions through discrete transistor circuits to the integrated circuit version to be described in this paper. The single-channel *SLC*TM-1 system will be described in some detail and followed by a less detailed description of the eight-channel *SLC*TM-8 system which shares many of the integrated circuit techniques used in the *SLC*-1 system. The role of these analog systems in the loop plant can be inferred from the advantages they offer, which are outlined toward the end of the paper.

II. THE *SLC*-1 SINGLE-CHANNEL CARRIER SYSTEM

2.1 *General description of operation*

The simplicity of single-channel carrier systems which derive via carrier techniques an additional channel on an existing voice pair has long been recognized.^{2,3} Over the years, they have progressed through vacuum tube and discrete transistor versions to the *SCL*-1 system which uses custom integrated circuits. This system, like all previous systems, basically consists of two modems: one at the central office (CO) and the other at the subscriber's premises. Figure 1 shows the system configuration which also shows a low-pass filter to prevent carrier frequencies from entering the physical customer's telephone. A carrier frequency of 76 kHz is used in the direction to the subscriber and 28 kHz in the opposite direction.

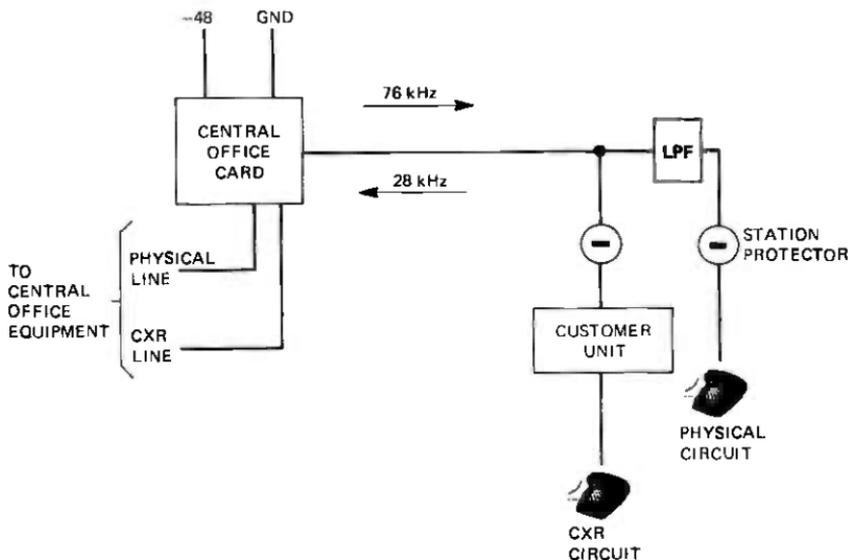


Fig. 1—*SLC*[™]-1 system configuration.

The block diagram of the CO modem is shown in Fig. 2. In the idle state no carriers are transmitted. When the derived customer goes off-hook the 28-kHz transmitter in the subscriber unit is turned on and the signal received at the CO modem is bandpass filtered, amplified, envelope detected and applied to the hybrid through an audio filter-amplifier combination. The output of the envelope detector also controls the loop

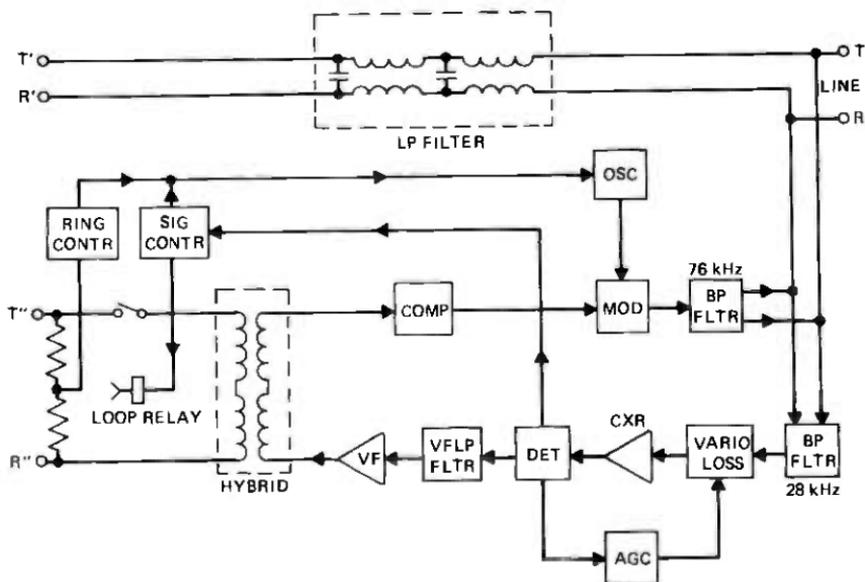


Fig. 2—*SLC*[™]-1 CO modem.

relay which repeats loop closure and dial pulses. *TOUCH-TONE*[®] calling signals are transmitted just as speech signals. The CO transmitter is turned on either by the received 28 kHz, so that it can return dial tone, or by a ringing signal applied to the modem by the CO equipment. The ringing signal gates the transmitter on and off at the 20-Hz rate. Audio signals are compressed by a syllabic compressor before being applied to the transmitter.

In the subscriber terminal, shown in block form in Fig. 3, modulation and demodulation are similar to the CO modem except for interchange of transmit and receive frequencies and the syllabic expander in the receive path. The 28-kHz signal transmitted by the subscriber unit is level controlled by the received 76-kHz signal so that carrier signals received by the CO modems tend to be at the same level and unequal level crosstalk problems are minimized. When the phone is on-hook, 20-Hz modulation on the received 76-kHz carrier activates the ringing generator circuit which applies ringing to the phone. The electronics and the telephone at the subscriber end are powered by a six-cell rechargeable nickel-cadmium battery. During the idle states of the physical and derived telephones, the battery is charged by a dc-to-dc converter that draws about 3 mA from the line. The physical arrangements of the CO and subscriber terminals are described in a companion paper.⁴

Custom linear integrated circuits are used in the CO and subscriber modems. These integrated circuits have made possible for the first time the use of a high performance syllabic compandor that meets trunk carrier objectives. The compandor, which is used only in the high fre-

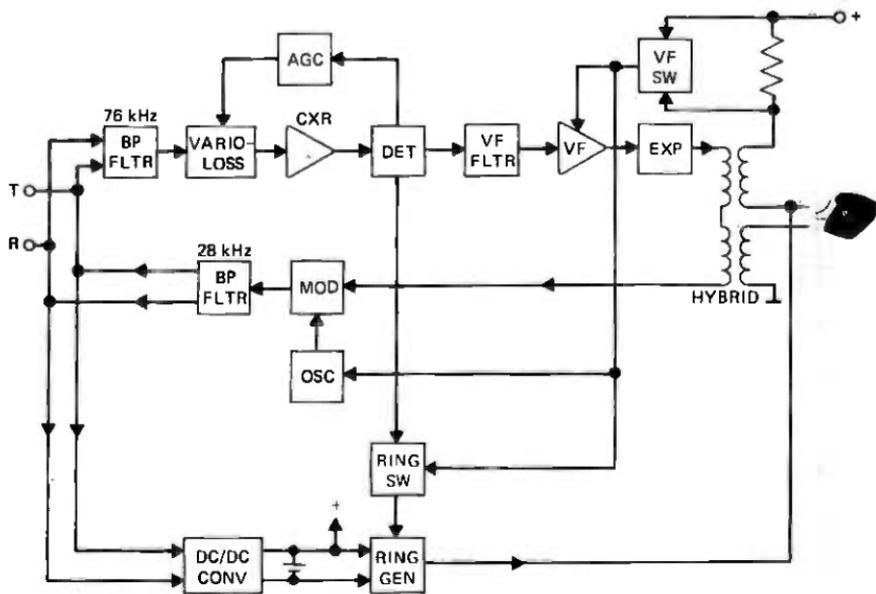


Fig. 3—SLC™-1 subscriber modem.

quency direction, allows lower receive levels, permitting a range of 18 kft on any nonloaded resistance designed loop without excessive bridge taps. Among other benefits that arise from the use of silicon integrated circuits, which can accommodate complex circuitry on a single SIC, are low signal distortion without sacrificing signaling speeds, close control of channel loss and wide operating temperature range. Each of the modems uses four silicon integrated circuits. The transmitter, receiver, and compressor or expander functions are provided by one chip each, the control and miscellaneous functions are built into the fourth chip. Since the control functions are different at each end, a total of five silicon integrated circuit designs are used.

2.2 Circuit description

2.2.1 Receiver

The receiver performs the functions of automatic gain control, envelope direction, audio filtering and amplification. It also detects the presence of the carrier signal to generate control signals for ringing or loop closure, and provides cross-control current for the transmitter. All of the above receiver functions are incorporated into one IC chip. The functional schematic diagram for the receiver circuit is shown in Fig. 4. The input signal is fed to a balanced variolossler which is followed by a balanced-input carrier frequency amplifier. The amplified signal goes through an envelope detector, which consists of a high-gain amplifier

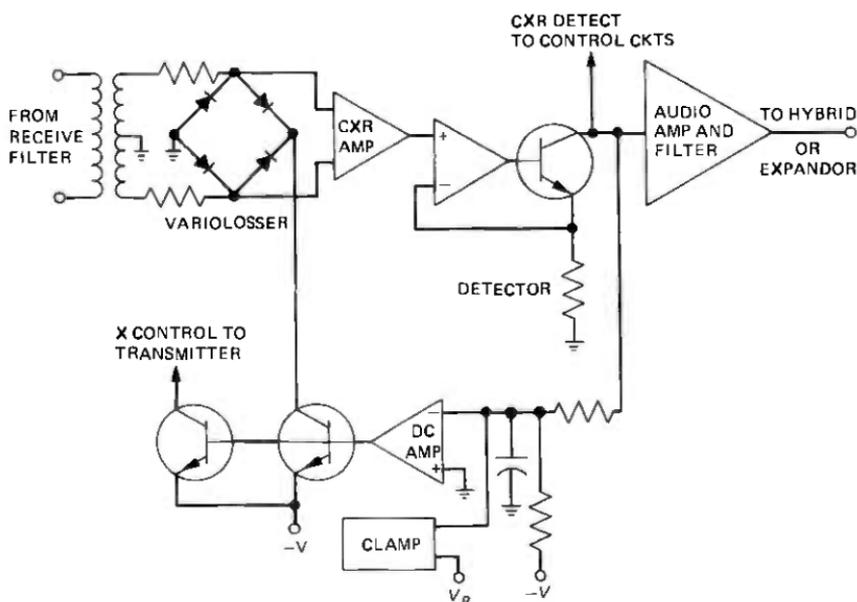


Fig. 4—Receiver circuit.

with a transistor in the feedback that acts as a rectifier. The V_{be} of the rectifying transistor is absorbed in the high loop-gain; therefore the current in the collector of the transistor is a precise half-wave rectified version of the input signal. The carrier component of the detected signal is filtered by a third-order active filter, which also acts as an amplifier and driver for the hybrid, and the demodulated audio appears at the output. For automatic gain control (AGC), the dc component of the rectified signal is applied to a high-gain dc amplifier, the output of which controls the current in the variolossor diodes.

Through the use of a custom-designed IC, it has been possible to achieve high performance economically. For example, the AGC holds the audio output to within ± 0.2 dB for about 55 dB of carrier level variation at the input. Thanks to a precise detector and high loop gain, the audio output level, for a given modulation index, is determined primarily by a few discrete resistors, hence resulting in excellent temperature stability. The receiver also features low harmonic distortion, about 40 dB down for 50 percent modulation at 1 kHz. The capacitor that precedes the dc amplifier affects both the response time of the AGC and audio harmonic distortion. One has to choose a large capacitor to reduce the amount of the residual audio signal in the variolossor control current, which would produce second harmonic distortion in the audio signal. However, this would also slow down the AGC response time. The problem has been resolved by using a clamp circuit which in the absence of a carrier signal holds the input to the dc amplifier close to its steady-state value (just above ground), hence reduces the AGC response time considerably.

The same receiver chip is used in both the central office and the remote terminal units. To minimize the battery drain in the remote terminal unit, in the idle condition when the telephone set is on-hook, the audio portion of the receiver circuit is kept off, and the receiver draws only 900 μ A from the battery.

2.2.2 Compressor

The compressor used in the high-frequency direction is a 2:1 syllabic compressor, which reduces the probability of crosstalk interference. Together with a wide AGC range, the compressor extends the transmission range to 53 dB of loss at 76 kHz, and hence covers essentially all nonloaded loop plant.

The compressor uses a new design, which represents a marked departure from prior art in its implementation. Both the compressor and the expander use the same custom-designed IC chip. Each circuit consists of one IC chip and several discrete resistors and capacitors. The functional block diagram is shown in Fig. 5. The high gain of the operational amplifier and negative feedback force the output signal of the two differential pairs to be of the same amplitude and opposite phase (input

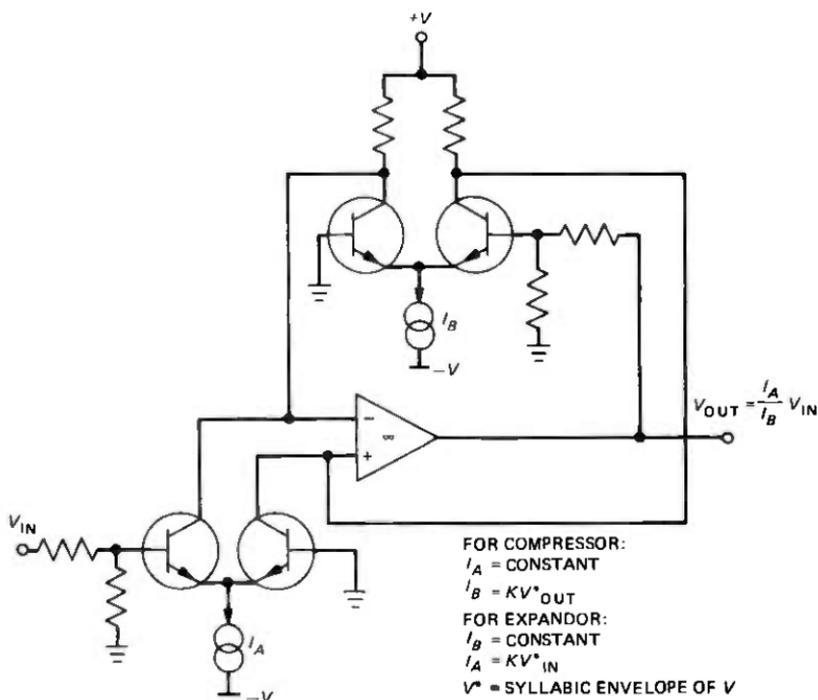


Fig. 5—Comandor circuit.

of the operational amplifier to be zero), hence it can easily be shown that

$$V_{OUT} = \frac{I_A}{I_B} V_{IN} \quad (1)$$

For the compressor circuit I_A is a constant dc, and I_B is proportional to the syllabic envelope of V_{OUT} , which is obtained by rectifying and filtering the output signal. For the expander, I_B is a constant dc and I_A is made proportional to the syllabic envelope of V_{IN} .

This circuit arrangement offers several novel features. First, the gain of the circuit is essentially temperature independent, since the temperature dependent factor in the gain of the feedback differential pair is canceled by an identical factor in the differential pair which is the forward path. For 100°C change in temperature (-40°C to +60°C), the output variation is about ± 0.1 dB. This is of particular importance in the loop plant where the customer equipment can be expected to undergo wide temperature variations. Also, the circuit of Fig. 5 relies on the gain of differential pairs which is well defined in terms of the currents I_A and I_B . The gain of this circuit is essentially determined by discrete resistors. This results in very stable compressor and expander characteristics and excellent comandor tracking (less than ± 0.2 dB over 70-dB range). In

contrast, conventional designs employ the temperature dependent ac impedance of diodes for variolossers. Moreover, the ac impedance of diodes as a function of the dc control current is not so well characterized⁵ and is a variable from one unit to another.

The subjective testing of the compandor has revealed no noticeable degradation in the speech quality, or the hush-hush effect, sometimes associated with compandors. It offers about 28 dB of effective compandor advantage (in the presence of speech). The performance of this compandor well exceeds the recommendation of CCITT⁶ for trunks. High performance in syllabic compandors used in loop carrier systems is most desirable, since they can be in cascade with other syllabic compandors in the telephone network.

2.2.3 Transmitter

A simplified schematic diagram of the transmitter is shown in Fig. 6. It consists of a carrier frequency LC oscillator and a differential pair modulator. The current I_1 is dc with the audio modulating signal superimposed on it. Through the use of an operational amplifier and discrete resistors, the modulation index (ratio of the ac to dc component in I_1) is set with great accuracy. Also, because of the high loop gain of the operational amplifier, the current I_1 is essentially free of harmonics of the modulating audio input, even for modulation levels approaching 100 percent. The modulator, which consists of the differential pair Q_1, Q_2 , can be used either as a linear or switching modulator, by grounding pins

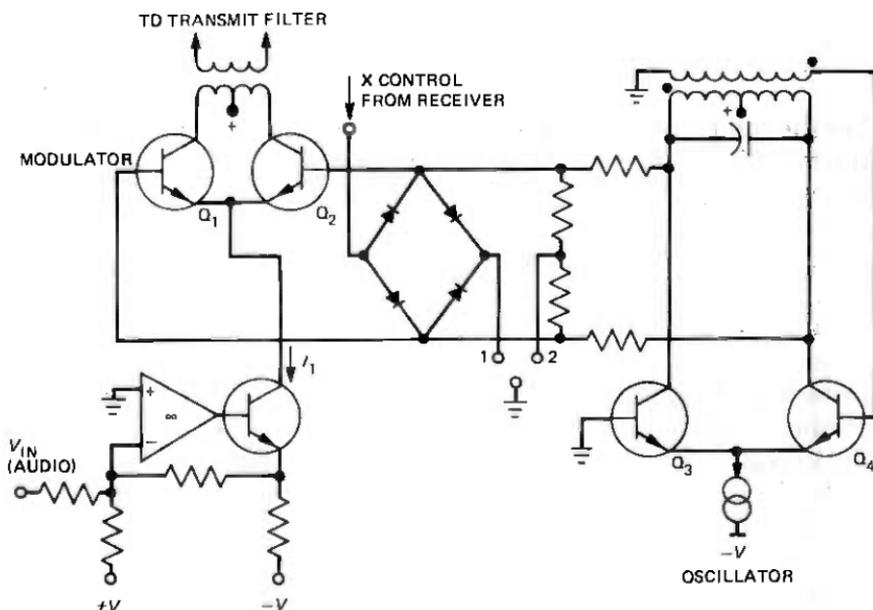


Fig. 6—Transmitter circuit.

1 or 2, respectively. In the remote terminal unit, where the cross-control function is necessary, the modulator is used in its linear mode. The cross-control current from the receiver controls the ac impedance of the diode bridge, and hence the carrier level at the bases of Q_1 , Q_2 and the output level of the transmitter, without affecting the modulation index. The diode bridge also conveniently serves to cancel the gain variation of Q_1 , Q_2 due to temperature, and to reduce carrier frequency distortion, by predistorting the signal. In the central office unit, where the output level of the transmitter is to remain constant, the modulator is used in the switching mode. In this condition the diode bridge is not conducting, and there is sufficient carrier signal at the bases of Q_1 and Q_2 to switch the current I_1 back and forth between Q_1 and Q_2 .

The oscillator circuit uses a differential pair gain stage, where the output is transformer-coupled to the input, in the positive feedback sense. The swing at the base of Q_4 is of sufficient magnitude to switch the current I_2 completely between Q_3 and Q_4 . Hence, the collector currents of Q_3 and Q_4 are square waves, which after being filtered by the LC tank produce sinusoidal voltage at the collectors. The amplitude of the oscillation is determined mainly by I_2 and the load resistance across the collectors, and can be set accurately. The frequency stability is determined almost entirely by the LC product and the active circuit has negligible effect on it.

The entire transmitter circuit is incorporated in one IC chip, and the same chip is used both in the central office and the remote terminal units. In summary, the transmitter exhibits good level and frequency stability, linearity, and accurate modulation index. It operates satisfactorily from -40°C to $+60^\circ\text{C}$.

2.2.4 Ringing generator

For ringing, the 20-Hz ringing signal is detected in the central office unit and the 76-kHz transmitter is turned on and off at 20-Hz rate. At the remote terminal receiver the envelope of the incoming carrier is detected, and the resulting 20-Hz signal is applied to the ringing generator. A simplified functional block diagram of the ringing generator is shown in Fig. 7.

The 20-Hz ringing signal from the receiver is used to turn a 50-kHz oscillator on and off. The resulting 50-kHz pulse train, which is modulated by the ringing signal, is up-converted by the switching transistor Q_2 and the transformer. The output of the transformer, which is a high voltage 50-kHz signal, is envelope detected by the diode CR1 and the capacitor. Hence, the voltage across the capacitor is a high voltage 20-Hz square wave capable of ringing the phone. During the half-cycle that the voltage across the capacitor is high, the ringing current flows through the forward biased zener CR3, the ringer in the phone, and back through

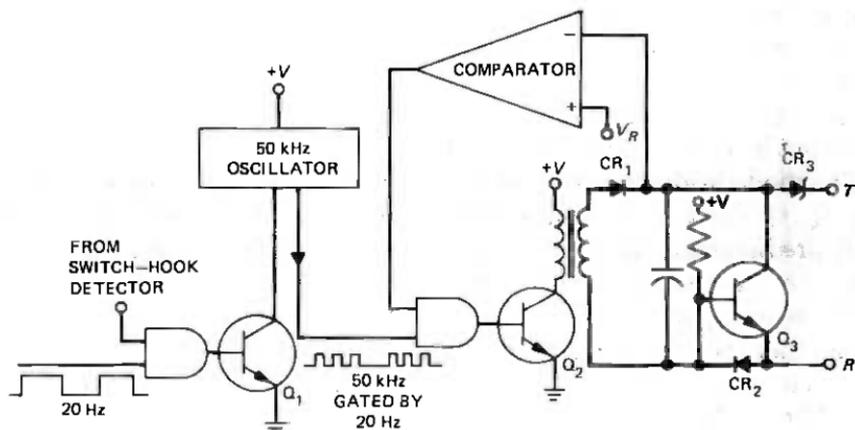


Fig. 7—Ringing generator circuit.

the forward biased diode CR2, which keeps Q_3 off. In the low half-cycle, the current flows back through the reverse biased zener CR3, and the transistor Q_3 , which is saturated in this condition and provides a low impedance path. A high voltage reverse biased diode, not shown in the diagram, isolates the ringing signal from the switch-hook detector circuit. The ringing voltage is regulated by gating off the input to the up-converter as soon as the output voltage exceeds a reference level V_R . In the off-hook condition the ringing generator is inhibited by the gate preceding Q_1 .

This ringing scheme has several advantages. First, by using a high frequency up-converter it avoids bulky 20-Hz transformers. Also, since the 20-Hz switching is done on the low-voltage side of the converter, only one high voltage transistor is necessary. The ringing generator can ring at least three phones, over a temperature range of -40°C to $+60^{\circ}\text{C}$.

All of the components in Fig. 7 to the left of Q_2 , plus the control functions and part of the battery charger, are incorporated into one IC chip.

2.2.5 The battery charger

The battery, which consists of six nickel-cadmium cells, is trickle-charged from the line. The battery charger shown in Fig. 8 draws a small dc current from the line (about 3 mA), and through the use of a simple and efficient switching type converter, charges the battery at a much higher rate.

The input resistors prevent the dc loading of the line, and the diode bridge guards against tip and ring reversal. The oscillator produces a current pulse train, which drives the switching transistor Q_1 on and off into saturation. During the interval that Q_1 is saturated, current drawn from the line builds up in the inductor. When the transistor is turned

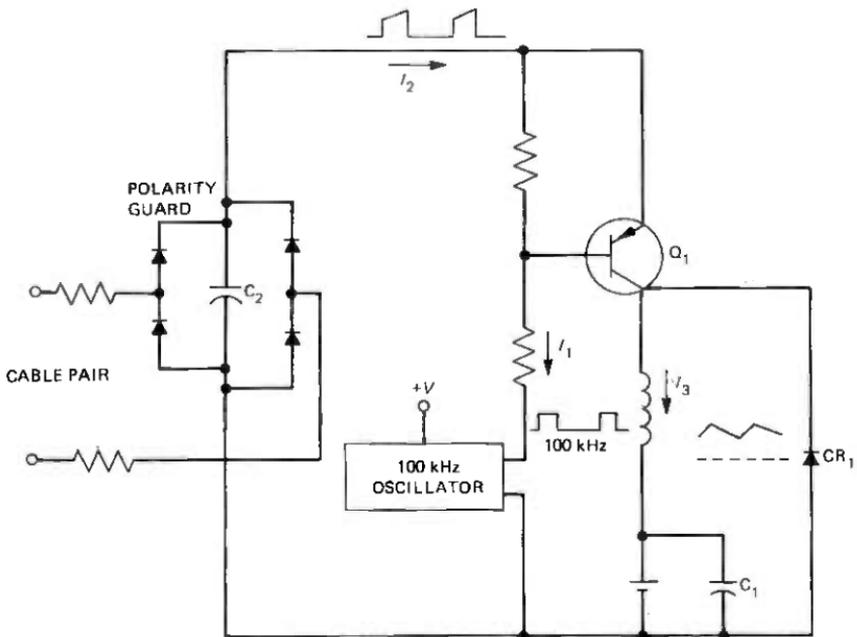


Fig. 8—*SLC*[™]-1 battery charger circuit.

off, current continues to flow in the inductor through the diode CR_1 at a decreasing rate. Hence, the battery is charged during both intervals. The ac component of the current I_2 is filtered by the capacitor C_2 , and its average dc component is drawn from the line. Note that the battery is charged approximately at the peak rate of I_2 , which is much larger than its average value drawn from the line. The ratio of the charging current to the line current is determined by the duty cycle of the oscillator.

2.2.6 Testing compatible remote terminal

The current drawn by the battery charger makes the physical pair look leaky when tested for leakage between the conductors. When the normal *SLC*-1 system is used with ESS offices, the leakage tests performed prior to call completion can indicate excessive leakage and the call might not be completed. In addition, routine insulation tests from an automatic line insulation test set or from a test desk do not provide very useful information on the condition of the cable pair. To overcome these problems a test compatible subscriber unit has been developed. In this version the battery charger senses the onset of a test on the cable pair and disconnects the battery charger from the line for a period of about ten seconds. During this time interval, the subscriber unit looks like a high resistance, greater than 800 k Ω , and useful information on leakage between the conductors can be obtained.

III. THE SLC-8 MULTICHANNEL CARRIER SYSTEM

3.1 General system description

Analog multichannel carrier systems, like single channel systems having their beginning in the trunk plant, were first used in the subscriber plant in the 1950s. The type-P carrier system⁷ designed for the loop plant provided, via DSB-AM techniques, four channels on a single pair of wires. Even though the circuits used transistors, a variety of technological problems made it too expensive except for use on very long rural loops. In the 1960s several manufacturers introduced multichannel analog carrier systems that have been more economical to use. The SLC-8 system represents another large step in the evolution of these systems, since the reliability, performance and economies made possible by the use of custom linear integrated circuits are exploited for the first time.

The SLC-8 system uses one pair of wires to derive eight voice channels. The frequency allocation is shown in Fig. 9, which is essentially the same as the REA allocation to allow for the coexistence of SLC-8 system with other analog systems in the same cable. The CO terminal consists of eight modems and a common circuit pack. The common circuit pack has, in addition to all the circuits necessary to interface the modems with the carrier line, a -48 V to ± 135 V dc/dc converter that powers the remote terminal and the repeaters. The ± 135 Vdc is applied to the line in a metallic mode along with the multiplexed signals. Repeaters using directional filters are used along the carrier line at approximately four-mile intervals on 22-gauge cable. Any combination of wire gauges is allowed in a repeater span and the spacing should correspond to 35 dB of loss at 112 kHz, which is a generally accepted standard for analog loop carrier systems. The remote terminal arrangement can be one of two types. The "lumped" arrangement shown in Fig. 10 has a remote terminal consisting of eight modems and a common power converter that draws power from the line and provides all the required dc voltages for the modems. The other type is the "distributed" arrangement shown in Fig. 11 where there are as many as eight separate remote terminals each housing a modem and a power supply. The distributed system allows the use of the system on very sparsely populated routes since a remote terminal can be connected to the carrier line anywhere along its whole length. In both the

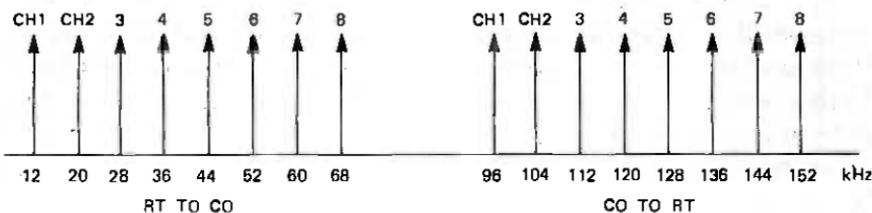


Fig. 9—Frequency allocation for SLC™-8 system.

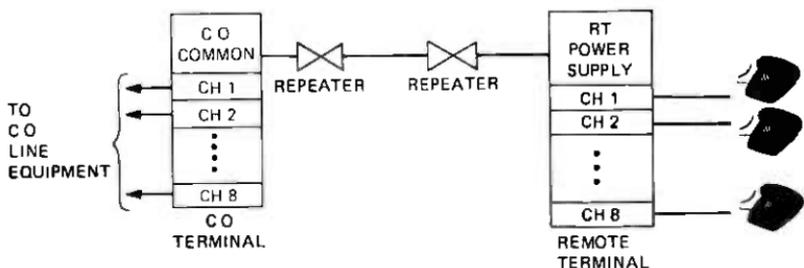


Fig. 10—Lumped SLC™-8 system.

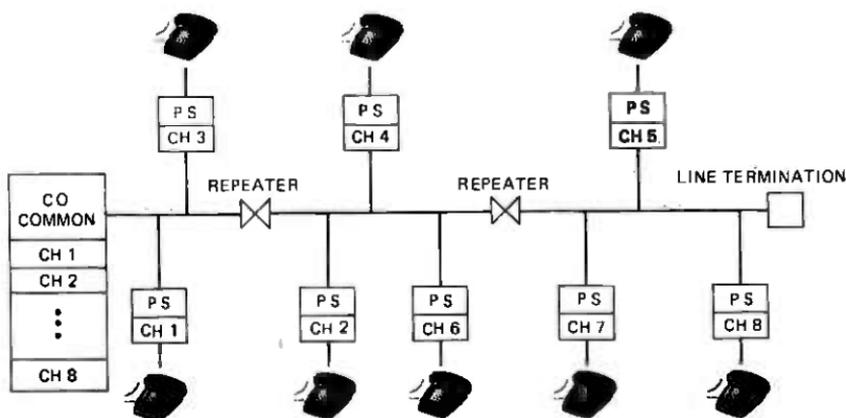


Fig. 11—Distributed SLC™-8 system.

lumped and distributed systems the repeaters and remote terminals are completely powered from the CO up to a maximum carrier line resistance of 2400 ohms. This resistance limit corresponds to approximately 28 miles on 19 gauge and 14 miles on 22 gauge from the CO. For the very few routes that are longer than this allowable conductor resistance, other methods of powering, such as remote ac power with battery backup or a parallel pair for additional dc power from the CO, must be provided.

3.2 Principles of operation

The block diagram of the CO modem is shown in Fig. 12. The similarity of this block diagram to that of the SLC-1 CO modem shown in Fig. 2 is obvious. The basic difference is that the CO to RT carriers are always on and ringing information is transmitted to the RT by modulating the carrier with a 2-kHz tone that is gated at the 20-Hz CO ringing frequency. Also, the receiver works into an expander instead of directly driving the CO line equipment, since companding is used in both directions of transmission. The single-party CO modem uses four custom integrated circuits, one each for the receiver, expander, compressor and the com-

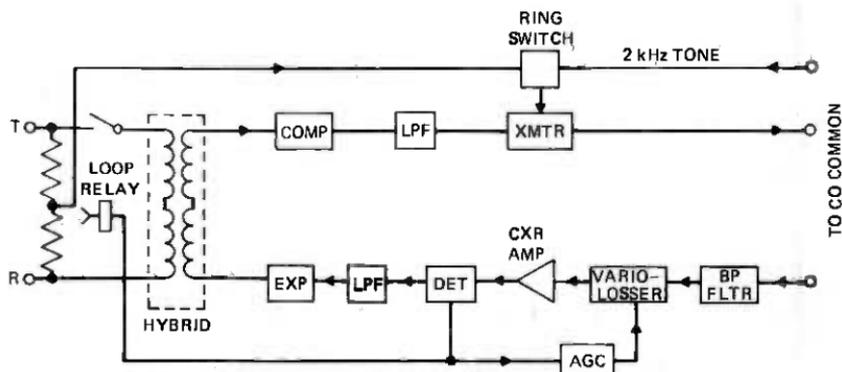


Fig. 12—*SLC*[™]-8 single-party CO modem.

bin functions of transmitting and controls. The transmitter and controls circuit is the only one specifically designed for the *SLC*-8 system; the other three are identical to the ones used in the *SLC*-1 system. Differences in levels in the two systems are accommodated by changes in the values of discrete resistors and capacitors external to the integrated circuits. The multiparty modems use an additional integrated circuit to perform the more complex control functions necessary for both two-party ANI (Automatic Number Identification) and four-party fully selective services.

The frequency allocation in Fig. 9 allows the use of a common band-pass filter to eliminate harmonics of all carriers for the high group as shown in the block schematic of the common circuit pack in Fig. 13. The low group signals received from the line are low pass filtered, preamplified and distributed to the modem cards. The common circuitry also contains the 2-kHz audio tone generator which is used by all the modems

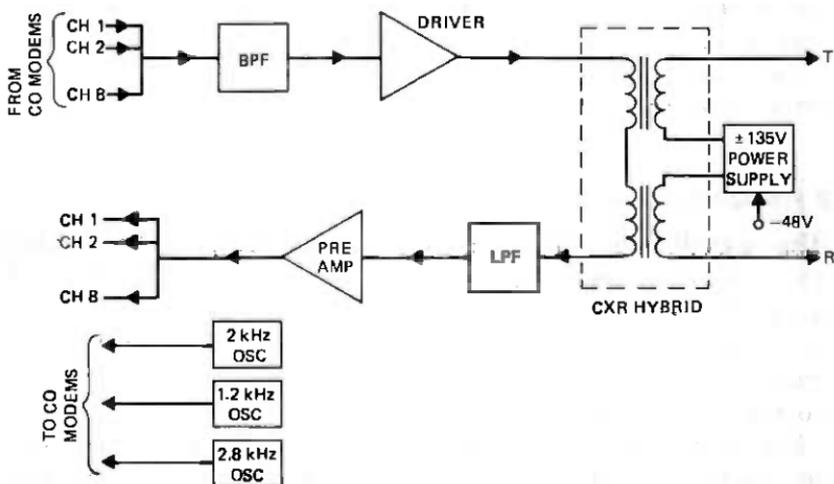


Fig. 13—*SLC*[™]-8 CO common circuit.

to transmit ringing information, as mentioned earlier. For multiparty service the ringing information as to whether the tip or ring party is to be rung is indicated by the presence or absence of a 2.8-kHz tone and the information as to whether it is positive or negative superimposed, by the presence or absence of a 1.2-kHz tone. These 2.8-kHz and 1.2-kHz tone oscillators, which are utilized only by the multiparty modem cards, are also included in the common card. The -48 V to $\pm 135\text{ V}$ dc/dc converter is a switching type operating at a frequency of 82 kHz, which is exactly the center of the low and high groups to minimize interference problems. The circuitry needed to regulate this supply and provide short circuit protection is built into a custom integrated circuit. The same integrated circuit is also used in the repeater and remote terminal power supplies.

The single party modems used in the lumped and distributed remote terminals are identical; a block diagram is shown in Fig. 14. The differences between this modem and the *SLC-1* subscriber modem shown in Fig. 3 are the added compressor in the transmit path and the way ringing is generated. Ringing voltage is obtained by chopping the 180 Vdc provided by the power supply card at a 20-Hz rate. The 20-Hz signal is obtained from the received gated 2 kHz on the carrier so that the ringing frequency is exactly the same as at the CO. These modems use a total of five linear integrated circuits, one each for the receiver, expander, compressor, transmitter and controls. The transmitter, compressor, and expander circuits are the same as the ones used in *SLC-1*, the receiver is very similar to the one in *SLC-1*, but the controls circuit is specifically designed for *SLC-8*. As in the multiparty CO modem, an additional integrated circuit is used for multiparty control functions.

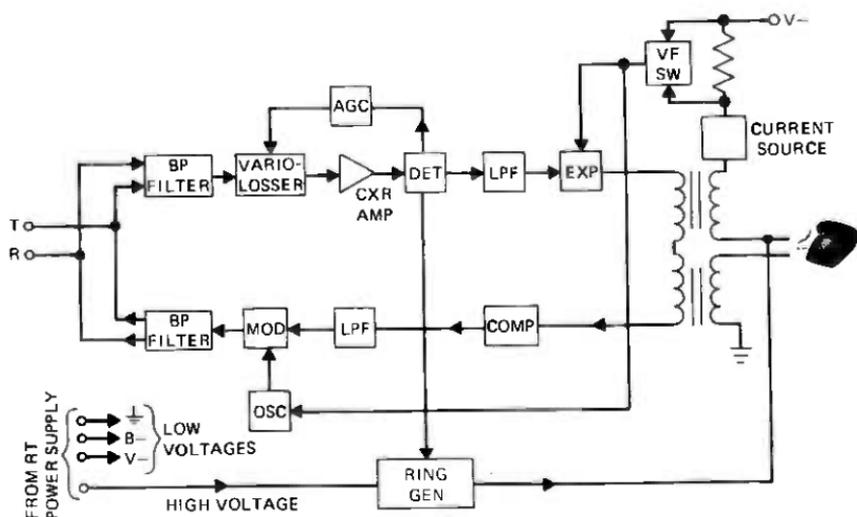


Fig. 14—*SLC*[™]-8 single-party RT modem.

The common power supply used in the lumped remote terminal is shown in block form in Fig. 15. The ± 135 Vdc on the carrier line is fed into two dc/dc converters through a low-pass filter and a polarity guard. One of the converters provides 180 Vdc to the modems for conversion into ringing signals when needed. The other converter provides required voltages for the modem circuits and approximately 16 volts for powering the telephone sets. Like the CO converter, these converters are also switching-type operating at 82 kHz. The power supply in the distributed remote terminal has only one dc/dc converter that provides all the required voltages to the modem. The converters in both the lumped and distributed terminals use the same integrated circuit as in the CO converter for control.

The functional diagram of the repeater is shown in Fig. 16. The high group signals from the CO direction are passed through the high-pass filter level adjusted by a variollosser and preamplifier combination. The shaping network shapes the signals so as to compensate for a nominal repeater span. The line driver drives the line through another high pass filter. The variollosser is controlled by the output of the line driver so that the high group output on the field side of the repeater is similar to the levels originating at the CO. The low group side of the repeater is similar to the high group side except that the variollosser in this direction is controlled by the same control current as in the high group variollosser to preequalize for the

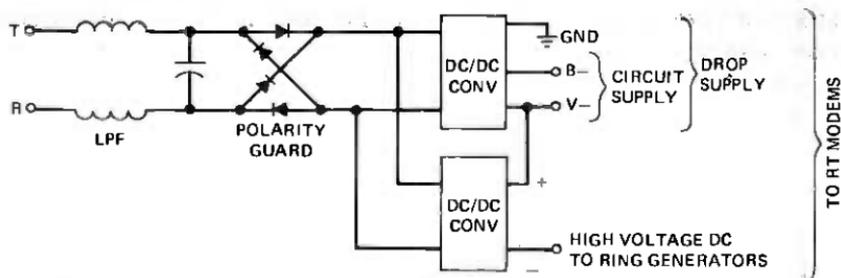


Fig. 15—RT common power supply.

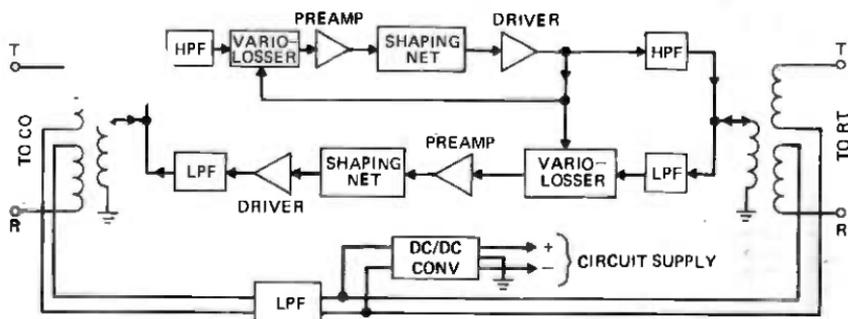


Fig. 16—SLC™-8 repeater.

repeater section on the CO side of the repeater. The directional filters are conventional LC filters. The loss requirements of these filters are determined not just by the stability requirement of the repeater but also by the required stability of the gain shape in each direction and inherent noise in the input stages of the preamplifiers. The filters introduce some nonlinear phase shifts in the carrier channels close to the cut-apart region. These nonlinear phase shifts result in linear and nonlinear distortions in the baseband signal and cannot be allowed to accumulate over many repeaters. Phase equalization in cascade with each repeater is used when the total number of repeaters used on a carrier line is more than three, and all eight baseband channels have to meet the standard transmission objectives for frequency response.

The repeater uses a total of five integrated circuits, one in the power supply that converts the ± 135 volts on the line to the voltages needed for the circuit, one for gain control and preamplification in each direction and two line drivers. The integrated circuits used in each direction of transmission are the same types but use different external components.

IV. ADVANTAGES OF ANALOG CARRIER SYSTEMS IN THE LOOP PLANT

All the advantages of analog carrier systems which fall into the broad categories of ease of engineering and installation, flexibility, and simple maintenance arise from the low bandwidths needed for transmission, absence of complex common equipment, ability to use one pair of wires for both directions of transmission and the availability of a high performance syllabic compandor.

The impairments to carrier systems caused by crosstalk between cable pairs can be very effectively circumvented by analog carrier systems. The low bandwidth requirements of these systems allows the use of two separate bands of frequencies for the two directions of transmission without serious increase in the cable loss which not only makes these systems immune to NEXT but also makes operation on a single pair of wires possible. The other significant crosstalk, FEXT, is overcome by the use of syllabic compandors. It has been shown⁸ that a compandored DSB analog carrier system when installed, even in small 25-pair cables with no pair selection, has insignificant intelligible crosstalk problems due to FEXT. This insensitivity to the two major crosstalk mechanisms in twisted pair cable gives rise to wide repeater spacings (approximately four miles on 22 gauge) and very simple carrier line engineering. Once the bridge taps and load coils are removed, the repeaters are located at 35 dB loss at 112 kHz as computed from cable records. Field measurements of loss at repeater locations are made basically to locate unrecorded bridge taps or load coils. Because of the wide repeater spacings

few long routes need more than three repeaters, two being the average. This minimization of the number of repeaters in a system decreases the number of repeater sites along a route even if there are many systems used along a route.

Installation of these systems is greatly simplified by the lack of complex common circuitry and interconnection within a terminal. Common circuitry is essentially limited to power supplies and is built into the system. Each CO terminal has its own $\pm 135\text{V}$ line supply and each terminal, which occupies only half a shelf,⁴ needs only -48V from the CO which is fed through a dedicated fuse in a fuse and alarm panel. The CO installation, therefore, essentially consists of mounting the shelf, connecting -48 volts, and cabling the carrier line and demultiplexed channels to the main distribution frame.

The plug-in repeaters used along the carrier line have no field adjustments just as all other parts of the system. The remote terminals, lumped or distributed, are completely powered from the CO up to a cable resistance of 2400 ohms, a particularly important advantage where power distribution is subject to long outages. Most applications on long routes fall within this resistance limit. This powering capability comes from several reasons. One is that an eight-channel system can be located nearer to the customers being served without any significant system fill problems and this decreases the power needed for the telephones. The second reason is that the multiplexing and demultiplexing operations are done by passive LC filters which need no power. Complete CO powering of these systems is a definite advantage since no coordination of installation activities with the power company is needed. Standby batteries for powering the system during power outages, which are generally considered as high maintenance items are also eliminated.

The minimal engineering and installation effort needed for these systems provide flexibility: a decrease in planning interval needed allows rapid response to service requests and growth by small increments, a system at a time or even a channel at a time, which decreases initial investment and takes out the risks generally inherent in growth forecasts. Temporary applications, even in urban areas, are encouraged since in most cases fewer than three repeaters are used and it is relatively easy to remove a system from service and reuse it on another route.

Since the only significant common equipment in a terminal are power supplies, the reliability of analog carrier should in principle be excellent. The lack of complex organization greatly simplifies troubleshooting and craft training.

V. THE FUTURE OF ANALOG CARRIER IN THE LOOP PLANT

The single-channel carrier system described can be installed and removed with very little craft training and practically no disruption of

service to the existing service on the physical wire pair. Any time-domain multiplexing technique to allow the transmission of two channels on a wire pair requires modulation of both channels. This essentially involves two sets of electronics as opposed to one set of electronics for a frequency division system that preserves baseband transmission. Hence, the cost advantage of these analog single-channel systems is likely to continue for a long time.

In case of the multichannel system the ease of engineering, installation, and maintenance, for reasons given earlier, are hard to surpass. The seemingly small number of channels, eight, on a pair of wires gives about the same pair gain ratio as the larger digital systems that need four pairs of lines including spare carrier lines. It is interesting to note that even though it is possible to increase the number of channels from eight by increasing the bandwidth on the carrier line, or going to a single sideband system, it is in most cases undesirable for several reasons. The ability to power the repeatered line, remote terminal and the standard telephones at a given ohmic distance from the CO vanishes as the number of channels is increased and the need for a spare repeatered line can arise, resulting in a loss of some of the important advantages of these systems. From these general arguments it appears that the number of channels per system is not likely to be increased in future designs.

The basic analog carrier channel can provide better than 10 dB noise performance and is transparent to most voice band data signals. The extension of integrated circuit technology to analog systems as it has been done in the *SLC-1* and *SLC-8* systems has eroded the cost differences between analog and digital modems which provided the original impetus for digital systems. Further developments such as the use of mechanical filters and low-power phase-locked loop techniques can benefit these systems even more and their attractiveness will continue as long as there is a demand for small cross-section systems, or a need to work between an analog local office and a standard telephone set.

VI. ACKNOWLEDGMENTS

The author, as it frequently happens while reporting on significant development projects, is just the scribe singled out of a group of people, and of the large group of people responsible for the development of *SLC-1* and *SLC-8* systems acknowledges the contributions of P. P. Bohn, B. S. Bosik, J. H. Derby, D. E. Stone, R. Toumani, F. P. Tuhy, Jr., and T. G. Wilson, Jr., to the system and circuit designs reported in this paper.

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Loop Plant Electronics:

Digital Loop Carrier Systems

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This article examines the reasons for developing large pair-gain digital carrier systems, describes two such systems that were developed for Bell System application—namely, the SLM™ Subscriber Loop Multiplexer System and the SLC™-40 Subscriber Loop Carrier System—and discusses in detail several circuit designs used in the SLC-40 System. The SLM system is described and its application history is discussed. Also, the experience gained from the SLM development and its impact on the development of SLC-40 and the Loop Switching System (LSS) is reviewed. A detailed description of the SLC-40 equipment and system operation is presented next. The introduction of this system into the operating telephone companies is also covered. The final section, which discusses circuit innovations that make the SLC-40 system possible, includes details on channel unit dc signaling and the multiplexer.

I. INTRODUCTION

The loop electronics overview to this issue¹ examines the need for pair gain systems in the loop plant and traces the history of these systems. This article examines briefly the reasons for developing large pair-gain digital carrier systems developed for the Bell System, the SLM™ Subscriber Loop Multiplexer System and the SLC™-40 Subscriber Loop Carrier System; it then finally discusses, in detail, design breakthroughs which made the SLC-40 system possible.

In the late 1960s, P. A. Gresh of Bell Laboratories and C. D. Howe of AT&T² studied the long subscriber route characteristics of the Bell System and concluded that the optimal size for a single carrier system would be between 75 and 100 lines. This size system could serve the

largest amount of long route growth and save the most capital compared to a cable design. This analysis influenced the design of the *SLM* system, the first Bell Laboratories design of a large pair-gain digital system.

A digital approach was chosen in 1967 when the *SLM* system design began, since a digital system offered the potential for high transmission quality and low cost with newly available integrated circuits. Further, digital systems offered the potential of lower cost by using time division concentration at the remote terminal, and could use the T1 trunk carrier hardware for the digital line.

The *SLM* system, first introduced in the Bell System in 1971, is a combination carrier and concentrator system that has an 80-line capacity. The *SLM* system uses delta modulation encoding. The line concentration designed into the *SLM* system provided for 80 subscriber lines switched to 24 digital channels. Adding full access of all subscriber lines to the carrier channels assured that the probability of blocking would not exceed 0.5 percent more than four times per year.

The T1 trunk digital carrier line was chosen as the basis for the transmission medium for the *SLM* system because of its proven design and good operational experience. Dependable, low cost repeaters and apparatus were already available, eliminating development time and expense. Maintenance methods and test equipment for T1 carrier were already available and proven. Finally, the use of the latest integrated circuit and thin film technologies in the *SLM* system permitted addition of desirable service features into the circuit designs while retaining the potential for low cost.

SLM designers were strongly influenced by the expectation that a multiple remote terminal system, with each terminal having a capacity of 40 lines, could be more useful to the operating companies. In this configuration, simple per-channel adaptive delta modulation codecs (see Ref. 4) offered lower costs than shared PCM codecs duplicated in each remote terminal.

In 1974 the *SLC-40* system was introduced into the Bell System (Fig. 1). The *SLC-40* system was a result of the experience gained from the *SLM* system and from breakthroughs in delta modulation encoding which permitted 40 channels to be transmitted over a single T-carrier line, compared to 24 channels for the *SLM* system. The *SLC-40* system design resulted in a simplified, unconcentrated, high pair gain, low cost system.

After descriptions of both systems are presented in the next two sections, considerably more detail is devoted to those innovations in the *SLC-40* system which have made it a low cost, high performance system which has made major penetration into the rural plant in the Bell System.

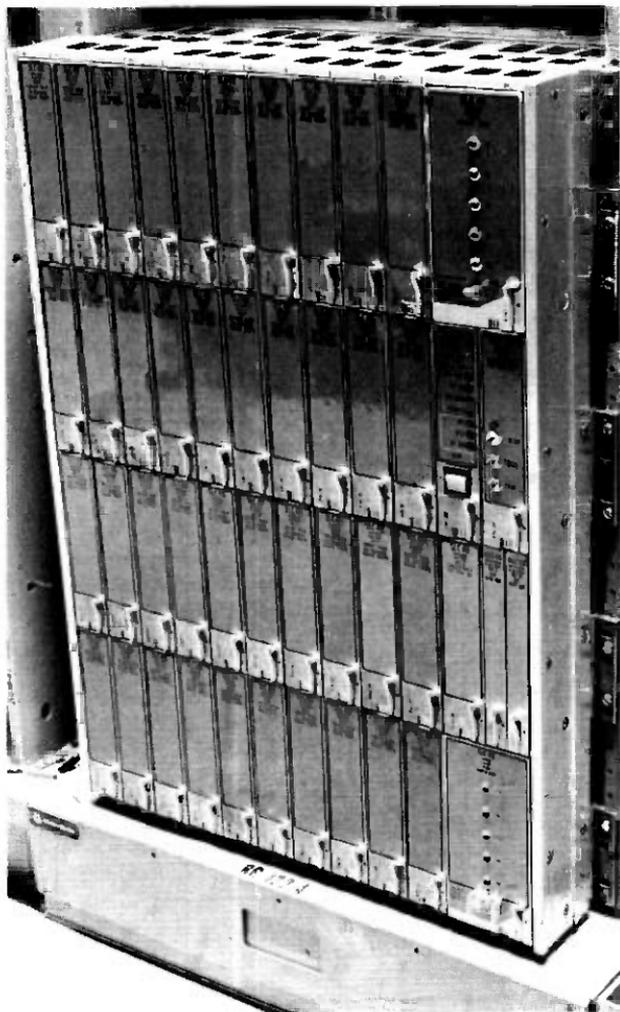


Fig. 1—The SLC™-40 central office terminal.

II. SUBSCRIBER LOOP MULTIPLEXER

2.1 Description

The *SLM* system is a digital carrier system that combines switching and carrier to serve 80 customer lines over a T1-type digital line. The system is capable of providing single-party, two-party ANI, multiparty, and coin service. A block diagram of the *SLM* system is shown on Fig. 2.

SLM circuitry was designed to be installed on a “per-line” basis as much as possible. The use of per-line equipment at the remote terminal assures that simple, rapid reparability is possible and also minimizes operating company investment when fewer than eighty customers are served. The customers can be grouped at several locations and still have access to all 24 channels.

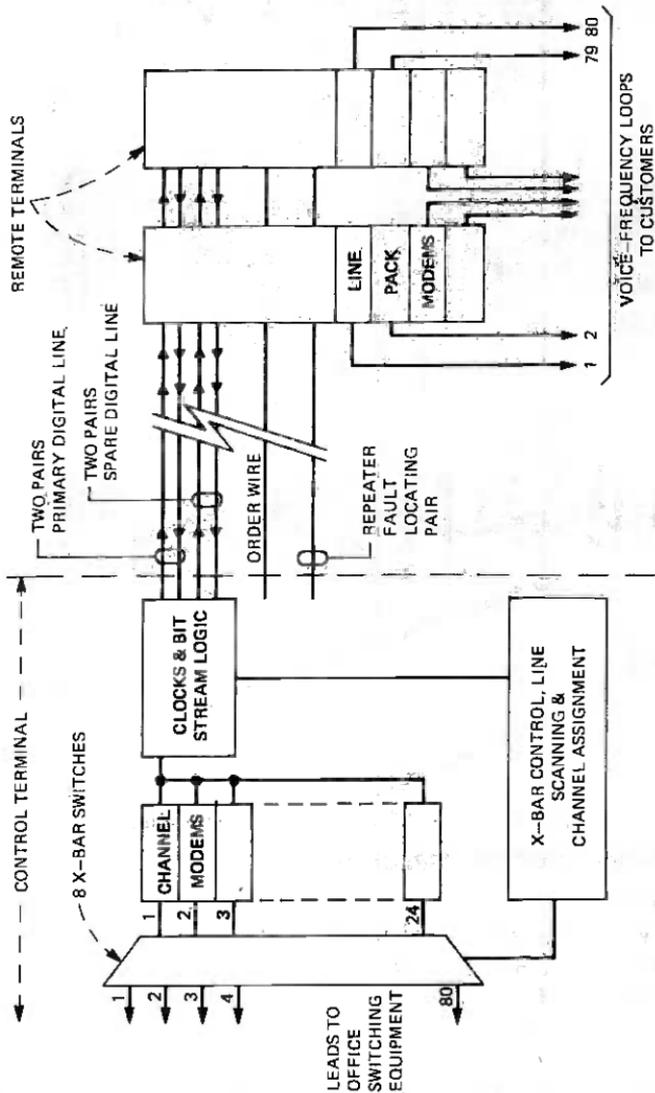


Fig. 2—Block diagram of subscriber loop multiplexer.

To achieve a high pair gain and a low cost per line served, the *SLM* system concentrated 80 lines onto 24 channels. All lines at all remote terminals were provided full access to all channels to assure a low probability of blocking.

The central office expansion from 24 channels to 80 lines is performed by an 80 by 24 miniature crossbar switch. The remote terminal concentration is performed by assignment of one of the 80 lines to one of the 24 channels transmitted over the digital line. The line pack at the remote terminal performs many important functions for each customer's line. It connects the line to the assigned channel, converts analog voice signals to digital signals and vice versa, and performs all line signaling functions such as off-hook detection, dial pulse or *TOUCH-TONE*[®] transmission, ringing detection and ring trip detection. By including most circuit functions in the line pack, the remote terminal common equipment can be reduced. The line packs can be added only when needed, thereby reducing initial system costs.

The *SLM* system was designed with ease of maintenance as an important feature. By inserting both line signaling and transmission onto the line pack, a per-line trouble can be quickly located, and if the trouble is within the *SLM* system, the line pack can be easily replaced. A per-line testing capability is built into the system that can be accessed from a test desk. This allows diagnosis of the reported trouble prior to dispatching a repairperson.

Extensive alarms of common system troubles were also provided. Automatic equipment checks all channels daily, removes bad channels from service and signals the trouble condition. Common system problems, including channel failure, can usually be repaired by plug-in card replacement.

Several features make the digital line portion of the system reliable. The digital line is monitored for errors, and if the error rate exceeds a threshold, a spare line is automatically switched into service. Also, a remote terminal or a defective portion of the digital line can be disconnected by looping back the digital line at the most distant remote terminal that could sustain service. This keeps the customers in the unaffected portion of the system in service. Additional details on loop electronics maintenance are described elsewhere in this issue.³

To attain maximum versatility, the *SLM* system can be operated with up to six remote terminals, as long as the total number of subscriber lines does not exceed eighty. This versatility can be an excellent hedge against uncertain growth.

The designers of the *SLM* system made extensive use of the most recent developments of integrated circuits and thin film substrates to serve many functions at low cost. Their use was essential to provide the logic for delta modulation at low cost and very low power levels. Low power

consumption is very important for the remote terminal standby battery design, allowing use of very low maintenance nickel-cadmium batteries.

Additional details on the characteristics and use of the *SLM* system appear in Refs. 4, 5, and 7.

2.2 Experience

A field experiment of the early *SLM* design was conducted in Lawrenceville, Georgia in 1968. The designs were then modified and improved, and the first operational field trial of *SLM* hardware was conducted by South Central Bell Telephone Company in Brandon, Mississippi from October 1971 to October 1972. The first telephone customers were served by this system in December 1971. The Brandon trial verified that the hardware design was satisfactory and that the concentration feature maintained good service with no known cases of blocked calls.

A second trial by Northwestern Bell Telephone Company was held in Duluth, Minnesota to evaluate performance in a cold climate. This trial was a full operational trial; it checked all procedures and documentation in addition to checking the improved hardware designs. Northwestern Bell employees installed and maintained the systems throughout the trial. This system was made operational in December 1972 and the first customers were loaded on the system in February 1973.

The use of the *SLM* system for pair gain applications grew in the operating telephone companies until 1975 when the *SLC-40* system became available. Since the simplified *SLC-40* system could serve in similar pair-gain applications and had a significant cost advantage over the *SLM* system, manufacture of the *SLM* system was discontinued in 1977.

Though no more *SLM* systems will be manufactured, the operating *SLM* systems are providing high quality service with low maintenance. The *SLM* system was a pioneer in digital pair gain systems in the Bell System. It has given the operating telephone companies confidence in the ability of current pair gain systems such as *SLC-40* and *LSS*⁶ to provide reliable, high quality service and to save construction capital investments. It also showed that a system could be designed using concentration that would not limit service due to the traffic.

The experiences and observations gained in the development and use of *SLM* have been useful in selecting features and concepts for more recently developed pair gain systems for subscriber use such as the Loop Switching System (*LSS*) and the *SLC-40* system, which will be described in detail later. Some of the more important lessons learned were:

(i) T-type digital lines will perform satisfactorily in the loop plant and can be properly installed and maintained by available craftspeople.

Operation of these lines and their low-cost T1 hardware have proven very practical in the subscriber cable environment. The design of these digital lines, supported by computerized techniques, can be readily handled by operating company engineers. T-carrier technology was chosen for the digital line of *SLC-40*. Similar satisfactory performance has been observed on the *SLC-40* systems.

(ii) A well-designed concentrator, with a very low probability of blocking, will perform well in the subscriber loop plant and give excellent customer service. An intracalling feature at the remote terminal is not necessary to provide good service. Traffic studies of *SLM* systems in service showed that the assumptions used for *SLM* traffic design were conservative even in a suburban environment where telephone activity should be higher than in rural communities. These observations supported the conservative traffic design approach that was subsequently designed into the Loop Switching System (LSS).

(iii) Several maintenance concepts were reinforced by *SLM* experiences. The need for a spare digital line was confirmed by maintenance tracking studies.³ Plug-in maintenance was found to be easily performed. Per-line equipment aided in quickly finding single customer problems. Common alarms associated with a small number of common circuit packs helped to simplify system troubleshooting. These features were directly implemented into the *SLC-40* design.

(iv) The feature of allowing up to six remote terminals per *SLM* system did not turn out to be economically viable. The extra cost involved in establishing each remote terminal more than offset the advantages of extra flexibility. Almost all *SLM* systems were equipped with just two remote terminals; only a few had three remote terminals. No more than three were ever used.

(v) Finally, *SLC-40* was the result of continuing work to improve upon the encoding algorithms used in *SLM*. In the course of this work it was discovered that suitable speech quality could be obtained with less than 40 kilobits per second sampling. This made possible a simple carrier system with much lower "getting started" cost than *SLM*, a desirable attribute for application on low-density small-cross-section routes. As it turned out, the IC technology then available made the new *SLC-40* system lower cost than the *SLM* system at all sizes and led to the phase-out of the *SLM* system.

III. THE *SLC-40* SYSTEM

The *SLC-40* system is a digital subscriber carrier system that uses delta modulation to derive 40 full-time voice frequency channels from a single digital line, operating at 1.544 megabits/second over two cable pairs. This section will provide a functional description of the *SLC-40* system and will describe introduction of the system to the operating telephone companies.

The *SLC-40* system (shown in Fig. 3) can best be described through its three main components: The Central Office Terminal (COT), the Remote Terminal (RT), and the Digital Line.

3.1 The COT

The COT equipment of an *SLC-40* system is composed of a central office (CO) channel bank assembly, a jack panel, and a fuse and alarm panel, and is designed to be bay-mounted into an existing or newly placed central office. One 11-foot 6-inch CO frame can accommodate the necessary equipment for up to four central office terminals.

The COT equipment (shown in Fig. 1) provides an interface between the digital signals transmitted over the digital line and the subscriber originated voice frequency signals that appear at CO line equipment terminals and are connected to the main distributing frame. Each channel bank is connected to a main and a spare digital line at the jack panel. The jacks provide access for performance testing of both the channel bank and the digital lines. The channel banks are powered by normal -48 Vdc and optional 130 Vdc office battery via fuses in the fuse and alarm panel, which also contains relays that, when operated, output system alarms to the office. Power sharing resistors that are utilized by the optional current regulator units to power the digital line are also located within the fuse and alarm panel. One jack panel and one fuse and alarm panel provide connecting facilities for four CO channel bank assemblies.

A CO channel bank assembly must be equipped with two types of plug-in apparatus. These are the common circuit packs and the per-line circuit packs, called COT channel units. All seven of the common circuit packs are necessary for proper system operation, while the 40 channel units may be added one at a time to satisfy the demand caused by continued growth in customer service.

The common circuit packs plug into connectors that are mounted within the two right most columns of the four shelf channel bank assembly. The common units include a low voltage power unit that converts office battery into a +5 volt and -8 volt supply for TTL logic and the analog circuits of the channel units, and a line-feed power unit that converts office battery into a current-regulated voltage supply for the digital line. A line interface unit couples either the main or the spare digital line to the line-feed power unit and the digital multiplexer circuit packs, while generating the transmit and the receive clock signals used at the COT. The choice of either main or spare digital line operation is determined within the maintenance unit by monitoring line errors and other system status indicators. The three remaining common circuit packs contain the multiplexer/demultiplexer circuits. The multiplexer/demultiplexer provides parallel-to-serial conversion of the

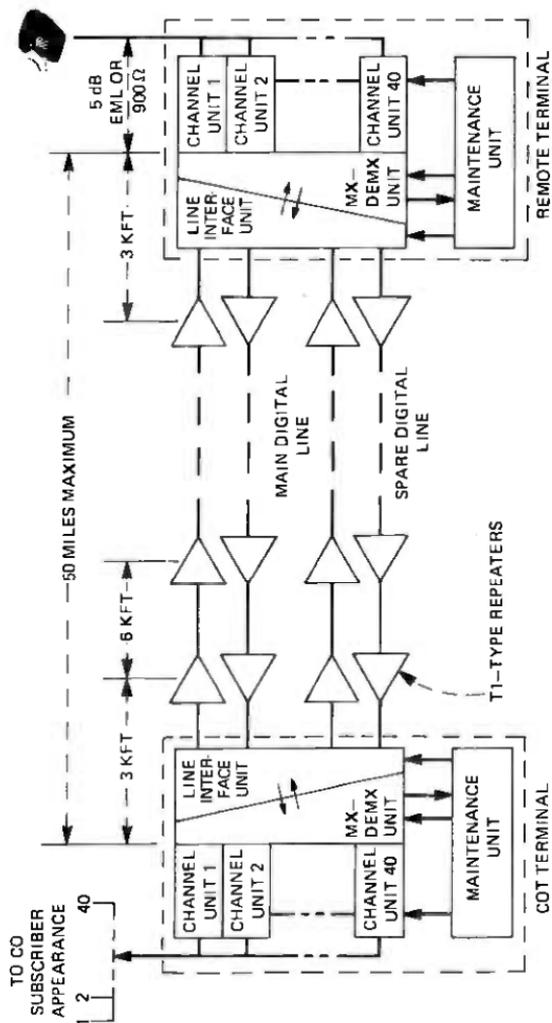


Fig. 3—Typical *SLC*[™]-40 system.

channel pack output data signals into a transmit data stream and also provides serial-to-parallel conversion of the receive data stream into 40 individual channel pack input data signals. The multiplexer also controls the synchronization of data transmission between the COT and the RT. This function is called framing.

The channel units perform three basic functions: ringing, transmission, and signaling. Each COT channel unit can detect the presence of a ringing voltage applied by the office at the CO subscriber line appearance. When this occurs, the COT channel unit outputs a special digital code that is transmitted to the corresponding RT channel unit. The RT channel unit detects this code and, if the subscriber's station set is not off-hook, applies ringing. The kinds of ringing voltage that can be applied to the subscriber are determined by the type of COT and RT channel units installed. There are three codes of COT channel units, selected according to service requirements; i.e., single-party service, two-party service with ANI, and multiparty service with full selective ringing.

The transmission of voice frequency signals by the COT channel unit is facilitated by the hybrid circuit. The hybrid circuit separates transmit and receive signals, thereby allowing them to be processed individually. The channel unit transmitter low-pass filters the CO signal from the hybrid and encodes it using delta modulation into digital data.⁸ The encoder sampling is timed by transmit clock signals from the multiplexer. These signals also cause the data to be output to the multiplexer on the transmit data bus. Independent of the channel unit transmitter, the receive data bus transfers data from the demultiplexer to the channel unit receiver. This data is input to the channel unit at the proper time by a receive clock signal. The data is then decoded by a delta demodulator, filtered, and amplified to standard audio levels. The hybrid circuit directs the resultant voice frequency signal towards the CO.

The COT channel unit also decodes signaling information from the received data. When a subscriber's station set is on-hook, the RT channel unit transmits all ones data to the COT channel unit. There, a dial pulse receiver checks the incoming data for the presence of continuous ones and, if present, forces the dial pulse repeat relay to release. This causes a dc open circuit towards the CO. If the subscriber's station set goes off-hook, a delta modulation pattern (random ones and zeros) rather than continuous ones is transmitted to the COT channel unit. This break in the all-ones data sequence is detected by the dial pulse receiver which causes the dial pulse repeat relay to operate. Thus, dc loop current is drawn from the CO, indicating that the subscriber's station set is off-hook. Dial pulsing, which can be viewed as a series of on-hook and off-hook conditions, is treated in the same manner as loop supervision.

3.2 The RT

The remote terminal of the *SLC*-40 system is configured for both cabinet and frame mounting. The RT cabinet is designed to handle applications where moderate subscriber service growth would normally require feeder cable reinforcement. The RT frame is used when several remote terminals are required at one location or an RT is to be installed inside a building.

The RT cabinet apparatus consists of a three-section weatherproof aluminum cabinet (see Fig. 4) that contains a four-shelf RT channel bank assembly, primary lightning protection, jack access to a main and a spare digital line, a battery compartment that is equipped with a heater, a ringing supply, and an ac powered battery charger. This cabinet can be either pole or pedestal mounted for use with aerial and buried plant.

The RT channel bank assembly is designed to accept 40 RT channel units and seven common units, in a configuration similar to the COT channel bank assembly. The RT common units perform the same functions as those used at the COT, with the major operational difference being that the transmit and receive clocks are both derived from the incoming digital line signal. Located beneath the RT channel bank assembly, another shelf mounts the ringing supply and the battery charger plug-in units. The battery charger uses 117 Vac to develop a nominal

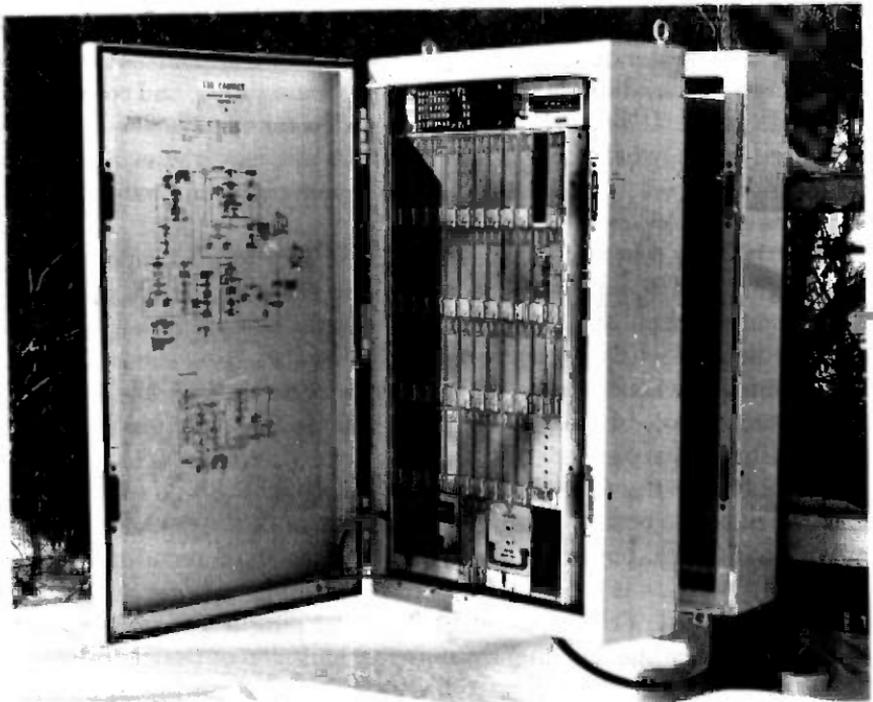


Fig. 4—The *SLC*[™]-40 remote terminal cabinet.

-43 Vdc that is used to charge the flooded nickel-cadmium battery string, power the plug-in apparatus, and supply talk current. This choice of battery voltage allows dc supervision on subscriber loops to 900 ohms. When ac power is lost, the reserve power stored in the battery string will sustain system operation for at least 10 hours.

The RT frame equipment consists of a 7-foot frame that is factory wired with two RT channel bank assemblies, a common ringing supply, and a fuse, filter, and jack panel. The RT frame must be housed in a PBX-type building environment that includes a signal grade -48 Vdc power supply equipped with stand-by batteries, a cross-connect facility, and primary lightning protection for the voice frequency and carrier pairs. The fuse, filter, and jack panel contains fusing and a filter used to connect the external power supply to the channel bank assemblies and the ring supply. This panel also provides jack access to the digital line, an order wire, and an optional fault-locate pair.

3.3 The digital line

The digital line of the SLC-40 system is configured such that apparatus from the T1 carrier system can be utilized. Two-way repeaters are used to regenerate a digital bipolar signal at the T1 line rate of 1.544 megabits/second. The repeaters, typically spaced at 6-kilofeet intervals, are simplex powered over the cable pairs of the digital line.

Two types of repeater apparatus are available for construction of the digital line. The standard T1 configuration includes apparatus cases plus plug-in repeaters, adapters, coil cases, fault-locate filters, and connectors. In addition, the 217-type repeater is provided for applications in aerial or buried plant, where only a few repeaters are required at a repeater point and where it is desired to mount repeaters within existing non-pressurized enclosures.

The 217-type repeater consists of a single 209-type T1 repeater, primary lightning protection via four dual-gap gas tubes, and a connectORIZED weatherproof housing. This repeater may be removed from the digital line for testing or replacement by disconnecting the stub cable. Provisions for a fault-locate filter are not made in this case since the maintenance procedure for this repeater utilizes the quick-access feature to perform a binary search of the digital line with a portable test set.

Depending on the overall length and gauge of the digital line, the line is powered either entirely from the COT, from both the COT and the RT, or with additional powering from remote power-feed terminals. When multiple power spans are required, a power looping repeater is necessary at each junction. This repeater isolates the simplex current of each powering span, while enabling through transmission of the digital signal. The SLC-40 system can operate over either a standard 140 mA T1 line or the new 60 mA low power T1 line, when equipped with the proper

power and line interface units. A spare digital line is required with a cabinet-mounted RT.

When an RT frame is used, the two remote terminals in the frame can each have a main and a spare digital line, share a spare digital line, or be connected to a T1 Outstate system, which provides either a 1 for 5 or 1 for 11 protection line switch for the *SLC-40* systems.

The 1 for 2 spare line sharing option without T1 Outstate is implemented by two relays, one located at the COT jack panel and the other in the RT fuse, filter, and jack panel. These relays are wired so that the spare digital line is connected to system 2 through the normally closed contacts. If system 1, the priority system, needs the spare line, it operates the relays, thereby connecting it to the spare line through the normally open contacts.

To use digital lines provided by the T1 Outstate equipment, special line interface units are required for the *SLC-40* system. The COT and the RT frame must be wired with the "no spare" option and need not be equipped with line-feed power units, as the digital line is powered through office repeaters located within the span terminating module of the T1 Outstate system. Also, special sensing and switching plug-in units are required at the span terminating module. These units pass bipolar violations that occur on the digital line, thus permitting normal operation of the *SLC-40* multiplexing and maintenance apparatus (see Section 4.2.2).

3.4 Operating company introduction

A field trial model *SLC-40* system was first used at Ligonier in the Western Area of Bell of Pennsylvania. The first customers were placed on the system in March 1974. This trial was not just a hardware trial, but rather a full operational trial in which Bell of Pennsylvania staff constructed, installed and maintained the system, using preliminary documentation in the standard Bell System format. All aspects of the trial were successful. Minor hardware and documentation modifications were made for the twelve introduction systems that were produced in late 1974 and early 1975.

The original single party capability has been expanded to include multiparty (four-party) and two-party ANI service. The channel packs for both of these offerings underwent field trial in C&P of West Virginia, multiparty in the summer of 1975 and two-party ANI in the spring of 1976. Both of these trials were successful and these channel packs are now available for telephone company use.

Operating telephone companies requested manufacture of *SLC-40* remote terminal equipment in a frame for use inside a building. This equipment has use where a building already exists or where a building can be built for a wire center deferral. Also, in cases where a large number

f remote terminals are collocated, it may be feasible for cost or appearance to use a small building. Obvious savings are available by eliminating costly cabinets and sharing batteries and battery charging equipment.

The frame-mounted *SLC-40* system developed to satisfy this need was first used in Winterset, Iowa in a small central office building that had once held a small switching machine. Equipment cutover was completed in the fall of 1976 with no difficulties. This location was also used in early 1977 to field trial extended range channel packs, capable of service out to 1600 ohms beyond the RT. The frame-mounted remote terminal has had good acceptance by the operating telephone companies.

IV. CIRCUIT DESIGNS THAT ARE UNIQUE TO THE *SLC-40* SYSTEM

4.1 DC signaling: simulating a metallic pair

SLC-40 channel units must transmit dc signaling* as well as voice frequency signals. Since the delta modulator cannot transmit these dc signals, special detectors on each channel unit must recognize when the signals are present, and then command the channel unit at the distant terminal to simulate the signal. The carrier loop thus performs like the metallic loop it replaced, so neither the central office equipment nor the subscriber's station set need be modified for operation with the *SLC-40* system.

Single-party telephone service requires two kinds of dc signaling: off hook and ringing. When a subscriber served by the *SLC-40* system goes off hook, an off-hook detector in the RT channel unit senses this condition and commands its corresponding COT channel unit to operate a relay. When operated, this relay draws dc current from the central office, just as the station set would do if it were on a metallic pair. Dial pulsing is treated like a series of on hook and off hook transitions. To ring a single-party phone, the central office applies a 20-Hz voltage on the ring conductor. The COT channel unit, therefore, has a ringing detector which commands the RT channel unit to switch ringing voltage onto the subscriber's ring conductor.

To provide *SLC-40* service to single-party subscribers at lowest cost, RT and COT channel units are available that are capable of transmitting only the signaling required for single-party service. More complex signaling circuits are required to provide service on multiparty (ONI) lines or on two-party (ANI) lines, and RT and COT channel units with these capabilities are available at added cost.

The following sections describe in detail some of the signaling circuits used in *SLC-40* channel units.

* The term "dc signaling" includes 20-Hz ringing, because the 20-Hz frequency is so low compared to VF signals.

4.1.1 Single-party RT off-hook detector

The current design of single-party RT channel unit uses the circuit illustrated in Fig. 5 to power the subscriber's station set, apply ringing, and detect off-hook.⁹ The circuit works on loops up to 1600Ω, with up to five ringers,* and it follows dial pulses accurately even under adverse conditions. The circuit has two modes of operation, determined by whether relay K1 is operated or released, and each mode will be described separately.

With relay K1 released, the 48V battery powers the loop through current limiting resistors R1 and R2, and the comparators are used as a two-stage off-hook detector. Resistors R3–R7 form a bridge network, with resistor values chosen such that the output of the bridge ($V_1 - V_2$) is negative when the "metallic current" $(I_R - I_T)/2$ exceeds 14 mA. The bridge resistor values are also chosen to reject "longitudinal currents" $(I_T + I_R)/2$, caused by 60-Hz induction from nearby power lines, that might otherwise interfere with the off-hook detection.

To avoid wrong numbers, the off-hook detector must not distort the duration of the dial pulses it detects. The circuit utilizes the inductance of hybrid coil "B" for this purpose. After dial pulse transitions (opens or closures) the metallic current does not achieve its steady state value instantly. Instead, its rate of change is slowed by the inductance and capacitance of the subscriber's ringer. If the off-hook detector sensed only the level of current, it would distort the duration of dial pulses under limiting conditions. To prevent this, the bridge is made sensitive to both

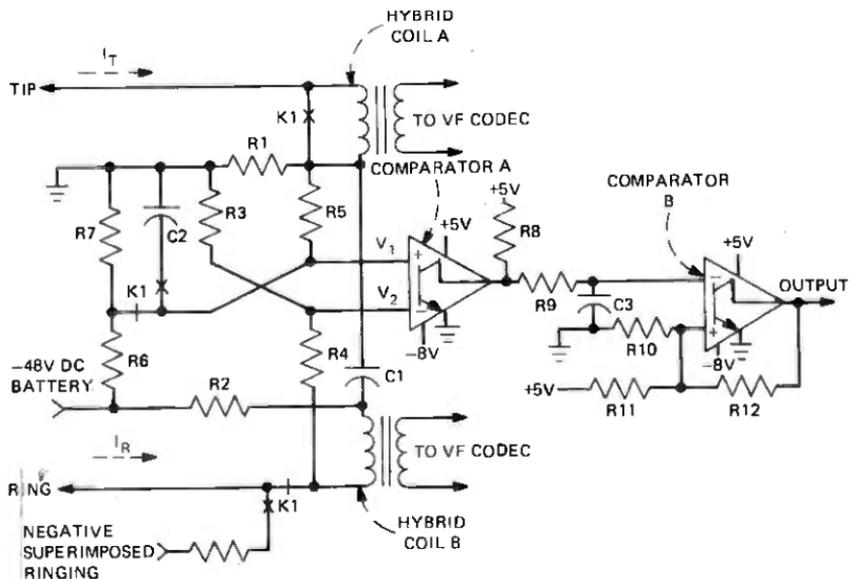


Fig. 5—Single-party off-hook detector.

* For loop greater than 900Ω, the maximum number of bridged ringers is 4.

the level and rate of change of metallic current. The changes in current that occur during dial pulsing develop a voltage transient across the inductance of hybrid coils "A" and "B," and the transient on coil "B" is coupled (via R4) to comparator A. Hence, comparator A's output detects dial pulse transitions with little delay.

The second stage of the off-hook detector (comparator B and associated components) transmits legitimate output transits from comparator A, but it blocks false transitions that can occur under certain conditions. One example of such false transitions occurs if the subscriber has several ringers. The metallic current then may briefly oscillate following the break transition of dial pulsing. This could cause comparator A to detect "split pulses" (more than one make and break transition per dial pulse). False transitions could also occur if one side of the subscriber's carbon block protector faults to ground on a loop with 60-Hz longitudinal voltage (induced from a nearby power line). The faulted protector can then cause a significant 60-Hz metallic current to flow in the loop, which would switch comparator A on and off at a 60-Hz rate. False transitions such as these should not be transmitted to the central office channel unit, where they would be repeated to the central office.

The negative input of comparator B connects to capacitor C3, which charges to +5 Vdc when comparator A detects an absence of metallic current and it discharges to 0 Vdc when metallic current flows. Comparator B changes state when the voltage on C3 crosses the threshold voltage established at its positive input by R10, R11, and R12. Resistor R12 provides hysteresis on the threshold voltage, and the thresholds are chosen such that the filter circuit delays both make and break transitions by 14 msec. Since the make and break transitions are equally delayed, the filter circuit does not significantly distort the duration of dial pulses. If comparator A detects a brief split pulse, the filter circuit prevents this false transition from appearing at the output of comparator B (although this distorts the dial pulse by an acceptably small amount). The hysteresis prevents the output of comparator B from changing states faster than a 48-Hz rate, so 60-Hz switch hook closures cannot possibly be transmitted to the central office.

The second mode of the single-party signaling circuit occurs when ringing is applied to the subscriber by operating relay K1. In addition to switching a negative superimposed 20-Hz ringing voltage onto the "ring" conductor, the relay reconfigures the input to comparator A to detect off hook during ringing (ring trip). In this mode, the ringing current is returned via the tip conductor and R1 to ground. R5 and C2 low-pass filter the ac ringing return voltage, such that V1 does not cross the negative threshold voltage established at V2, while the subscriber is on hook. Consequently, comparator A's output remains high. When the subscriber goes off hook, a dc component flows in the ringing return

current, causing V1 to cross the threshold voltage, forcing the output of comparator A low. After 14 msec, the output of comparator B switches high. This releases relay K1 (removing ringing from the loop) and enables the dial pulse detector to detect off hook.

4.1.2 *Multiparty ringing detector*

This section describes the ringing detector used in the current design of the COT channel unit for multiparty service.¹⁰ The requirement that the COT channel unit detect when the central office applies ringing was previously discussed for single-party service. For multiparty service, the ringing detector must also distinguish which of four different kinds of ringing the central office is applying: 20-Hz voltage superimposed on either positive or negative dc, and applied to either the tip or ring conductor. The multiparty COT channel unit uses the circuit shown in Fig. 6 to detect the presence and polarity of ringing on the ring conductor. An identical detector is used for the tip conductor. These circuits detect ac-dc ringing of 65 to 110 vrms, 17 to 23 Hz, superimposed on ± 30 to ± 60 Vdc. The net distortion of the duration of the ringing burst is less than 100 msec, and it will not falsely detect ringing as a result of dial pulsing or from up to 40 Vrms of induced 60-Hz voltage on tip and ring. The impedance of this circuit exceeds $5M\Omega$ when measured by a central office's automatic line insulation test, so the central office does not falsely interpret the SLC-40 loop as leaky and in need of maintenance.

The 20-Hz detector can be subdivided (as shown in Fig. 6) into a high pass filter, an amplitude to duty cycle converter, and a duty cycle detector. The high pass filter blocks dc while passing 20 Hz. The amplitude to duty cycle converter consists of Q1, Q2, and Q3, and associated passive components. Transistor Q2 turns on during positive ringing peaks, and Q3 turns on during negative peaks. Since the Q2 and Q3 collectors are tied together, capacitor C2 charges and discharges at a 40-Hz rate when ringing is present. The duty cycle when Q2 or Q3 conducts increases as the 20-Hz amplitude increases.

The duty cycle detector prevents transients on the ring conductor from being falsely interpreted as ringing. When actual ringing is present, the duty cycle when Q2 or Q3 conducts is large enough to discharge C2 below the threshold voltage on the positive input to comparator B. The comparator output then switches high, indicating that the central office has applied ringing. Resistor R10 provides hysteresis to avoid multiple output transitions when ringing is applied.

The polarity detector uses zener diodes CR3 and CR4 to isolate C3 from the voltage on the ring conductor when ringing is not applied. When ringing is applied, C3 charges positive through the diodes if the ringing is positive superimposed, or negative if ringing is negative superimposed. Comparator B senses the polarity on C3 to determine the polarity of

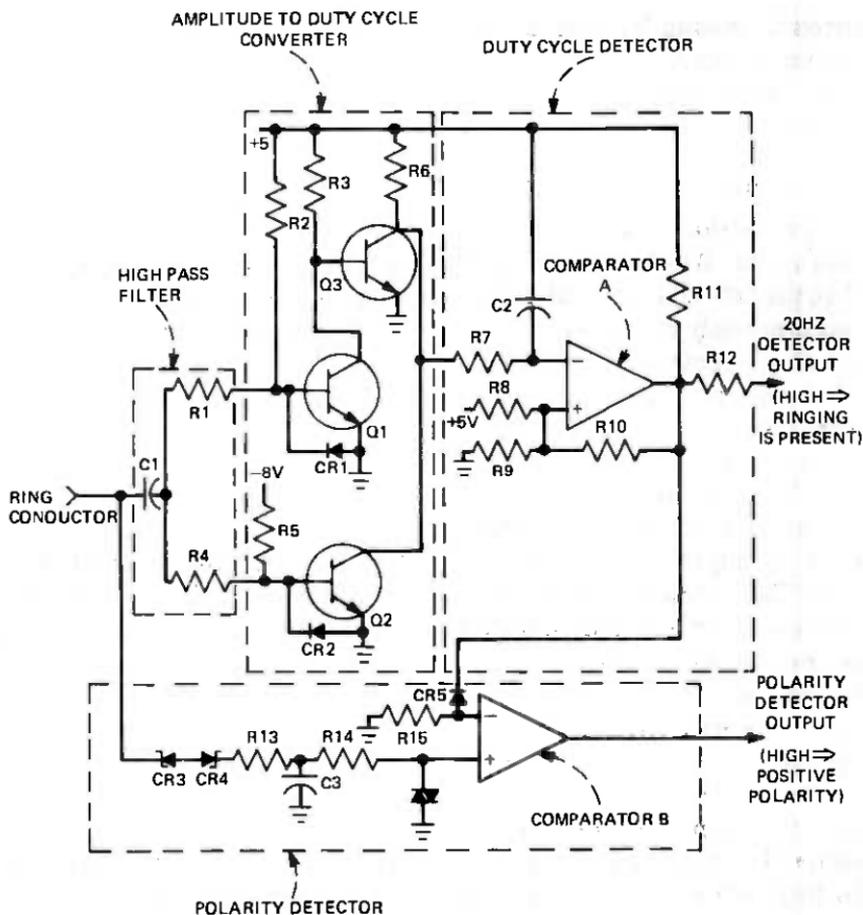


Fig. 6—Multiparty ringing detector.

ringing. The polarity detector operates faster than the 20-Hz detector when ringing is applied, and it releases slower when ringing is removed. This choice of timing prevents the momentary connection of ringing to the wrong party when ringing is applied or removed. Additional circuitry (not shown) prevents momentarily ringing the wrong party during re-rtive calls.

4.1.3 Applying multiparty ringing at the RT

The state of a COT channel unit's ringing detector must control the application of ringing at its corresponding RT channel unit, and the SLC-40 system used a simple but effective technique to do this.¹¹ For multiparty service, the ringing detector has five possible states, and each of these states causes a unique four bit serial "ringing code" to be transmitted continuously to the RT channel unit (while the subscriber is on hook). The five states and their ringing codes are listed in Table

I. Since ringing is applied only while the subscriber is on hook and the delta modulator is used only while off hook, the ringing codes and the coder can share the 37.7-kb/s data path that links the RT channel unit to its COT channel unit.

The multiparty RT channel unit selectively operates three relays in response to the ringing code, to apply the appropriate ringing condition to the subscriber. These relays are shown in Fig. 7, and Table I indicates the conditions when they are operated. Relay K1 (when operated) applies ringing on the ring conductor. Relay K2 applies ringing on the tip conductor, and relay K3 selects positive or negative superimposed ringing. When no ringing is to be applied, all three relays are released.

The five ringing codes are selected to be easily decoded into operate commands to the three relays. The reader can verify that the decoding circuit (Fig. 8) continuously provides the appropriate operate commands as the ringing code shifts through the shift register. This selection of ringing codes also prevents a defective logic gate in common equipment from unintentionally applying ringing on all channels simultaneously, since the decoder inhibits all relays if it receives continuous ones or zeros. The channel unit disables the relays when it senses off-hook. The shift register, decoding circuit, and other circuitry are contained within a custom integrated circuit.

Table I — Multiparty ringing states and ringing codes

COT ringing detector state	Ringing code	RT relays to be operated
No ringing detected	1111	None
Negative polarity detected on ring	1010	K1
Negative polarity detected on tip	1110	K2
Positive polarity detected on ring	0001	K1, K3
Positive polarity detected on tip	1100	K2, K3

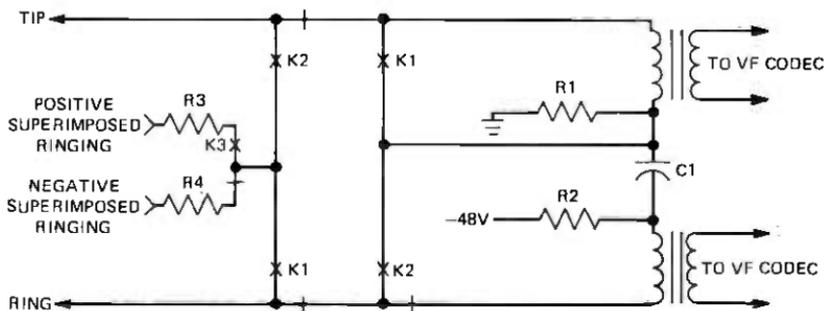


Fig. 7—Ringing relays for multiparty RT channel unit.

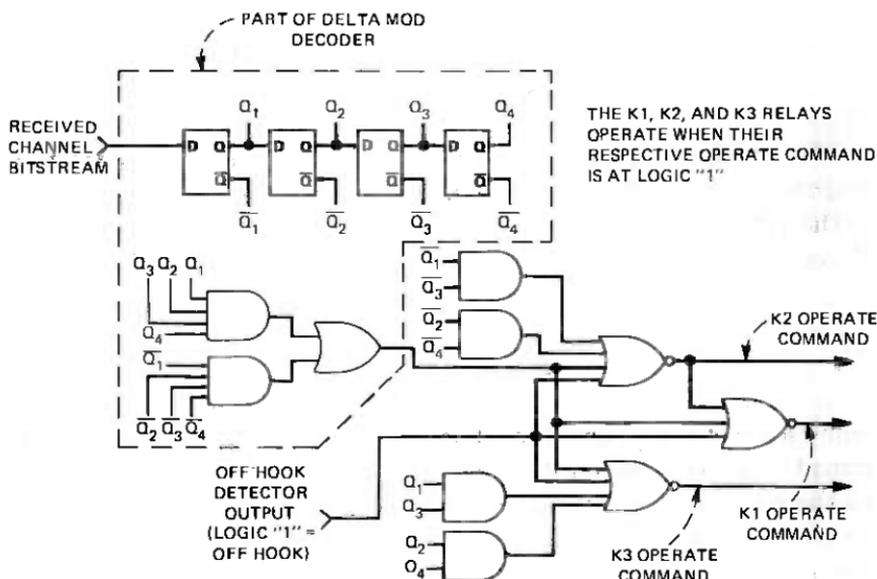


Fig. 8—Multiparty ringing code decoder at the RT.

4.1.4 Channel units for two-party ANI service

RT and COT channel units are also available to serve subscribers on two-party lines equipped for Automatic Number Identification (ANI). The tip party station set on such a line has an identifying resistance to ground (typically 2.65 k Ω) while off hook that allows the central office to determine whether the tip or ring party has initiated a call. Central offices test ANI loops for the identifying ground either between dialed digits or before and after dial pulsing, to assure the reliability of the charging scheme.

The two-party ANI channel unit at the RT tests for tip party ground under command from the COT channel unit.¹² The first test occurs automatically when the subscriber initially goes off hook. The COT channel unit does not repeat this off hook to the central office until the first test is complete. The COT channel unit commands subsequent tests whenever the central office performs a tip party ground test. The circuit that senses when the CO makes a tip party ground test is illustrated in Fig. 9. Relay contact K1 closes to simulate off hook to the central office. When a central office tests for tip party ground, loop current is interrupted, and the COT channel unit senses this condition as an interruption in loop current flow through the optical coupler's LED.

The COT channel unit commands the RT channel unit to test for tip party ground by transmitting a burst of continuous logic 0 for 250 msec. When the RT channel detects this burst, it shorts tip and ring to -48V and checks for longitudinal dc current. If the longitudinal current is sufficient to indicate a tip party ground, the RT channel unit returns a

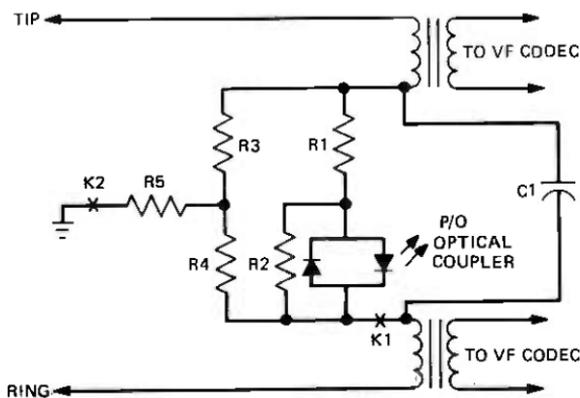


Fig. 9—Channel unit interface to central office for ANI service.

burst of logic 0 to the COT. If dc longitudinal current does not flow (i.e., ring party is off-hook) the RT transmits the delta modulator output instead of logic 0. The COT channel unit examines its received bitstream and operates relay K2 if it receives a burst of logic 0 at the end of the test. The K2 relay contact simulates a tip party ground to the central office, and this indication is maintained for the duration of the call, unless a subsequent test indicates a switch in parties.

4.2 SLC-40 multiplexer: a digital design that is tailored for loop plant operation

The SLC-40 multiplexer/demultiplexer provides parallel-to-serial and serial-to-parallel data conversion of the transmit and receive data, respectively, for the channel units and the maintenance unit. The framing circuits of the COT and the RT multiplexer, independent of data generated by the channel units, synchronize these data conversions by monitoring the receive serial data, the frame bit, and bipolar violations. If a loss of synchronization is detected, each multiplexer inhibits data transmission and initiates a "handshake" routine that reestablishes synchronization. The multiplexer also maintains a minimum density of ones in the serial bit stream that is output to the digital line. This function is called zero suppression.

The multiplexer/demultiplexer functions are implemented with the TTL logic circuitry. These circuits are packaged in three common plug-in circuit packs at the COT and at the RT. The transmit and the receive circuit packs used at each terminal are identical. A functional circuit pack is used to specialize the multiplexer for use at the COT or the RT.

The channel unit to multiplexer data interface is provided by a system of ten transmit and ten receive data buses. Each bus connects a column of channel units to the multiplexer. The data on each bus is gated in (receive) or out (transmit) of a channel unit by the channel clocks. These

clock signals are generated by the multiplexer with a separate clock phase connected to each row of channel units.

4.2.1 The frame structure

Each frame produced by the *SLC-40* multiplexer contains 164 sequential bits of serial data that is output to the digital line at the T1 rate of 1.544 megabits/second. The frame is composed of four groups of 40 channel unit data bits and four "housekeeping" bits, as shown in Fig. 10. Every channel unit produces one data bit for transmission in each group, thus the channel units are clocked at 37.66-kHz. The M1 and M2 housekeeping bits transfer system status information between the maintenance units at a 18.83 kilobit/second rate. The remaining two housekeeping bits are produced by each multiplexer and are used to insure that the COT and the RT multiplexer/demultiplexers are synchronized, i.e., in frame.

The frame data sequence is generated by the parallel-to-serial data converter from the channel unit transmit data on the ten data transmit buses. This data conversion is controlled by the transmit frame counter at the COT. At the RT, since the transmit clock is slaved to the receive clock, the receive frame counter can also perform as the transmit frame counter. Thus, at the RT, there exists a fixed relationship between the receive and the transmit frames, unlike the COT, which can accommodate any phase relationship resulting from delay attributed to the length of the digital line. Each receive frame counter controls a serial-to-parallel data converter that demultiplexes the serial bit stream onto the ten receive data buses that supply receive data to the channel units.

The transmit and the receive frame counters are each configured from a four-bit binary counter, a four-bit shift register and two flip-flops. The binary counter, which is clocked at the T1 rate, traverses a 41-state mini-cycle consisting of three 10-state micro-cycles and one 11-state micro-cycle. The state of the shift register controls the length of each micro-cycle and is advanced once per micro-cycle. The two flip-flops are

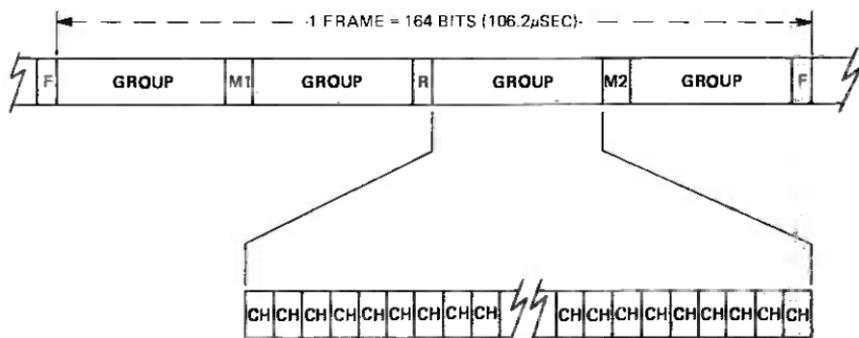


Fig. 10—The *SLC*[™]-40 frame.

connected as a modulo-4 counter that changes once per mini-cycle of the binary counter and selects the housekeeping bit that is to be inserted into the frame during that mini-cycle. Together these four mini-cycles form a 164-state macro-cycle defining the full period of the framing counter and the length of a frame.

Each state of the frame counter shift register produces a channel unit clock signal. These four clock phases control the transmission of data between the multiplexer and the channel units over the transmit and receive data buses. Each channel unit transmitter utilizes two nonadjacent phases of the transmit channel clock. The first clock phase times the channel unit digital encoder sampling and the second phase enables the output of this data onto a transmit data bus. Two transmit and two receive clock phases are distributed across each row of ten channel units. The primary and secondary role of each clock phase is reversed at every other channel unit.

During the frame counter shift register mini-cycle, the binary counter is clocked 41 times. By advancing the shift register state at half-clock intervals on alternate shifts, symmetrical channel clock phases are produced, which are 10 , $10^{1/2}$, 10 , and $10^{1/2}$ T1 clock intervals in length. This symmetry makes possible the above-mentioned clock pairing.

The position of each channel unit's data within a group of the frame differs by five time slots between the COT-produced frame and the RT-produced frame. This five time slot offset in channel unit data position within a frame allows a unique test of the COT to be conducted when the COT digital line is looped at the jack panel. This test allows an analog test signal to verify the correct operation of the clocks and data buses at the channel units.

4.2.2 Misframe detection

The multiplexer generates the frame (F) and the random (R) housekeeping bits and inserts them into the transmit data frames (see Fig. 10). The value of the F-bit is determined by taking a modulo-2 sum over the preceding 164 transmitted bits. The resultant parity data is transmitted by F at the end of the frame. The value of F is randomized by the R-bit that is output by a pseudorandom data generator at a rate of one bit per frame. The R-bit is always included in any sum over 164 bits, thereby preventing bits other than the F-bit from consistently exhibiting the value of F.¹³

The loss of synchronized data transmission, a misframe, is determined in the receive demultiplexer by taking a modulo-2 sum over the preceding 164 received data bits and the received frame bit. This sum should be zero; if not, a parity violation has occurred. The detection of two parity violations in any three successive frames forces the receive demultiplexer to initiate the reframing process.

Receive data errors that are caused by noise or crosstalk on the digital line are detected by the multiplexer as bipolar violations. When a bipolar violation is detected during the preceding frame or at the frame bit, the resultant frame parity violation is not accumulated. Thus the misframe detection time increases as the digital line error rate increases.

4.2.3 Reframing

When either the COT or the RT detect that the system is out of frame, they initiate a reframing process. This "handshake" process is accomplished by the cooperative transmission of special synchronization sequences between the COT and the RT. The terminals alternate in their progression through this series of reframing states, where each state is signaled by the transmission of a special data sequence. These special data sequences contain all ones except at the R-bit and the F-bit time slots, which may have a value of zero or one. This results in a fast reframe, because very few frames of parity testing are necessary to verify the frame bit location in the received data stream.

Each multiplexer has two status conditions, out of frame (OOF) and out of sync (OOS), that must be satisfied to complete each step of the reframing process. During normal system operation, both the COT and the RT multiplexers are not out of frame ($\overline{\text{OOF}}$) and out of sync. A multiplexer is not out of sync ($\overline{\text{OOS}}$) during the reframe process when it locates the frame bit in the received special code, and the received data does not satisfy the parity test. Table II shows the transmitted code sequences and identifies each multiplexer's status during individual steps of the reframe process that occurs after the RT multiplexer has detected an out of frame condition. This reframe process lasts for only eight or ten frames if the digital line is error free during the reframe time interval.

Table II — Reframe sequence when misframe is detected at RT

COT multiplexer status	Transmitted data sequence*		RT multiplexer status	Transmitted data sequence		Number of frames†
	R	F		R	F	
$\overline{\text{OOF}} \cdot \text{OOS}$	Normal		$\overline{\text{OOF}} \cdot \text{OOS}$	Normal		—
$\text{OOF} \cdot \text{OOS}$	Normal		$\text{OOF} \cdot \text{OOS}$	1 1		2
$\text{OOF} \cdot \text{OOS}$	Alternate 1's and 0's	0	$\text{OOF} \cdot \text{OOS}$	1 1		1 or 3
$\text{OOF} \cdot \text{OOS}$	Alternate 1's and 0's	0	$\text{OOF} \cdot \overline{\text{OOS}}$	1 0		1
$\text{OOF} \cdot \overline{\text{OOS}}$	0	0	$\text{OOF} \cdot \overline{\text{OOS}}$	1 0		2
$\text{OOF} \cdot \text{OOS}$	0	0	$\text{OOF} \cdot \text{OOS}$	Normal		2
$\text{OOF} \cdot \text{OOS}$	Normal		$\text{OOF} \cdot \text{OOS}$	Normal		—

* Special reframe sequences are all ones except at the F and R housekeeping bits as shown.

† This count assumes that no bipolar violations occur during the reframe time.

4.2.4 Zero suppression

A zero suppression circuit acts on the serial data stream by forcing a one whenever the minimum ones density is not sufficient to guarantee proper signal regeneration by the digital line. This is seldom necessary with the *SLC-40* system, however, since each channel unit outputs all ones while not in use by a subscriber, thereby producing a data bit stream with a high density of ones.

The zero suppression circuit utilized in the *SLC-40* multiplexer forces ones into the output serial data stream such that there are at least $N-1$ ones in any $8N-1$ successive data bits.¹⁴ Thus the minimum long term (N large) density of ones is $1/8$, and it is possible to divide the output data bit stream into eight bit or less blocks, each of which contains at least a single one. Thus a one is forced only when 15 or more adjacent time slots of the frame contain zeros, which requires the 15 channel units associated with these time slots to be active. Under these conditions, a one is forced with a probability of 2^{-15} (approximately 3×10^{-5}), since the probability of a one in any active time slot is assumed to be 0.5 for an active channel. This introduces a negligible impairment to the channel unit decoded signal.

A four-bit binary counter, a flip-flop, and some connecting logic are used to implement the *SLC-40* zero suppression algorithm that is shown in the state transition diagram of Fig. 11. The counter state and the input data value are changed simultaneously by the transmit clock, thus the

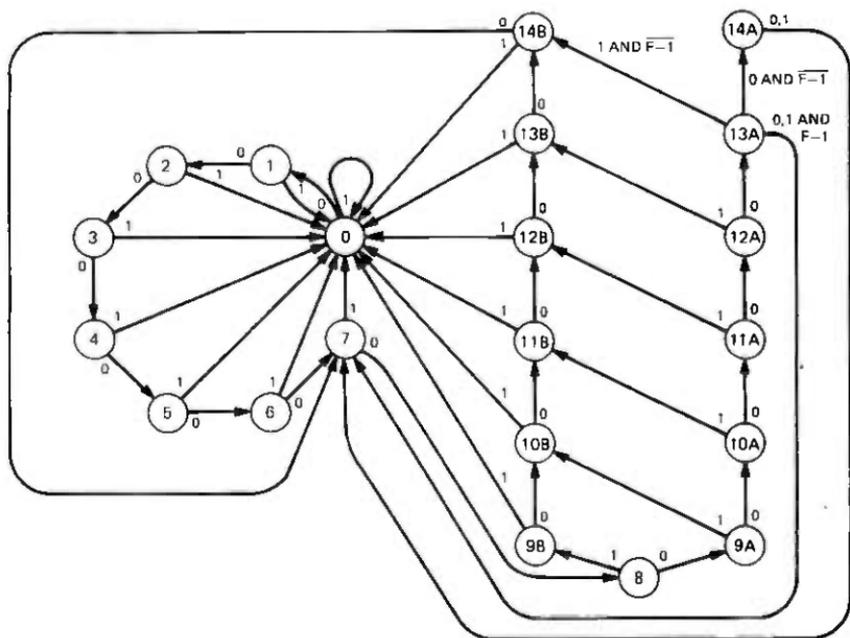


Fig. 11—State transition diagram for the *SLC*[™]-40 zero suppression algorithm.

state transitions shown are timed by the transmit clock. The next state is determined by the input data value, and the output data equals the input data unless a one is forced. The circuitry also "looks ahead" to prevent a forced one at the F-bit location in the frame. This is accomplished by detecting a frame bit time slot minus one pulse, F-1, when the zero suppression circuit is in state 13A. This forces a one at the next transition, and the correct frame bit is output in the following time slot.

4.2.5 Control of logic-signal-induced noise within the channel bank assembly

In a system realized with sequential logic like the SLC-40 system, logic load current flow is impulsive and consolidated at clock transition times. This current flow through the interconnect and power supply wiring can couple noise signals into other circuits.

To control this interference, a printed wiring board backplane is used to distribute logic signals and power to each row of channel units. The data bus and channel clock wiring is routed from the multiplexer to wire-wrap pins on each backplane over physically separated paths. This isolates the open-collector logic driven transmit data buses from crosstalk caused by the receive data buses and the channel unit clocks. Also, the multiplexer plug-in units are centrally located within the common units, thereby reducing the average length of the logic signal wires. This system configuration results in a readily manufacturable design with controlled noise characteristics, thus ensuring that the critical timing of the logic signals is maintained.

To reduce the transient current flow in the logic power wiring, and thus minimize radiation to adjacent circuitry, a heavy gauge pair of wires is utilized to connect the low voltage power unit to the individual backplanes. The multiplexer is powered through leads that are connected to the nearest backplane resulting in a minimum length logic signal return path between the multiplexer and the channel units. In addition, each plug-in unit contains a power supply decoupling capacitor, and the total effect of all these capacitors is to produce a distributed filter that eliminates the flow of high frequency currents on the power wiring.

4.2.6 Multiplexer/demultiplexer to digital line interface (including system maintenance function)

The line interface unit under control of the maintenance unit couples the multiplexer/demultiplexer and the line-feed power unit to either the main or the spare digital line. The maintenance unit selects the proper line through an algorithm that has as inputs the statuses of the digital line and the terminals. When a trouble condition persists, a two second clock advances the state of this algorithm in an attempt to restore the system to operation.

The current supplied by the line-feed power unit to the line interface unit is connected through a relay to center-taps on the line side of the transmit and the receive transformers. Normally, this current simplex powers the main digital line, but when the maintenance unit causes the relay to operate, the spare line is powered.

The transmit clock signal that the line interface generates causes the parallel-to-serial data converter in the multiplexer to output one new data bit of the serial bit stream for each clock pulse. The transmit clock also controls the conversion of this data into a bipolar code that the transmit transformer couples onto the powered digital line.

The line interface unit also equalizes and regenerates the low level bipolar signal on the receiving side of the digital line and converts this signal into TTL compatible positive and negative data rail unipolar signals. These signals, along with the recovered receive clock, are supplied to the demultiplexer which uses the two rail data to detect bipolar violations.

The maintenance unit integrates the bipolar violation data it receives from the demultiplexer. If errors occur at a sufficiently high rate or other trouble indicators exist for two seconds, the maintenance unit forces the line interface units to transfer system operation to the spare digital line. Data sent over the maintenance channel causes both terminals to switch at the same time. Should operation on the spare line not clear the trouble condition within two seconds, the RT line interface unit loops the receive digital signal back towards the COT. This loop-back test condition lasts for two seconds, then the maintenance unit returns the system to main digital line operation. The resulting "system out" state of the maintenance units causes a major alarm at the CO as the system is inoperative. The "system out" state also inhibits channel unit operation, thereby preventing false off-hook indications at the COT or ringing at the subscriber.

When the *SLC-40* system is equipped with the most recent vintage maintenance units, a main or a spare digital line retry occurs once after either line fails. This retry allows the digital line four minutes to clear, which normally is sufficient if the trouble condition was caused by accidental contact with the line during work activity in the cable. The system does not continuously retry the main and the spare digital lines, as this would hamper trouble-shooting efforts on the defective digital line. Also, the resultant line powering transients might disturb other digital lines on the same cable route.

V. SUMMARY

The two large pair-gain digital carrier systems developed for the Bell System, first, the *SLM* system and, subsequently, the *SLC-40* system, have proven that this kind of system can be operated successfully in the

loop plant of operating telephone companies. Both systems have provided quality transmission and high reliability with low maintenance, while deferring or eliminating capital expenditures for cable, conduit and central office construction.

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Loop Plant Electronics:

The Loop Switching System

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A new system for achieving more efficient use of cable pairs, the Loop Switching System (LSS), has been developed which uses a microcomputer to control the call processing, traffic measurement and display, system maintenance, and manual testing. The LSS, which works with any central office, switches 96 subscriber lines onto 32 voice frequency trunk pairs by means of a graded multiple space division switching network at the remote terminal (RT) and expands the trunks back into the individual line appearances at the central office terminal (COT) located within a central office. An additional 96 lines and 32 trunks can be added to the LSS, either at the same RT or a second RT on a different cable route, using the same COT common control equipment. Control of the RT is via two standard voice-frequency pairs used for a full-duplex data link. This four-wire data link, operating at 1250 bits per second, is used to transmit concentrator connection orders from the COT to the RT, service request information from the RT to the COT, and other information pertaining to LSS maintenance and alarms. This paper describes the system features, operational characteristics, and circuit designs used in the LSS system.

I. THE HISTORY OF LINE CONCENTRATORS

The concept of using subscriber line concentrators to reduce the cost of subscribers loops has been considered for many years. As early as 1908, development engineers proposed the use of remote line concentrators.¹ Line concentrator design effort²⁻⁸ by the Bell System and other companies throughout the world has continued ever since on both electro-mechanical and, more recently, all electronic versions. Designs for use with specific central offices avoid the expansion stage of switching in the central office terminal needed by universal line concentrators which work

with any central office. They, therefore, can achieve lower costs in a more limited market area. However, past field experience has been with concentration of the universal type.

Early universal line concentrators suffered from the technical difficulties of maintaining complex switching equipment remote from the central office. The harsh outside plant environment also created a number of electrical and physical reliability problems. The traffic capacity was inadequate to handle the wide variations in traffic generated by small groups of subscribers, resulting in an excessive number of blocked calls. Consequently, the use of line concentrators by the telephone companies fell into disfavor.

A new breed of electronic concentrators has solved these basic problems and began to reverse these attitudes. Introduced in the early 1970's was the Subscriber Loop Multiplex (*SLM*[™]) system,⁹ designed by Bell Laboratories and manufactured by Western Electric. The *SLM* system concentrates 80 subscriber lines on 24 T1 carrier derived channels. To date, 190 *SLM* systems have been installed.

The *SLM* system demonstrates that electronic concentrator systems for use in the outside plant are reliable and can be maintained by the telephone companies. Trouble report rate studies of customers served by the *SLM* system show that the service is comparable to that of customers served by standard voice frequency loops.

One of the most significant results of the *SLM* field experience is that there have been no traffic problems. The *SLM* system is conservatively designed to handle more than twice the customer traffic level compared to previous concentrators.

The experience gained from the *SLM* development formed the basis for the development of the *SLC*[™]-40 carrier system¹⁰ and the Loop Switching System (*LSS*), the latter being the subject of this paper. Both the *SLC*-40 system and the *LSS* system provide service more economically than the *SLM* system. However, the *SLM* system played an important role in establishing the practicality of both digital transmission and modern electronically controlled line concentration in the loop plant.

II. THE *LSS* SYSTEM

This section describes the features of the *LSS* system and the reasons for certain design decisions. Figures 1 and 2 illustrate the *LSS* configuration for one-RT and for two-RT operation.

2.1 *LSS* applications

The *LSS* system provides feeder cable relief in both permanent and temporary applications. Generally, permanent applications will be to serve growth in suburban and rural areas with clusters of a hundred or

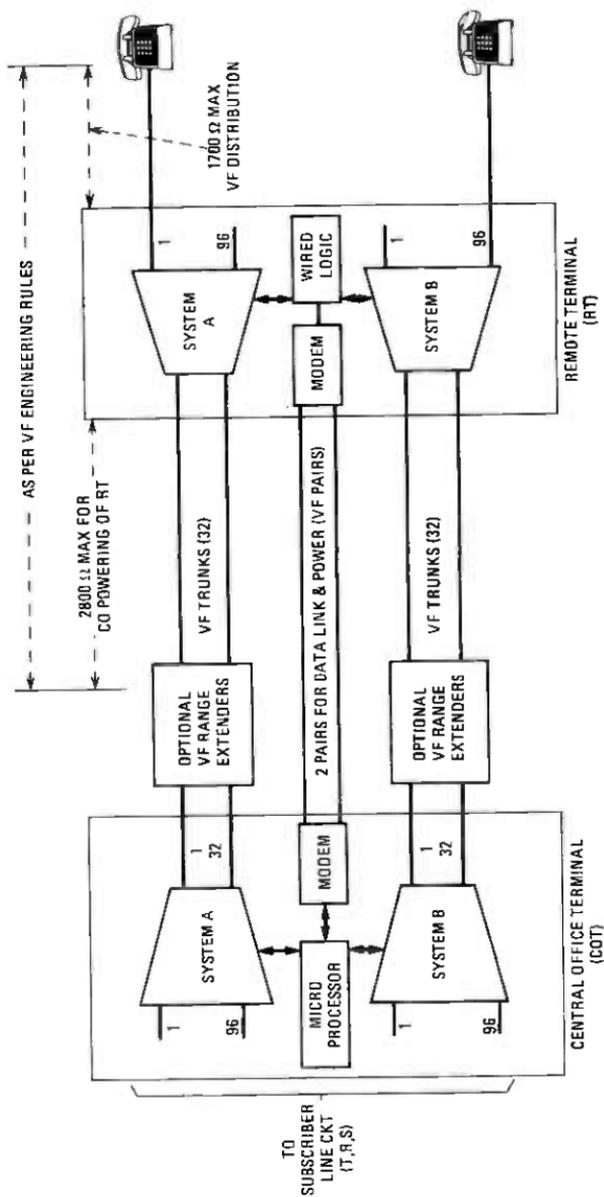


Fig. 1—Typical LSS system layout, one RT.

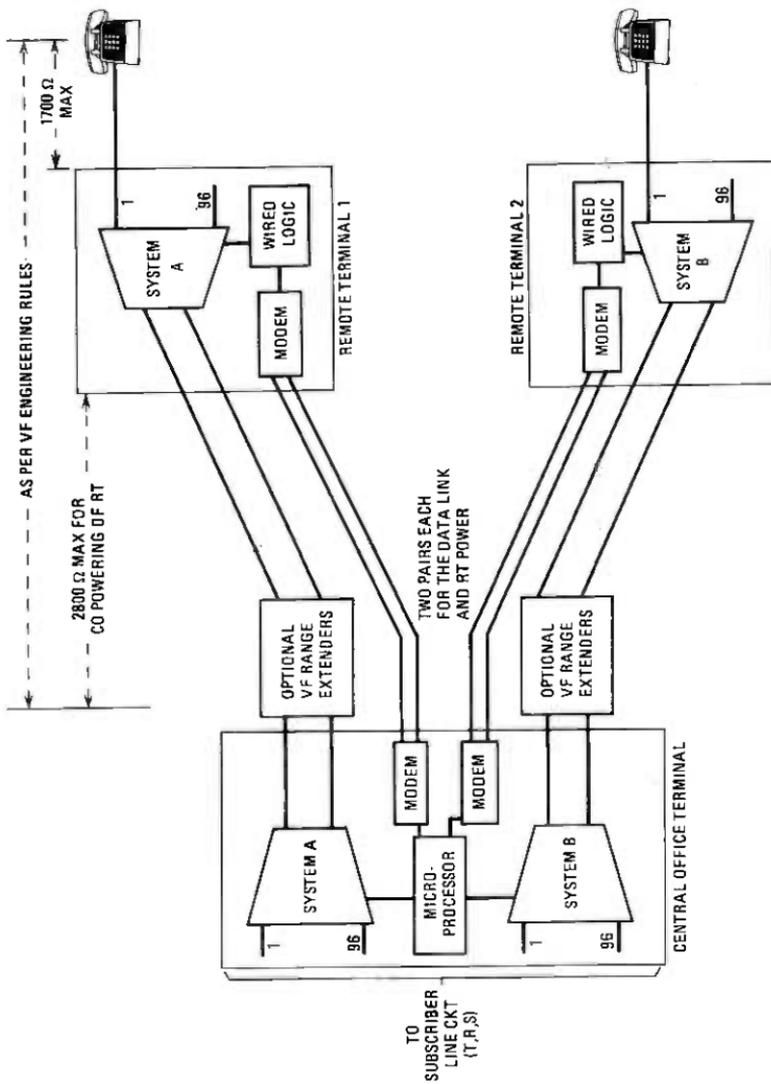


Fig. 2—Typical LSS system layout, two RTs.

more customers. Mobile home parks, rural subdivisions, and resort areas typically have that characteristic. Temporary application to defer cable and/or structure will be found in suburban and rural areas. Another application will be for wire center deferral, particularly in conjunction with subscriber carrier for the trunks.

2.2 Central office interface

The LSS system is designed as a universal line concentrator to interface with step-by-step (SXS), No. 5 Crossbar (#5XB), and ESS central offices. Sleeve lead control chosen for the LSS system has the strong advantage of compatibility with automatic line insulation testing (ALIT) and repair service bureau testing of subscriber lines with the local test desk or the Line Status Verifier (LSV). Systems with ringing detectors and loop current detectors cannot be tested without ringing phones.

Interfacing with SXS and #5XB central offices is accomplished by terminating tip, ring, and sleeve on the LSS central office terminal (COT) for each subscriber line. Interfacing with ESS offices also requires ESS signal distributor applique circuits to provide sleeve leads and ESS remote master scanner applique circuits for transmitting trunk busy information from LSS to ESS. To associate these ESS circuits with specific subscriber lines, ten words per line are required in the line translator program store of No. 1 ESS and a lesser number of words in No. 2 ESS and No. 3 ESS.

2.3 Facility requirements

In its simplest application, LSS uses ordinary voice frequency cable pairs for trunks and the distribution plant beyond the RT. This avoids any plant conditioning, achieves the lowest cost, and is compatible with existing and future repair service bureau functions. The distribution pairs and the station set together can have a resistance of 2000 ohms limited only by service request detection by the RT.

The LSS system adds into the subscriber loop only relay contacts and 40Ω of resistance to protect the contacts from surges. Ringing signals and battery are supplied by the central office and not by the LSS system. Therefore, the total subscriber loop length from the COT to the customer station is limited by the standard signaling and transmission requirements for subscriber loops from the serving office. Range extension devices¹¹ such as the 5A REG or the 2A REG can be used on the trunks of the concentrator to obtain additional range in a manner similar to nonconcentrated loops.

Carrier systems such as the SLC-40 system also may be used to provide the concentrator trunks for situations requiring still greater pair gains, i.e., when feeder cables are too small to provide the necessary trunk pairs in multiple LSS installations. However, the economics of this combination will be somewhat less attractive than the LSS system alone.

2.4 System powering arrangements

A basic design goal was to power the RT from the COT for a distance of up to 2800Ω of loop resistance (80 kft of 22-gauge cable). This distance was chosen to be consistent with voice frequency engineering rules for loops with range extenders with gain (REGs). Using $+135$ and -135 Vdc from the central office, the highest dc voltages permitted, a simplex powering arrangement using two pairs of wires can deliver about 13 watts to the remote terminal. A new dc-to-dc converter was designed that is 80 percent efficient and derives a useful low voltage power of ten watts for the circuits in the remote terminal. This limitation of ten watts of power requires a latching switching network and CMOS and low-power TTL logic circuits in the common control.

The simplex power for the RT is generated in the COT from a current limited (95 mA) ± 135 Vdc 30-watt converter operating from the -48 Vdc central office battery. Alternatively, this simplex power may be obtained from the ± 130 Vdc common systems central office battery supply in conjunction with a 95-mA series regulator. For this latter case, the RT powering range is 1800Ω of loop resistance (900Ω of simplex resistance) instead of 2800Ω because of the lower available voltage and the voltage drop in the auxiliary series regulator.

When the RT is too far from the COT to be supplied from the central office or if the trunks and data link are carrier derived, the RT may be powered from a locally available -48 Vdc common systems supply in a small hut or building or from commercial 117 Vac 60-Hz power brought into the cabinet. If commercial power is used, batteries are required to maintain equipment operation for approximately 10 hours after a commercial power failure.

The COT of the LSS is powered from a low voltage converter which delivers about 100 watts to the COT at ± 5 Vdc and -24 Vdc from the -48 Vdc central office battery.

2.5 Data link

The same two pairs used to power the remote terminal are also used for a full duplex data link. Modems similar to those used in 202 type data sets convert the low level logic signals to 1250 bits per second frequency shift keying (FSK) analog signals for transmission over the data link. This bit rate allows the use of standard voice grade cable pairs with no special conditioning for the maximum distances envisioned.

2.6 Maintenance

A major design objective was a trouble rate (troubles per 100 stations per month), service availability (downtime), and maintenance costs for customer lines served by the LSS system similar to that of equivalent cable service.¹² To achieve this goal, the LSS is provided with important

maintenance features including continuous monitoring of system performance, alarm displays at the COT which indicate system status and any defective plug-in units, and automatic troubleshooting procedures which aid in maintaining and restoring service. Some of the routine maintenance functions performed automatically by LSS include the following:

(i) Each time a trunk is assigned to a line, the trunk assignment is verified, and the trunk and switching network are tested for leakage paths worse than 50 k Ω and for continuity. Failure of the test initiates a new trunk assignment and causes a minor alarm.

(ii) Data transmission between the COT and RT is continuously checked for accuracy.

(iii) Once a day, all relays are operated and released to insure that they are working satisfactorily.

(iv) On a routine basis, much of the per-line and common control circuitry is tested for proper performance.

If any of these tests fail, an appropriate alarm is brought in.

The following troubleshooting procedures are implemented automatically when the routine maintenance tests show a failure:

(i) Should the LSS be unable to process a call, the system transfers automatically from the main data link pairs to an alternate set of pairs consisting of LSS trunk pairs 31 and 32. If it is not possible to communicate with the RT via either the main or alternate data link pairs, all of the critical common control plug-in units in the COT are tested automatically, and the appropriate plug-in and central office alarms are raised.

(ii) When COT power is restored after a failure or a critical plug-in unit is replaced, the LSS automatically reinitializes itself by disconnecting all idle lines and reconnecting all active lines (sleeve grounded).

(iii) When a line unit is plugged in at the COT all lines on that unit are disconnected in order to ensure that the switching network is in an initial state.

The following manual tests can be performed at the LSS central office terminal:

(i) When any of the common control units in the COT are replaced, the automatic troubleshooting procedures can be initiated manually to verify that the new units are functioning.

(ii) The leakage and continuity tests for trunks and the switching network can be initiated manually on a desired line for all the trunks accessed by that line.

(iii) Any trunk can be removed from service by operating a trunk make-busy switch.

- (iv) The number of busy trunks can be read out on a numeric display.
- (v) The trunk connected to a specific working line can be read out.
- (vi) All indicator lights can be tested.

Remote interrogation of the alarm indications can be made over the DDD network by the use of an optional COT maintenance unit. The types of alarms are COT, RT1, and RT2 major and minor. The alarm status is conveyed by means of a series of six sequential tones either modulated or unmodulated depending on the alarm state.

At the remote terminal, a leakage and continuity test can be initiated manually for the switching network and the idle lines and trunks associated with a selected line unit.

2.7 Traffic capacity, monitoring, and administration

The LSS is designed to provide a traffic handling capability similar to that of the *SLM* system. Specifically, for 6 ccs per line and 25 percent intracalling, the probability of blocking is 0.5 percent when the LSS is fully loaded. Both rural and suburban residential traffic measurements indicate that this level of traffic and intracalling is reached only on the four busiest hours of the year.¹³

The traffic characteristics of the LSS are designed far more conservatively than the 10-trunk, 50-line 1A concentrator. For example, in order to achieve the same traffic performance as the LSS, the 1A would have to be limited to about 22 lines instead of 50.

The LSS traffic monitoring capability is similar to that of the *SLM* system. The peak hourly traffic and the total number of calls blocked since the storage registers were last reset is displayed on a test and display unit at the COT. By means of the optional traffic unit, an office traffic usage recorder register can be connected to the LSS for remote monitoring of LSS traffic. However, frequent traffic measurements are really only justified during the initial loading procedures.

Traffic administration procedures are similar to those already approved for *SLM*. In brief, the LSS can be loaded with 70 main stations initially. Additional loading is based on the weekly peak traffic for four weeks. It is expected that in over 90 percent of the applications of LSS, the traffic administration procedures will permit fully loading each LSS system with 96 lines on 32 trunks.

After these initial procedures, blocked calls are monitored automatically. A minor alarm comes on if the number and frequency of blocked calls exceeds service objectives.

The same high level of traffic performance could have been engineered using intra-switching and fewer trunks. Studies showed that at 25 percent intracalling this approach would have cost more per net pair gained

for the type of switching network used in the LSS, would have degraded badly were the intracalling rate lower than predicted, would not allow LSS lines to utilize central office custom calling features and would have required local rather than central office powering of the RT. Therefore, intracalling is not provided by LSS in spite of a certain emotional appeal.

2.8 The switching network

2.8.1 Configurations

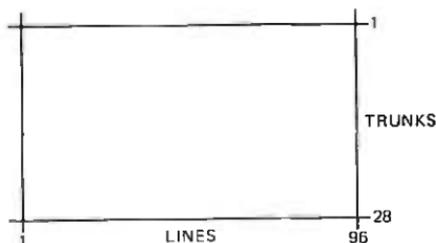
There are three basic types of switching networks which are candidates for use in line concentrators: one-stage, two-stage, and graded multiple.¹⁴ These networks are shown in Figs. 3, 4, and 5.

For the traffic levels described in Section 2.7, computer simulation shows that the two-stage design shown in Fig. 4 is just as efficient as a one-stage switch. Twenty-eight trunks are required for each, yet the two-stage network requires only three-eighths as many crosspoints as the one-stage switch.

For the graded multiple network of Fig. 5, four lines have access to seven trunks; the trunks are multiplied to other groups of four lines in a manner maximizing the traffic capacity. But because of the limited access of the lines to the trunks, it is less efficient than a one- or two-stage network. Hence, 32 trunks must be used instead of 28 for the same traffic capacity again as determined by computer simulation.

However, the graded multiple shown in Fig. 5 has $\frac{2}{3}$ of the number of crosspoints compared to the two-stage network of Fig. 4, and $\frac{1}{4}$ of the crosspoints of the one-stage network. The lower cost of the graded multiple network, due to the lesser number of crosspoints and simpler control, offsets the traffic inefficiency, equivalent to four trunk pairs. Hence, the fully loaded price per pair gained of a graded multiple concentrator is about the same as a concentrator with a two-stage network.

The graded multiple network has an advantage for low growth applications because of a lower getting started cost. The reason is that the



$$\text{CROSSPOINTS} = 96 \times 28 = 2688$$

Fig. 3—One-stage network.

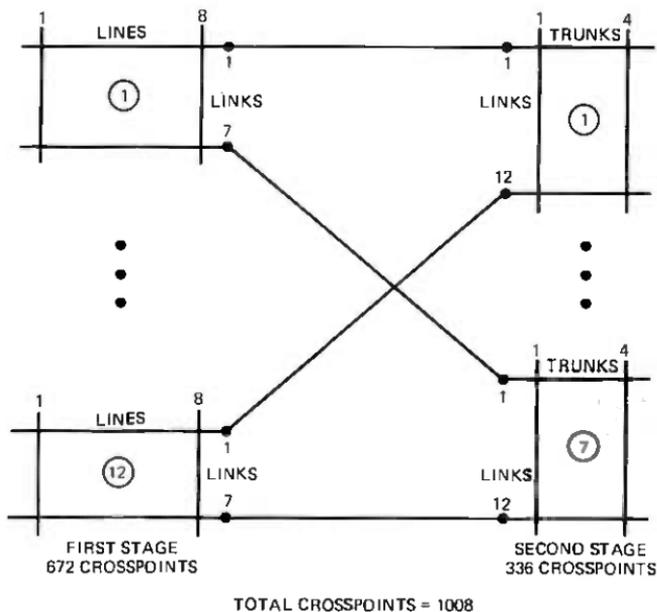


Fig. 4—Two-stage network.

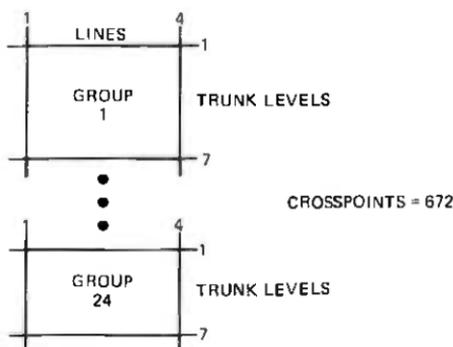


Fig. 5—Graded multiple.

entire second stage of the two-stage network plus the control circuitry must be installed initially. Only the first stage of the two-stage network but all of the graded multiple network may be added as a function of line growth. Also, the graded multiple can be organized into modules for easy maintenance, with all the crosspoints for eight lines on a single plug-in unit. Hence, the LSS uses the graded multiple network of Fig. 5.

The seven trunks out of a total of 32 trunks accessed by each line group are called level 1, level 2, . . . , level 7. Assignment priority is sequential in level, with level 1 having the highest priority and level 7 the lowest. A trunk wired to a given level in one line group is multiplexed to only the same level of other groups. Six trunks are used for level 1 and level 2, with

each trunk multiplied between four line groups. Four trunks each are used for levels 3 through 7 with each of these trunks multiplied between six line groups. The lowest priority level is almost never assigned except when traffic overload is imminent. In order to equalize trunk usage, every 24 hours the priority sequence alternates between levels 1, 2, 3, 4, 5, 6, 7 and levels 1, 2, 7, 6, 5, 4, 3.

Trunks are tested for leakage and continuity prior to connection to avoid assigning a defective trunk (see Section 2.6). If a trunk should test good but is defective for reasons other than leakage or continuity, e.g., noise, a service problem may develop during light traffic conditions. To avoid this "killer trunk" problem, assignment priorities alternate between level 1 and level 2 after every call. This arrangement prevents always assigning a defective level 1 trunk to the same line making successive originations.

2.8.2 Crosspoints

Switching crosspoints may be broadly classed as either the semiconductor type or the electromechanical type. For use in a line concentrator subject to a rather harsh environment, e.g., lightning surges, power line crosses, ringing voltage, and dc currents, only the electromechanical type of crosspoints can meet the environmental requirements without the use of special per line circuitry.

Both sealed and nonsealed types of electromechanical crosspoints are available. The sealed types offer protection from dirt and humidity but the ones available today from Western Electric are somewhat less rugged compared to the nonsealed types; the sealed contacts should not be used to switch dc in the transmission path and have one-tenth the surge current capability.

An example of sealed crosspoints is the remreed switch used in ESS systems. The remreed switches consist of a matrix of magnetically latching reed type make contacts sealed in a glass envelope. These sealed contacts contain some water vapor to help prevent the hard gold contacts from sticking at high temperatures. But the water vapor presents a finite probability of contact freezing at the low temperatures to which a concentrator RT may be subjected. Hence, remreed switches in the form available today were eliminated from further consideration for LSS.

The nonsealed category of crosspoints includes wire spring and flat spring relays, crossbar switches, and stepping switches. Of these, the Western Electric miniature magnetically latching wire spring relay was clearly the best and was chosen as the switching crosspoint for LSS. This relay is about one cubic inch and contains six transfer contacts. The latching feature is consistent with the low power requirement for a batteryless, simplex-powered RT. These relays can handle 30 A surge currents; hence, they are far more rugged than any of the sealed reed relays.

Extensive testing¹⁵ has been performed on these relays at room temperature in the presence of dust, moisture, and various gases such as sulfur dioxide and hydrogen sulfide. Other tests were performed at high humidity over a -40°C to $+70^{\circ}\text{C}$ temperature range. These tests clearly establish that adequate protection from such an environment is obtained when the relays are mounted on a printed wiring board attached to a die cast aluminum frame with an aluminum cover. Gasketing around the connectors insures that particles greater than 20 microns are filtered out.

2.8.3 The LSS switching network

As shown in Fig. 6, three of these multicontact relays (K1, K2, K3) wired in a tree configuration provide each line with access to one of seven trunks. The eighth port of this tree network is connected to a per-line service request detector at the RT and a ring trip and overflow circuit at the COT. Overflow is applied to a line in the COT when all of the seven accessible trunks are busy. A fourth relay (K4), a nonlatching miniature flat spring relay, serves as a network isolating relay. This relay isolates the line from the switching network during the switching of the three per-line relays in order to avoid objectionable transients on the line and on working trunks. Contacts on the K4 relays at the RT and COT also are used to check the quality of the assigned trunk and the switching network just prior to connecting the line. This preconnection trunk testing ensures that a defective trunk is not assigned to a subscriber.

In order to confirm that the desired line relays have been operated (or released) and that a line is connected to the desired trunk, an additional contact on each of the relays K1, K2, and K3 provides a voltage on one of three corresponding confirmation buses if the relay is latched while relay K4 is operated. The confirmation buses are multiplied to all line networks. The concentrator common control interrogates these confirmation buses and initiates corrective action if the actual trunk connection differs from the intended connection.

As shown in Fig. 7, one end of each of the three network relay coils for each line is connected to three common drive buses. The other end of each of these relay coils is connected to three common return buses through make contacts on the K4. A specific line relay network is connected to the six common buses by simply operating the desired line isolating relay (K4).

Reliable operation of the switching network is obtained by carefully controlling the latching and releasing currents for the latching relays. One set of three current regulators supply the latching current of $-140\text{ mA} \pm 15\text{ percent}$ for 16 ms for the K1, K2 and K3 relays. Another set of three current regulators supply the releasing current of $+35\text{ mA} \pm 15\%$ for 52 ms. The appropriate combinations of latch and release current

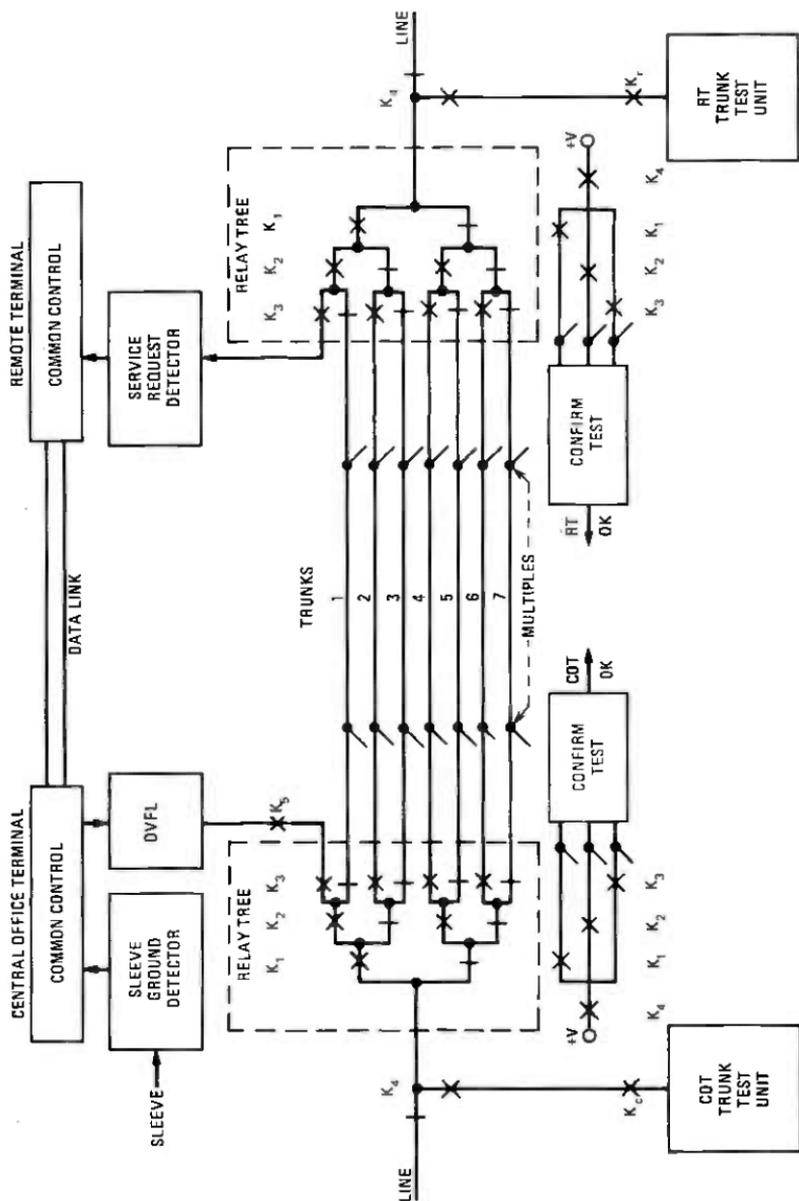


Fig. 6—The LSS switching network.

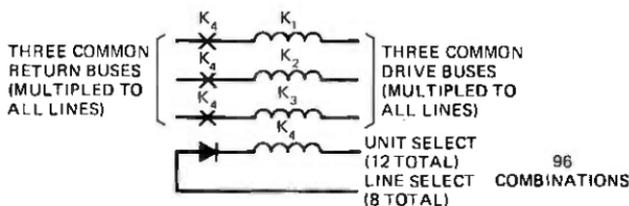


Fig. 7—Switching network control for one line.

are supplied to the three relays depending upon the desired trunk connection contained in a 3-bit trunk code supplied by the processor. Sensing circuits within this "network controller" monitor relay drive current. Incorrect current indications result in a major alarm and a network controller failure indication on an alarm unit.

The K4 relay is also operated by means of a current regulated to $-32 \text{ mA} \pm 10 \text{ percent}$. The maximum operate time is less than 32 ms. The relay releases in less than 52 ms after the drive current is removed since it is nonlatching. The appropriate K4 relay (one out of 192) is connected to this drive signal by means of a decoding network in the network controller which receives an 8-bit line number from the processor.

III. SYSTEM OPERATION

3.1 Call processing

A stored program controller at the COT implements the call processing algorithm and communicates with a relatively simple wired logic processor at the RT via the data link. The basic function of the common control circuitry is to connect a trunk in accordance with the results of an RT scan of the service request detectors and a COT scan of the sleeve lead status. A trunk is disconnected only when a sleeve lead is ungrounded.

Three types of orders are transmitted by the processor to the RT: a scan order, a network order, and an activate order. Scanning is performed at all times except when interrupted during the connection and disconnection of a trunk. The response of the RT to each of these orders is discussed in turn.

When the RT receives a scan order, the RT transmits the number of the line requesting service and, for redundancy, its binary complement. If differences occur between the line number and its complement, the COT rejects the scan result and initiates another scan order.

When a network order is received by the RT, it is stored in a register and also is returned to the COT for comparison with the original transmission. If there are no errors, the COT transmits an activate order to the RT which triggers the RT into implementing the network order, e.g., connecting a trunk to a line.

After the trunk is connected to a line, the RT returns the confirmation

answer to the COT. This confirmation answer, which indicates the actual line number and trunk level connected to the line, is compared with the original network order for consistency. Corrective action is initiated if differences exist.

If two RTs are connected with one COT via two separate data links, the COT scans first one RT and then the other for call originations. If an origination is discovered in one RT, the necessary network order is transmitted to that RT only. Scanning then resumes with the other RT.

Data transmission between the COT and RT is asynchronous, i.e., information is not transmitted in predetermined and regular time slots. Instead, orders and answers to orders are recognized by special preambles. These orders and answers are shown in Fig. 8.

The timing associated with the LSS call processing algorithm is shown in Fig. 9. For a terminating call, a trunk is connected within about 300 ms after the sleeve lead is grounded. For an originating call, a trunk is connected within about 400 ms after a station set goes off-hook. Originating call times include 100 ms for the service request detector; this delay is caused by a filter used to reject 60 Hz longitudinal signals.

3.2 COT processor

The LSS is controlled by PROCON, a microprocessor manufactured by Western Electric. The LSS uses the 8-bit data, 24-bit instruction PROCON with a 500-kHz clock. Every instruction is fetched and executed in one clock period ($2 \mu\text{s}$).

To control LSS, 5700 words of ROM (instructions) and 512 words of RAM (data) are required. Approximately 2000 words of program instruction are used for basic call processing and 3,700 words are used for automatic trouble locating, manual testing, alarming, and traffic measurements.

As shown in Fig. 10, the separate 8-bit data input and output buses of PROCON are combined into an 8-bit I/O data bus by means of hardware external to PROCON. Data is transferred between PROCON and the peripheral units by means of this I/O bus.

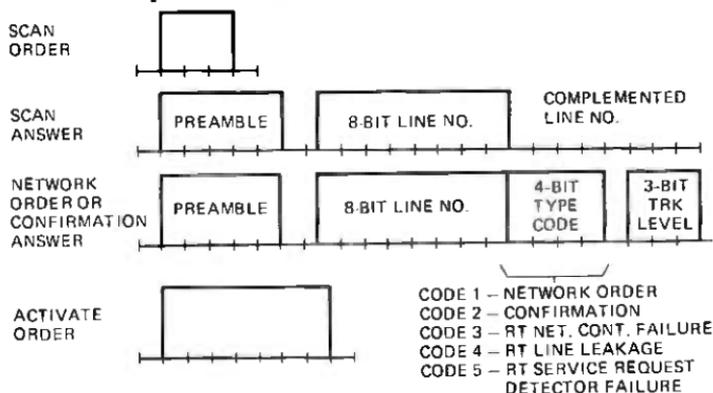
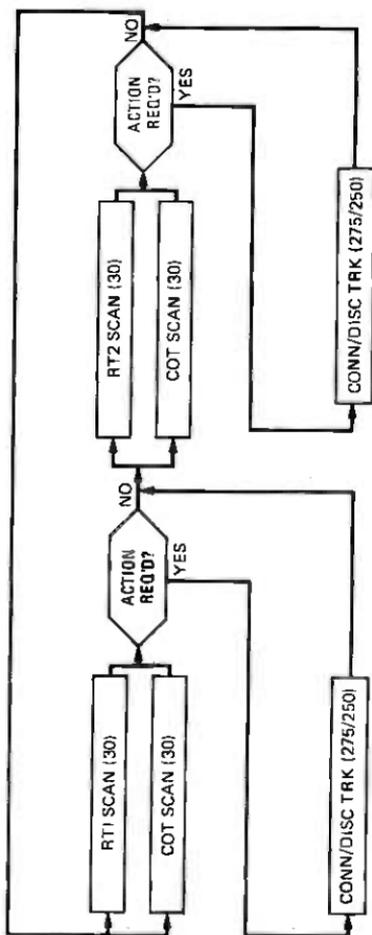


Fig. 8—Data organization for orders and answers.



NOTE: ALL NUMBERS IN MILLISECONDS (MS)

CONNECTION TIME * 275 - 335

DISCONNECT TIME 250 - 280

*FOR CALLS ORIGINATING AT THE RT, ADD APPROXIMATELY 100MS FOR THE TIME DELAY OF THE SERVICE REQUEST DETECTOR. IF TRUNKS ARE DERIVED FROM SLC-40 OR USE THE 5A RANGE EXTENDER, ADD THE TIME DELAY CONTRIBUTED BY THESE UNITS.

Fig. 9—Call processing times.

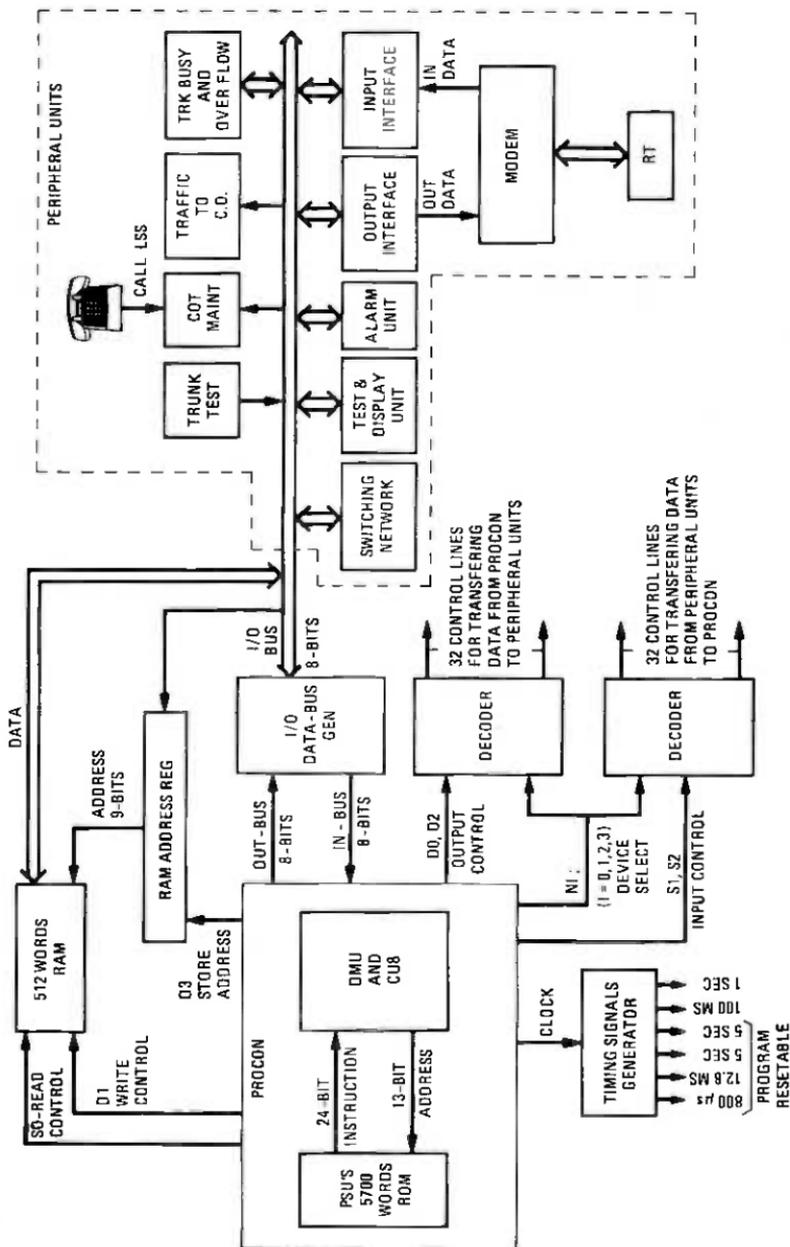


Fig. 10—Block diagram showing communication paths between PROCON, COT units, and the RT.

Thirty-two output control signals are used to transfer data selectively via the I/O bus from PROCON to various other peripheral units. The PROCON destination signals D0 and D2 and four device selection signals N0, N1, N2, N3 are used to generate these 32 control lines by means of a decoder external to PROCON.

A different set of 32 input control signals are used to transfer data selectively from the peripheral units to PROCON via the I/O bus. These control signals are generated from the PROCON source signals S1 and S2 and the same four device selection signals used for the output control.

The 512 word RAM is divided into an upper and lower half, corresponding to the ninth bit of the 9-bit RAM address. As long as successive RAM addresses are confined to either the upper or lower half of the RAM, the RAM can be addressed with 8 bits (one PROCON instruction) plus the ninth bit previously stored. But if the RAM address changes from one-half of the RAM to the other, two instructions are required to generate the 9-bit RAM address. After the RAM address is stored in the RAM address register, the RAM is then accessed in one instruction which enables either the S0 read control signal or the D1 write control signal.

A timing signal generator external to PROCON generates all of the timing signals necessary for the operation of LSS. This generator is driven by the PROCON clock and is controlled by the PROCON stored program.

3.3 RT wired logic processor

A wired logic processor at the RT is under the direct control of the COT microprocessor via the interconnecting data link. CMOS logic was chosen to minimize power drain. A stored program processor at the RT was not possible because of power drain with available technology.

When the processor recognizes the identifying preamble for the network order, the 16-bit network order, which follows immediately, is read into a 16-bit shift register and also is returned to the COT for verification. Upon receipt of an activate order from the COT indicating that the network message in the shift register is indeed correct, the line number and trunk level are transmitted in parallel from the shift register to the network controller for execution and also to the scanner. After execution, the processor receives the 3-bit confirmation code from the corresponding per-line switching network circuitry and inserts it into the trunk code position in the shift register. The line number is read back into the register from the scanner. The type code is changed and the contents of the register is serially transmitted to the COT.

After the execution of the network order, but just before the K4 line relay is released, i.e., after the K1, K2, K3 switching network relays have operated or released as required, the processor interrogates the status

of the service request detector for that line via the scanner. If the network order is the type which assigns a trunk and hence disconnects the service request detector from the line, the service request detector output should show an on-hook condition (test state 1). If, on the other hand, the network order is the type which disconnects a trunk from the line and hence reconnects the service request detector to the line, the service request detector output should show an off-hook condition before the K4 relay is released (test state 2), and an on-hook condition after the K4 relay is released (test state 3). This off-hook condition is caused by the resistance provided by the RT trunk test circuit connected to tip and ring of the line via the K4 line relay. If any of the above three tested states of the service request detector are inconsistent, the processor indicates this fact by altering the normal 4-bit "type code" portion of the 16-bit confirmation message returned to the COT. The COT will raise the appropriate alarm if test state 1 or 2 is in error or will enter the line number in the leakage memory if test state 3 is in error. This leakage memory is discussed in greater detail in Section 3.10.

The tests of the service request detector described above also serve to test the network controller decoder and the scanner multiplexer. When a network controller, a scanner, or a line unit fails, at first it may not be readily apparent which unit has failed and only an RT line unit failure minor alarm is raised. Eventually, the COT microprocessor will be able to deduce the specific failure by correlating subsequent trouble conditions, and the major alarm for a failed network controller or scanner, or both, will be added to the minor alarm for the line unit failure.

When the RT processor receives a scan order from the COT, the processor enables the RT scanner. The scanner sequentially interrogates the service request detectors for originating calls or specific RT alarms represented as a nonused line number and transmits to the processor the line code of the first line found which requires service. The processor inserts the 8-bit line code and its complement into the shift register in parallel and serially transmits this line code to the COT.

When the COT microprocessor is not transmitting network or scan orders to the RT, auxiliary COT hardware continuously transmits the activate order to the RT, thereby preventing a locally powered RT from switching to the alternate data link pairs.

3.4 Scanning at the RT

The RT scanner contains an 8-bit counter which generates the line numbers in sequence. Three bits of each line number are used to interrogate one of eight service request detectors mounted on one line unit or various alarms and other maintenance functions contained in the maintenance unit. Five bits are used to choose the output from one of 24 line units or from the maintenance unit. The line number generator

is enabled by the RT processor when a scan order is received from the COT, and is disabled when a service request is encountered. The contents of the line number generator is transferred to the RT processor for transmission to the COT. Scanning resumes at the next line number in the sequence when another scan order is received from the COT. If no service requests are detected in the length of time it takes to generate all line numbers, the line number generator is disabled and the processor transmits to the COT the no-request-for-service code of 255. If the scanner fails to scan, the line code 254 is transmitted instead, causing a major alarm in the COT.

3.5 Scanning at the COT

Generation of line numbers for scanning the sleeve lead status at the COT is accomplished within the microprocessor itself. By means of external multiplexing hardware similar to that used at the RT, the 8-bit line number generated by the microprocessor is used to interrogate the status of one of up to 192 sleeve ground detectors. Analogous to the RT, each COT line unit contains eight sleeve lead detectors and a one-of-eight selector with the outputs of these selected by a common multiplexer.

The scanning of all 192 sleeve leads at the COT is completed in the 30 ms period required to send a scan order to an RT and receive back the scan answer. It is necessary to scan for terminating calls this rapidly in order to provide the proper control interface with ALIT in SXS and #5XB offices. When a terminating call is detected, a control signal is passed to the ALIT system which causes it to reduce the rate of line tests. The slower testing rate provides time for the LSS system to connect a trunk and permit a valid ALIT test to be completed through to the subscriber's station set. Every new sleeve ground is interpreted as a possible ALIT test. Since ALIT tests are usually performed late at night when the traffic is light, the few subscriber calls on LSS lines should have little adverse affect on ALIT testing speeds.

3.6 Data modems

Data modems similar in design to those used in 202 type data sets are used at both the COT and the RT. The modems are designed for up to 24 dB of loss in the data link. This amount of loss permits the use of up to 3600 ohms of loaded 19, 22, 24, or 26 gauge pairs, or a mixture thereof. Additionally, the facility may also consist of low-loss channels derived from a digital voice-grade carrier system such as the SLC-40 system.

Circuitry is provided within the COT modem to permit automatic looping of the FSK analog output of the modulator to the input of the demodulator. This loop-back is used during automatic troubleshooting procedures to test modem operation.

3.7 Data link switching

In the event of a data link failure, a means is provided for automatically interchanging the data link pairs with two of the level five trunks of the concentrator. The level five trunks are, on the average, the least used trunks and the probability of interrupting a customer on a level five trunk is minimal.

Interchanging the data link pairs at the COT is under the control of the microprocessor. This same function is performed at the RT by the data link switching circuit. If the RT is powered over the data link pairs, the data link senses on which pairs the power is present and connects those pairs to the data modem. If the RT is locally powered, the data link switch hunts between the data link pairs at a 2.5-second rate when a hunt signal is received from the processor. The processor generates this hunt signal when a scan order or an activate order is not received at least every 400 ms. In order to prevent false hunting when the COT microprocessor has failed, or when the microprocessor has entered the troubleshooting phase, activate orders are continuously generated by COT hardware and transmitted to the RT.

3.8 Blocked calls

When all of the seven trunks accessed by a line are busy, the K5 relay (see Fig. 6) is operated and connects a ring trip and overflow circuit at the COT. If a terminating call arrives during the time when all trunks are busy, the LSS circuitry trips and ringing and supplies the overflow or fast busy signal in both SXS and #5XB offices. In ESS offices, a blocked call is never completed through to the LSS system. This ring trip and overflow function is performed by ESS. The fact that all of the LSS trunks are busy is obtained from another contact on the K5 relay via an ESS remote master scanner applique circuit. One such applique circuit is required for each group of four lines which share the same seven trunks.

If an originating call is blocked by the LSS system, dial tone is delayed until a trunk becomes available.

3.9 Testing the trunks and the switching network

The trunk test units at the COT and the RT are used for testing the trunk and the switching network for longitudinal leakage, metallic leakage, dc continuity, and ac transmission just prior to cutting through a line. The K4 line relay (see Fig. 6) is operated at the COT and the RT to isolate the line from the switching network and prevent clicks on working trunks when the K1, K2, and K3 relays are operated or released. After 70 ms, the Kc relay at the COT operates and the COT trunk test unit is connected through to the switching network and the trunk. The Kr relay at the RT is not operated until after the leakage measurements are completed at the COT. The trunk is an open circuit at the RT during the leakage measurements.

The longitudinal leakage test is made by connecting tip and ring at the COT to a -24 Vdc supply for 40 ms to condition the trunk. After the conditioning period, the leakage current flowing is integrated for exactly $\frac{1}{60}$ second to cancel out the effects of power line induction. At the end of the integration period, a flip-flop is set to a "1" state (test failure) if the integration result exceeds a threshold equivalent to 50 k Ω of leakage.

The longitudinal leakage measuring circuit is designed to function with up to 50 mA of induced 60 Hz longitudinal current flowing into a 100 Ω impedance to ground. The measuring circuit can also tolerate a power-line frequency error of up to 0.5 percent.

The next test to be performed is the metallic leakage measurement. A floating -24 Vdc source is connected between tip and ring for 30 ms. At the end of this 30 ms period, a second flip-flop is set to a "1" (test failure) if the current flowing exceeds a threshold equivalent to a metallic leakage of 50 k Ω . The metallic current due to power line induction is not expected to exceed 0.1 mA ac.

The last two trunk tests, i.e., the ac loss test and the dc continuity test, are both made at the same time. A 2100-Hz tone is connected to tip and ring at the COT. A dc voltage of 24 V is also connected to tip and ring.

After the Kr relay at the RT operates, the RT trunk test unit is connected to the trunk. The input circuitry for this unit completes the path for the dc continuity test. The COT dc continuity circuit indicates a pass condition only if the dc resistance of the loop is less than 50 k Ω . The circuitry, and hence the threshold, is the same used for the metallic leakage test and the test result is the logical complement.

The 2100 Hz tone transmitted from the COT is detected by the RT trunk test unit and causes a 660 Hz oscillator at the RT to turn on for about 40 ms. The 660 Hz tone transmitted from the RT is detected at the COT. The ac transmission test passes if the transmission loss at 2100 Hz and 660 Hz does not exceed 20 dB.

The results of the four trunk tests are combined logically to produce a pass or fail indication.

3.10 Service disconnection and the "leakage" memory

If service is discontinued for an LSS line, the tip, ring, and sleeve jumpers for this line are removed at the MDF and the line placed on intercept. It is not necessary to remove the corresponding jumpers at the RT cross-connecting terminal.

If a low resistance condition should occur on an LSS line beyond the RT which has been disconnected at the MDF, a trunk will be assigned to this line in the normal manner. Since the sleeve lead is not connected, the trunk will be disconnected within five to ten seconds. When the trunk is disconnected, the service request detector is reconnected to the line

at the RT and will show an off-hook condition, contrary to the normal case. The fact that the sleeve was not grounded and that the service request detector still shows an off-hook condition causes the line number to be put into the "leakage" memory. Sixteen such memory locations are provided for system A and sixteen for system B. Any line appearing in the leakage memory will be semipermanently assigned a trunk. This procedure prevents a trunk from being repetitively connected and disconnected to an off-hook line for which the sleeve is not grounded, and marks this trunk as available when necessary during high traffic conditions. Once a day this leakage memory will be cleared automatically.

If a working line develops a trouble beyond the RT which is recognized as an off-hook by LSS, causing a trunk to be assigned, but is not recognized as an off-hook by the central office, the situation is identical to that described above for an off-hook nonworking line. The line number will be put into the leakage memory and a trunk will be semipermanently assigned. Eventually, this condition will be identified by the central office during routine leakage testing.

3.11 Permanent signals

For SXS and #5XB offices, a line with a permanent signal will be assigned a trunk in the normal manner and will remain connected to this trunk for the duration of the condition. To LSS, the permanent signal condition appears like a normal call because the sleeve is grounded by the central office. A sufficiently large number of permanent signals will cause excessive blocking during high traffic conditions. For ESS offices, a permanent signal line will enter a "high-and-wet" condition after about two minutes. During the high-and-wet condition, the sleeve lead is not grounded. Consequently, in ESS offices a permanent signal line is treated as a leaky line by LSS. The line number is placed in the leakage memory, and a trunk is semipermanently assigned, thereby avoiding repetitive connection to the ESS recorded announcement trunk. Trunks connected to lines in the leakage memory can be used for other lines during high traffic conditions.

IV. EQUIPMENT DESCRIPTION

As shown in Fig. 11, the COT of the LSS requires a maximum of 61 inches of vertical space on a 23-inch wide miscellaneous bay and consists of up to four assemblies which are interconnected by connectorized cables. The 17-inch common control assembly consists of a fuse and alarm panel, two power units, PROCON, and an 8-inch shelf for 13 common control plug-in units.

The common control assembly for RT2 consists of a 4-inch high shelf which accommodates a power unit and a modem and is required whenever a second RT is provided.

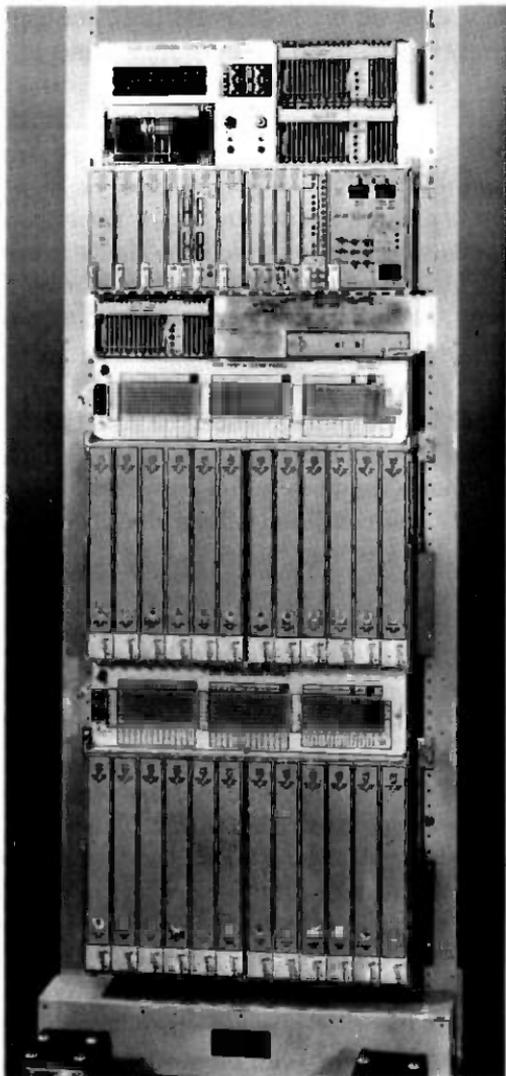


Fig. 11—COT of the LSS.

The 20-inch line unit assembly consists of a 14-inch high tray, which accommodates up to twelve 8-line line units and a 6-inch high field of terminals which provide test access to the lines, trunks, and other pairs. All external distributing frame connections for these wires are connectorized.

A second line unit assembly is used for the second set of 96 lines.

As shown in Fig. 12, the RT of the LSS is mounted in a cabinet 48 inches high, 29 inches wide, and 13 inches deep and consists of up to four assemblies which are interconnected by connectorized cables. These assemblies can also be mounted on a 23-inch wide miscellaneous bay.

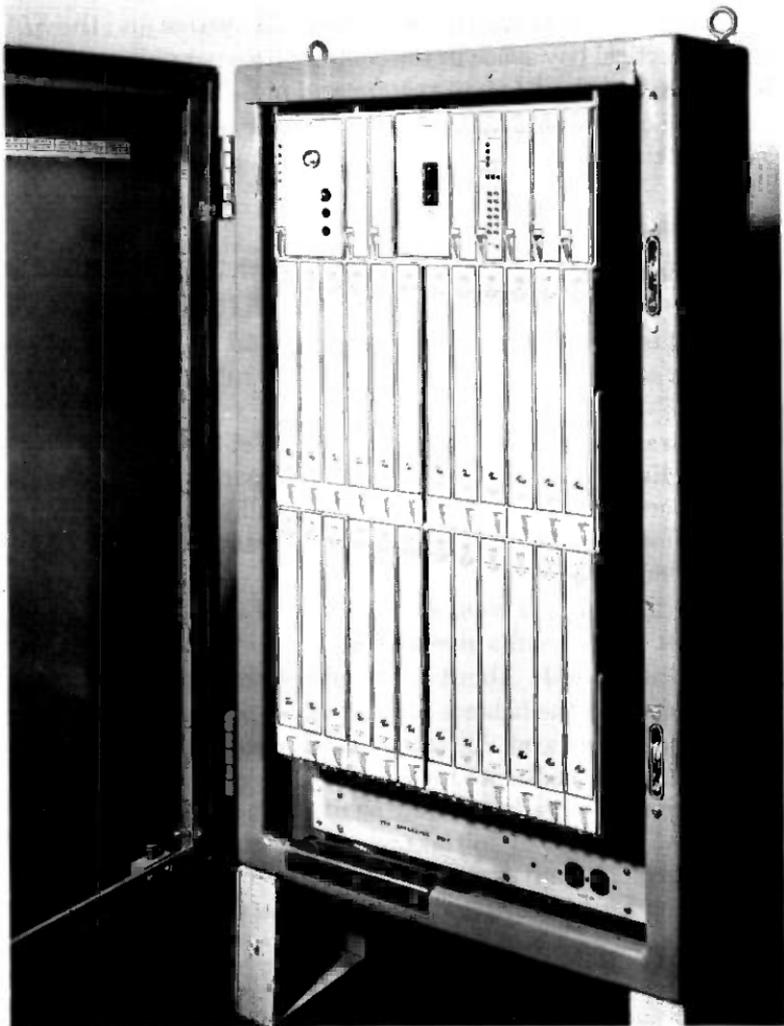


Fig. 12—RT of the LSS.

The common control assembly consists of an 8-inch high shelf for eight common control plug-in units.

The line unit assembly consists of a 14-inch-high shelf which accommodates up to twelve 8-line line units. It is equipped with gas tube protectors for the trunks, lines, data link and order wire pairs.

A second line unit assembly is added for a second set of 96 lines.

A 3-inch panel contains a thermostatically controlled heater and a duplex power outlet and is required only when it is necessary to control the humidity level within the cabinet.

A connectorized stub cable is used to interconnect the lines, trunks,

data link pairs, and the order wire pair with a cross-connecting terminal or a distributing frame.

Comparing the size of the LSS with the *SLM* system and the *SLC-40* system, the vertical bay space in the central office per pair gained for the LSS is half that of the *SLC-40* system and one-third that of the *SLM* system. For a cabinet mounted RT, the volume per pair gained for the LSS is one-fifth that of both the *SLC-40* system and the *SLM* system.

V. FIELD TRIAL

The LSS field trial began during August 1976 near San Diego, California, in the Pacific Telephone and Telegraph Company. The two-RT arrangement is used to provide service for up to 192 customers served by both SXS and No. 1 ESS. The system was fully loaded in January 1977.

The peak traffic carried by the LSS between February and May of 1977 is 4.7 ccs per line. The traffic level is about 20 percent below the design criteria of 6 ccs per line. Only one blocked call occurred during these entire four months, which is completely consistent with theory for this peak traffic load.

The reliability of LSS is measured in terms of service to the customers served by LSS. Early results give a trouble report rate of 4.2 reports per 100 stations per month. About 2.8 troubles were caused by non-LSS-related problems. Of the balance of 1.4 troubles, incorrect administration of central office sleeve leads for ESS customers served by LSS contributed 0.7 trouble. Test procedures for verifying sleeve lead connections have since evolved which should mitigate against such errors. The remaining 0.7 trouble was caused by initial LSS programming errors which have since been corrected and system outages caused by intermittent solder connections on several early models of plug-in units.

VI. SUMMARY

The Loop Switching System described in this paper utilizes a micro-computer which permits a wide variety of operational and maintenance features to be included in the system which would not have been possible if a hard wired logic approach had been used. By storing the history of system performance, a more accurate and thorough diagnosis of system trouble is possible and a more flexible call processing algorithm can be implemented. In addition, the turnaround time on correcting system bugs and adding features is greatly reduced when such changes can be implemented by software modifications.

The LSS system is currently in production. The first LSS systems were shipped by Western Electric to Pennsylvania and South Dakota in mid-1977.

VII. ACKNOWLEDGMENTS

The LSS system is the result of the joint efforts of and consultations with persons too numerous to mention by name from over 20 different departments within Bell Laboratories, Western Electric, and AT&T.

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Loop Plant Electronics:

Physical Design Considerations

By R. W. HENN and D. H. WILLIAMSON

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The introduction of electronics into the loop plant has created the need for mounting electronic systems in the outdoors as well as in the central office. This article details the physical design considerations and requirements for the plug-in circuit boards, cabinets, closures, and hardware, and their interconnection to the telephone plant. Of particular interest is the product-line diversity from the small voice frequency range-extension devices and analog carrier systems to the large digital carrier and voice frequency loop switching systems. Each requires unique electrical and mechanical partitioning to optimize performance. Other critically important considerations are the cost-performance tradeoffs in a cost-sensitive market; the analysis, testing and evaluation of the system in environmental extremes of temperature, humidity, and atmospheric pollutants; the need to be compatible with existing loop plant hardware; and, lastly, the human factors considerations necessary to provide an effective interface with the telephone craftsman. Specific designs are discussed which illustrate the practical problems encountered in meeting these requirements. In addition, the future thrust of loop electronics is discussed.

I. INTRODUCTION

Although electronics technology has been used for many years in the central offices and toll plant of the Bell System, its use in the loop plant is in its infancy. The first Western Electric loop electronic range-extension product, the 2A, was installed in central offices in 1968. Since that time, a complete family of loop electronic products has been developed, and its introduction has generated new challenges for physical design.

Unlike most Bell System electronic systems, certain elements of loop electronics hardware must be designed to mount outdoors (commonly referred to as the "outside plant") as well as in the central office. The central office is generally a controlled environment and is largely standardized from a facilities and craft point of view. The outside plant environment, on the other hand, has more variety in facilities including aerial, underground, and buried plant; a multiplicity of cable and cable apparatus with which the electronics must interconnect; and large variety in crafts: splicers, installers, linepersons, and repairpersons.

The aim of this paper is to highlight the central office and outside plant physical design of loop electronics products currently being manufactured by the Western Electric Company, Incorporated.

The loop electronics product line is diverse. Small carrier and voice frequency systems and large carrier and concentrator systems must be economically and ruggedly designed and packaged. Typically, the volume occupied at a remote terminal site varies from 135 cubic inches for a single channel analog carrier system to 16 cubic feet for a 40-channel digital carrier system. Since most loop electronic systems so far have been installed on rural routes, physical design has concentrated on above-ground mounting in the outside plant. This paper will deal primarily with these designs. As loop electronics penetrates the suburban and urban markets, the aesthetic pressure for out-of-sight plant will create a need to design for belowground installation.

II. PHYSICAL DESIGN REQUIREMENTS AND CONSTRAINTS

2.1 Overview

Providing basic electronic functions (logic, modulation, amplification, memory) while meeting all specified performance requirements is the key to any electronic system design. The physical designer must insure that requirements are not violated in partitioning the structure into various subassemblies, interconnecting the subassemblies, and packaging the system for mounting. This implies decisions that interact with reliability and cost in an extremely dynamic and cost-sensitive market.

2.2 Specific requirements¹

2.2.1 Environment

The outdoor environment requires design to ambient temperature extremes of -40 to 120°F while accommodating both solar heating and internal heat generation by the electronics. Equipment must operate in ambient relative humidities up to 100 percent. The electronics must be protected from airborne particulates and pollutants including salt, dirt, hydrocarbons and industrial smog.

2.2.2 Human factors

The extremely labor-intensive loop plant requires special attention to a person's abilities to perform simple installation, operational, and maintenance tasks. The goal is to develop systems in which the hardware complements the skills of the people using them. This is a necessity in loop electronics because the hardware is being introduced into a working cable plant where job classifications are rigidly defined, and new tools, hardware, and methods frequently require changes in established procedures.

2.2.3 Compatibility

The need to interface with existing telephone hardware requires compatibility in many areas. The necessity for electrical compatibility at input and output terminations is obvious. Less obvious is the need to fit standardized equipment formats in the central office and need to interconnect new electronic equipment to existing apparatus.

2.2.4 Flexibility

Within the demanding loop electronics market it is necessary to provide flexibility for new features. Product obsolescence must be continually evaluated and dealt with. Plug-in assemblies and modular techniques are required to simplify changes and modification and to facilitate introduction of lower-cost silicon integration techniques as new technologies develop.

2.2.5 Manufacturing

As a product line, loop electronics systems, although rapidly enjoying acceptance, are comparatively low runners compared to other transmission and switching systems. This has encouraged the use of existing hardware so that system costs can be reduced by piggybacking onto a large volume runner.

III. SYSTEM PHYSICAL DESIGN DESCRIPTIONS

3.1 Analog systems

The *SLC*TM-1 single channel carrier and *SLC*-8 multichannel carrier systems are described in a preceding article.² Both systems require central office and remote terminal equipment installations. The remote terminal ends of these systems differ significantly from each other in size and mounting locations. The *SLC*-1 mounts on the customer's premises; the *SLC*-8 in outside plant cabinets.

3.1.1 Central office equipment

The central office terminal equipment for *SLC-1* and *SLC-8* is illustrated in Fig. 1. Central office equipment design, for both, is relatively standard. The design format is simply described as a printed wire board plug-in (called a circuit pack) that plugs into a bay-mounted shelf. The *SLC-1* and *SLC-8* shelves mount in 23-inch-wide unequal flange, bulb



SLCTM-1



SLCTM-8

Fig. 1—Central office terminal equipment.

angle, or duct type frames. Mounting in wider bays like the ESS bay, for example, requires special mounting brackets.

The shelf designs for both systems take advantage of existing high-runner transmission products. *SLC-1* uses a standard sheet metal shelf and separable backplane design developed for the F Signaling and Metallic Facilities Terminal systems. The steel shelf and backplane are illustrated in Fig. 2. To achieve low hardware costs, the typical individual card frame holder and faceplate were eliminated from the circuit pack. Instead the printed wire board slides into a new, low-cost plastic track that snaps into the standard mounting shelf. An attractive, blue, brushed aluminum common front cover replaces twelve individual circuit pack faceplates. Circuit pack cost is kept low by using single-sided printed wire boards and custom Western Electric Transmission Equipment Dips (TEDs). The TEDs are unique in that thin film resistors are functionally trimmed during TED manufacture to eliminate costly hand selection of discrete resistors during circuit pack assembly. The circuit packs are interconnected to the central office battery and equipment frame by wire wrapping to a Western Electric 928-type, gold finger, plug-in connector mounted to the F-signaling backplane. To save connector costs, the 928D connector was designed with contacts on one side only to match the single sided printed wire board.

The *SLC-8* central office terminal consists of a bay-mounted shelf and a fuse and alarm panel. Nine circuit packs comprise one system: eight channel units and one power supply. *SLC-8* takes advantage of a new, low profile line interface hybrid transformer for increased shelf packaging density, allowing 18 circuit packs per shelf. The standard D4 system shelf that is used has the required slot pitch. Figure 3 illustrates the equipment components. The D4 shelf is a die cast aluminum structure designed for frameless circuit packs. Its high-volume production lets *SLC-8* share the economies of scale. To establish a family appearance with *SLC-1*, a similar common cover is used instead of individual faceplates. The *SLC-8* circuit packs utilize the Western Electric 963B-20 socket connector, replacing the traditional gold finger plug-ended printed wire board. The *SLC-8* backplane is a single-sided printed wire board that requires no hard wiring. The 3/32 epoxy glass backplane mounts directly to the D4 shelf. Also shown is the required fuse and alarm panel.

3.1.2 *SLC-1* subscriber premises apparatus

Both indoor- and outdoor-mounted subscriber premises electronic packages have been designed for *SLC-1*. To achieve a small package, using standard low-cost electrical components, a novel manufacturing technique was introduced in which a rigid epoxy printed wire board is folded in the final manufacturing step to reduce the frontal area of the

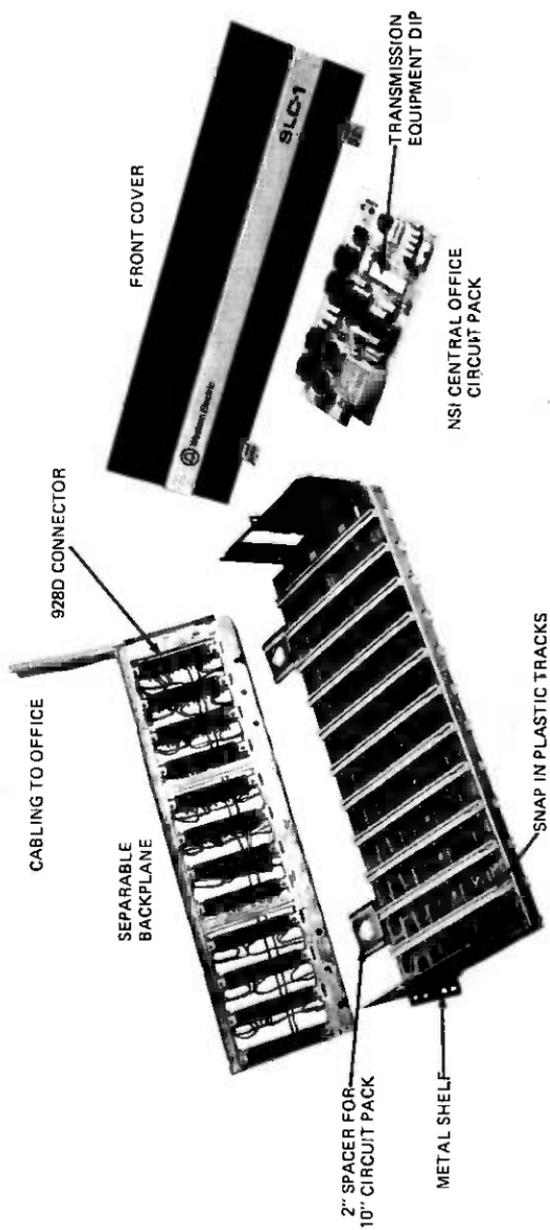
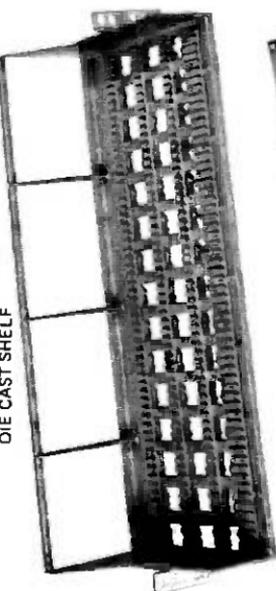
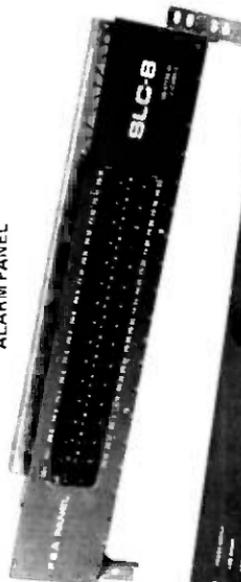


Fig. 2—*SLC*[™]-1 central office shelf components.

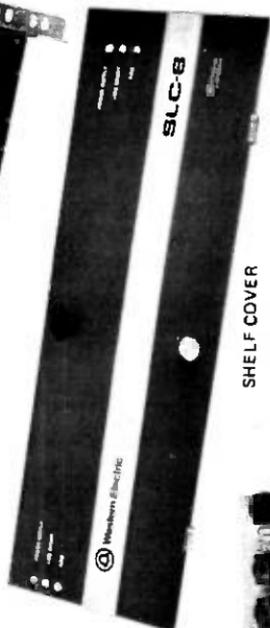
STANDARD D4 ALUMINUM
DIE CAST SHELF



FUSE AND
ALARM PANEL



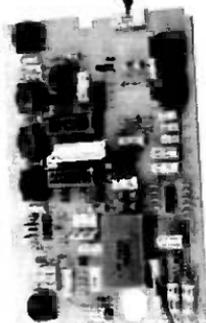
SHELF COVER



SINGLE-SIDED
PRINTED WIRE
BACKPLANE



LOW PROFILE HYBRID
TRANSFORMER



TYPICAL CHANNEL
CIRCUIT PACK

963B-20
CONNECTOR

Fig. 3—SLC™-8 central office components.

package. Figure 4 illustrates the technique. The printed wire board is manufactured and tested flat, then sheared apart and folded. A flexible ribbon cable provides electrical connection between the two pieces. In addition, snap-on plastic posts provide mounting for a nickel-cadmium battery that powers the unit.

To further reduce costs of the indoor-mounted unit a standard gray plastic cover is used to protect the electronics. The cover is injection molded from a polyvinyl chloride (PVC) resin and is illustrated in Fig. 5. It is in common use in other Bell System applications.

The outdoor mounted unit required a custom design for the harsher external environment. A vented, bell-type-housing closure is injection molded from an ABS*/PVC blended material already being used for other loop closures (see Section 3.4.2). Material cost, required detail, weatherability, and aesthetics determined this selection. Injection molding a one piece closure with a 6.5-inch-deep cavity required careful mold design and balanced plastic flow. The closure is illustrated in Fig. 5.

The ABS/PVC thermoplastic was selected for these characteristics:

(i) Good weatherability, including ultraviolet light (UV) and oxidative stability. A dark color (charcoal gray) with a high carbon black content provides the UV protection.

(ii) High impact strength and flame retardancy at reasonable cost.

(iii) Acceptable tolerance to solvents, sprays and paints commonly used in a household or by a craftsman.

(iv) Acceptable high temperature performance in ambient temperature extremes in the United States. Actual solar heating temperature increases to 187°F were recorded in the Southwest desert on the top surface of the closures, without distortion.

The same circuit pack is used in both the indoor and outdoor units. A standard epoxy-base solder resist cover coat (PC401†) protects the printed wire paths. The coating serves two functions. It protects the wire paths from contamination during manufacture and it aids the wave soldering of the circuit pack. Short term, high humidity (95 percent RH and 90°F) chamber tests and field trials comparing solder resist coated and noncoated circuit packs have demonstrated superior functional performance of the solder resist coated printed wire boards.

To assist the craftsman during installation, the outdoor unit integrates the electronics and a standard station protector (123A1A protector block) within a single closure. Because the electronics are preterminated

* Acrylonitrile butadiene styrene.

† Western Electric specification WL2333.

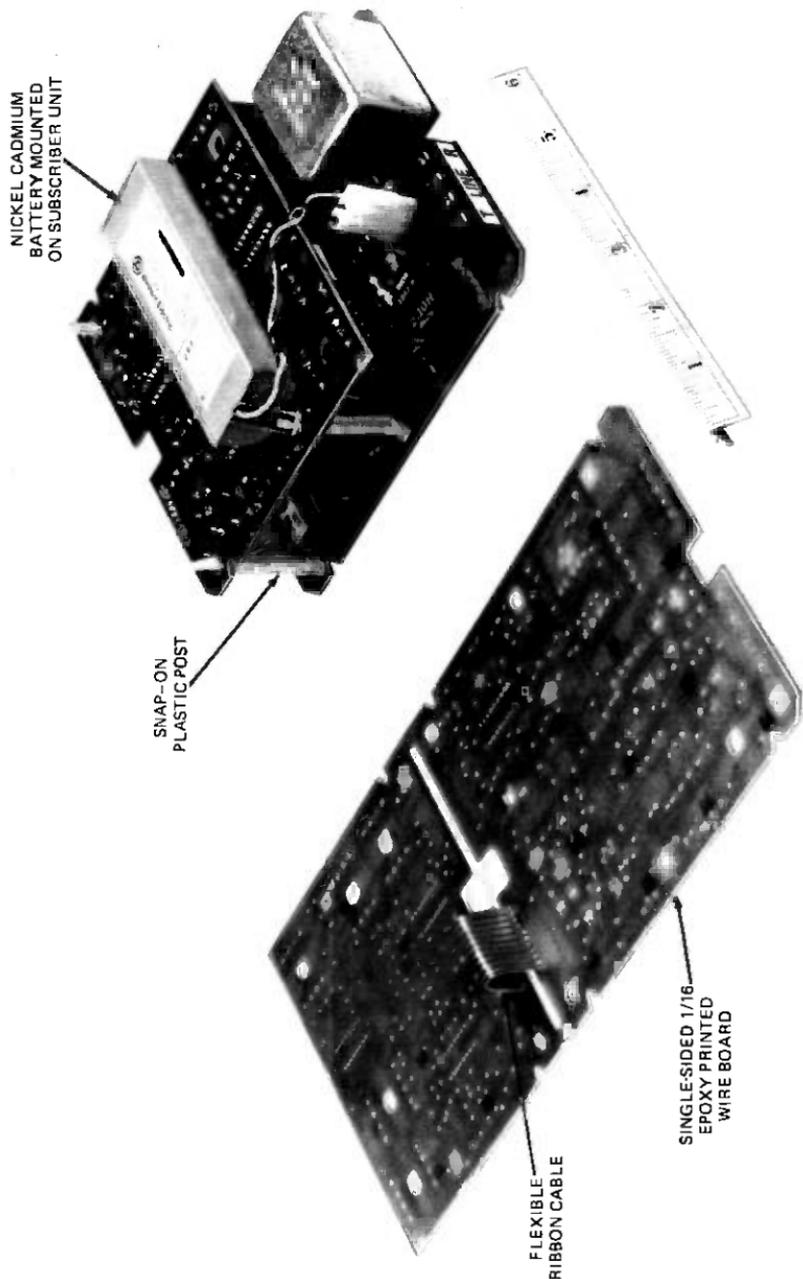


Fig. 4—SLC™-1 subscriber circuit pack.



Fig. 5—*SLC*[™]-1 subscriber premises electronics.

at the factory, the installer need terminate only the drop wire and station wire as in a standard main station installation (see Fig. 6). The mounting plate locates the closure-mounted protector unit at the same level as the old protector to avoid the need for piecing out and splicing existing station wiring.

The *SLC*-1 hardware also includes a small isolation filter mounted in series on the VF cable pair. Because of its similarity to the voice frequency 28A ringer isolator mounting, filter physical design details are discussed in Section 3.4.2.

To further help the craftsperson during installation, the *SLC*-1 subscriber electronics have been packaged in a specially designed carton with simplified installation and troubleshooting instructions. These packages are illustrated in Fig. 5.

3.1.3 *SLC*-8 remote terminal apparatus

The *SLC*-8 remote terminals can serve groups of eight customers from one location or are distributed to serve sparsely populated rural areas.

The physical design of the "lumped" system is built around an eight-channel modular housing. The system is modular to permit grouping remote terminals in a variety of sizes for flexibility in growth and application. *SLC*-8 lumped installations use existing loop plant pedestals and cabinets. Standard cable terminating apparatus is used

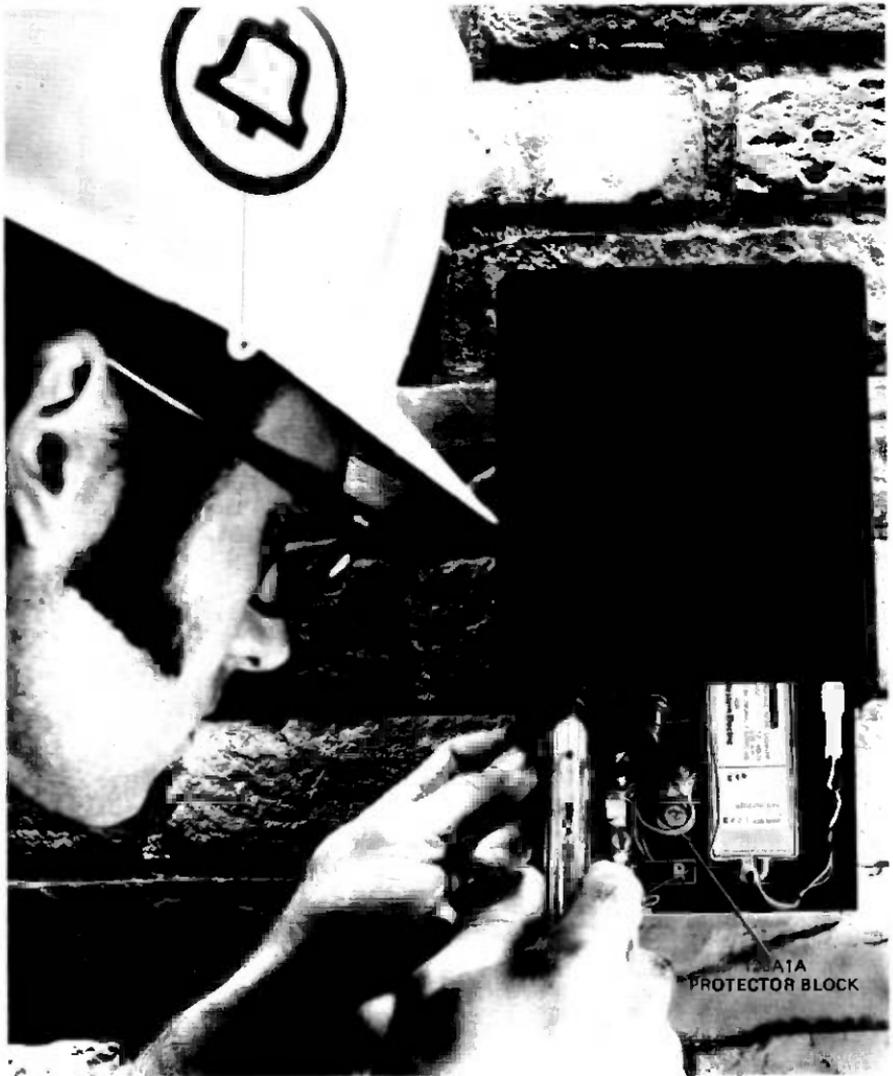


Fig. 6—SLC™-1 main station termination using standard 216B tool.

to connect the electronics to the cable plant. This design approach achieves craft and hardware compatibility and reduces the need to learn new techniques.

Four lumped remote terminal installations are illustrated in Fig. 7. Each comprises a modular circuit pack housing, cabinet, and interconnect hardware.

The lumped, modular circuit pack housing is illustrated in Fig. 8. The housing is injection foam molded from a polycarbonate plastic. The foam molding process was selected primarily for the low pressure molding technique it employs. Low pressure tooling is less expensive than that

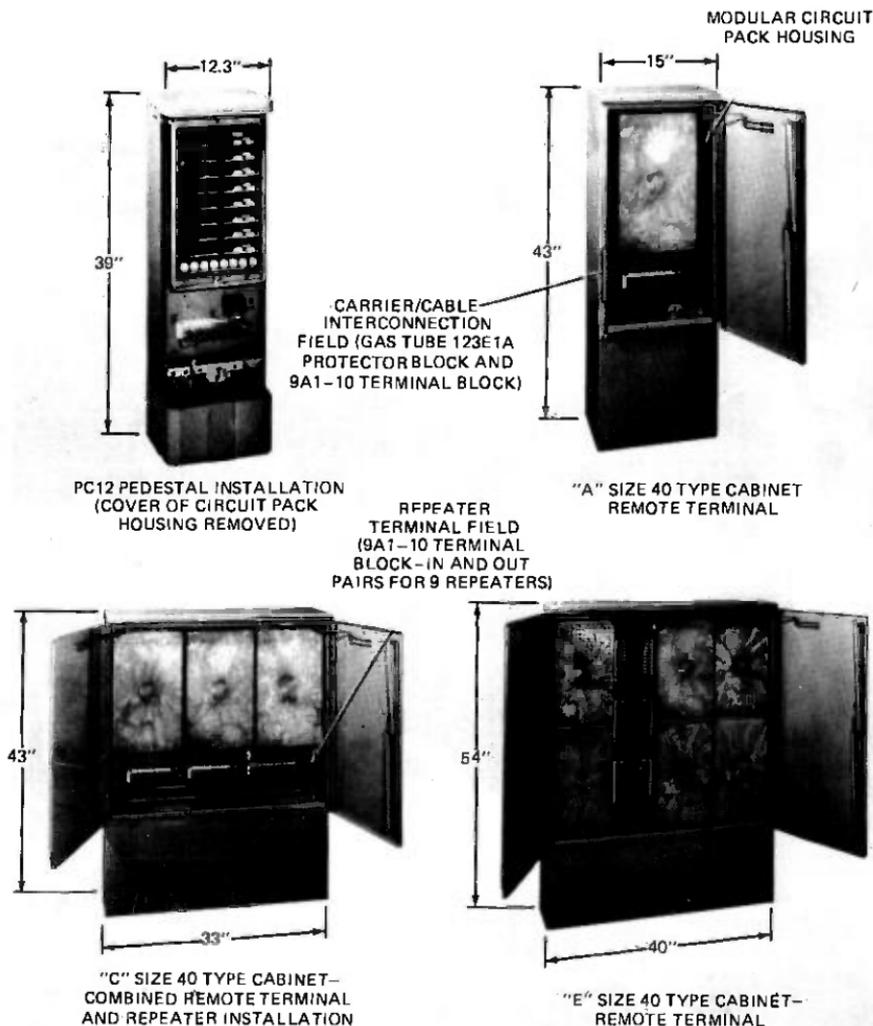


Fig. 7—SLC™-8 lumped remote terminal hardware.

for normal injection molding. Because the molding press must react against only the expanding plastic, the internal mold pressures are in the 200-400 psi range. Smaller tonnage presses and less expensive aluminum tooling can be used than for standard injection molding. Low pressure molding also reduces molded-in stresses. This produces parts with less distortion and permits more intricate detail for large parts.

Injection molding reduces secondary operations during manufacture by permitting functional details to be molded in. For example, tracks for the circuit packs, mounting bosses and holes to mount the printed circuit backplane, a molded-in hinging arrangement for the door, and details for locating and supporting accessory hardware can be molded.

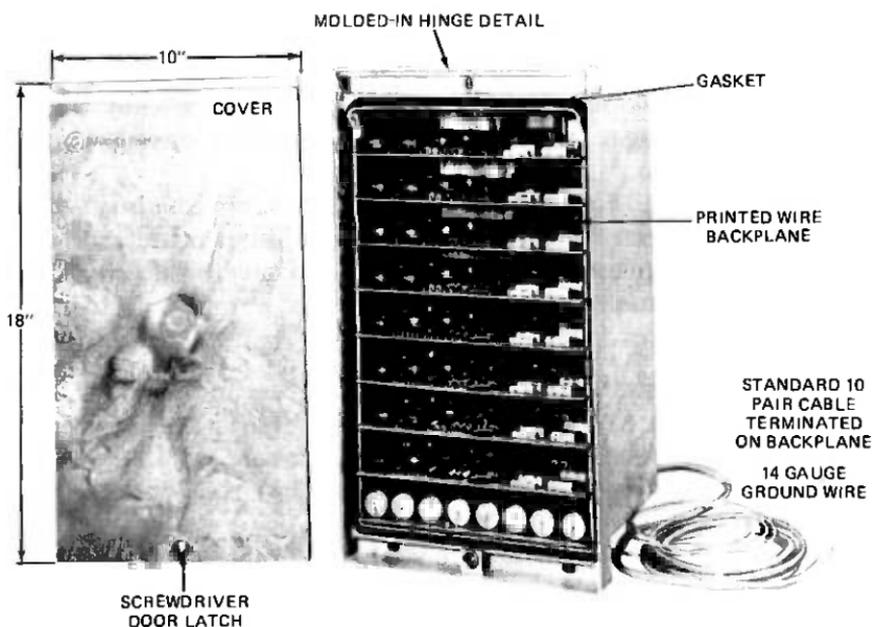


Fig. 8—SLC™-8 circuit pack housing.

The multiplicity of molded-in detail requires careful attention to tolerance limits. Of special concern was the circuit pack fit in the molded tracks and the mating with the rigid 963B-20 backplane connector.

A polycarbonate molding compound was selected because of its inherent dimensional stability, low shrinkage rate, toughness, impact strength, and fire retardancy. Fire retardancy is important because the modular housing will occasionally be mounted inside buildings.

Foamed parts produce a swirl finish that may be objectionable. Surface finish in this application was not critical, however, because the housing mounts inside a cabinet.

The lumped housing is gasketed with an EPDM (ethylene propylene diene monomer) closed-cell rubber foam for protection from dust, industrial pollutants and liquid water. This material combines low closing force and long term dimensional recovery characteristics after compression, even under accelerated high temperature testing. In addition, EPDM rubber is significantly more ozone resistant than neoprenes, providing longer service life.

The unit is not sealed, but breathes with the atmosphere through a small opening in the bottom. The door latch requires a screw driver for entry. A twisted wire, paired cable provides for connection to the cable plant.

The remote terminal housings mount in the standard PC12 Bell System pedestal and the new family of 40-type cabinets designed for Serving

Area Interfaces (SAIs) and Rural Area Interfaces (RAIs). The design philosophy for using existing cabinets includes:

(i) Need for flexibility: *SLC-8* is a small channel carrier system for which the operating companies require a variety of mounting arrangement and sizes.

(ii) Low cost and craft compatibility: *SLC-8* takes advantage of existing Western Electric and AT-Specification closures manufactured in reasonably high quantities. Craftspeople who have been using these closures in conventional cable installations need no new skills to install them for electronics applications.

The PC12 pedestal is used without modification and provides an inexpensive installation for a single lumped system. The pedestal is galvanized steel, finished with a green polyester coating. The pedestal provides the gross mechanical protection for the circuit packs and the interconnect hardware.

A series of 40-type cabinets have been coded specifically for *SLC-8*. Special mounting brackets have been added to support the circuit pack housings and the accessory hardware. These cabinets are fabricated from heavy gauge galvanized steel, and finished with an electrostatically deposited polyester coating. The cabinets offer pole or pedestal mounting for one, three, and six lumped systems. They provide substantial mechanical protection for the electronics. The pedestal models require concrete mounting pads.

Interconnection of the electronics to the outside cable plant is engineered for craft skill compatibility. For example, standard binding post terminating blocks provide termination and test access of the carrier pair and the eight derived channels. The carrier pair is protected by a standard 123-type protector block. All wire terminating and splicing is done with standard loop plant hardware.

In addition to the remote terminal installations, repeater installations have been designed with the same building blocks. Provision for in- and out-carrier pairs and gas tube protection on the circuit packs are the only differences. Each circuit pack housing holds nine repeaters. Repeaters are available in the PC12 pedestal and in the A and C size 40-type cabinet versions. Figure 7 illustrates a combined repeater and remote terminal installation.

Distributed capability is provided with a custom designed closure, coded in two versions, one that holds one remote terminal channel and the other for two repeaters. The housing is a one-piece metal closure, fabricated and finished like the 40-type cabinet, with galvanized steel and polyester paint finish. The internal structure is a foam-molded detail that provides support for the circuit packs. Craft access to the circuit packs is provided through one end. Termination access is from the other

end. The door design, locking, and gasketing are similar to the lumped housing. The structure is vented through one opening in the housing. The housing measures 12" × 12" × 5" and pole-mounts at eye level. Standard binding post hardware is used as in the lumped housing. Figure 9 illustrates the design details.

3.2 Digital systems

The present digital subscriber carrier system is the *SLC-40* system which has been described in a previous article.³ The principal physical parts of the system are a central office terminal, a remote terminal, and an interconnecting digital repeated line.

3.2.1 Central office terminal

The central office terminal for the *SLC-40* system, shown in Fig. 10, consists of a channel bank, fuse/alarm and jack panels, and a number of printed wiring board plug-in units. As is true for most loop electronics systems, the bank design format is taken from an existing high production volume system. The shelf, connector system, and circuit-pack board outline are identical to the D3 channel bank system except for the height of the board. To avoid the costs of painting and marking faceplates for each plug-in unit, decals are used to give the specific designations required.

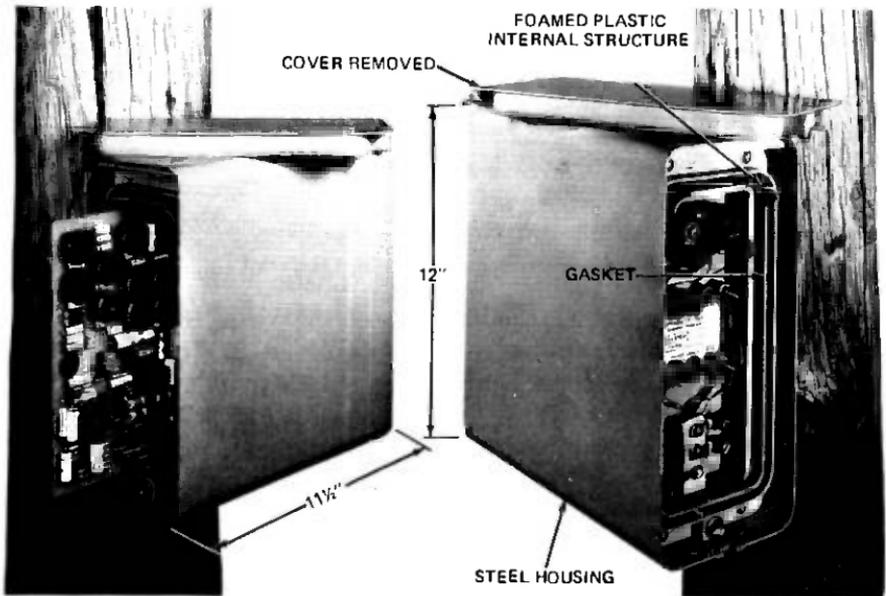


Fig. 9—*SLC*[™]-8 distributed remote terminal configuration; holds one *SLC*-8 channel.

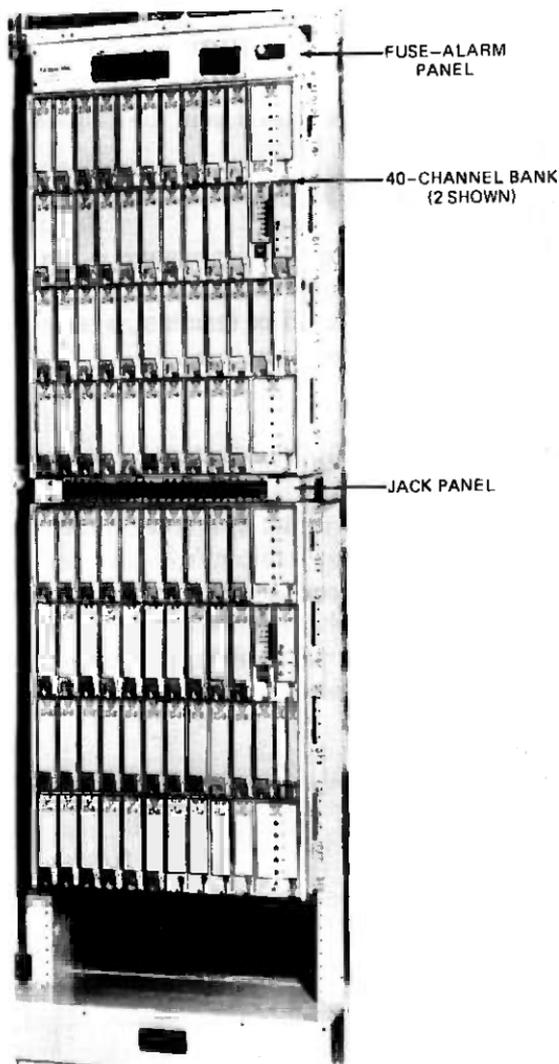
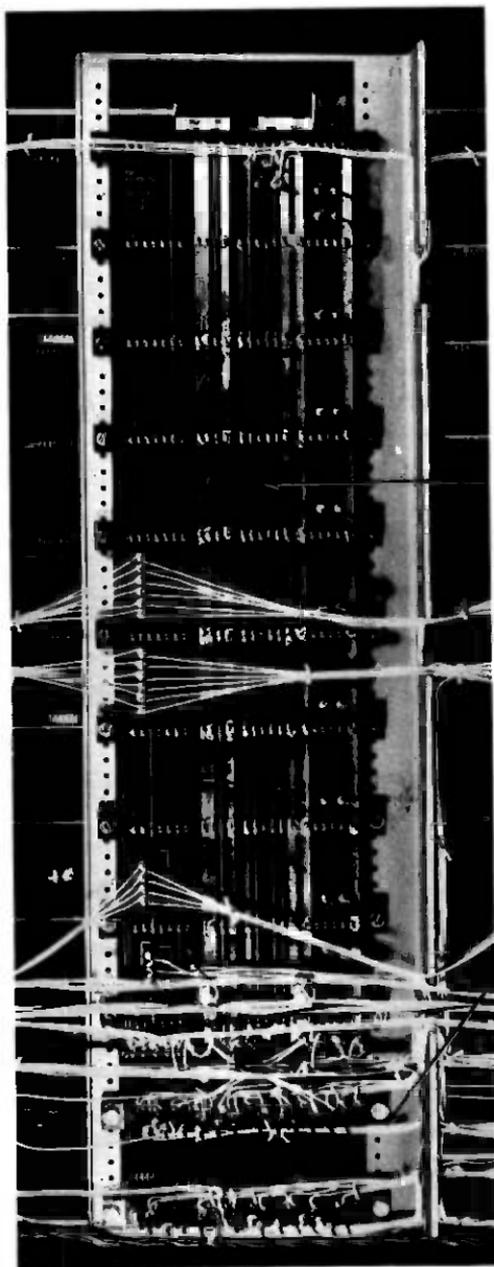


Fig. 10—SLC™-40 central office terminal.

As can be seen in Fig. 11, the interconnecting wiring in the system is a combination of conventional wire wrapped leads and printed wiring board backplane connections. Each of the common function circuit packs (multiplexer, power supply, etc.) requires unique wiring. Wire wrapped connecting leads are used for interconnecting these nonrepetitive circuit packs. There is, however, a great deal of commonality among the forty channel units that are used in each bank. Here, a simple double-sided printed wiring backplane is used with connectors having terminals suitable for mass soldering. The same printed backplane is used for both ends (COT and RT) of the system to further promote the advantages of



PRINTED WIRING
BOARD

WIRE-WRAPPED
LEAD

Fig. 11--SLC™-40 bank backplane wiring.

consistency and simplification of manufacture. Wire wrap terminals are provided for most of the external connections from the COT.

The bank and panel assemblies for the *SLC*-40 system are designed to be front mounted into a 23-inch equipment frame having a 5-inch front guardrail to upright dimension. This allows the use of such assemblies in virtually all of the older electromechanical Class 5 offices which often have bulb angle or other old style frames. Further, the COT equipment is only available as panel or bank assemblies, and not pre-mounted on equipment frames. This is done to minimize the Western Electric stocking inventory. It is necessary for Western Electric to be able to ship equipment directly from stock to achieve a 10-week delivery interval as opposed to the more conventional interval of 26 weeks or more.

3.2.2 Remote terminal

Two basic configurations are available for the *SLC*-40 system remote terminal. The RT may be mounted inside a building in a more or

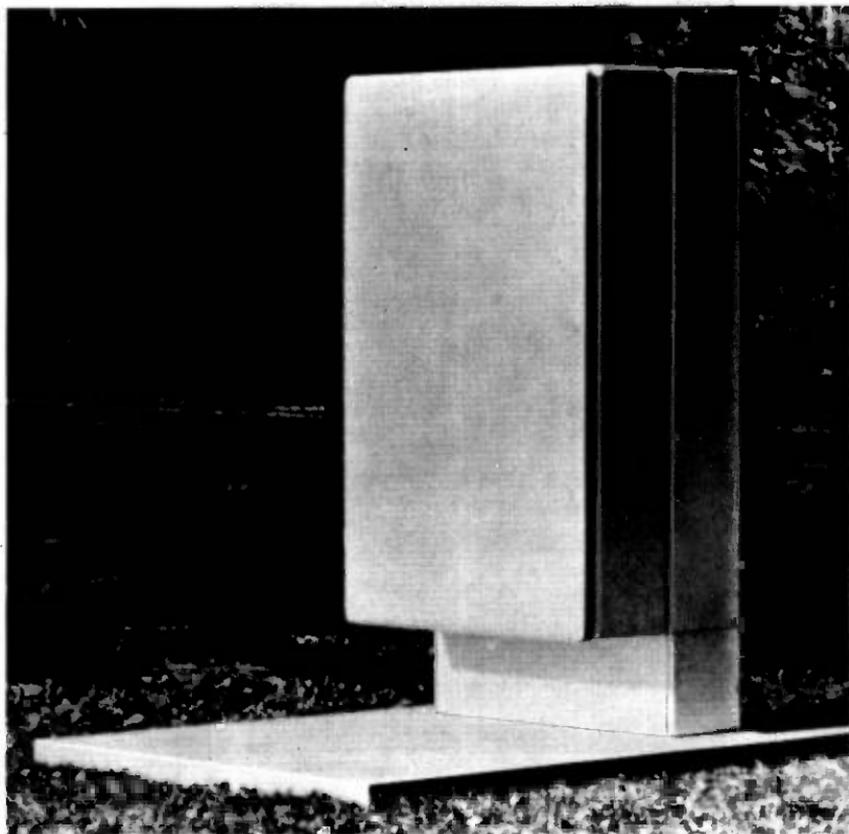


Fig. 12—*SLC*[™]-40 cabinet remote terminal.

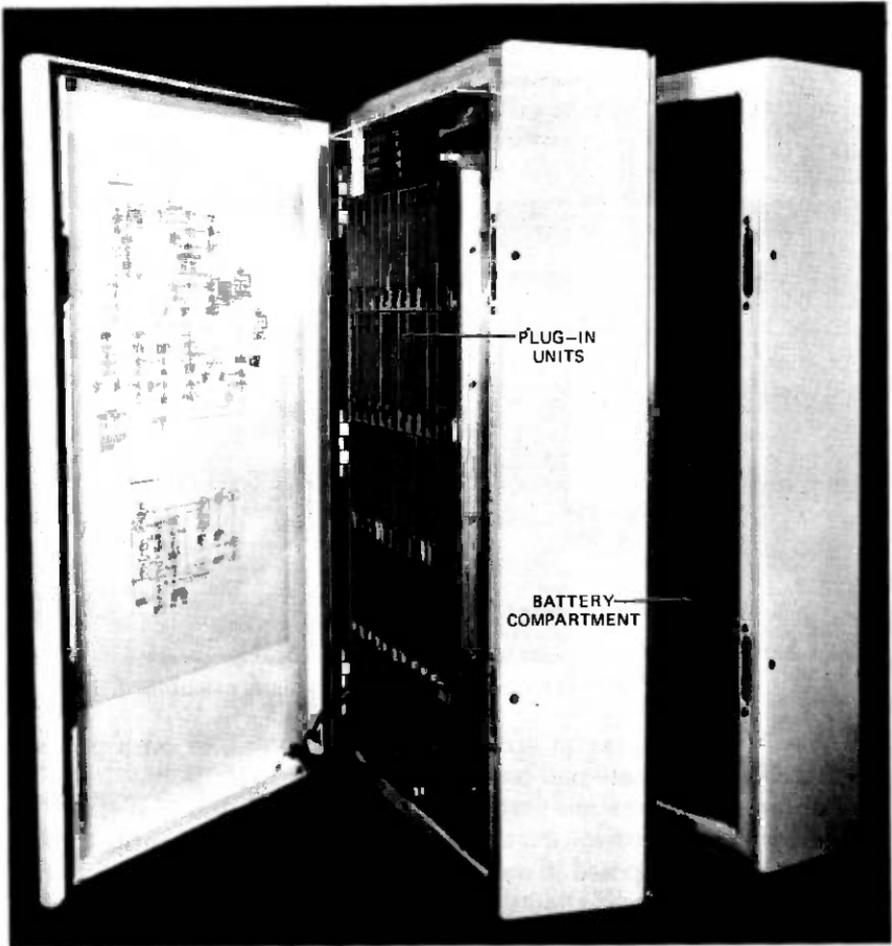


Fig. 13—SLC™-40 RT cabinet, both sections open.

less conventional equipment approach, or a cabinet may be used for outdoor mounting.

The RT cabinet system as shown in Fig. 12 is a rather large (4 ft. high, 16 ft³ volume) unit. It operates from commercial ac power, has basic power conditioning and ringing generator capabilities, and contains a battery for reserve operation against the loss of commercial power. Finally, the RT performs the operations required of a digital channel bank.

The cabinet itself is a formed aluminum structure with hinging in two places as illustrated in Fig. 13. The front door allows access to the front of the bank to permit maintenance of the plug-in units. Further, the entire cabinet is hinged at about its midpoint to give access to both the rear wiring of the channel bank, and also the back cabinet section which houses the reserve batteries. The cabinet is capable of being mounted

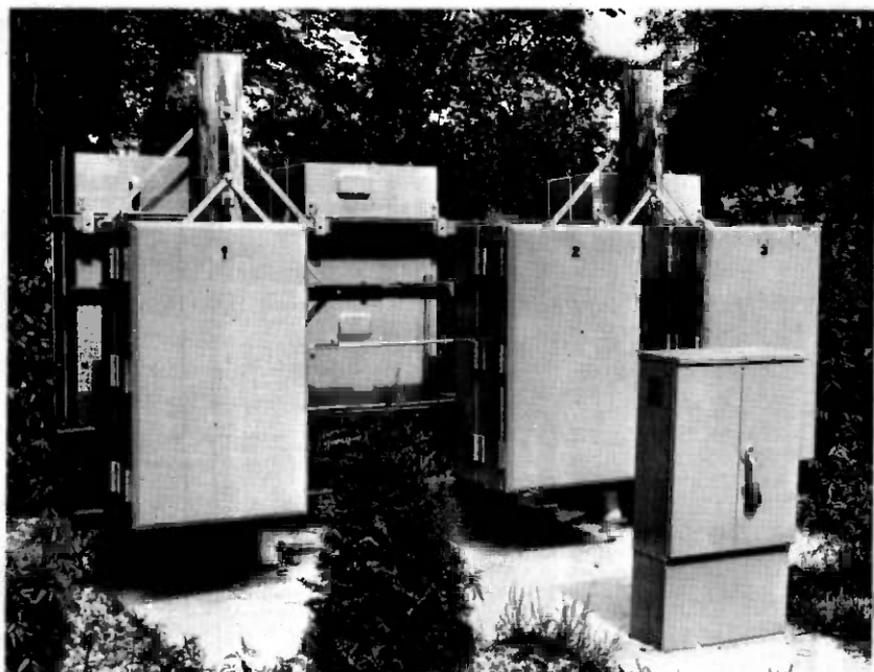


Fig. 14—*SLC*[™]-40 RT cabinet, multiple cross-arm mounting.

on a pole, on a crossarm structure between poles, or on a pedestal mounted to a concrete pad (see Figs. 12 and 14).

For large systems such as this one, it has been found that it is often more economic to provide environmental protection to the entire RT with one structure, as opposed to protecting smaller subsections of the system. For this reason, the *SLC*-40 cabinet was designed to provide such protection. The cabinet has completely gasketed doors, provisions for diverting water which might penetrate a damaged gasket away from the electronics, as well as drains and vents to prevent humidity extremes inside the cabinet. Walls are capable of withstanding 22 caliber rifle fire. Also, the *SLC*-40 RT dissipates approximately 100 watts inside the cabinet which serves to markedly decrease local relative humidity conditions, and hence reduce corrosion potential.

The same internal heating which helps prevent corrosion, however, can be troublesome in very hot sections of the United States. To further compound the thermal situation, solar heating must be allowed for. For the *SLC*-40 RT, a white exterior finish was used on the cabinet to minimize the amount of solar heating. White paint has the fortunate combination of properties which give high thermal reflectance for solar radiation, yet high thermal emissivity for lower-temperature long-wave radiation.

The design of the RT channel bank is very similar to the COT. In fact, many of the common control plug-in units are used in both ends of the

system. A fifth shelf is used with the RT cabinet to mount the required ringing generator and rectifier.

The nickel-cadmium batteries used to provide reserve for the system require a number of design provisions. A heating system is required for the batteries to prevent extreme loss of capacity at cold temperatures. Small amounts of hydrogen are given off during normal float charging conditions, and this must be vented to the outside of the RT cabinet. The batteries must be accessible for periodic watering and cleaning, and electrolyte spills from leaking cells should be isolated from circuit ground to prevent excessive heating from fault currents.

The batteries are mounted within a separate enclosure in the rear of the RT cabinet as shown in Fig. 15. The enclosure is fabricated from steel, since this material is not attacked by the potassium hydroxide battery electrolyte. A silicone rubber resistance wire heater is bonded to the exterior of the door of the battery compartment. The battery temperatures are controlled by bi-metal switch type thermostats which are also mounted outside the compartment, but are closely coupled to the battery case temperature via a relatively low thermal impedance path offered by the conductance of the supporting shelf.

The battery compartment door is gasketed to separate hydrogen from the RT electronics. The battery compartment is separately vented to the

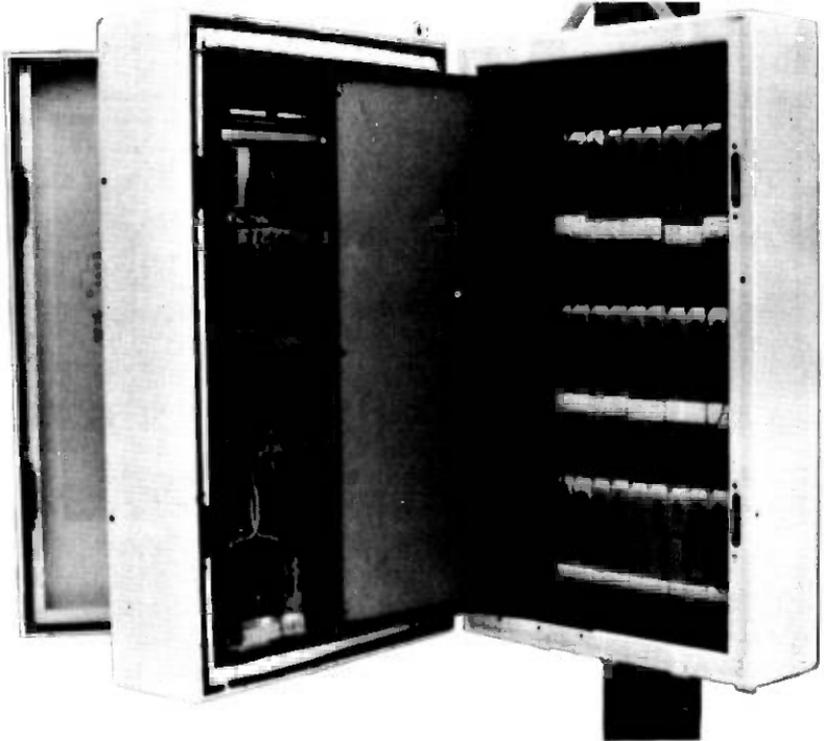


Fig. 15—*SLC™*-40 RT cabinet, battery compartment door open.

outside of the RT cabinet by two ports. The size and vertical separation of the ports are designed to allow buoyancy forces from density differences to induce sufficient flow to insure a nonexplosive state inside the compartment. Normal wind-induced siphoning further aids the dissipation of the hydrogen-rich air.

The battery compartment is electrically insulated from ground, and electrolyte flow paths are provided to conduct leaking electrolyte out of the RT cabinet, thus preventing fault current problems.

As mentioned earlier, a frame version of the RT is also available for inside mounting in customer premises equipment rooms or in small huts.

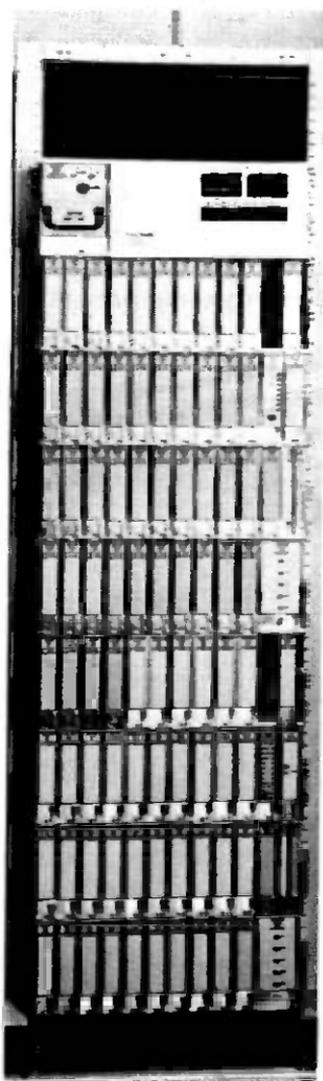


Fig. 16—SLC™.40 frame remote terminal.

As can be seen in Fig. 16, two 40 channel banks are mounted on a 7-foot frame with a common fuse/filter/ringing generator panel. The design format is identical to the cabinet-mounted RT bank. A separate source of dc power must be provided for the frame RT.

3.2.3 Digital line

A simplified digital line troubleshooting routine has been evolved for the *SLC-40* system to ease maintenance for those areas unfamiliar with T1 fault location techniques. The simplified routine requires sequential visits to a number of repeater sites to change out and test the repeaters. Conventional repeaters and cases can be used with this routine, but a quick disconnect scheme was deemed more desirable. For this reason, and to enable a more economic installation of only a few repeaters at a site, a self-protected weather resistant repeater design was developed. This design, shown in Fig. 17, consists of a standard T1 repeater housed in a sealed box with gas tube protectors, and a quick disconnect type waterproof aircraft connector. As shown in Figs. 18 and 19, these repeaters are housed in small aerial or pedestal closures which give the appearance of conventional telephone plant apparatus, and allow easy access to the repeaters mounted inside.

3.3 Line concentrator system

This section deals with the Loop Switching System (LSS) which is also the subject of another article.⁴ Again, the major physical entities consist of terminals for the central office and remote ends of the system.

3.3.1 Central office terminal

The LSS COT, illustrated in Fig. 20, is composed of three shelf assembly designs and a number of plug-in circuit packs. The shelf and plug-in unit format is taken from an earlier system (the Subscriber Loop Multiplexer system) which was compatible with the physical requirements for LSS. Thus, initial manufacturing capital outlay is minimized. Again as in the case for the *SLC-40* COT, only shelf assemblies are offered to minimize the number of orderable units for stocking simplicity and to minimize the ordering interval.

The primary common control assembly is a combination of a shelf for plug-ins and a power-fuse-alarm-microprocessor panel. The two entities are combined into a single mechanical unit to simplify the interconnecting wiring required. Connectorized cables are used to interconnect this assembly to the other LSS shelf assemblies. Due to the variety of gauges and destinations, more conventional wire-wrap and screw-down terminations are used for the wires that connect to the central office power and alarm circuits.

A second common control shelf is required to house the power supply

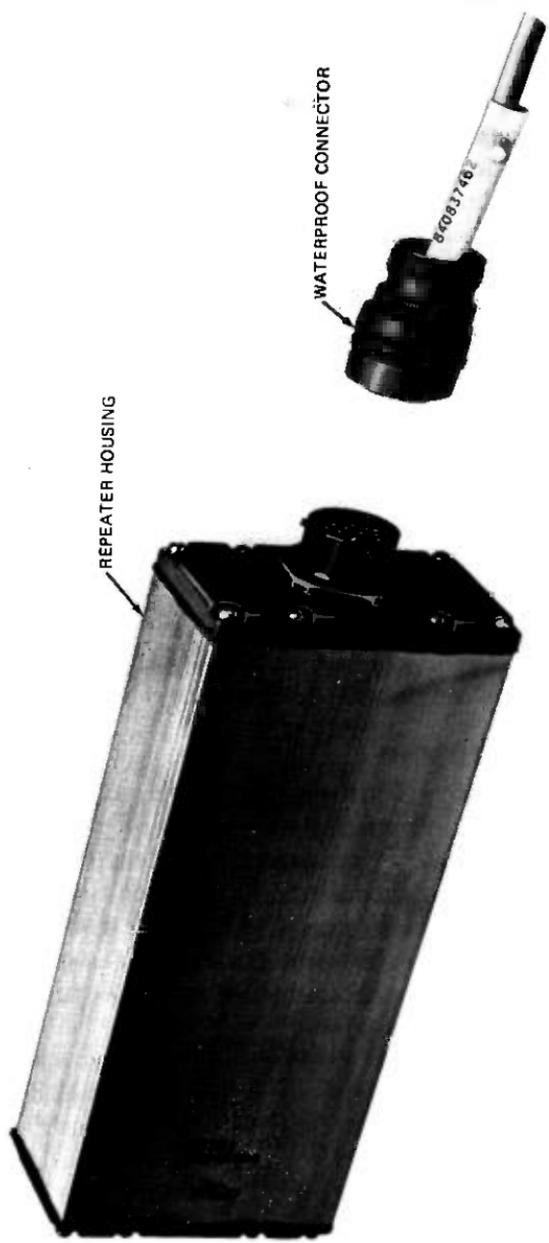


Fig. 17—SLC™-40 quick-disconnect repeater.

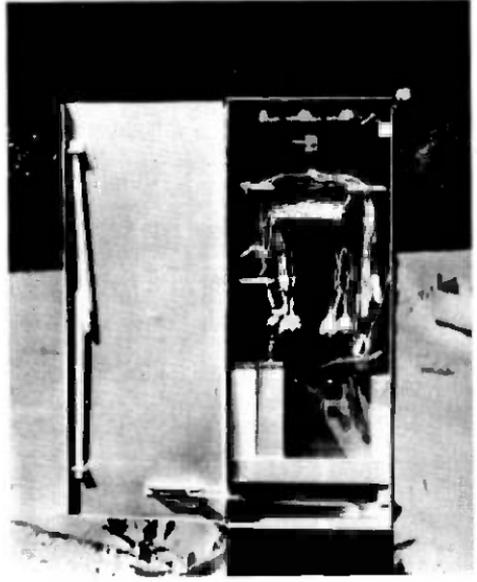
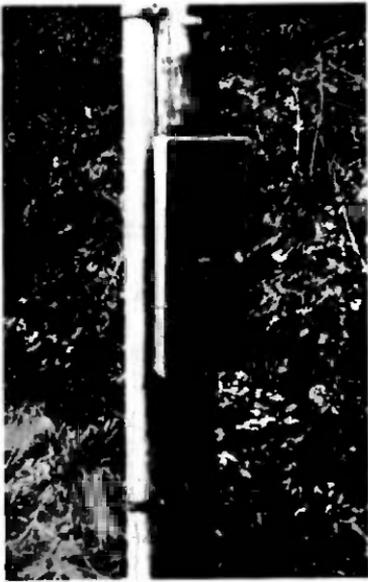


Fig. 18—*SLC*[™]-40 pole-mounted repeater enclosure.



Fig. 19—*SLC*[™]-40 pedestal-mounted repeater enclosure.

and modem for a second remote terminal. This assembly connects to the system via connectorized cables to the primary common control, and is only provided when a double RT operating mode is used.

A pin-and-socket-type connector arrangement is used for the circuit packs on LSS. This is the 963-type connector mentioned earlier in the *SLC*-8 description. The socket part of the connector, shown in Fig. 21, is mounted on the circuit pack and is heat-staked and mass-soldered to the printed wiring board. Multiple numbers of 20 contact connectors are used on each unit depending on the number of connections required.

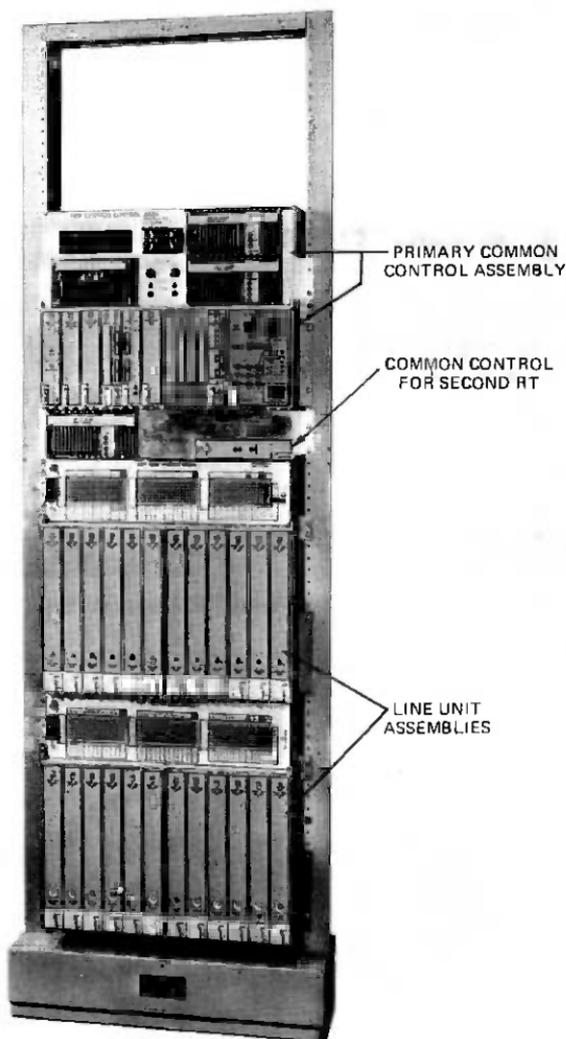


Fig. 20—LSS central office terminal.

The pin half of the connector, Fig. 22, is essentially a 0.025-inch-square post which is force fitted into a plated-through hole in a printed wiring board backplane. One end of the pin plugs into the connector socket, while the other end can be used as a wire-wrap terminal. Each group of 20 pins is provided with a plastic guide block to both physically protect the pins from damage and to provide alignment guidance between the pins and the socket connectors during plug-in engagement. Sequential engagement of circuit ground before any other connections is accomplished by making these pins longer. This prevents voltage breakdown of ungrounded devices.

Printed paths are used to provide much of the interconnecting

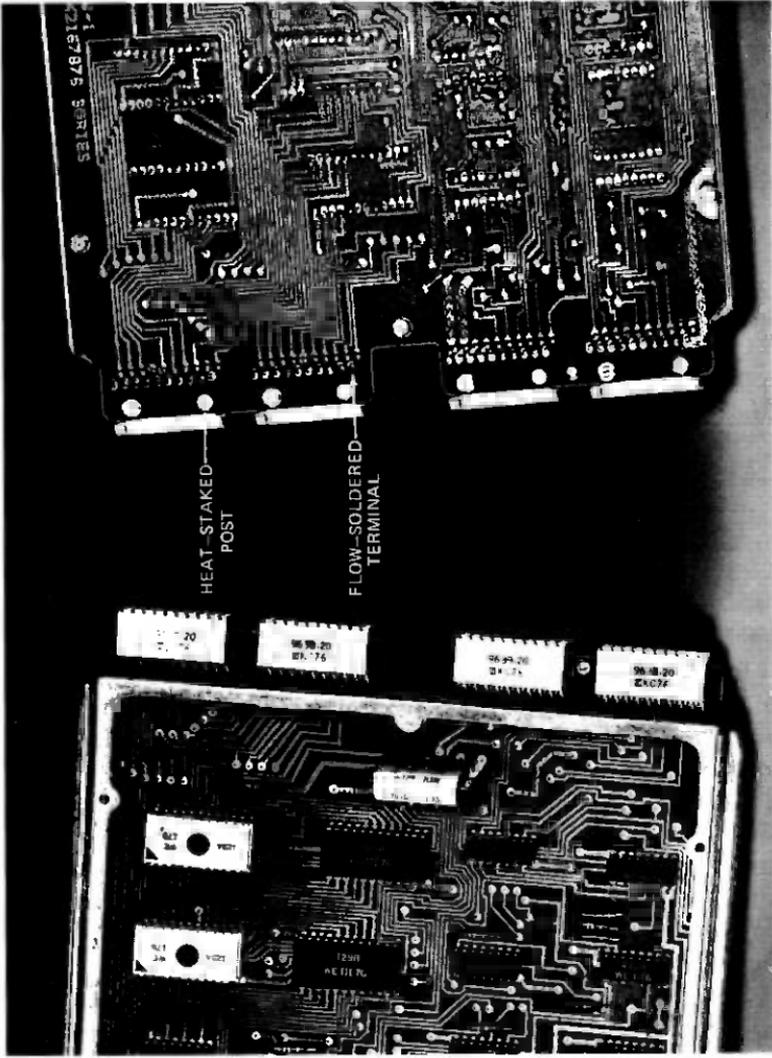


Fig. 21—LSS circuit pack connector.

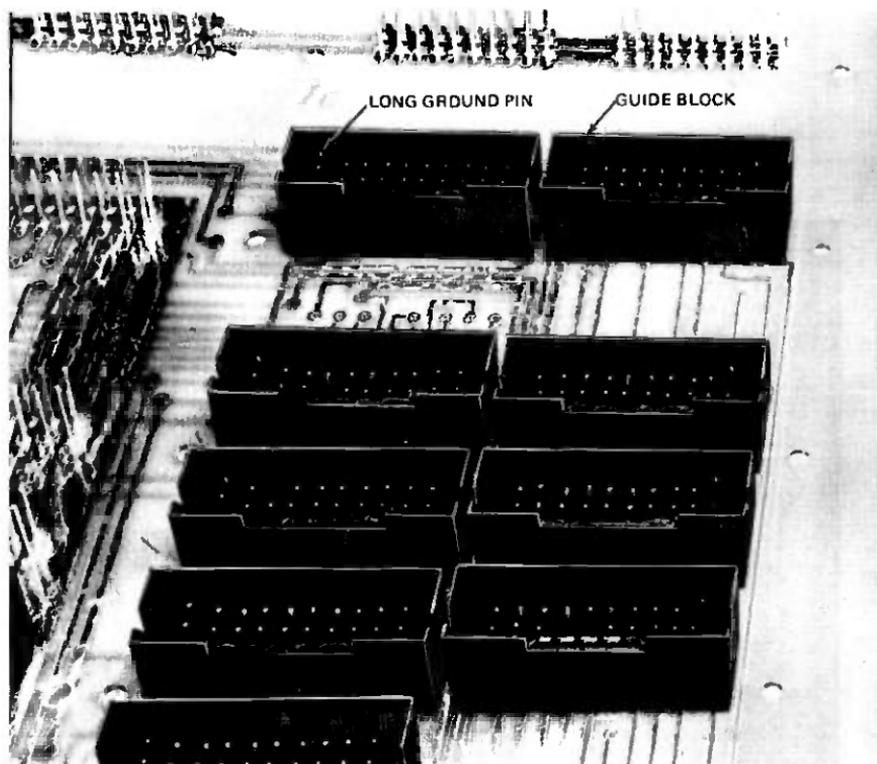


Fig. 22—LSS backplane connector pins.

intrashelf wiring required on the various shelves. Specifically, this accounts for more than 30 percent of all the factory-applied wiring for the system. All of the backplane printed boards are limited to only two wiring levels to avoid the expense of more costly multilayered boards.

The third shelf assembly design for the COT is the line unit assembly which mounts twelve identical line unit circuit packs. These circuit packs contain the relays that provide the switching network for a 96-line, 32-trunk system. These rather large, 14-inch-high, plug-in units have the components for eight subscriber lines on a single printed wiring board. The use of a single large board for eight lines instead of two or more smaller boards results in a substantial decrease in the number of connector contacts and interconnecting wires required for the switching network.

Testing access is provided to all line, trunk, and data link pairs through a compact field of terminals. These may be connected to a larger set of test terminals, as shown in Fig. 23, through a probe cord which is part of the assembly. External connections of lines, trunks and other control leads are also made from this assembly. These connections are all connectorized to allow ease of installation and replacement, and to simplify

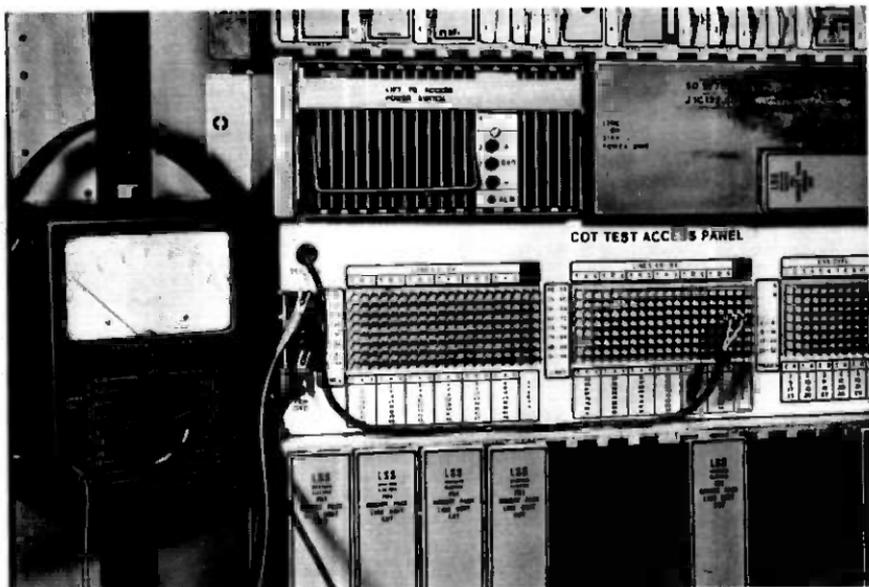


Fig. 23—LSS test access panel.

manufacturing testing. The connectors used mate with standard key telephone type connectorized cables to further simplify installation.

3.3.2 Remote terminal

The LSS remote terminal electronics can be mounted in a cabinet (Fig. 24) or on a conventional equipment frame for inside applications. In either case, the same two basic shelf assembly designs are required.

The RT common control assembly and the RT line unit assembly both use hardware and wiring formats which are identical to their COT counterparts. As in the COT, one common control assembly is used with either one or two line unit assemblies for a 96- or 192-line system, respectively. These assemblies by themselves constitute a functional, self-contained system.

The cabinet is a derivation of the *SLC-40* system cabinet. The center section and door were retained, and the large rear section is replaced with a rear door. This yields a cabinet of about 10 ft³ volume with a height of 4 feet. Mounting arrangements include both pedestal and crossarm versions.

The protective capabilities of the RT cabinet are identical to those discussed for the *SLC-40* system with one exception. The differing characteristic is that the LSS dissipates only about 13 watts in the RT cabinet. This means that LSS is not as self-protecting against humidity as is the *SLC-40* system. However, this has been somewhat compensated for by using a green exterior finish on the cabinet as opposed to the white finish used for the *SLC-40* cabinet. The green finish has a higher solar

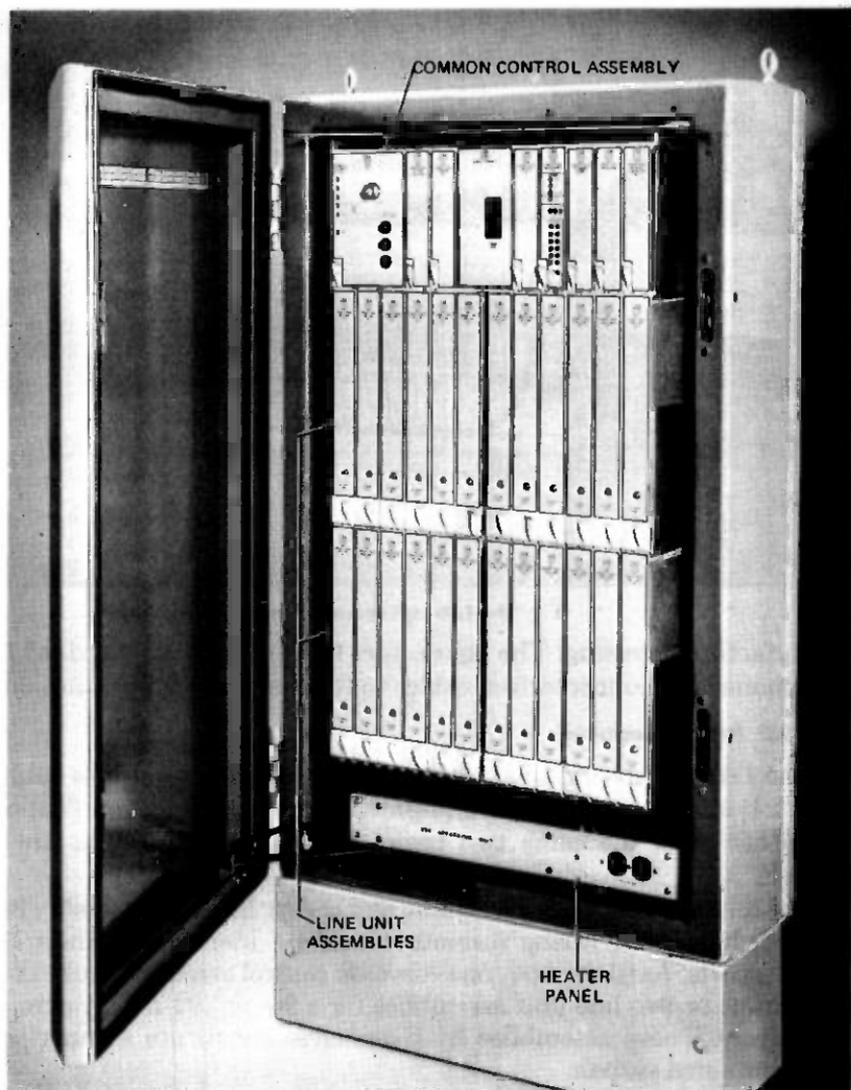


Fig. 24—LSS cabinet remote terminal.

absorptivity than a white finish and results in somewhat higher average internal cabinet temperatures than would be the case for a white cabinet, along with accompanying lower relative humidity values. RT corrosion is only expected to be significant in areas where very high average relative humidities exist with high temperature. Such areas as the Southern United States, particularly in some sections of the Gulf Coast states, may require the use of a cabinet heater panel to provide the same degree of humidity control that exists in other systems.

The heater panel is powered from commercial 120 volt ac power, and as a result requires additional installation cost to provide such service.

Since the maximum persisting dew-point temperature in the continental United States is under 80°F,⁵ the heater panel is designed to provide a temperature rise of 10° to 20°F for all ambient temperatures under about 100°F. The heater shuts off at ambients greater than 100°F to minimize power consumption and to prevent unnecessary heating at high temperatures. The 10° to 20°F temperature rise decreases the internal relative humidity to levels that greatly reduce corrosion rates.

Another influence on the physical design of the LSS has been the experience gained from prior systems that used open-contact switching networks in outdoor cabinets. Visions of dust, corrosion products, and freezing condensate causing blocked contact conditions have occasioned some rather strong reactions against any similar switching network. The switching relays in LSS have been protected against and extensively tested⁶ for such problems. It has been found that a substantial degree of protection is provided by enclosing the relays in a closely fitting, but not sealed, enclosure.

The line unit circuit pack design, Fig. 25, provides a protective relay

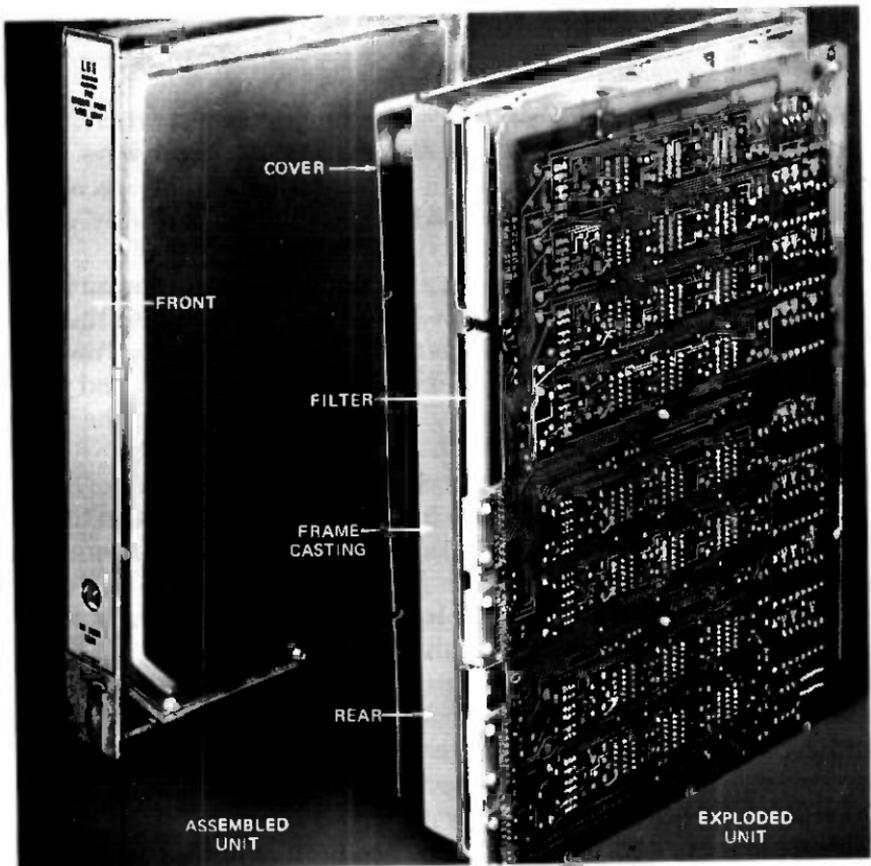


Fig. 25—LSS line unit circuit pack.

enclosure with a minimum of mechanical parts and accompanying cost. One side of the enclosure is formed by the printed wiring board that mounts the relays. Mass soldering of the components to the board results in the closing of virtually all through holes. An aluminum frame casting, to which the printed board is mounted, forms the next four sides. The rear surface has pockets to allow filter material to be held in compression between the frame and the board surface. The filter allows a vented enclosure in deference to outgassing from the relay coils, and at the same time excludes the larger sized dust particles and debris that have the potential to cause contact blockages. The sixth side of the enclosure is provided by a formed aluminum cover which is attached to the board frame.

The line unit circuit packs used in the COT are identical to the RT units except that the dust filters are omitted. The cover is retained for the COT units to provide an additional measure of mechanical handling protection.

3.4 Voice frequency electronics

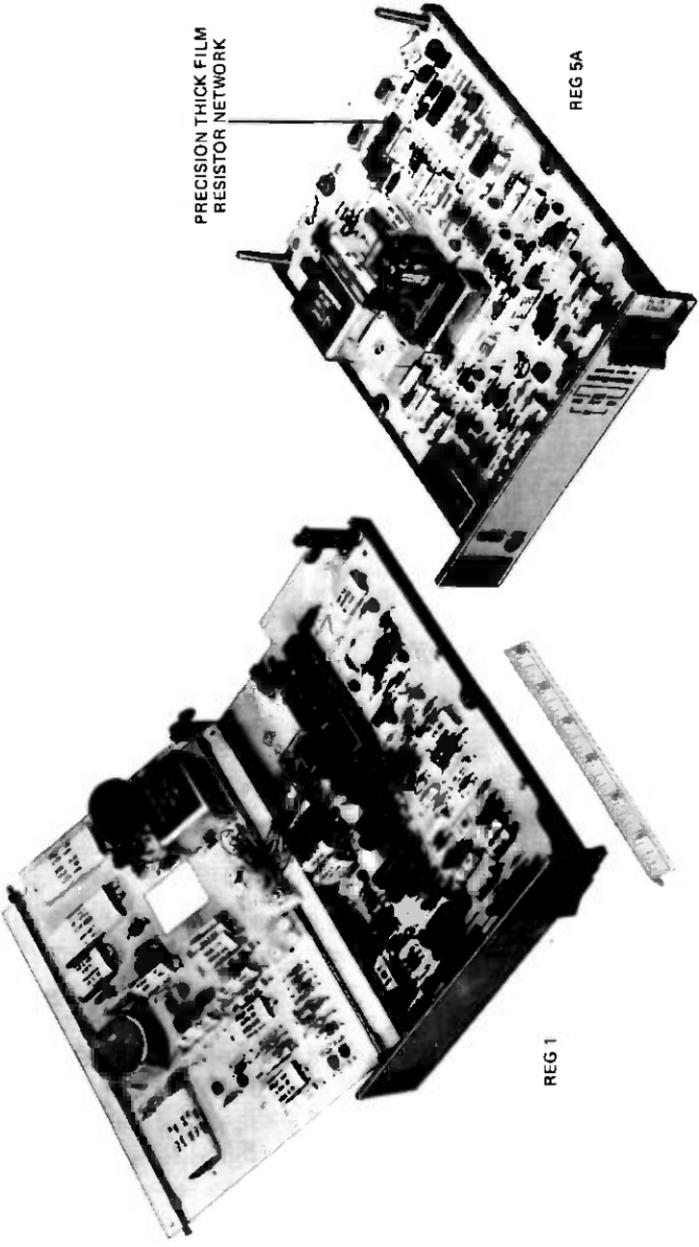
3.4.1 Range Extension with Gain (REG)

Range Extension with Gain is described in a previous article.⁷ The REG system differs from other loop electronic products in two ways. First, it consists of equipment mounted in the central office only; second, it has an extremely high annual production volume, currently about 250,000 circuit packs/year.

A REG system consists of a circuit pack that mounts in a transmission equipment bay. A REG is a per/line device and is wired into the loop between the switching equipment and the main distributing frame. It is powered by a circuit pack power supply mounted in the bay and wired to a bay-mounted fuse, alarm, and filter panel powered from the -48V central office battery.

REG has been available since 1972. Since that time, a number of circuit improvements and cost reductions have been made. Each REG has remained interchangeable with earlier models. Today the 5A REG is a single-sided printed wire board, contrasting with the first model, which required two printed wire boards. It was originally designed in the F-Signaling system format to take advantage of commonality in manufacturing facilities. Figure 26 illustrates the old and the new REGs. Circuit pack size is 8" high by 10" deep. To maintain compatibility with earlier REGs, the standard aluminum die-cast card frame is still used. Except for a precision thick-film resistor network, all electrical components are the traditional discrete variety.

Traditionally, REG bay equipment has been manufactured in factory-wired arrangements, although separate shelves are also available.



NEW

OLD

Fig. 26—REG circuit packs.



Fig. 27—REG bays, South Knoxville, Tennessee, central office.

These include 11'-6" and 9' bulb angle bays and 7' and 11'-6" unequal flange bays. A typical bay arrangement is illustrated in Fig. 27. An 11'-6" bay serves up to 165 REGs. Interconnection of the circuit pack to the CO cabling is through a 908A gold finger connector mounted on the shelf. Intershelf and intrashelf connections are hard-wired using the wire-wrap terminals on the rear of the Western Electric 908A connector.

Through mid-1977 the standard one-piece die-cast F-Signaling shelf was used. With the introduction of the new F-Signaling steel split shelf (see Section 3.1 on *SLC-1*), the REG bay has been reconfigured into a modular format. This approach has reduced Western Electric manufacturing costs and added flexibility in telephone company ordering options. The approach permits shelves of equipment to be incrementally ordered and plugged together in the field.

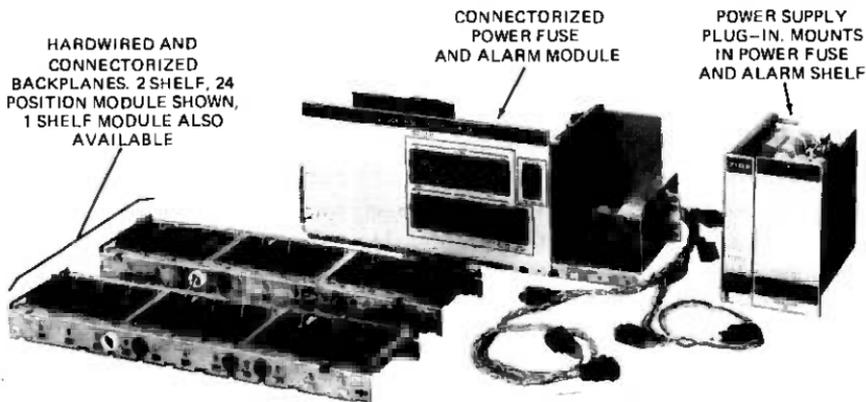


Fig. 28—Modular REG bay components.

The REG bay has now been structured into three modular units. One unit combines the 71A power supply, filter circuit and fuse and alarm panel into a standard F-Signaling shelf mounting. A connectorized cable interconnects it to each shelf of REG circuit packs. The other two units are hard-wired, connectorized backplanes that also mate with standard steel shelves. The modular building blocks are illustrated in Fig. 28.

The modular packages can be combined into any REG bay offering. Typically, a telephone company can select a vacant bay area in a central office and equip it with a multiplicity of steel shelves. Backplane units and power supply/fuse and alarm panel can be ordered as required, mounted on the shelf or bay and plugged together.

The modular format provides Western Electric with the capability to substantially reduce floor space allocated for manufacture and storage of bays. More automation can be introduced at a bench handling smaller units.

3.4.2 Ringer Isolator and small closure design

The other segment of the voice frequency product line is the ringer isolator. A previous article⁷ describes the need and usage of this device. The current Western Electric product is the 28A ringer isolator. The unit is a solid state circuit encapsulated in a small weatherproof housing which mounts next to the subscriber protector block.

The design objectives for the 28A were to produce a device with improved electrical performance compared to its predecessor, the 11A. That was accomplished with a reduction of size and cost by a factor of two.

Since the unit required weatherproofing to permit mounting on the outside as well as the inside of the subscriber premises, and because of its low price, it was decided to encapsulate it.

The 28A circuit comprises 25 components mounted on a 1.5 × 3" single-sided, epoxy glass, printed wire board. Design guidelines for the

printed wire board manufacture by machine insertion were carefully observed and worked to the limits. A polyurethane encapsulation compound, blended from a polycin and vorite mixture, is used for encapsulation. This is the same compound used for the SLC-1 1119A isolation filter. A plastic case, similar to the one used on the 1119A, was designed for the 28A. The 28A and the 1119A are compared in Fig. 29. In production, both units are injection molded from the same tool.

The ABS/PVC blended thermoplastic, used on the 28A and SLC-1 outdoor closure, was evaluated for high-temperature performance in Yuma, Arizona. Tests demonstrated the superior performance of the charcoal grey ABS/PVC blend over a 100 percent PVC thermoplastic, which deformed under continuous high solar exposure. Materials evaluation also indicated excellent adhesion of the polyurethane encapsulant to the ABS/PVC thermoplastic.

The 28A is terminated at the protector block with a standard, two-pair D station wire cable pigtail. This use of D station wire presents a familiar interface to the telephone company installer. A pigtail design was selected during field trial to avoid the use of terminals. The terminals would have required an additional craft installation operation. Their presence also creates a potential corrosion site. A snap-on cover carried over from the 1119A filter design provides convenient storage for any excess pigtail.

The pigtail arrangement could not be used on the 1119A filter, however, because of the wide variation in mounting location on a telephone pole or a subscriber's house. In addition, the filter requires termination

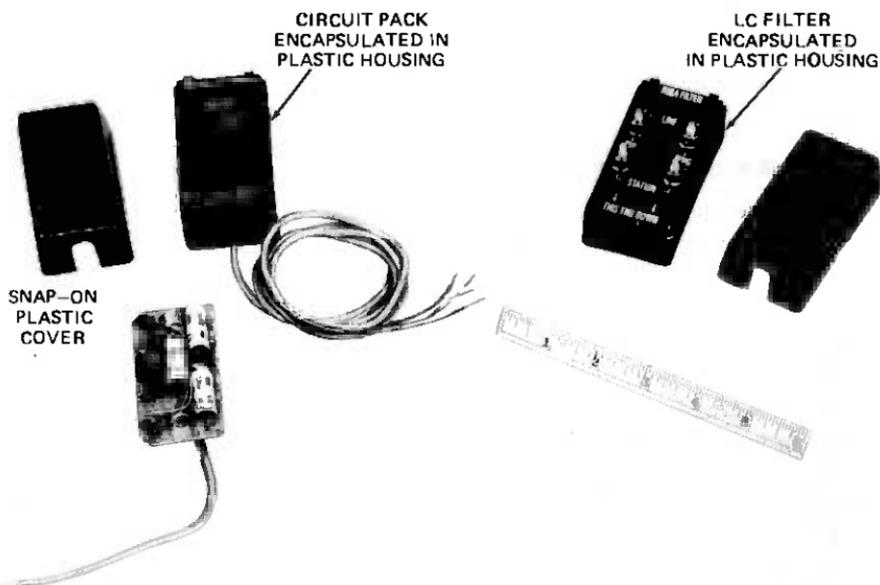


Fig. 29—Encapsulated electronic packages.

to drop wire (20 gauge copper coated steel) or station wire (22-24 gauge copper).

IV. ENVIRONMENTAL TESTING AND EVALUATION

The introduction of electronics in the outside plant requires both short-term and long-term product testing. Previous sections in this article have mentioned short-term testing programs conducted during product development that evaluated new materials and electronic component applications.

Detailed analysis and experimentation have also been done in the following areas:

- (i) Solar absorptance of paints and thermal analyses of large electronic cabinets.
- (ii) Modeling and experimental testing in real life environments of the temperature rise in small closures and outside plant pedestals and cabinets.
- (iii) Moisture and humidity testing in outside plant electronic structures.
- (iv) Effects of pollutants, dust and aerosols on wire spring relay components.

In addition to these short-term tests, long-term remote testing sites have been established in Thibodaux, Louisiana and Lake Quinault, Washington. Lake Quinault, at the edge of the Washington rain forest, provides a constantly high humidity (80 to 100 percent RH), with temperatures ranging from 30° to 90°F. Thibodaux provides a relative humidity of 75 to 90 percent with an annual temperature range of 25° to 95°F.

These sites have been used since early 1976 to evaluate terminals, printed wire board coatings, encapsulating compounds, and plastics.

The tests are a valuable aid to the laboratory environmental testing. Real-life effects of insects and direct and airborne debris provide insights into closure design, terminal layout, and printed wire board protection.

The Thibodaux site is shown in Fig. 30. Facilities are available for mounting closures on wall surfaces. Cabling is provided into a test shed for test access and powering. New electronic devices are installed at the sites to provide real time information on field performance.

V. PHYSICAL DESIGN FOR FUTURE LOOP ELECTRONICS

We have described in part some of the highlights and philosophies of loop electronics physical designs. With the advent of microprocessor chips and Large Scale Integration (LSI) technologies, lower cost, miniaturized loop electronic systems will permeate the urban and suburban

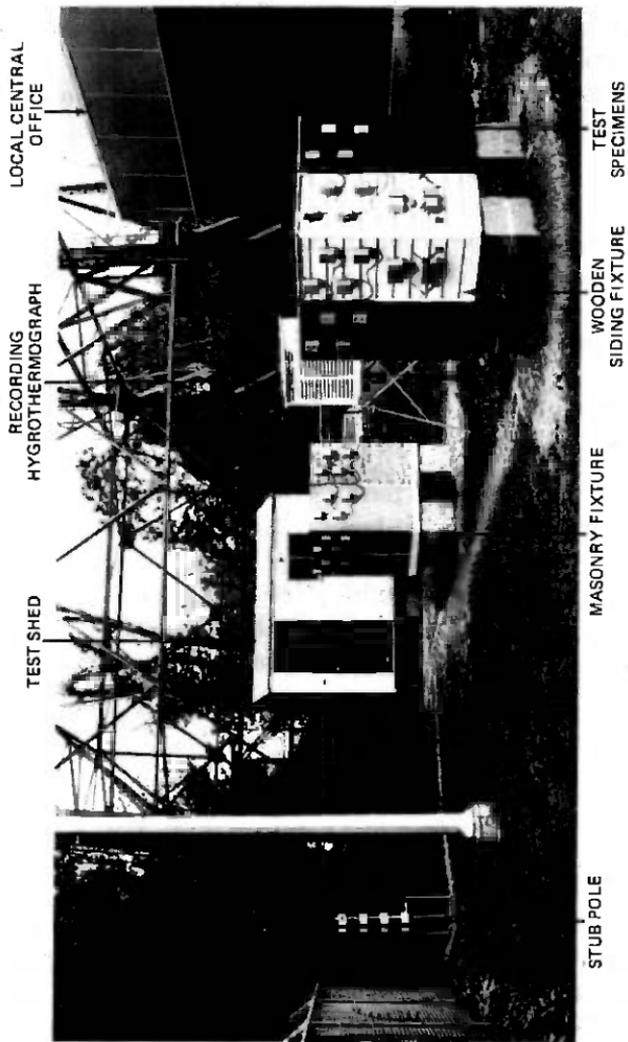


Fig. 30—Remote environmental test site, Thibodaux, Louisiana.

loop plant market. Greater flexibility will be required at remote terminal sites providing for increased suburban growth rates and greater rearrangement and change of customer lines. Furthermore, increased concentration of electronics serving suburban customers will require order of magnitude decreases in equipment size and more aesthetic packaging.

New package designs for the next generation of loop electronics systems are currently under exploratory evaluation.

VI. ACKNOWLEDGMENTS

Physical design and testing of loop electronic equipment and apparatus are the direct result of the combined efforts and talents of the technical staff and technical assistants of the Loop Electronics Facilities Department. We wish to acknowledge and thank the many staff members whose individual ideas, designs, and contributions form the backbone of work summarized in this article.

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