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The Aerovox Research Worker

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Measurement of Inductance

By the Engineering Department, Aerovox Corporation

THE measurement of inductance can be accomplished in a manner that is similar to the measurement of capacitance, that is, by the impedance method or the comparison method. The impedance method involves the use of an ammeter. voltmeter and, preferably, a wattmeter. The comparison method requires some form of standard inductance or capacitance and a bridge circuit. The measurement of inductance by the impedance method will be discussed first.

Inductance is that property of a circuit which restricts the flow of an alternating current and prevents the change of current flow. Inductance also alters the time phase relation of the current and voltage in a circuit. An ideal inductance will have the voltage drop across it 90° ahead of the current flowing through it. Any resistance or losses in the inductance show up as a phase angle of less than 90°. As in a condenser, all losses are lumped and considered as equivalent series resistance. This means that any inductance can be considered as an ideal inductance having no resistance and a series resistance. The diagrams of Figure 2 give the circuit and vector relations of an inductance.

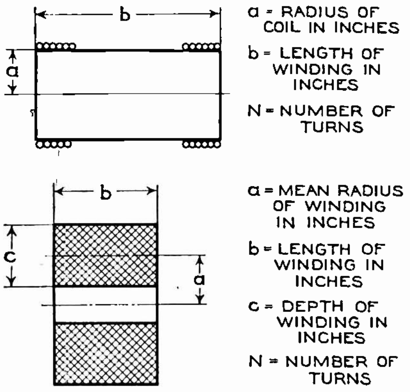

Figure 1

INDUCTANCE FORMULAE

The inductance of a coil may be calculated to any degree of exactness desired, provided the proper formula is used. The Bureau of Standards bulletin No. 169 is a compilation of a large number of these formulae with examples worked out to indicate degree of accuracy and exactness. Some of the simpler formulae will be given here.

The simplest of the more exact formulae for single layer solenoids is Nagoaka's:

$$L = 0.02596 \frac{KD^2N^2}{l} \text{ MICROHENRIES}$$

The factor K is an end correction factor and is 1 for a coil whose length l is infinite compared to its diameter D. N is the number of turns of the coil. The values of K are given in the following table as a function of the ratio D/l. All dimensions are in inches.

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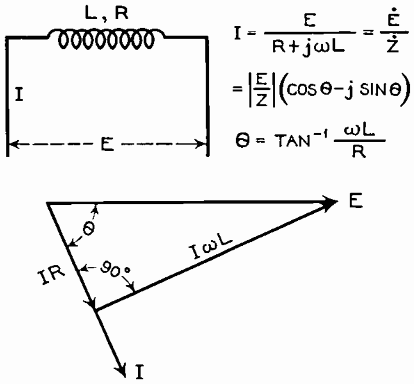
D/l	K	D/l	K
0.00	1	0.90	0.711
0.05	0.979	1.0	0.688
0.10	0.959	1.1	0.667
0.15	0.939	1.2	0.648
0.20	0.920	1.4	0.611
0.25	0.902	1.6	0.580
0.30	0.884	2.0	0.526
0.35	0.867	2.5	0.472
0.40	0.850	3.0	0.429
0.45	0.834	3.5	0.394
0.50	0.818	4.0	0.365
0.55	0.803	4.5	0.341
0.60	0.789	5.0	0.320
0.65	0.775	6.0	0.285
0.70	0.761	7.0	0.258
0.75	0.748	8.0	0.237
0.80	0.735	9.0	0.219
0.85	0.723	10.0	0.203

A group of emperical formulae developed by H. A. Wheeler is of great help as a relatively simple and fairly accurate method of calculating the inductance of coils.



$$I = \frac{E}{R+j\omega L} = \frac{\dot{E}}{\dot{Z}}$$

$$= \left|\frac{E}{Z}\right|(\cos\theta - j\sin\theta)$$

$$\theta = \tan^{-1}\frac{\omega L}{R}$$

Figure 2

For single layer solenoids whose length is greater than 0.8 its radius is:

$$L = \frac{a^2N^2}{9a+10b} \text{ MICROHENRIES}$$

a = RADIUS OF COIL

with an accuracy of 1% as checked by Nagoaka's formula. Multi-layer coils can be calculated by the formula:

$$L = \frac{0.8\,a^2N^2}{6a+9b+10c} \text{ MICROHENRIES.}$$

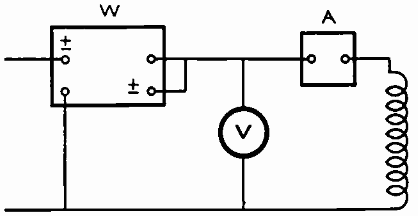
a = MEAN RADIUS OF COIL; b = LENGTH OF COIL; c = DEPTH OF WINDING IN INCHES.

The diagrams of Figure 1 give the values of a, b, c.

Iron core coils can not be calculated directly as the inductance of such a choke depends on the magnitude of the applied voltage and the current that flows in the winding. A magnetization curve of the core iron is required for the computations which are of the cut and try type.

AIR CORE COILS

The inductance of any coil can be found by measuring the current that flows through the coil when an alternating voltage is impressed across the coil. The ratio of the voltage to the current is the impedance of the coil:



$$R = \frac{P}{I^2} \text{ OHMS} \qquad Z = \frac{E}{I} \text{ OHMS}$$

$$X = \sqrt{Z^2 - R^2} \text{ OHMS}$$

$$L = \frac{X}{2\pi f} \text{ HENRIES OR .00265X HENRIES AT 60 CPS}$$

Figure 3

$$Z = \frac{E}{I} \text{ OHMS}$$

If the losses in the coil are small or if the resistance is negligible, the reactance X is equal to the impedance Z and

$$L = \frac{X}{2\pi f} \text{ HENRIES}$$

If a more accurate determination is required, the d.c. resistance can be measured with an ohmmeter or a Wheatstone bridge.

$$L = \frac{\sqrt{\left(\frac{E}{I}\right)^2 - R^2}}{2\pi f} \text{ HENRIES}$$

The same results can be obtained from the readings of an ammeter, wattmeter and voltmeter connected as shown in Figure 3. The equivalent series resistance of the coil is

$$R = \frac{P}{I^2} \text{ OHMS}$$

The impedance, reactance and inductance can be found from the equations given above. The three voltmeter method of measuring can be used to determine the constants of a coil. The coil to be measured is connected as in Figure 4. The readings of current, voltage of the coil and of a known resistor in series with the coil are taken. From these readings the equivalent series resistance and the inductance are found as follows:

$$Z_C = \frac{E_C}{I} \text{ OHMS} \qquad L = \frac{Z_C \sin\theta}{2\pi f} \text{ HENRIES}$$

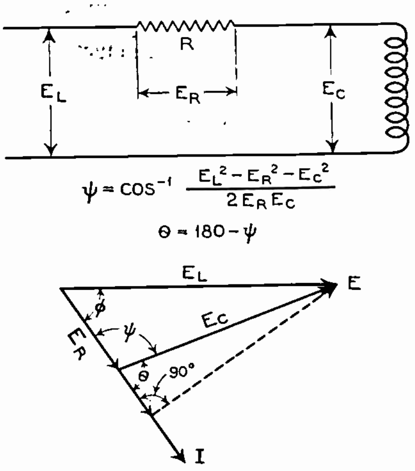
$$Z_L = \frac{E_L}{I} \text{ OHMS} \qquad R_C = Z_C \cos\theta \text{ OHMS}$$

$$\psi = \cos^{-1}\frac{Z_L^2 - Z_C^2 - R^2}{2Z_CR} \qquad I = \frac{E_R}{R} \text{ AMPERES}$$

$$\theta = 180 - 4$$

IRON CORE COILS

The measurement of iron core coils requires that the magnitudes of the currents and voltages be taken into account. The inductance of an iron core coil is not constant, but depends on the current flowing through it.



$$\psi = \cos^{-1}\frac{E_L^2 - E_R^2 - E_C^2}{2E_RE_C}$$

$$\theta = 180 - \psi$$

Figure 4

Choke coils that carry a uni-directional current should be measured under operating conditions. The circuits used are exactly the same as the circuits given for air core coils, except for a provision for a polarizing current. Figure 5 shows a circuit that can be used for choke coils which require a polarizing current. The condenser is put in as a blocking condenser to keep the d.c. from flowing through the line, and the choke in series with the battery is required to prevent the coil to be measured from being short-circuited by the battery.

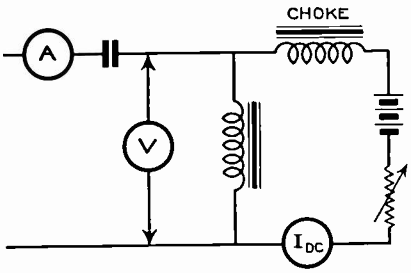


Figure 5

The inductance of such an iron core coil as in Figure 5 may vary over a 4 to 1 range as the polarizing current is varied from zero to its maximum operating value. Obviously, when the rating of any choke coil is given, the d.c. polarizing value must be stated.

RADIO FREQUENCY COILS

By radio frequency coils is meant coils of such size and construction that the resonant frequency or natural period of the coils is in the radio frequency part of the spectrum. This will include coils having inductances of 100 millihenries to coils having an inductance of 1 microhenry or less.

Coils of 1 millihenry or higher can be measured at power frequencies using the ammeter-voltmeter method if a reliable low-range low-current voltmeter is available. A vacuum-tube voltmeter is an ideal instrument for this purpose. Coils with an inductance of less than 1 millihenry can best be measured at radio frequencies, using the methods outlined and the circuit of Figure 4. The meters should be accurate at the frequencies used. Thermo-couple and vacuum-tube voltmeters are the most desirable for this purpose. Extreme care must be taken to prevent stray coupling from the oscillator to the coil being measured.

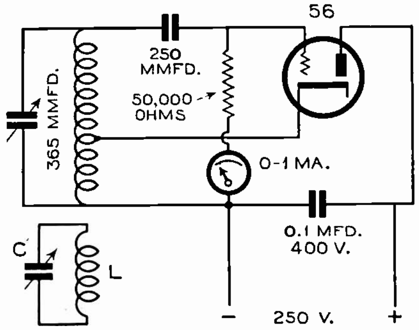


Figure 6

Another method of finding the inductance of a radio frequency coil is to resonate the coil with a calibrated condenser. An oscillator or a signal generator is used as a source of voltage of known frequency. The coil to be measured is loosely coupled to the oscillator and the circuit is tuned to resonance by a variable condenser connected across the coil. Resonance is best indicated by the dip of grid current of the oscillator. In Figure 6 the coil L can be coupled closely for the preliminary adjustment, but as resonance is reached the coupling is decreased until the dip of the grid meter is barely perceptible. The inductance of the coil is given by:

$$L = \frac{1}{4\pi^2 f^2 C} \text{ HENRIES } (C \text{ IN FARADS})$$

or if C is in microfarads L is given in microhenries.

The frequency of the oscillator can be determined very accurately by beating against the carrier of a broadcast station.

This method does not determine the true inductance of the coil, but its apparent inductance at the frequency used. The apparent inductance of the coil is larger than the geometric inductance because of its distributed capacitance. A refinement of the resonance method is to measure the capacity required to resonate the coil at two different frequencies. Thus:

$$L = \frac{1}{4\pi^2 f_1^2 (C_1 + C_0)} \text{ HENRIES}$$

$$L = \frac{1}{4\pi^2 f_2^2 (C_2 + C_0)} \text{ HENRIES}$$

C_0 IS THE DISTRIBUTED CAPACITANCE OF THE COIL AND THE WIRING.

OR, $$L = \frac{1}{4\pi^2 (C_1 - C_2)} \left(\frac{1}{f_1^2} - \frac{1}{f_2^2} \right) \text{ HENRIES}$$

If f_2 is the second harmonic of f_1, the above equation reduces to

$$L = \frac{3}{16\pi^2} \frac{1}{(C_1 - C_2) f} \text{ HENRIES}$$

Care should be taken to measure coils at radio frequencies far below their resonant frequency.

Radio frequency choke coils act similar to electrically long transmission lines. The impedance of such a choke of single winding is given by:

$$Z = Z_0 \tanh \gamma \text{ OHMS}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

γ = ANGLE OF THE CHOKE WHICH IS A FUNCTION OF FREQUENCY

Thus as the frequency is varied the impedance is highly inductive at low frequencies increasing to a very high value just before the natural period of the coil is reached and then becoming capacitive with a decreasing impedance as the frequency is increased. This cycle is repeated at each harmonic of the natural frequency. (See Figure 7). The efficiency of radio frequency choke coils can be improved by connecting a number of small coils in series. Therefore the measurement of this type of coil is a function of the frequency and unless the coil is measured at the exact frequency at which it is used, the inductance determined has but little meaning.

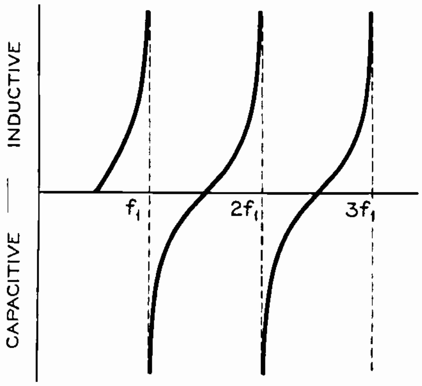


Figure 7

A coil wound with #18 enamelled wire in several layers had an inductance of 102.3 microhenries when measured at 60 cycles by the impedance method, an inductance of 102.6 microhenries at 1000 cycles; but at 500 KC the measured inductances, using the resonance method with a calibrated condenser, was 623 microhenries.

ERRATUM NOTICE

Attention has been called to an error in the April issue of the Aerovox Research Worker.

The vector diagram of Figure 3 is not the correct diagram for the circuit shown in Figure 2. The correct diagram is given below:

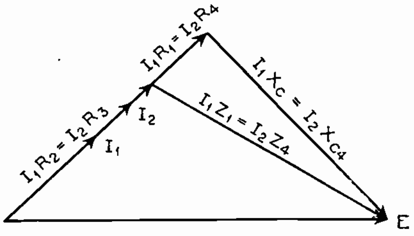


Fig. 3A
Substitute for Figure 3 (April,'38)

Since the voltage across the detector is zero, the voltage drop across R_2 must equal the voltage drop across R_3. This condition can occur only when the currents in the two branches, I_1 and I_2 are in phase. The magnitudes are not necessarily the same. As the currents are in phase and the voltage across the detector is zero, the voltage drops across Z_1 (C_1-R_1) and Z_4 (C_4-R_4) must be equal. The voltage drop across C_1 is equal to the voltage drop across C_4 and the voltage drop across the equivalent series resistance R_1 is equal to the voltage drop across the resistance R_4.

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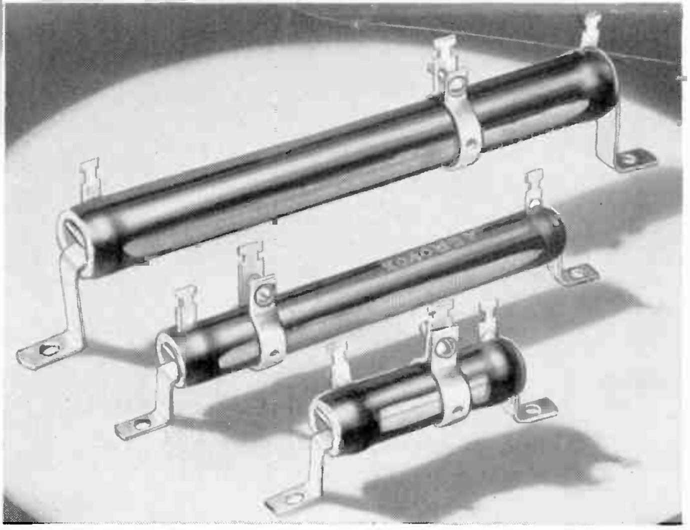

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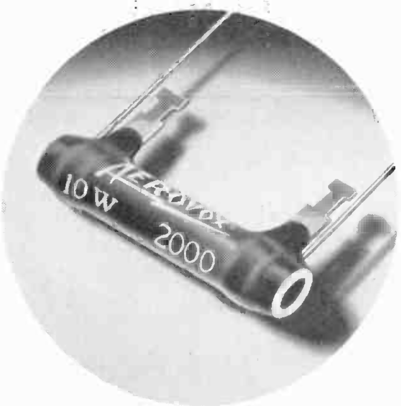

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